Master of Science Thesis

Formation flight in civil aviation

Development of a decentralized approach to formation flight routing

C.M.A. Verhagen, BSc.

17th of July, 2015

Faculty of Aerospace Engineering · Delft University of Technology

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For obtaining the degree of Master of Science in Aerospace Engineering at Delft University of Technology

C.M.A. Verhagen, BSc.

17th of July, 2015

Faculty of Aerospace Engineering · Delft University of Technology

Delft University Of Technology
Department Of
Air Transport Operations

The undersigned hereby certify that they have read and recommend to the Faculty of Aerospace Engineering for acceptance a thesis entitled **"Formation flight in civil aviation"** by **C.M.A. Verhagen, BSc.** in partial fulfillment of the requirements for the degree of **Master of Science**.

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Summary

A model has been developed to demonstrate the value of a decentralized approach to formation flight routing. In the considered scenario, formation flight is an in-flight option. While centralized approaches are well covered in the literature, anticipating the use of formation flight before take-off, decentralized approaches have received less attention. Several examples do exist, noting that only one completely decentralized method was encountered. A completely decentralized approach is herein defined as one that does not use any kind of pre-flight formation flight planning or routing restrictions.

While flying, each aircraft may communicate with others when they breach each other's communication range. During these communications, a formation flight strategy is formulated and the flights have to choose directly if they wish to commit to it. This method of obtaining fuel savings characterizes this work. Its compactness allowed for a broad evaluation of the potential of a decentralized approach within the project time frame. The sub-optimality of this greedy method enhances the value of any positive findings from this research, since it is likely that a global optimum has not yet been found.

The decentralized approach that was developed during this project shows promising results when considering the efficiency with which large scenarios can be evaluated. A typical simulation in the performed transatlantic case study, containing 347 flights, took less than 6 minutes on a standard PC. Additionally, the capability of the model to sustain obtained savings when flights are delayed has been shown. The ability of flights to perform multiple consecutive formation flight segments is new to the research area. The latter significantly increases the usage rate of formation flight in the considered scenarios.

It is noted that, in any current publication on formation flight implementation, the estimation of overall fuel savings strongly depends on assumptions. As such, the added value of this research is of a more relative nature. Based on the conservative assumption that any formation member other than the formation leader experiences a fuel flow reduction of 10%, the resulting routes and used formations were studied. The maximum obtained overall fuel savings estimation is 4.3% in a case study on 347 transatlantic flights. These savings were found when a combined objective was used, taking into account both fuel consumption and additional flight time. More significantly, formation sizes ranging from 2 to 6 aircraft were commonly used. A formation flight usage rate of 73%, with respect to total flight time, was recorded during the same simulation.

Acknowledgements

When I took the time to look back on my project, the first thing I realized was that I had a very good time. I enjoyed the project, the people, the results and dare I admit it, even the reading.

I hereby warmly thank everyone that either made this project possible or contributed to it in any way. In particular:

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All others, feel thanked, you are.

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Nomenclature

C_t	Thrust coefficient	$[-]$
C_L	Lift coefficient	$[-]$
C_D	Drag coefficient	$[-]$
CR	Radius of communication	$[km]$
dt	Time step size	$[minutes]$
$M - value$	Parameter to control fuel flow	$[km/\sqrt{kg}]$
R	Range/segment length	$[km]$
S	Wing surface area	$[m^2]$
T_A	Thrust available	$[kN]$
V	Flight speed	$[m/s]$
V_{min}	Minimum flight speed	$[m/s]$
V_{max}	Maximum flight speed	$[m/s]$
w_A	Geometric routing weight of flight from origin A	$[-]$
w_B	Geometric routing weight of flight from origin B	$[-]$
w_C	Geometric routing weight of formation flight segment	$[-]$
W	Aircraft gross weight	$[kg]$
W_0	Aircraft gross starting weight	$[kg]$
W_1	Aircraft gross end weight	$[kg]$
α	Angle used in synchronization method	$[deg]$
β	Angle used in synchronization method	$[deg]$
ρ	Free stream density	$[kg/m^3]$
θ_f	Formation angle	$[deg]$
$Point\ A\ and\ B$	Origin A and B	
$Point\ C\ and\ D$	Destination C and D	
$Point\ E$	Dummy origin E	
$Point\ F$	Dummy destination F	
$Point\ J$	Joining point	
$Point\ J'$	Relocated joining point	
$Point\ J_{new}$	Joining point that enables synchronization	
$Point\ J_{old}$	Former joining point	
$Point\ S$	Splitting point	
$Point\ S'$	Relocated splitting point	

