1 CSC 349 A Ass. 6 Xiaoqing Zhang V00904789

1(a). $x_0 = 0$, $x_1 = 2h$, $x_2 = 3h$

$$P(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} f(x_0) + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} f(x_1) + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} f(x_2)$$

$$= \frac{(x-2h)(x-3h)}{(-2h)(-3h)} f(0) + \frac{x(x-3h)}{2h(-h)} f(2h) + \frac{x(x-2h)}{3h(h)} f(3h)$$

$$= \frac{x^2-5xh+6h^2}{6h^2} f(0) - \frac{x^2-3xh}{2h^2} f(2h) + \frac{x^2-2hx}{3h^2} f(3h)$$

1(b). $\int_0^{3h} p(x)\,dx$

$$= \int_0^{3h} \left[\frac{x^2-5xh+6h^2}{6h^2} f(0) - \frac{x^2-3xh}{2h^2} f(2h) + \frac{x^2-2hx}{3h^2} f(3h)\right] dx$$

$$= \int_0^{3h} \frac{x^2-5xh+6h^2}{6h^2} f(0)\,dx - \int_0^{3h} \frac{x^2-3xh}{2h^2} f(2h)\,dx + \int_0^{3h} \frac{x^2-2hx}{3h^2} f(3h)\,dx$$

$$= \frac{f(0)}{6h^2}\left[\frac{x^3}{3} - \frac{5x^2h}{2} + 6xh^2\right]_0^{3h} - \frac{f(2h)}{2h^2}\left[\frac{x^3}{3} - \frac{3x^2h}{2}\right]_0^{3h} + \frac{f(3h)}{3h^2}\left[\frac{x^3}{3} - \frac{2hx^2}{2}\right]_0^{3h}$$

$$= \frac{f(0)}{6h^2}\left[9h^3 - \frac{45h^3}{2} + 18h^3\right] - \frac{f(2h)}{2h^2}\left[9h^3 - \frac{27h^3}{2}\right] + \cancel{\frac{f(3h)}{3h^2}\left[9h^3 - 9h^3\right]}$$

$$= \frac{f(0)}{6h^2}\left(\frac{9h^3}{2}\right) - \frac{f(2h)}{2h^2}\left(\frac{-9h^3}{2}\right)$$

$$= \frac{3h}{4} f(0) + \frac{9h}{4} f(2h)$$

1(c). $h = 0.12$, $f(0) = 0.5$, $f(2h) = 0.50727$

$$\int_0^{0.36} f(x)\,dx \approx \frac{3}{4}(0.12)(0.5) + \frac{9}{4}(0.12)(0.50727)$$

$$= 0.045 + 0.1369629$$

$$= 0.1819629$$

$$|\varepsilon_t| = \left|\frac{0.1819695 - 0.1819629}{0.1819695}\right| \approx 0.003627\% < 1\%$$

2(a). Let $I = \frac{5}{9} f(-\sqrt{\frac{3}{5}}) + \frac{8}{9} f(0) + \frac{5}{9} f(\sqrt{\frac{3}{5}})$ and $I \approx \int_{-1}^{1} f(x)\,dx$

① Let $f(x) = 1$: $(d=0)$

$\int_{-1}^{1} 1\,dx = [x]_{-1}^{1} = 2$

$\frac{5}{9}(1) + \frac{8}{9}(1) + \frac{5}{9}(1) = \frac{18}{9} = 2$

they are equal

② Let $f(x) = x$: $(d=1)$

$\int_{-1}^{1} x\,dx = [\frac{x^2}{2}]_{-1}^{1} = \frac{1}{2} - \frac{1}{2} = 0$

$\frac{5}{9}(-\sqrt{\frac{3}{5}}) + 0 + \frac{5}{9}(\sqrt{\frac{3}{5}}) = 0$

they are equal

③ Let $f(x) = x^2$: $(d=2)$

$\int_{-1}^{1} x^2\,dx = [\frac{x^3}{3}]_{-1}^{1} = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$

$\frac{5}{9}(-\sqrt{\frac{3}{5}})^2 + 0 + \frac{5}{9}(\sqrt{\frac{3}{5}})^2 = \frac{5}{9}(\frac{3}{5}) + \frac{5}{9}(\frac{3}{5}) = \frac{2}{3}$

they are equal

④ Let $f(x) = x^3$: $(d=3)$

$\int_{-1}^{1} x^3\,dx = [\frac{x^4}{4}]_{-1}^{1} = \frac{1}{4} - \frac{1}{4} = 0$

