CSC 226 Assignment 1

1. (a) $7! = 5040$

(b) J C J C J C J $\quad 3! \cdot 4! = 6 \times 24 = 144$

(c)
C C C _ _ _ _
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$5! \cdot 3! = 120 \times 6 = 720$

(d)
C C C J J J J
J J J J C C C

$3! \cdot 4! \cdot 2 = 6 \times 24 \times 2 = 288$

2. (a)
$$\binom{n}{2} + \binom{n-1}{2} = \frac{n!}{2!(n-2)!} + \frac{(n-1)!}{2!(n-1-2)!}$$
$$= \frac{n!}{2!(n-2)!} + \frac{(n-1)!}{2!(n-3)!}$$
$$= \frac{n(n-1)}{2} + \frac{(n-1)(n-2)}{2}$$
$$= \frac{n^2 - n + n^2 - 3n + 2}{2}$$
$$= \frac{2n^2 - 4n + 2}{2} = n^2 - 2n + 1 = (n-1)^2$$

(b) Binomial theorem:
$$(x+y)^n = \binom{n}{0}x^0y^n + \binom{n}{1}xy^{n-1} + \binom{n}{2}x^2y^{n-2} + \dots + \binom{n}{n}x^ny^0$$
$$= \sum_{k=0}^{n}\binom{n}{k}x^ky^{n-k}$$

$$(1+x)^n - \binom{n}{1}x(1+x)^{n-1} + \binom{n}{2}x^2(1+x)^{n-2} - \dots + (-1)^n\binom{n}{n}x^n$$
$$= \binom{n}{0}(-x)^0(1+x)^n + \binom{n}{1}(-x)^1(1+x)^{n-1} + \binom{n}{2}(-x)^2(1+x)^{n-2} + \dots + \binom{n}{n}(-x)^n(1+x)^0$$
$$= [(-x) + (1+x)]^n \quad \text{by Binomial Theorem}$$
$$= 1^n$$
$$= 1$$

3. (a) $\binom{4+32-1}{32} = \binom{35}{32} = 6545$

(b) since $x_i > 0$, means $x_i \geq 1$, let $y_i = x_i - 1$, $y_i \geq 0$
$$x_1 + x_2 + x_3 + x_4 = 32$$
$$(y_1+1) + (y_2+1) + (y_3+1) + (y_4+1) = 32$$
$$y_1 + y_2 + y_3 + y_4 = 28$$
$$\binom{4+28-1}{28} = \binom{31}{28} = 4495$$

4. Let x_1 = first day send out resumes number

x_2 = first day and second day send out total number

x_3 = 1st day, 2nd day and 3rd day send out total number

$\vdots$

x_{42} = total 42 days send out resumes number

$\therefore 1 \le x_1 < x_2 < x_3 < x_4 < \dots < x_{42} \le 60$

$\therefore (x_1+23) < (x_2+23) < (x_3+23) < \dots < (x_{42}+23) \le 83$

There are $42 \times 2 = 84$ numbers between 1 to 83, by Pigeonhole Principle, at least two are equal

$\therefore$ There is a period of consecutive days during which you send out exactly 23 resumes.

5. (A, R_1) and (B, R_2) are posets. R on $A \times B$ by

$((a,b),(x,y)) \in R$ if $(a,x) \in R_1$ and $(b,y) \in R_2$.

R is reflexive: $((a,b),(a,b)) \in R$ since $(a,a) \in R_1$ and $(b,b) \in R_2$ and R_1, R_2 are reflexive

R is antisymmetric: if $((a,b),(x,y)) \in R$ and $((x,y),(a,b)) \in R$

then $(a,x) \in R_1$ and $(b,y) \in R_2$ and $(x,a) \in R_1$ and $(y,b) \in R_2$

then (a,x) and $(x,a) \in R_1$ and (b,y) and $(y,b) \in R_2$

then $a = x$ and $y = b$

$\therefore (a,b) = (x,y)$

R is transitive: if $((a,b),(x,y)) \in R$ and $((x,y),(m,n)) \in R$

then $(a,x) \in R_1$ and $(x,m) \in R_1$ and $(b,y) \in R_2$ and $(y,n) \in R_2$

then $(a,m) \in R_1$ and $(b,n) \in R_2$

then $((a,b),(m,n)) \in R$

$\therefore$ R is transitive

$\therefore$ R is a partial order.