Chapter 1

Introduction

Over the past decades, formation flight has become a recognized method when considering possibilities to increase the fuel efficiency of civil aviation. Compared to other efficiency measures, such as innovative aircraft designs, formation flight requires limited amounts of new technology, as implementing it would mainly be an operational change. Therefore, formation flight is an attractive potential efficiency measure, as it could be implemented by airlines without the requirement for new equipment and related investments.
By examining the flight behaviour of birds, as done by Lissaman in 1969 [21], a general understanding of the (aero)dynamics of formation flight was developed. As the potential for fuel savings became more apparent, flight tests were conducted to confirm this, i.e. by Ray et al in 2002 [29] and more recently by Flanzer et al. in 2014 [12]. The latter used two military C-17 aircraft and reported achieved fuel savings of 5-10% for the trailing aircraft, increasing with mission length.

To make use of the acquired knowledge, the practical implementation of formation flight was considered. A literature study was performed in order to explore this research area. In 2003, Ribichini et al. presented their approach to formation flight routing for UAV's [31]. Their approach used a greedy algorithm and it considered formation flight as an in-flight option. This locally coordinated use of formation flight characterizes what will be called a *decentralized approach*. In later years, the focus in the research area turned to the routing and scheduling of an entire fleet based on origins and destinations and introducing formations where this would contribute to an overall decrease in fuel consumption. Such an approach will be called a *centralized approach*, since formations are pre-determined on a network level. A leading example of such a centralized approach was published by Xu et al. in 2014 [37]. They presented a case study in which 150 flights of the Star alliance could save around 7% on their combined fuel burn on routes over the Atlantic Ocean, if they planned to fly in small formations (1,2 or 3 aircraft).

This research aims to estimate the value of a decentralized approach, inspired by Ribichini et al., with respect to a centralized approach. Global optimization has disadvantages, such as its vulnerability to delays and computational inefficiency, as illustrated by Xu et al. [37]. Theoretically, however, global optimization could provide estimations regarding the

highest possible fuel savings on a certain set of routes. Recent work is limited in applying global optimizations, due to exponentially increasing computation times with increasing formation and network sizes.
When a decentralized approach is considered, the threats of delays and computational limitations are potentially circumvented. This research aims to elaborate on this, while facing the challenge of obtaining results that allow for comparison of the two approaches.

During the development of this work, result visualisation was often used as a verification tool. After a simulation, it was known for every aircraft, at every time step, where and how it was flying. With the recorded flight data, it was possible to generate videos of the performed flights. As such, the behaviour of each simulated aircraft could be quickly evaluated. When developing any new function of the model, the results could be verified by looking at the effect on the behaviour of aircraft. Thanks to a light weight simulation design, it was possible to obtain a video of a small to medium sized scenario in a matter of seconds. In this report, these movies can not be shown. Instead, flown trajectories as well as several snapshots of video-frames will be provided in Chapter 7.

This report provides an overview of the research area in Chapter 3. Chapters 4 and 5 provide the simulated operational concept and the implemented fuel consumption calculation method, respectively. In Chapter 6, key elements of the developed model are presented. Consecutively, Chapter 7 provides the main results of this thesis, presented in a case study of 347 transatlantic flights. Chapter 8 contains the conclusions and recommendations.

Chapter 2

Description of the research area

The interest in possible energy savings through formation flight has brought forward many publications over the past decades. In the past twenty years, as fuel prices and airline competitiveness increased, formation flight as a fuel saving mechanism for civil aviation was suggested. Studies on potential savings and operational implementation followed. This chapter presents an overview of the publications that are in the center of the research area, as a means to more narrowly introduce the topic of this thesis.

A general search on the topic of using formation flight in aviation was conducted. Among the first encountered documents were the flight test results of Ray et al. obtained in 2002 [29]. In 2011, the feasibility of formation flight as a fuel saving mechanism for civil aviation as well as potential gains were positively discussed by Herinckx et al. [15]. They published a graph which illustrates the increasing theoretical benefits of formation flight as formations increase in size. This graph is given in Figure 2.1. It was graphically enhanced before it was included in this section.

