CSC 349A
Assignment 1
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1(a).

```
function Euler(m,c,g,,t0,v0,tn,n)
fprintf('values of t     appoximatins v(t)\n')
fprintf('%8.3f',t0),fprintf('%19.4f\n',v0)
h=(tn-t0)/n
t=t0;
v=v0;
for i=1:n
    v=v+(g-c/m*v)*h;
    t=t+h;
    fprintf('%8.3f',t),fprintf('%19.4f\n',v)
end
```

1(b).

```
>> Euler(86.2,12.5,9.81,0,0,12,15)
values of t     approximations v(t)
   0.000          0.0000
   0.800          7.8480
   1.600         14.7856
   2.400         20.9183
   3.200         26.3396
   4.000         31.1319
   4.800         35.3684
   5.600         39.1133
   6.400         42.4238
   7.200         45.3502
   8.000         47.9372
   8.800         50.2240
   9.600         52.2456
  10.400          54.0326
  11.200          55.6123
  12.000          57.0088
```

1(c).

```
>> Euler(86.2,12.5,3.71,0,0,12,15)
values of t     approximations v(t)
   0.000          0.0000
   0.800          2.9680
   1.600          5.5917
   2.400          7.9110
   3.200          9.9612
   4.000         11.7737
```

```
   4.800           13.3758
   5.600           14.7921
   6.400           16.0441
   7.200           17.1508
   8.000           18.1292
   8.800           18.9940
   9.600           19.7585
  10.400           20.4343
  11.200           21.0318
  12.000           21.5599
```

1(d).

```
>> g=9.81;
>> m=86.2;
>> c=12.5;
>> t=12;
>> v=(g*m/c)*(1-exp(-(c*t/m)))

v =

   55.7775

>> Euler(86.2,12.5,9.81,0,0,12,15)
values of t     approximations v(t)
   0.000            0.0000
   0.800            7.8480
   1.600           14.7856
   2.400           20.9183
   3.200           26.3396
   4.000           31.1319
   4.800           35.3684
   5.600           39.1133
   6.400           42.4238
   7.200           45.3502
   8.000           47.9372
   8.800           50.2240
   9.600           52.2456
  10.400           54.0326
  11.200           55.6123
  12.000           57.0088
>> E=abs((55.7775-57.0088)/55.7775)

E =

    0.0221
```

2(a).

```
function Euler2(m,k,g,t0,v0,tn,n)
fprintf('values of t      approximations v(t)\n')
fprintf('%8.3f',t0),fprintf('%19.4f\n',v0)
h=(tn-t0)/n;
t=t0;
v=v0;
for i=1:n
    v=v+(g-k/m*v*v)*h;
    t=t+h;
    fprintf('%8.3f',t),fprintf('%19.4f\n',v)
end
```

2(b).

```
>> Euler2(73.5,0.234,9.81,0,0,18,72)
values of t      approximations v(t)
   0.000             0.0000
   0.250             2.4525
   0.500             4.9002
   0.750             7.3336
   1.000             9.7433
   1.250            12.1202
   1.500            14.4558
   1.750            16.7420
   2.000            18.9714
   2.250            21.1374
   2.500            23.2343
   2.750            25.2572
   3.000            27.2019
   3.250            29.0655
   3.500            30.8456
   3.750            32.5408
   4.000            34.1505
   4.250            35.6748
   4.500            37.1143
   4.750            38.4705
   5.000            39.7450
   5.250            40.9402
   5.500            42.0587
   5.750            43.1033
   6.000            44.0770
   6.250            44.9832
   6.500            45.8252
   6.750            46.6063
```

7.000	47.3300
7.250	47.9995
7.500	48.6182
7.750	49.1894
8.000	49.7161
8.250	50.2013
8.500	50.6480
8.750	51.0588
9.000	51.4363
9.250	51.7831
9.500	52.1013
9.750	52.3933
10.000	52.6609
10.250	52.9062
10.500	53.1309
10.750	53.3366
11.000	53.5249
11.250	53.6971
11.500	53.8547
11.750	53.9988
12.000	54.1305
12.250	54.2509
12.500	54.3608
12.750	54.4613
13.000	54.5531
13.250	54.6369
13.500	54.7134
13.750	54.7833
14.000	54.8471
14.250	54.9053
14.500	54.9584
14.750	55.0069
15.000	55.0512
15.250	55.0915
15.500	55.1284
15.750	55.1620
16.000	55.1926
16.250	55.2206
16.500	55.2461
16.750	55.2693
17.000	55.2905
17.250	55.3099
17.500	55.3275
17.750	55.3436

```
  18.000          55.3583
```

