(1)

CSC 349A Ass. 4 Xiaoqing Zhang V00904789

1.(a). $Ax^{(4)} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \Longrightarrow x^{(4)} = \left[\begin{array}{cccc|c} 0 & -2 & -2 & -4 & 0 \\ -1 & -1 & 1 & 0 & 0 \\ 2 & 4 & -2 & 0 & 0 \\ 1 & 1 & -1 & 0.5 & 1 \end{array}\right]$

step 1 : $k=1$, $p=3$, $\therefore$ row 1 and row 3 switch

$\therefore \left[\begin{array}{cccc|c} 2 & 4 & -2 & 0 & 0 \\ -1 & -1 & 1 & 0 & 0 \\ 0 & -2 & -2 & -4 & 0 \\ 1 & 1 & -1 & 0.5 & 1 \end{array}\right]$

forward elimination a_{21}, a_{31}, a_{41}

$m_{21} = \frac{a_{21}}{a_{11}} = \frac{-1}{2}$ $\quad m_{31} = 0$ $\quad m_{41} = \frac{1}{2}$

$\underline{E_2 = E_2 - m_{21}E_1}$: $a_{21} = a_{21} - m_{21}a_{11} = -1 - (\frac{-1}{2})(2) = 0$

$a_{22} = a_{22} - m_{21}a_{12} = -1 - (-\frac{1}{2})(4) = 1$

$a_{23} = a_{23} - m_{21}a_{13} = 1 - (-\frac{1}{2})(-2) = 0$

$a_{24} = a_{24} - m_{21}a_{14} = 0$

$b_2 = b_2 - m_{21}b_1 = 0$

$\underline{E_3 = E_3 - m_{31}E_1}$: since $m_{31} = 0$, E_3 no change

$\underline{E_4 = E_4 - m_{41}E_1}$: $a_{41} = a_{41} - m_{41}a_{11} = 1 - (\frac{1}{2})(2) = 0$

$a_{42} = a_{42} - m_{41}a_{12} = 1 - (\frac{1}{2})(4) = -1$

$a_{43} = a_{43} - m_{41}a_{13} = -1 - (\frac{1}{2})(-2) = 0$

$a_{44} = a_{44} - m_{41}a_{14} = 0.5$

$b_4 = b_4 - m_{41}b_1 = 1$

$\therefore$ new matrix = $\left[\begin{array}{cccc|c} 2 & 4 & -2 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & -2 & -2 & -4 & 0 \\ 0 & -1 & 0 & 0.5 & 1 \end{array}\right]$

Step 2 : $k=2$, $p=3$, $\therefore$ row 2 and 3 switch

$\therefore \left[\begin{array}{cccc|c} 2 & 4 & -2 & 0 & 0 \\ 0 & -2 & -2 & -4 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0.5 & 1 \end{array}\right]$

forward elimination a_{32}, a_{42}

$m_{32} = \frac{a_{32}}{a_{22}} = \frac{-1}{2}$ $\quad m_{42} = \frac{a_{42}}{a_{22}} = \frac{1}{2}$

$\underline{E_3 = E_3 - m_{32}E_2}$: $a_{32} = a_{32} - m_{32}a_{22} = 1 - (-\frac{1}{2})(-2) = 0$

$a_{33} = a_{33} - m_{32}a_{23} = 0 - (-\frac{1}{2})(-2) = -1$

$a_{34} = a_{34} - m_{32}a_{24} = 0 - (-\frac{1}{2})(-4) = -2$

$b_3 = b_3 - m_{32}b_2 = 0$

$E_4 = E_4 - m_{42} E_3$: $a_{42} = a_{42} - m_{42} a_{22} = -1 - (\frac{1}{2})(-2) = 0$

$a_{43} = a_{43} - m_{42} a_{23} = 0 - (\frac{1}{2})(-2) = 1$

$a_{44} = a_{44} - m_{42} a_{24} = 0.5 - (\frac{1}{2})(-4) = \frac{1}{2} + 2 = \frac{5}{2}$

$b_4 = b_4 - m_{42} b_3 = 1$

$\therefore$ new matrix :
$$\left[\begin{array}{cccc|c} 2 & 4 & -2 & 0 & 0 \\ 0 & -2 & -2 & -4 & 0 \\ 0 & 0 & -1 & -2 & 0 \\ 0 & 0 & 1 & \frac{5}{2} & 1 \end{array}\right]$$

Step 3 : $k = 3$, $p = 3$. no change

Forward elimination a_{43}, $m_{43} = \frac{a_{43}}{a_{33}} = -1$

$E_4 = E_4 - m_{43} E_3$: $a_{43} = a_{43} - m_{43} a_{33} = 1 - (-1)(-1) = 0$

$a_{44} = a_{44} - m_{43} - a_{43} = \frac{5}{2} - (-1)(-2) = \frac{1}{2}$

$b_4 = b_4 - m_{43} b_3 = 1$

$\therefore$ new matrix =
$$\left[\begin{array}{cccc|c} 2 & 4 & -2 & 0 & 0 \\ 0 & -2 & -2 & -4 & 0 \\ 0 & 0 & -1 & -2 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 1 \end{array}\right]$$

