1

CSC 349A Assignment 2

Xiaoqing Zhang
V00904789

1(a).

S_m	f_1	f_2	f_3	f_4	S_e	e_1	e_2
0	2	0	0	3	0	0	4

positive positive

$(04)_5 = 0\times5^1 + 4\times5^0 = 4$

$0.2003\times5^4 = (2003)_5$
$= 2\times5^3 + 3\times5^0$
$= 250+3$
$= 253$

∴ the exact decimal value is 253

1(b).

S_m	f_1	f_2	f_3	f_4	S_e	e_1	e_2
1	1	0	0	4	0	0	3

negative positive

$-0.1004\times5^3 = (-100.4)_5 = -(1\times5^2 + 4\times5^{-1})$
$= -(25+0.8) = -25.8$

∴ the decimal value is -25.8

1(c).

S_m	f_1	f_2	f_3	f_4	S_e	e_1	e_2
0	1	0	0	0	1	4	4

(positive) $\underbrace{1\ 0\ 0\ 0}_{\text{smallest}}$ (negative) $\underbrace{4\ 4}_{\text{smallest}}$

the base 5 #, using above form 01000144

$(44)_5 = 4\times5^1 + 4\times5^0 = 24$

∴ $(0.1000\times5^{-24})_5 = 1\times5^{-1}\times5^{-24} = (3.3554432\times10^{-18})_{10}$

∴ the 8 base-5 digits # is 01000144
the base-5 # is 0.1000×5^{-24}
the decimal value is 3.3554432×10^{-18}

1(d).

	Reminder
25/5	0
5/5	0
1/5	1

	Reminder
125/5	0
25/5	0
5/5	0
1/5	1

$\boxed{(25)_{10} = (100)_5 = (0.100\times5^3)_5}$ $\boxed{0\,1\,1\,0\,0\,0\,0\,0\,3}$

$\boxed{(125)_{10} = (1000)_5 = (0.1000\times5^4)_5}$ $\boxed{0\,1\,1\,0\,0\,0\,0\,0\,4}$

For the next of 0.1000×5^3 is 0.1001×5^3

∴ gap is $0.1001\times5^3 - 0.1000\times5^3 = (0.0001\times5^3)_5$

∴ $(0.0001\times5^3)_5 = (1\times5^{-1})_{10} = (0.2)_{10}$

∴ the size of the gap is 0.2

2(a).

$fl(-2c) = fl(-2\times 1.234) = fl(-2.468) = -2.468$

$fl(b^2) = fl((-78.99)^2) = fl(6239.4201) = 6239$

$fl(4a) = fl(4\times 1.2) = fl(4.8) = 4.8$

$fl(4ac) = fl(4.8\times 1.234) = fl(5.9232) = 5.923$

$fl(b^2-4ac) = fl(6239-5.923) = fl(6233.077) = 6233$

$fl(\sqrt{b^2-4ac}) = fl(\sqrt{6233}) = fl(78.949335) = 78.94$

$fl(b-\sqrt{b^2-4ac}) = fl(-78.99-78.94) = fl(-157.93) = -157.9$

$fl\left(\frac{-2c}{b-\sqrt{b^2-4ac}}\right) = fl\left(\frac{-2.468}{-157.9}\right) = fl(0.015630145) = 0.01563$

$fl(-b-\sqrt{b^2-4ac}) = fl(78.99-78.94) = fl(0.05) = 0.05$

$fl(2a) = fl(2\times 1.2) = fl(2.4) = 2.4$

$fl\left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right) = fl\left(\frac{0.05}{2.4}\right) = fl(0.0208333) = 0.02083$

$\therefore$ For formula $\frac{-2c}{b-\sqrt{b^2-4ac}}$ the root is 0.01563

For formula $\frac{-b-\sqrt{b^2-4ac}}{2a}$ the root is 0.02083

2(b).

For formula $\frac{-2c}{b-\sqrt{b^2-4ac}}$:

$|\varepsilon_t| = \left|\frac{0.01562594-0.01563}{0.01562594}\right| = 0.000259824 = 0.026\%$

For formula $\frac{-b-\sqrt{b^2-4ac}}{2a}$:

$|\varepsilon_t| = \left|\frac{0.01562594-0.02083}{0.01562594}\right| = 0.33303 9804 = 33.3\%$

2(c).

polynomial	(i) is more accurate	(ii) is more accurate
$0.01x^2-125x+0.05$	X	
$-0.3x^2+125x+0.025$		X

(2) CSC 349A Assig. 2 Xiaoqing Zhang V00904789

3(a). $f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots + \frac{f^{(n+1)}(\xi)}{(n+1)!}(x-a)^{n+1}$

$f(x) = \sqrt{x+3}$ $\quad f(1) = \sqrt{1+3} = 2$

$f'(x) = \frac{1}{2}(x+3)^{-\frac{1}{2}} = \frac{1}{2\sqrt{x+3}}$ $\quad f'(1) = \frac{1}{2\sqrt{1+3}} = \frac{1}{4}$

$f''(x) = -\frac{1}{4}(x+3)^{-\frac{3}{2}} = -\frac{1}{4(x+3)^{\frac{3}{2}}}$ $\quad f''(1) = -\frac{1}{4(1+3)^{\frac{3}{2}}} = -\frac{1}{4 \cdot 4^{\frac{3}{2}}} = \frac{-1}{32}$

$f'''(\xi) = -\frac{1}{4}(-\frac{3}{2})(\xi+3)^{-\frac{5}{2}} = \frac{3}{8(\xi+3)^{\frac{5}{2}}}$

$\therefore f(x) \approx 2 + \frac{1}{4}(x-1) - \frac{1}{64}(x-1)^2 + \frac{1}{16(\xi+3)^{\frac{5}{2}}}(x-1)^3$

3(b). $f(1.12) \approx 2 + \frac{1}{4}(1.12-1) - \frac{1}{64}(1.12-1)^2 \approx 2 + 0.03 - 0.000225$
≈ 2.029775

3(c). $\because 1 \le x \le 1.2$, ξ is between 1.00 and 1.12

$\therefore R_3 = \frac{1}{16(\xi+3)^{\frac{5}{2}}}(x-1)^3 \le \frac{(1.2-1)^3}{16(1+3)^{\frac{5}{2}}} = 0.000015625$
$= 1.5625 \times 10^{-5}$