CSC 349A   Ass. 5   Xiaoqing Zhang   V00904789

1(a). $f(x)=\sin^2(x)$, $x\in[0,2\pi]$, $\sin^2 0=0$, $\sin^2(\frac{2\pi}{3})=0.75$

$x_0=0$, $x_1=\frac{2\pi}{3}$, $x_2=\frac{4\pi}{3}$, $x_3=2\pi$, $\sin^2(\frac{4\pi}{3})=0.75$, $\sin^2(2\pi)=0$

$f(x_0)=0$, $f(x_1)=0.75$, $f(x_2)=0.75$, $f(x_3)=0$

$L_0(x)f(x_0)=0$ since $f(x_0)=0$

$$L_1(x)f(x_1)=\left[\prod_{j=0,j\neq 1}^{3}\frac{(x-x_j)}{(x_i-x_j)}\right]f(x_1)=\frac{(x-x_0)(x-x_2)f(x_1)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}$$

$$=\frac{x(x-\frac{4\pi}{3})(x-2\pi)\frac{3}{4}}{\frac{2\pi}{3}(\frac{2\pi}{3}-\frac{4\pi}{3})(\frac{2\pi}{3}-2\pi)}=\frac{(x^2-2\pi x-\frac{4\pi}{3}x+\frac{8}{3}\pi^2)\frac{3x}{4}}{\frac{16\pi^3}{27}}=\frac{81x^3}{64\pi^3}-\frac{135x^2}{32\pi^2}+\frac{27x}{8\pi}$$

$$L_2(x)f(x_2)=\left[\prod_{j=0,j\neq 1}^{2}\frac{(x-x_j)}{(x_i-x_j)}\right]f(x_2)=\frac{(x-x_0)(x-x_1)f(x_2)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}$$

$$=\frac{x(x-\frac{2\pi}{3})(x-2\pi)\frac{3}{4}}{\frac{4\pi}{3}(\frac{4\pi}{3}-\frac{2\pi}{3})(\frac{4\pi}{3}-2\pi)}=\frac{(x^2-2\pi x-\frac{2\pi x}{3}+\frac{4\pi^2}{3})\frac{3}{4}x}{\frac{4\pi}{3}(\frac{2\pi}{3})(-\frac{2\pi}{3})}$$

$$=\frac{\frac{3}{4}x^3-2\pi x^2+\pi^2 x}{-\frac{16}{27}\pi^3}=-\frac{81}{64\pi^3}x^3+\frac{27}{8\pi^2}x^2-\frac{27}{16\pi}x$$

$L_3(x)f(x_3)=0$ since $f(x_3)=0$

$$\therefore P(x)=\cancel{\frac{81x^3}{64\pi^3}}-\frac{135x^2}{32\pi^2}+\frac{27x}{8\pi}-\cancel{\frac{81x^3}{64\pi^3}}+\frac{27x^2}{8\pi^2}-\frac{27x}{16\pi}$$

$$=\frac{-135x^2}{32\pi^2}+\frac{54x}{16\pi}+\frac{108x^2}{32\pi^2}-\frac{27x}{16\pi}$$

$$=\frac{27x}{16\pi}-\frac{27x^2}{32\pi^2}$$

1(b).