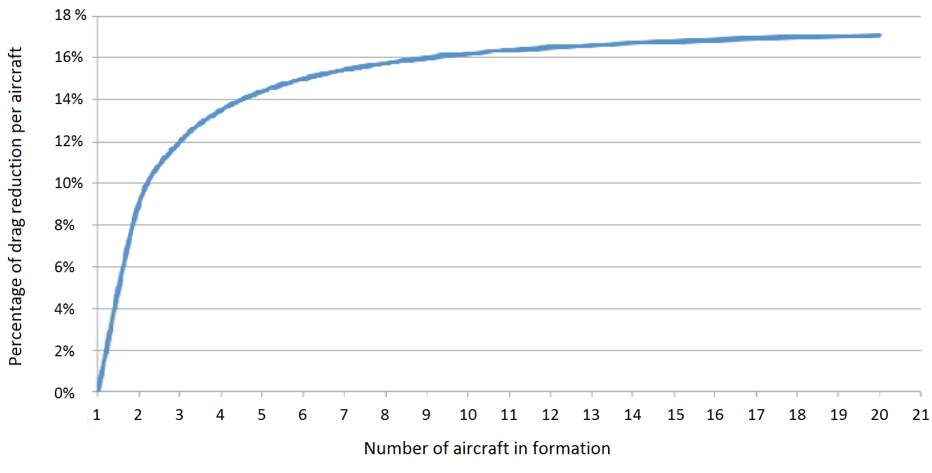


Figure 2.1: Theoretical relation between formation size and the obtainable drag savings per formation member, excluding the formation leader [15].

As more and more sources were collected, the search focussed on the implementation of formation flight in civil aviation as well as closely related subjects. Accordingly, the state of the art in the research area was identified. To illustrate this center of the research area, the research area map in Figure 2.2 was created. It depicts those of the encountered publications that surround or include key elements of the research topic. The connectors in the Figure, while representing citation connections between the different works, underline the coherence between the presented literature. As such, the research area map in Figure 2.2 presents a structured exposition of the literature that is most relevant in the area of interest. The remainder of this chapter covers this exposition.

In 2002, Ray et al. presented their results from formation flight tests with F/A-18 aircraft [29]. They reported an 18% fuel flow reduction for a trailing aircraft flying at Mach 0.56 at an altitude of 25.000 ft. Among other papers, including the work of Rao et al. on when two aircraft should form a formation [28], the results of Ray et al. inspired Ribichini et al. to do a theoretical study on the possible savings from practical implementation of formation flight in civil aviation, published in 2003 [31]. Ribichini et al. addressed the problem of routing flights in order to reduce overall fuel consumption, by allowing them to form formations when they judged this to be the most fuel efficient way to complete the remainder of their missions. Ribichini et al. focused on how an aircraft could decide en-route to join or start a formation. As formation flight always requires some detours which may nullify or even outweigh the potential benefits, a reliable method for analysing options was required. The approach in the paper of Ribichini et al. has a significant property that makes the paper one of the leading sources for this research: formation flight is not a conclusive part of the flight plan, it is an option. In this way, formations are only formed when they are believed to be beneficial, based on actual in-flight data and actual aircraft being near the same location and ready (and willing) to cooperate.

From 2004 to 2006, mainly theory exploring work was published, among which a view on how to generate formations and optimal trajectories presented by Raffard et al. [27] for a group of aircraft that wants to form a formation in-flight, whilst following a target or leader. The next work related to formation flight implementation was published in 2009 by Bower et al. and considered formation geometries and route optimisation as well as a simple five aircraft example, including estimated overall fuel savings around 10% [9]. As can be seen by following the connectors in Figure 2.2, this paper is cited by most of the hereafter discussed papers on formation flight implementation.

When discussing the implementation of formation flight in civil aviation, concerns on safety can not be neglected. Close formation flight requires accurate control to ensure safe operations. Most probably inspired by this notion, Ning et al. presented their work on the aerodynamics of extended formation flight in 2011 [23, 24, 25]. In these works, formations that have 10-50 wingspans of separation in flight direction between two consecutive aircraft are defined as extended formations. With the aircraft tracking errors as largest disturbance of possible drag reductions, they estimate possible maximum induced drag reductions (for the most benefiting formation member) of 30$\pm$ 3% and 40$\pm$ 6% for a two and three aircraft formation, respectively.

