CSC 226 Assignment 3

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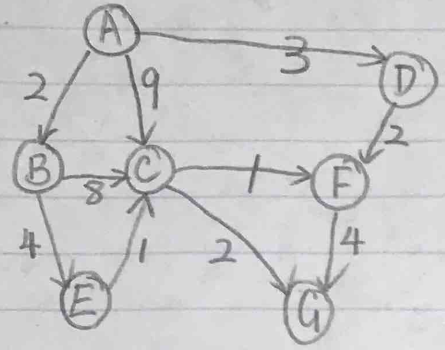
1. (C, G) (F, G) (B, E) (E, C) (C, F) (B, C) (D, F) (A, D) (A, C) (A, B)



Iteration 1:

A	B	C	D	E	F	G
0	~~∞~~ 2	~~∞~~ 9	~~∞~~ 3	∞	∞	∞

⇓

A	B	C	D	E	F	G
0	2	9	3	∞	∞	∞

Iteration 2:

A	B	C	D	E	F	G
0	2	~~9~~ 7	3	~~∞~~ 6	~~∞~~ ~~8~~ 5	~~∞~~ 11

⟹

A	B	C	D	E	F	G
0	2	7	3	6	5	11

Iteration 3:

A	B	C	D	E	F	G
0	2	7	3	6	5	~~11~~ 9

⟹

A	B	C	D	E	F	G
0	2	7	3	6	5	9

Iteration 4:

A	B	C	D	E	F	G
0	2	7	3	6	5	9

⟹

A	B	C	D	E	F	G
0	2	7	3	6	5	9

Iteration 5:

A	B	C	D	E	F	G
0	2	7	3	6	5	9

⟹

A	B	C	D	E	F	G
0	2	7	3	6	5	9

Iteration 6:

A	B	C	D	E	F	G
0	2	7	3	6	5	9

⟹

A	B	C	D	E	F	G
0	2	7	3	6	5	9

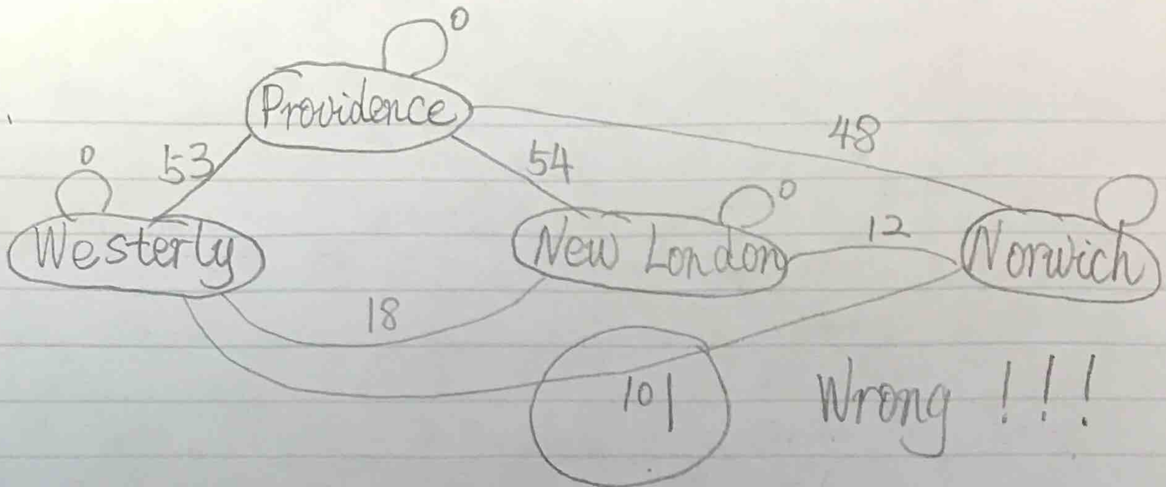
2. ① Initialize distance[] = {−∞, −∞ ..... } and dist[s] = 0
② Create a topological order for all vertices
③ Do following for every vertex u in topological order:
  Do following for every adjacent vertex v to u:
    if (distance[v] < distance[u] + weight(u,v))
      distance[v] = distance[u] + weight(u,v)

Auxiliary data structure used: topological order.