Step 4 : back substitution : $\frac{1}{2} x_4 = 1 \Rightarrow x_4 = 2$

$-x_3 - 2x_4 = 0$

$-x_3 - 4 = 0$

$\Longrightarrow x_3 = -4$

$-2x_2 - 2x_3 - 4x_4 = 0$

$-2x_2 + 8 - 8 = 0$

$\Longrightarrow x_2 = 0$

$2x_1 + 4x_2 - 2x_3 = 0$

$2x_1 - 2(-4) = 0$

$x_1 = -4$

$$\therefore x^{(4)} = \begin{bmatrix} -4 \\ 0 \\ -4 \\ 2 \end{bmatrix}$$

②

1(b). from (a) we have in the end the matrix is

$$\left[\begin{array}{cccc|c} 2 & 4 & -2 & 0 & 0 \\ 0 & -2 & -2 & -4 & 0 \\ 0 & 0 & -1 & -2 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 1 \end{array}\right]$$

$\because$ we interchange 2 times

$\therefore$ $\det A = (-1)^2 (2)(-2)(-1)(\frac{1}{2}) = 2$

2(a).

$x_1 \leftarrow b_1 / a_{11}$

$x_2 \leftarrow (b_2 - a_{21} \times x_1) / a_{22}$

for $i = 3$ to n do

$x_i \leftarrow (b_i - a_{i,i-1} \times x_{i-1} - a_{i,i-2} \times x_{i-2}) / a_{i,i}$

end

2(b). the number of divisions $= 1 + 1 + (n-3+1) = n$

the number of multiplications $= 1 + 2(n-3+1) = 2n-3$

the number of sabstractions $= 1 + 2(n-3+1) = 2n-3$

the number of the addition $= 0$

$\therefore$ grand total $= n + 2n - 3 + 2n - 3 = 5n - 6$

2(c).

```
function x = fs(A,b)
n = size(A,1);
x(1,1) = b(1,1)/A(1,1);
x(2,1) = (b(2,1)-A(2,1)*x(1,1))/A(2,2);
for i=3:n
    x(i,1) = (b(i,1)-A(i,i-2)*x(i-2,1)-A(i,i-
1)*x(i-1,1))/A(i,i);
end
end
```

```
>> A=[1 0 0 0; 2 3 0 0; 4 5 6 0; 0 7 8 9];
>> b=[1;5;15;24];
>> fs(A,b)

ans =

     1
     1
     1
     1
```

3(a). $\begin{bmatrix} -0.2345 & 2.107 \\ 0.1234 & -1.115 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2.345 \\ 1.001 \end{bmatrix}$

$$\Downarrow$$

$$\left[\begin{array}{cc|c} -0.2345 & 2.107 & -2.345 \\ 0.1234 & -1.115 & 1.001 \end{array}\right]$$

Step 1: $k=1$, $p=1$, no change

forward elimination a_{21}

$fl(m_{21}) = \frac{fl(a_{21})}{fl(a_{11})} = \frac{fl(0.1234)}{fl(-0.2345)} = fl(-0.526226) = -0.5262$

$E_2 = E_2 - m_{21} E_1$: $fl(a_{21}) = 0$

$fl(a_{22}) = fl(fl(a_{22}) - fl(m_2 \times a_{12}))$

$= fl(-1.115 - fl((-0.5262) \times (2.107)))$

$= fl(-1.115 - fl(-1.1087034))$

$= fl(-1.115 + 1.109)$

$= -0.006$

$fl(b_2) = fl(fl(b_2) - fl(m_{21} \times b_1))$

$= fl(1.001 - fl((-0.5262) \times (-2.345)))$

$= fl(1.001 - fl(1.233939))$

$= fl(1.001 - 1.234)$

$= fl(-0.233)$

$= -0.233$

$\therefore$ new matrix: $\left[\begin{array}{cc|c} -0.2345 & 2.107 & -2.345 \\ 0 & -0.006 & -0.233 \end{array}\right]$

Step 2: Back substitution: $fl(x_2) = fl(-0.233 / -0.006)$

$= fl(38.83333)$

$= 38.83$

$fl(x_1) = fl(fl(-2.345 - fl(2.107 \times 38.83)) / -0.2345)$

$= fl(fl(-2.345 - 81.81) / -0.2345)$

$= fl(-84.16 / -0.2345)$

$= 358.9$

$\therefore \hat{x} = \begin{bmatrix} 358.9 \\ 38.83 \end{bmatrix}$

explain: We use Gaussian elimination with partial pivot to compute result, we get $\hat{x} = \begin{bmatrix} 358.9 \\ 38.83 \end{bmatrix}$

and exact solution $x = \begin{bmatrix} 345.404\ldots \\ 37.329\ldots \end{bmatrix}$,

there is a big difference between two results, so it's very difficult to determine an accurate computed solution to an ill-conditioned linear system.

3(b).

```
>> A=[-0.2345 2.107; 0.1234 -1.115];
>> cond(A)

ans =

   3.9304e+03
```