In 2012, the publications on practical implementation of formation flight are continued by the work of Xu et al. on the global optimization of large networks of flights based on current schedules. They allow for formations of up to three aircraft and estimate fuel savings for 150 flights of the transatlantic Star Alliance to be 6.8% [38]. Noteworthy is that

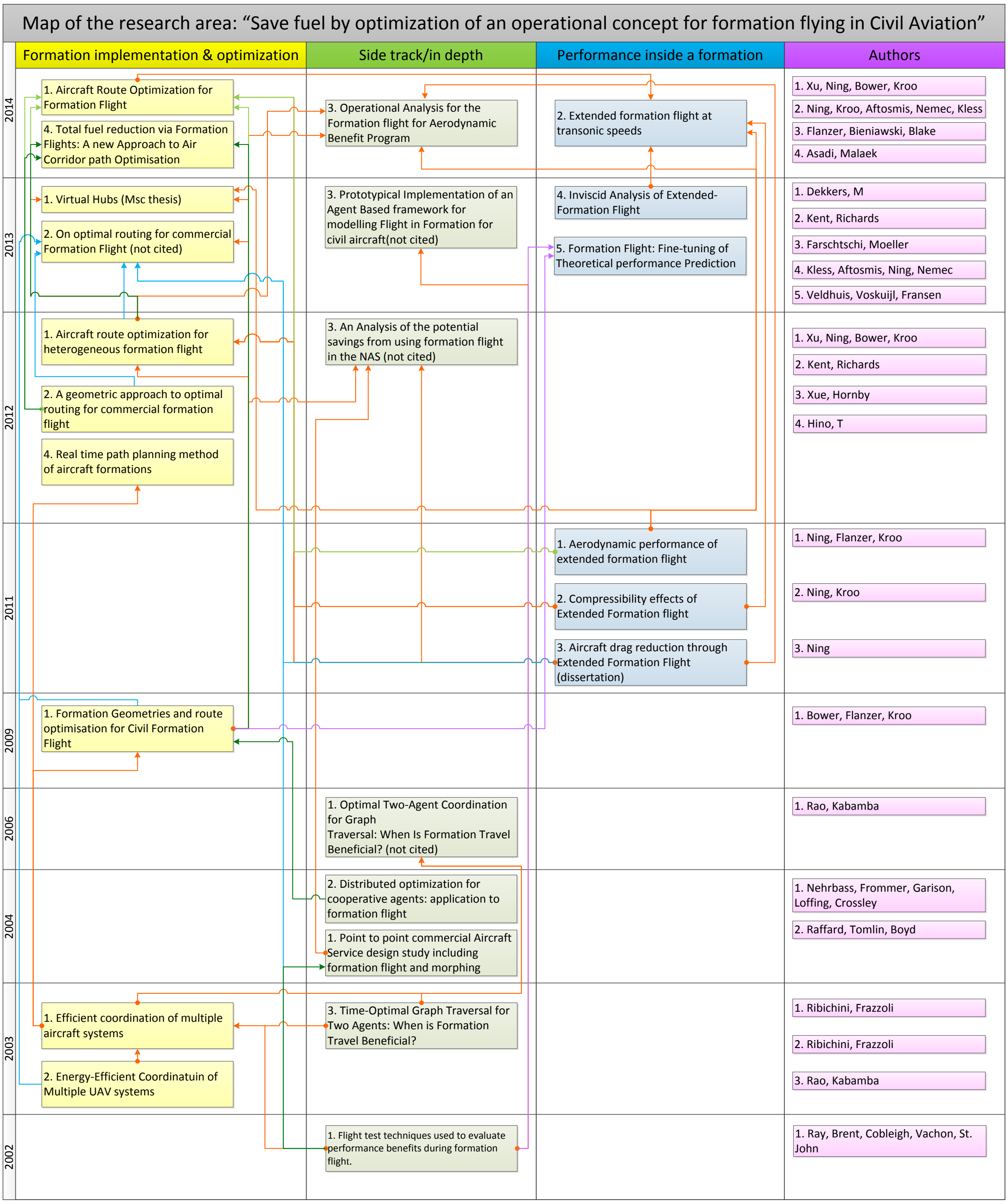


Figure 2.2: Map of the research area, showing the most relevant of the encountered publications as well as their citation connections.