Running Time:
- topological sort for vertex V and edges E is O(V+E) ⟺ step ②
- For every vertices O(V), it checks every adjacent vertices. Total adjcent vertices is O(E), So it runs O(V+E) times. ⟺ step ③

∴ the running time is O(V+E)

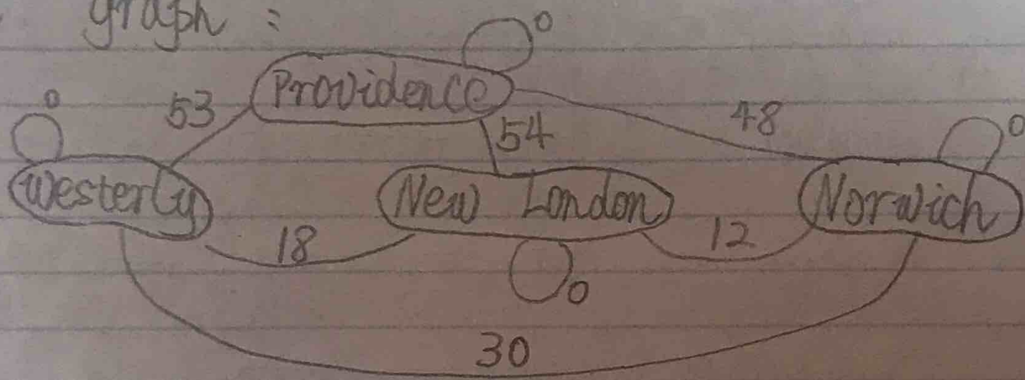
3.



Wrong !!!

From Westerly to Norwich shortest distance shouldn't be 101, we can go westerly – New London – Norwich, so the shortest distance is $18 + 12 = 30$

$\therefore$ correct graph :



4. ⓐ Boolean or : if there is 1 result is 1
Boolean and: if there is 0 result is 0
Because $M^2(i,j)$ is first take boolean and, then take boolean or, so if $M^2(i,j) = 1$ means there must be a path between vertex i and j and between i and j we can take one intermediate vertex which can be between 1 to n. If $M^2(i,j) = 0$ means there is no paths between vertex i and j.

4. ⓑ $M^2(i,j) = (M(i,1_1) \cdot (1_1, j)) + \dots + (M(i,n_1) \cdot M(n_1, j))$

$M^2(i,j) = (M(i,1_2) \cdot (1_2, j)) + \dots + (M(i,n_2) \cdot M(n_2, j))$

$M^4(i,j) = (M(i,1_1) \cdot M(1_1, 1_2) \cdot M(1_2, 1_3) \cdot M(1_3, j)) + \dots + (M(i,n_1) \cdot M(n_1,n_2) \cdot M(n_2,n_3) \cdot M(n_3, j))$

So $M^4(i,j) = 1$ means there must be a path between vertex i and j, and between i and j we can take three intermediate vertices in set $V(1 \dots n)$. $M^4(i,j) = 0$ means there is no paths between i and j.

4. ⓑ $M^k(i,j)=1$ means there must be a path between i and j, and between i and j we can take $k-1$ intermediate vertices in set $V(1\ldots n)$. $M^k(i,j)=0$ means there is no paths between i and j.

5. $$M(i,j) = \begin{cases} 0, & i=j \\ \text{weight}(i,j), & \text{if there is an edge between } i, j \\ \infty, & \text{otherwise} \end{cases}$$

According to question 4, we can see $M^2(i,j)=d$ means it takes the min. of all paths between vertex i and j, and taking one intemediate vertex which between 1 to n. So if $M^2(i,j)=d$, we can find the minimum path $=d$ between vertex i and j, and taking one intemediate vertex which between 1 to n.

According to question 4, we can see $M^k(i,j)=d$ means it takes the min. of all paths between vertex i and j, and taking $k-1$ intermediate vertices in set $V(1\ldots n)$. So if $M^k(i,j)=d$, we can find the minimum path $=d$ between vertex i and j, and taking $k-1$ intermediate vertices in set $V(1\ldots n)$.