2(c).

```
>> g=9.81;
>> m=73.5;
>> k=0.234;
>> t=18;
>> v=sqrt(g*m/k)*tanh(sqrt(g*k/m)*t)

v =

   55.3186

>> Euler2(73.5,0.234,9.81,0,0,18,72)
values of t     approximations v(t)
   0.000           0.0000
   0.250           2.4525
   0.500           4.9002
   0.750           7.3336
   1.000           9.7433
   1.250          12.1202
   1.500          14.4558
   1.750          16.7420
   2.000          18.9714
   2.250          21.1374
   2.500          23.2343
   2.750          25.2572
   3.000          27.2019
   3.250          29.0655
   3.500          30.8456
   3.750          32.5408
   4.000          34.1505
   4.250          35.6748
   4.500          37.1143
   4.750          38.4705
   5.000          39.7450
   5.250          40.9402
   5.500          42.0587
   5.750          43.1033
   6.000          44.0770
   6.250          44.9832
   6.500          45.8252
```

6.750	46.6063
7.000	47.3300
7.250	47.9995
7.500	48.6182
7.750	49.1894
8.000	49.7161
8.250	50.2013
8.500	50.6480
8.750	51.0588
9.000	51.4363
9.250	51.7831
9.500	52.1013
9.750	52.3933
10.000	52.6609
10.250	52.9062
10.500	53.1309
10.750	53.3366
11.000	53.5249
11.250	53.6971
11.500	53.8547
11.750	53.9988
12.000	54.1305
12.250	54.2509
12.500	54.3608
12.750	54.4613
13.000	54.5531
13.250	54.6369
13.500	54.7134
13.750	54.7833
14.000	54.8471
14.250	54.9053
14.500	54.9584
14.750	55.0069
15.000	55.0512
15.250	55.0915
15.500	55.1284
15.750	55.1620
16.000	55.1926
16.250	55.2206
16.500	55.2461
16.750	55.2693
17.000	55.2905
17.250	55.3099
17.500	55.3275

```
  17.750           55.3436
  18.000           55.3583
>> E=abs((55.3186-55.3583)/55.3186)

E =

   7.1766e-04
```

3. SEE next page

CSC 349 A $\boxed{\text{Ass. 1}}$ Xiaoqing Zhang V00904789

3. Using Mclaurin Series:

$n=1$: $f^* = 1-2 = -1$

$|\varepsilon_t| = \left|\frac{e^{-2}-(-1)}{e^{-2}}\right| = 8.389056 \approx 838.91\%$

$n=2$: $f^* = 1-2+\frac{2^2}{2!} = 1$

$|\varepsilon_t| = \left|\frac{e^{-2}-1}{e^{-2}}\right| = 6.389056 \approx 638.91\%$

$n=3$: $f^* = 1-2+\frac{2^2}{2!}-\frac{2^3}{3!} = -0.333333$

$|\varepsilon_t| = \left|\frac{e^{-2}-(-0.333333)}{e^{-2}}\right| = 3.463016 \approx 346.30\%$

$n=4$: $f^* = 1-2+\frac{2^2}{2!}-\frac{2^3}{3!}+\frac{2^4}{4!} = 0.333333$

$|\varepsilon_t| = \left|\frac{e^{-2}-0.333333}{e^{-2}}\right| = 1.463016 \approx 146.30\%$

$n=5$: $f^* = 1-2+\frac{2^2}{2!}-\frac{2^3}{3!}+\frac{2^4}{4!}-\frac{2^5}{5!} = 0.06666667$

$|\varepsilon_t| = \left|\frac{e^{-2}-0.06666667}{e^{-2}}\right| = 0.507396 \approx 50.74\%$

Using 1 over the Mclaurin Series:

$n=1$: $f^* = \frac{1}{1+2} = \frac{1}{3}$

$|\varepsilon_t| = \left|\frac{e^{-2}-\frac{1}{3}}{e^{-2}}\right| = 1.4630187 \approx 146.30\%$

$n=2$: $f^* = \frac{1}{1+2+\frac{2^2}{2!}} = \frac{1}{5} = 0.2$

$|\varepsilon_t| = \left|\frac{e^{-2}-0.2}{e^{-2}}\right| = 0.477811 \approx 47.78\%$

$n=3$: $f^* = \frac{1}{1+2+\frac{2^2}{2!}+\frac{2^3}{3!}} = 0.1578947$

$|\varepsilon_t| = \left|\frac{e^{-2}-0.1578947}{e^{-2}}\right| = 0.166693 \approx 16.67\%$

$n=4$: $f^* = \frac{1}{1+2+\frac{2^2}{2!}+\frac{2^3}{3!}+\frac{2^4}{4!}} = 0.142857$

$|\varepsilon_t| = \left|\frac{e^{-2}-0.142857}{e^{-2}}\right| = 0.055578 \approx 5.56\%$

$n=5$: $f^* = \frac{1}{1+2+\frac{2^2}{2!}+\frac{2^3}{3!}+\frac{2^4}{4!}+\frac{2^5}{5!}} = 0.1376147$

$|\varepsilon_t| = \left|\frac{e^{-2}-0.1376147}{e^{-2}}\right| = 0.0168427 \approx 1.68\%$

Conclusion: Compare these two approximations, we can see the $|\varepsilon_t|$ of 1 over the Mclaurin Series is less than the Mclaurin Series, so the 1 over the McLaurin Series is more accurate.