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>> x = [0:0.1:2*pi];
>> y1 = sin(x).^2;
>> y2 = 27*x/(16*pi)-27*x.^2/(32*pi.^2);
>> plot(x,y1,x,y2)
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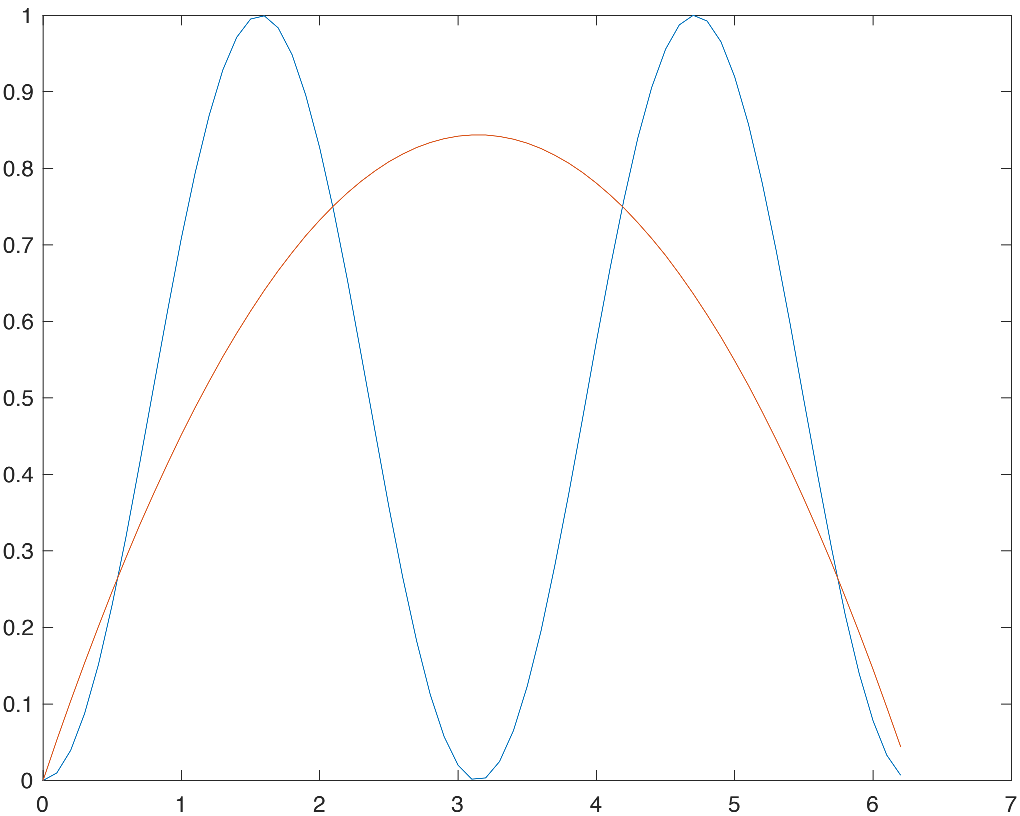


2. $x_0 = 0$, $x_1 = \frac{2\pi}{3}$, $x_2 = \frac{4\pi}{3}$, $x_3 = 2\pi$

$f(x_0) = 0$, $f(x_1) = \frac{3}{4}$, $f(x_2) = \frac{3}{4}$, $f(x_3) = 0$

(a) $S_0 = a_0 + b_0(x - x_0) + c_0(x - x_0)^2 + d_0(x - x_0)^3$

$= a_0 + b_0 x + c_0 x^2 + d_0 x^3$, $[0, \frac{2\pi}{3}]$

$S_1 = a_1 + b_1(x - x_1) + c_1(x - x_1)^2 + d_1(x - x_1)^3$

$= a_1 + b_1(x - \frac{2\pi}{3}) + c_1(x - \frac{2\pi}{3})^2 + d_1(x - \frac{2\pi}{3})^3$, $[\frac{2\pi}{3}, \frac{4\pi}{3}]$

$S_2 = a_2 + b_2(x - x_2) + c_2(x - x_2)^2 + d_2(x - x_2)^3$

$= a_2 + b_2(x - \frac{4\pi}{3}) + c_2(x - \frac{4\pi}{3})^2 + d_2(x - \frac{4\pi}{3})^3$, $[\frac{4\pi}{3}, 2\pi]$

(b) $S_0(x_0) = f(x_0)$

$a_0 = 0$ ①

$S_1(x_1) = f(x_1)$

$a_1 = \frac{3}{4}$ ②

$S_2(x_2) = f(x_2)$

$a_2 = \frac{3}{4}$ ③

$S_2(x_3) = f(x_3)$

$0 = a_2 + b_2(2\pi - \frac{4\pi}{3}) + c_2(2\pi - \frac{4\pi}{3})^2 + d_2(2\pi - \frac{4\pi}{3})^3$