$\frac{5}{9}(\sqrt{\frac{3}{5}})^3 + 0 + \frac{5}{9}(\sqrt{\frac{3}{5}})^3 = 0$

they are equal

⑤ Let $f(x) = x^4$: $(d=4)$

$\int_{-1}^{1} x^4\,dx = [\frac{x^5}{5}]_{-1}^{1} = \frac{2}{5}$

$\frac{5}{9}(-\sqrt{\frac{3}{5}})^4 + 0 + \frac{5}{9}(\sqrt{\frac{3}{5}})^4 = \frac{5}{9}(\frac{3}{5})^2 + \frac{5}{9}(\frac{3}{5})^2 = \frac{2}{5}$

they are equal

⑥ Let $f(x) = x^5$: $(d=5)$

$\int_{-1}^{1} x^5\,dx = [\frac{x^6}{6}]_{-1}^{1} = \frac{1}{6} - \frac{1}{6} = 0$

$\frac{5}{9}(-\sqrt{\frac{3}{5}})^5 + 0 + \frac{5}{9}(\sqrt{\frac{3}{5}})^5 = 0$

they are equal

⑦ Let $f(x) = x^6$: $(d=6)$

$\int_{-1}^{1} x^6\,dx = [\frac{x^7}{7}]_{-1}^{+1} = \frac{2}{7}$

$\frac{5}{9}(-\sqrt{\frac{3}{5}})^6 + 0 + \frac{5}{9}(\sqrt{\frac{3}{5}})^6 = \frac{5}{9}(\frac{3}{5})^3(2) = \frac{3}{25}(2) = \frac{6}{25}$

they are NOT equal

∴ the degree of precision of the quadrature formula $\frac{5}{9} f(-\sqrt{\frac{3}{5}}) + \frac{8}{9} f(0) + \frac{5}{9} f(\sqrt{\frac{3}{5}})$ is 5

2(b). $f(x) = e^{-x}\sqrt{x+2}$

$f(-\sqrt{\tfrac{3}{5}}) = e^{\sqrt{\frac{3}{5}}}\sqrt{2-\sqrt{\tfrac{3}{5}}} \approx 2.401831783$

$f(\sqrt{\tfrac{3}{5}}) = e^{-\sqrt{\frac{3}{5}}}\sqrt{2+\sqrt{\tfrac{3}{5}}} \approx 0.767709422$

$f(0) = e^{-0}\sqrt{2} \approx 1.41421356 2$

$$\int_{-1}^{1} e^{-x}\sqrt{x+2}\,dx \approx \tfrac{5}{9} f(-\sqrt{\tfrac{3}{5}}) + \tfrac{8}{9} f(0) + \tfrac{5}{9} f(\sqrt{\tfrac{3}{5}})$$

$$\approx \tfrac{5}{9}(2.401831783) + \tfrac{8}{9}(1.414213562) + \tfrac{5}{9}(0.767709422)$$

$$\approx 3.0179349$$

3(a).

```
function trap(a, b, maxiter, tol,f)
   m = 1;
   x = linspace(a, b, m+1);
   y = f(x);
   approx = trapz(x, y);
   disp('     m       integral approximation');
   fprintf(' %5.0f %16.10f \n ', m, approx);
   for i = 1 : maxiter
        m = 2.^i ;
        oldapprox = approx;
        x = linspace (a ,b,m+1 ) ;
        y = f(x);
        approx = trapz(x, y);
        fprintf(' %5.0f %16.10f \n ', m, approx);
        if abs(approx-oldapprox) < tol
           return
        end
   end
fprintf('Did not converge in %g iterations', maxiter)
```

3(b).

(1).

```
function y= f(x)
y=sin(1./x);
end

>> trap(0.1,3,20,10^(-6),@f)
     m       integral approximation
     1    -0.3143983004
      2     0.7147254605
      4     1.3447434609
      8     1.5589483255
     16     1.4776583126
     32     1.4679626280
     64     1.5197926883
    128     1.5355585774
    256     1.5386514853
    512     1.5393496800
   1024     1.5395196356
   2048     1.5395618423
   4096     1.5395723764
   8192     1.5395750089
  16384     1.5395756669
```

(2).

```
function y= f(x)
y=exp(3.*x)./sqrt(x.^3+1);
end

>> trap(0,1,20,10^(-10),@f)
     m      integral approximation
     1     7.6013096811
      2     5.9133433291
      4     5.4710046573
      8     5.3585418274
     16     5.3303053079
     32     5.3232385483
     64     5.3214713803
    128     5.3210295584
    256     5.3209191010
    512     5.3208914866
   1024     5.3208845830
   2048     5.3208828571
   4096     5.3208824256
   8192     5.3208823177
  16384     5.3208822908
  32768     5.3208822840
  65536     5.3208822823
  131072     5.3208822819
  262144     5.3208822818
  524288     5.3208822818
```