

**EEE 204: Numerical Techniques in Engineering**

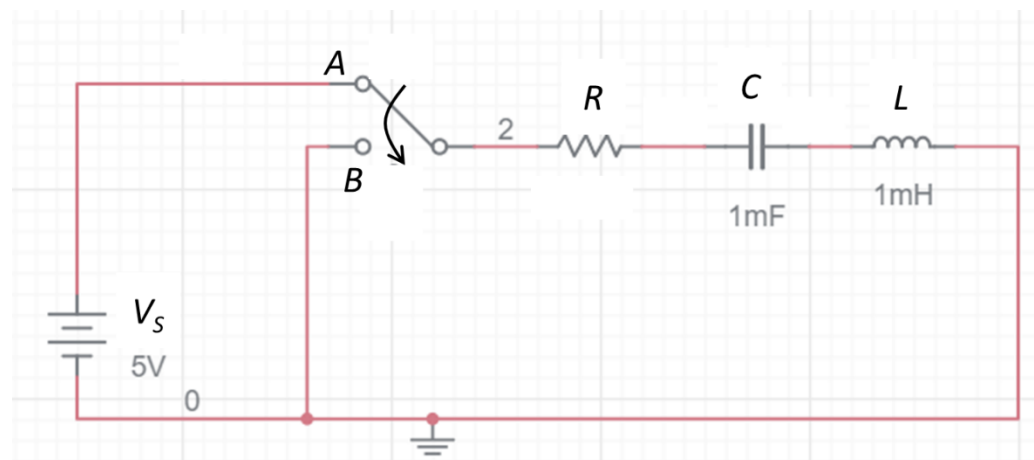
*Section (1)*

**Mid-2 Project**

*Total marks: 35. Rubrics for assessment given in a separate file.*

**Instructions:**

1. Deadline: Aug. 18, 2022 (submission of report + online submission + viva)
2. You can discuss with TA or instructor.
3. Students who copy will either withdraw the course or be sent to disciplinary committee.



The circuit was kept at position A for a long time. Then it switches from A to B at time  $t = 0$  triggering a transient response of voltage and current before reaching steady values. The goal of this project is to analyze the transient behavior of the system. The current  $i(t)$  through the circuit at  $t = 0$  is  $i(0) = 0$  mA, and at  $t = 50$  ms is  $i(50\text{ms}) = 132.56$  mA. The circuit parameters are:  $V_S = 5$  V,  $R = 0.1\Omega$ ,  $L = 1\text{mH}$ ,  $C = 1\text{mF}$ .

1. Setting up the equations:
  - a. Appropriately mark the directions (arrows and  $+/-$ ) of  $i$ ,  $v_R$ ,  $v_L$ ,  $v_C$  in the circuit.
  - b. Find the differential equation for the system and also write its boundary conditions.
  - c. Then derive the difference equation along with appropriate coefficient values.
  - d. Write the system equations in a matrix forms including the boundary conditions.

2. In MATLAB, write a code to solve the differential equation (for now you can set  $n = 1000$ ).
3. The exact solution can be found as follows:

$$\alpha = \frac{R}{2L} \text{ and } \omega_0 = \frac{1}{\sqrt{LC}}$$

$$s1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} \text{ and } s2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

$$A = -\frac{V_s}{L(s1 - s2)}$$

$$i_{exact}(t) = A \exp(s1 t) - A \exp(s2 t)$$

4. Find and plot the following:
  - a. Voltage across the inductor  $v_L(t)$ . [Hint: how is  $i$  related to  $v_L$ ?]
  - b. Instantaneous power  $P_R(t)$  dissipated in the resistor.
  - c. Instantaneous power absorbed by the inductor  $P_L$  vs  $t$ .
5. Visualizations and plots:
  - a. Plot the numerically solved  $i$  vs  $t$  for  $n = 100$ . Also plot the exact solution in the same plot. What is the amount of error  $E_n = \sqrt{\sum e_i^2/n}$ ?
  - b. Plot the numerically solved  $v_L$  vs  $t$  for  $n = 100$ .
  - c. Plot the numerically solved  $P_R$  vs  $t$  for  $n = 100$ . Also plot the exact solution in the same plot.
  - d. Repeat a, b, c for  $n = 200, 500, 1000, 5000$ . [ $n = 5000$  may take couple of minutes to run].
  - e. Comment on the accuracy of the solution for different  $n$ . Does the accuracy improve with  $n$ ? Why?
6. Find and plot  $E_n$  vs  $n$  for  $n = [75:25:1000 \ 2000 \ 3500 \ 5000]$ . (You may need to write another code with a loop for  $n$ ). Comment on the accuracy/error.

#### Bonuses:

1. First three students to complete [**Bonus 2pts**]
2. If you choose to do a higher difficulty level problem-set [**Bonus 3pts**]