$a_2 + \frac{2\pi}{3} b_2 + (\frac{2\pi}{3})^2 c_2 + (\frac{2\pi}{3})^3 d_2 = 0$ ④

(c) $S_1(x_1) = S_0(x_1)$

$a_1 = a_0 + b_0 \frac{2\pi}{3} + c_0(\frac{2\pi}{3})^2 + d_0(\frac{2\pi}{3})^3$

$a_1 - a_0 - \frac{2\pi}{3} b_0 - (\frac{2\pi}{3})^2 c_0 - (\frac{2\pi}{3})^3 d_0 = 0$ ⑤

$S_2(x_2) = S_1(x_2)$

$a_2 = a_1 + b_1(\frac{4\pi}{3} - \frac{2\pi}{3}) + c_1(\frac{4\pi}{3} - \frac{2\pi}{3})^2 + d_1(\frac{4\pi}{3} - \frac{2\pi}{3})^3$

$a_2 - a_1 - (\frac{2\pi}{3}) b_1 - (\frac{2\pi}{3})^2 c_1 - (\frac{2\pi}{3})^3 d_1 = 0$ ⑥

(d) $S_0'(x) = b_0 + 2c_0 x + 3d_0 x^2$

$S_1(x) = a_1 + b_1 x - \frac{2\pi}{3} b_1 + c_1(x^2 - \frac{4\pi x}{3} + (\frac{2\pi}{3})^2) + d_1(x^3 - 2\pi x^2 + \frac{4\pi^2 x}{3} - \frac{8\pi^3}{27})$

$S_1'(x) = b_1 + 2c_1 x - \frac{4\pi}{3} c_1 + 3d_1 x^2 - 4\pi d_1 x + \frac{4\pi^2}{3} d_1$

$S_2(x) = a_2 + b_2 x - \frac{4\pi}{3} b_2 + c_2(x^2 - \frac{8\pi x}{3} + (\frac{4\pi}{3})^2) + d_2(x^3 - 4\pi x^2 + \frac{16\pi^2 x}{3} - \frac{64\pi^3}{27})$

$S_2'(x) = b_2 + 2c_2 x - \frac{8\pi}{3} c_2 + 3x^2 d_2 - 8\pi x d_2 + \frac{16\pi^2}{3} d_2$

$S_1'(x_1) = S_0'(x_1)$

$b_1 + 2c_1(\frac{2\pi}{3}) - \frac{4\pi}{3} c_1 + 3d_1(\frac{2\pi}{3})^2 - 4\pi d_1(\frac{2\pi}{3}) + \frac{4\pi^2}{3} d_1 = b_0 + 2c_0(\frac{2\pi}{3}) + 3d_0(\frac{2\pi}{3})^2$

$b_1 - b_0 - \frac{4\pi}{3} c_0 - \frac{4\pi^2}{3} d_0 = 0$ ⑦

$S_2'(x_2) = S_1'(x_2)$

$b_2 + 2c_2(\frac{4\pi}{3}) - \frac{8\pi}{3} c_2 + 3d_2(\frac{4\pi}{3})^2 - 8\pi(\frac{4\pi}{3}) d_2 + \frac{16\pi^2}{3} d_2$

$= b_1 + 2c_1(\frac{4\pi}{3}) - \frac{4\pi}{3} c_1 + 3d_1(\frac{4\pi}{3})^2 - 4\pi d_1(\frac{4\pi}{3}) + \frac{4\pi^2}{3} d_1$

