1(a). $fl(\pi) = 3.142$

$fl(x - \pi) = fl(3.154 - 3.142) = 0.012$.

$fl((x-\pi)^2) = fl((0.012)^2) = fl(0.000144) = 0.000144$

$fl(\cos x) = fl(\cos(3.154)) = fl(-0.999923029) = -0.9999$

$fl(1 + \cos x) = fl(1 - 0.9999) = fl(0.0001) = 0.0001$

$fl\left(\frac{1+\cos x}{(x-\pi)^2}\right) = fl\left(\frac{0.0001}{0.000144}\right) = fl(0.694444) = 0.6944$

$|\varepsilon_t| = \left|\frac{0.4999359 - 0.6944}{0.4999359}\right| = 0.388831 \approx 38.88\%$

1(b). $f(x) = \cos x$ $f(\pi) = \cos \pi = -1$

$f'(x) = -\sin x$ $f'(\pi) = -\sin \pi = 0$

$f''(x) = -\cos x$ $f''(\pi) = -\cos \pi = 1$

$f'''(x) = \sin x$ $f'''(\pi) = \sin \pi = 0$

$f''''(x) = \cos x$ $f''''(\pi) = \cos \pi = -1$

$\therefore f(x) \approx -1 + 0 + \frac{1 \cdot (x-\pi)^2}{2!} + 0 + \frac{(-1)(x-\pi)^4}{4!}$

$\therefore \cos x \approx -1 + \frac{(x-\pi)^2}{2!} - \frac{(x-\pi)^4}{4!}$

$\therefore \cos x \approx -1 + \frac{(x-\pi)^2}{2} - \frac{(x-\pi)^4}{24}$

1(c). $f(x) = \frac{1+\cos x}{(x-\pi)^2} = \frac{1 + \left(-1 + \frac{(x-\pi)^2}{2} - \frac{(x-\pi)^4}{24}\right)}{(x-\pi)^2} = \frac{1}{2} - \frac{(x-\pi)^2}{24}$

$\therefore f(x) = \frac{1}{2} - \frac{(x-\pi)^2}{24}$

1(d). $f(x) = \frac{1}{2} - \frac{(x-\pi)^2}{24}$ at $x = 3.154$

data: $x = 3.154 \longrightarrow f(x) = 0.5 - \frac{(3.154-\pi)^2}{24} = 0.49999$

Perturb x by ε : $\hat{x} = x + \varepsilon$ with $\left|\frac{\varepsilon}{3.154}\right|$ small

$f(x) = \frac{1}{2} - \frac{(3.154 + \varepsilon - \pi)^2}{24}$

$= \frac{1}{2} - \frac{(0.012 + \varepsilon)^2}{24}$

$= \frac{1}{2} - \frac{0.000144 + 0.024\varepsilon + \varepsilon^2}{24}$

$= 0.499994 - 0.001\varepsilon - \frac{\varepsilon^2}{24}$

if $\left|\frac{\varepsilon}{3.154}\right|$ is small, then $f(x) \approx 0.49999$ since 0.001ε and $\frac{\varepsilon^2}{24}$ are also small

$\therefore f(x) \approx \frac{1}{2} - \frac{(3.154-\pi)^2}{24}$

$\therefore \{\hat{x}\} \approx \{x\}$ for all small ε, it's well-conditioned.

1(e). given data $x = 3.154 \longrightarrow 0.49999$

Pertubed problem, $\longrightarrow f(x) = \frac{1}{2} - \frac{(3.154-\pi)^2 + \epsilon^2 + 2\epsilon(3.154-\pi)}{24}$

$\hat{x} = 3.154 + \epsilon$

with $\left|\frac{\epsilon}{3.154}\right|$ small

if $\left|\frac{\epsilon}{3.154}\right|$ is "small", then

$f(x) \approx \frac{1}{2} - \frac{(3.154+\epsilon-\pi)^2}{24}$ and this

value is very close to
0.49999 for all small value of ϵ.

∴ There are no small values of ϵ for which $f(x) = \frac{1}{2} - \frac{(3.154+\epsilon-\pi)^2}{24}$ is close to 0.49999

∴ the computation is unstable

2(a).

```matlab
function  root = Bisect ( xl , xu , eps , imax, f,
enablePlot )
i=1;
fl=f(xl);
fprintf ( 'iteration      approximation \n')
while(i<=imax)
    xr=(xl+xu)/2;
    fprintf ( ' %6.0f %18.8f \n', i, xr )
    fr=f(xr);
    if(fr==0 || (xu-xl)/abs(xu+xl)<eps)
        root=xr;
        return
    end
    if(i==1|| i==2||i==4|| i==6)
        if(enablePlot==1)
            hold on;
            plot(xr,f(xr), '*g');
            plot(xl,f(xl), '*g');
            plot(xu,f(xu), '*g');
            hold off;
        end
    end
    i=i+1;
    if fl*fr<0
        xu=xr;
    else
        xl=xr;
        fl=fr;
    end
end
fprintf ( 'failed to converge in %g iterations\n',
imax )
end
```

2(b).

Q=20

g=9.81

B=3+y

Ac=3y+y^2/2

0=1-Q^2*B/(g*Ac^3)

1= Q^2*B/(g*Ac^3)

Q^2*B=(g*Ac^3)

400(3+y)=9.81*(3y+y^2/2)^3

1200+400y=9.81*(y^6/8+9y^5/4+27y^4/2+27y^3)

1200=9.81*(y^6/8+9y^5/4+27y^4/2+27y^3)-400y

2(c).

```
function y=f(x)
y=1-(400*(3+x))/(9.81*((3*x+(x^2)/2))^3);
end
```

>> Bisect(0.5,2.5,0.01,10,@f,1)

iteration   approximation

     1   1.50000000
     2   2.00000000
     3   1.75000000
     4   1.62500000
     5   1.56250000
     6   1.53125000
     7   1.51562500
     8   1.50781250

ans =