$b_2 - b_1 - \frac{4\pi}{3} c_1 - \frac{4\pi^2}{3} d_1 = 0$ ⑧

(e) $S_0''(x) = 2c_0 + 6d_0 x$

2. (e) $S_0''(x) = 2C_0 + 6d_0x$

$S_1''(x) = 2C_1 + 6d_1x - 4\pi d_1$

$S_2''(x) = 2C_2 + 6d_2x - 8\pi d_2$

$S_1''(x_1) = S_0''(x_1)$

$2C_1 + \cancel{6d_1\left(\frac{2\pi}{3}\right)} \cancel{- 4\pi d_1} = 2C_0 + 6d_0\left(\frac{2\pi}{3}\right)$

$2C_1 - 2C_0 - 4\pi d_0 = 0$ (9)

$S_2''(x_2) = S_1''(x_2)$

$2C_2 + \cancel{6d_2\left(\frac{4\pi}{3}\right)} \cancel{- 8\pi d_2} = 2C_1 + 6d_1\left(\frac{4\pi}{3}\right) - 4\pi d_1$

$2C_2 - 2C_1 - 4\pi d_1 = 0$ (10)

(f) $\because$ it's clamped bound

$\therefore S_0'(x_0) = f'(x_0)$ and $S_2'(x_3) = f'(x_3)$

$f(x) = \sin^2 x$

$f'(x) = 2\sin x \cos x$

$S_0'(x_0) = f'(x_0)$

$b_0 = 0$ (11)

$S_2'(x_3) = f'(x_3)$

$b_2 + 2C_2(2\pi) - \frac{8\pi}{3}C_2 + 3d_2(2\pi)^2 - 8\pi(2\pi)d_2 + \frac{16\pi^2}{3}d_2 = 0$

$b_2 + \frac{4\pi}{3}C_2 + \frac{4}{3}\pi^2 d_2 = 0$ (12)

Matrix:

$$
\begin{array}{c}
\begin{array}{cccccccccccc} a_0 & a_1 & a_2 & b_0 & b_1 & b_2 & C_0 & C_1 & C_2 & d_0 & d_1 & d_2 \end{array} \\
\left[\begin{array}{cccccccccccc|c}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{3}{4} \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{3}{4} \\
0 & 0 & 1 & 0 & 0 & \frac{2\pi}{3} & 0 & 0 & \left(\frac{2\pi}{3}\right)^2 & 0 & 0 & \left(\frac{2\pi}{3}\right)^3 & 0 \\
-1 & 1 & 0 & -\frac{2\pi}{3} & 0 & 0 & -\left(\frac{2\pi}{3}\right)^2 & 0 & 0 & -\left(\frac{2\pi}{3}\right)^3 & 0 & 0 & 0 \\
0 & -1 & 1 & 0 & -\frac{2\pi}{3} & 0 & 0 & -\left(\frac{2\pi}{3}\right)^2 & 0 & 0 & -\left(\frac{2\pi}{3}\right)^3 & 0 & 0 \\
0 & 0 & 0 & -1 & 1 & 0 & -\frac{4\pi}{3} & 0 & 0 & -\frac{4\pi^2}{3} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 1 & 0 & -\frac{4\pi}{3} & 0 & 0 & -\frac{4\pi^2}{3} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -2 & 2 & 0 & -4\pi & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 2 & 0 & -4\pi & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \frac{4\pi}{3} & 0 & 0 & \frac{4\pi^2}{3} & 0
\end{array}\right]
\end{array}
$$

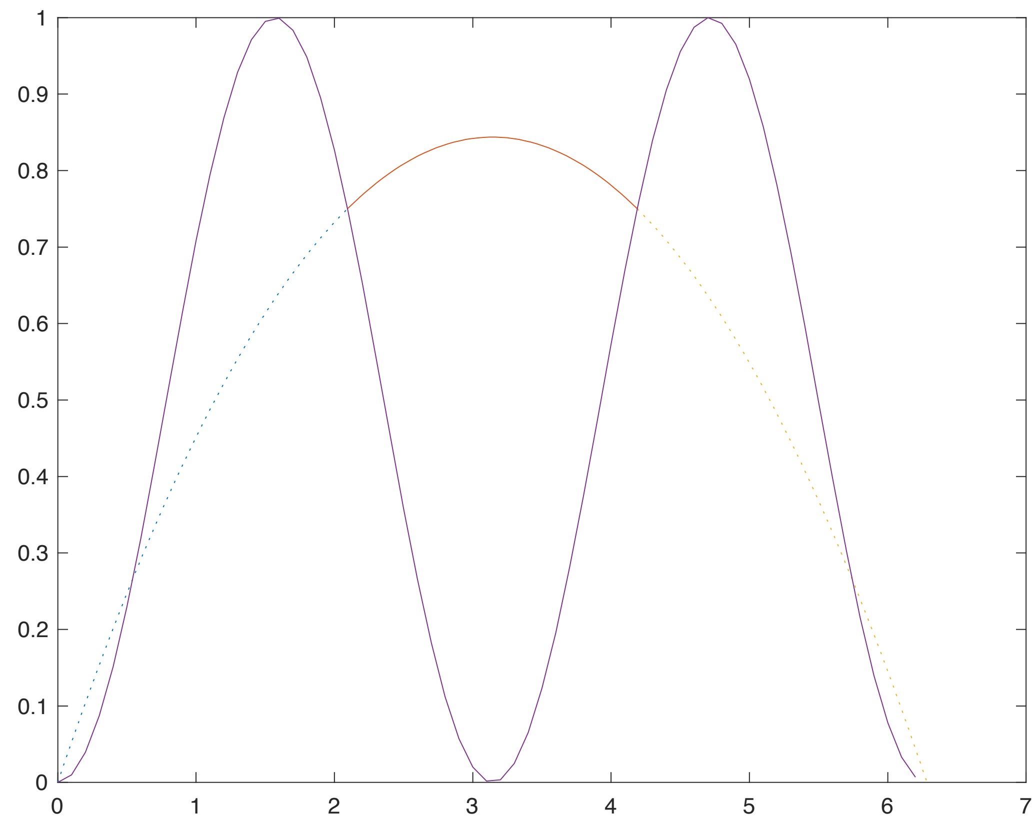
3(a).

```
>> format short
>> x=[0;2*pi/3;4*pi/3;2*pi];
>> y=[0;0;0.75;0.75;0;0];
>> pp = spline(x, y);
>> [b, c] = unmkpp( pp )

b =

     0    2.0944    4.1888    6.2832


c =

   -0.0816    0.3420         0         0
    0.0000   -0.1710    0.3581    0.7500
    0.0816   -0.1710   -0.3581    0.7500
```

S0(x) =-0.0816*x^3+0.3420*x^2;   (0<=x<=2.0944)
S1(x) =-0.1710 *(x-2.0944)^2+0.3581*(x-2.0944)+0.7500;
(2.0944<=x<=4.1888)
S2(x) = 0.0816* (x-4.1888)^3-0.1710 *(x-4.1888)^2-0.3581*(x-4.1888)+0.7500;
(4.1888<=x<=6.2832)

3(b).

```
>> x1=linspace(0,2*pi/3,50);
>> y1=c(1,1)*(x1-0).^3+c(1,2)*(x1-0).^2+c(1,3)*(x1-0)+c(1,4);
>> x2=linspace(2*pi/3,4*pi/3,50);
>> y2=c(2,1)*(x2-2.0944).^3+c(2,2)*(x2-2.0944).^2+c(2,3)*(x2-2.0944)+c(2,4);
>> x3=linspace(4*pi/3,2*pi,50);
>> y3=c(3,1)*(x3-4.1888).^3+c(3,2)*(x3-4.1888).^2+c(3,3)*(x3-4.1888)+c(3,4);
>> x=[0:0.1:2*pi];
>> y=sin(x).^2;
>> plot(x1,y1,':',x2,y2,'-',x3,y3,':',x,y,'-')
```



1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
1
2
3
4
5
6
7