they implement the previously discussed results of Ning et al. [23, 24, 25] on extended formation flight, to illustrate the possible value of extended formation flight to airlines. In the same year, Kent et al. presented their geometrical approach to finding flights that are eligible for formation flight, based on their origin and destination [17]. Similar to Xu et al., they determine all the possible formations and their corresponding routes that could

be used. The upper limit on formation size is now two aircraft. Accordingly, both authors apply integer optimisation to the acquired set of possible formations in order to obtain the combination of formations that minimizes the overall fuel consumption. Under the assumption of complete schedule flexibility, Kent et al. conclude that potential global fuel savings can be estimated at just over 10%. To obtain this figure, they allowed optimized formations of size two to be joined once again in formations of size four, for which a higher fuel efficiency is assumed.
In the same year, Xue et al. elaborated on grouping flights in formations along highways in the sky of the US [39]. These highways, named corridors, are constructed from great circle flight paths. Considering formations up to four aircraft, Xue et al. estimate that around 600 million dollars a year are to be saved on fuel, with respect to current flight plans, by the 1200 flights that are eligible for formation flight in the ten busiest corridors. When solo flights along great circle routes, which are currently not flown in the US, are compared to the use of the determined corridors, the estimated savings on fuel through the use of formation flight are approximately 320 million dollars annually. It is noted that, similar to Ribichini et al., Xue et al. form formations based on in-flight encountered opportunities to do so. However, the formation route up to the formation splitting point is constrained to the respective corridor.
Following a similar strategy of creating fixed points along which aircraft may be routed, Dekkers investigated the use of virtual hubs in 2013 [10]. He used a discrete event simulation to evaluate optimized virtual hub locations as well as the obtainable fuel savings from routing aircraft along these common points where they could be assigned to a formation. Both Xue et al. and Dekkers introduce a level of network organization in their flight planning. Therefore, they explore the middle grounds between centralized and decentralized approaches to the implementation of formation flight in civil aviation.

Also in 2013, Kent et al. extend their geometric approach with a differential fuel burn model based on a Breguet range equation as well as wind considerations in an example on 210 transatlantic routes [18]. Focussing more on theoretical aspects of formation flight, Kless et al. further explore the potential benefits from extended formation flight [20], while Veldhuis et al. consider the effects of i.e. positive sweep and winglets on drag reductions from formation flight [34]. The calculation model used by Veldhuis et al. predicts around 13% total drag decrease for a medium sized aircraft following an identical aircraft.
In 2014, Xu et al. published a continuation of their work in 2012, briefly mentioning the effect of delays on their estimated fuel savings. These savings seem to decrease linearly with the percentage of delayed participating flights. From flight tests with C-17 aircraft in extended formation flight, included in the work of Flanzer et al. [12], sustained fuel savings exceeding 10% for the trailing aircraft were autonomously achieved on flight segments with a duration of more than 90 minutes. Details on these specific flight tests can be found in the work of Bieniawski et al. [5]. Also in 2014, Asadi et al. studied the use of corridors, similar to Xue et al.[39] in 2012, in the Iranian airspace [4]. Moving from the second to the third column in Figure 2.2, the most recent piece of relevant literature contains the work of Ning et al., which states that flying at 1-3% lower Mach numbers, and slightly increasing the altitude, may increases the predicted overall obtainable drag savings through formation flight [26].

A dominating advantage of the approaches of Ribichini et al.[30] and Xue et al. [39] in comparison to global network optimizations (most other sources) is related to the certainty of efficient formations during implementation. If the entire formation flight is planned beforehand and a fraction of the participants is delayed or otherwise unable to join, the planned routes and formations are no longer optimal. The large scale planning approaches published up to this moment (i.e. Xu et al. in 2014 [37], Kent et al. in 2012 [18]) do not offer a resolution for this situation, whereas the method of Ribichini et al. provides an en-route decision scheme designed for flexibility and en-route optimization. The key difference between these two ways of approaching the implementation of formation flight in civil aviation can be found in the optimization perspective. While many sources introduce formations based on analysis of current networks and schedules, a few others try to find beneficial formation flight scenarios while the potential formation members are already flying. Studies by Raffard et al. in 2004 [27] and Hino in 2012 [16] offer alternative views on how aircraft could communicate about forming a formation.

Current literature considered, the research area can be expanded by studying a decentralized approach to the implementation of formation flight in civil aviation. Eligible topics for further research can be found in how Ribichini et al. have modelled communications between aircraft. They assumed only basic communications with the neighbouring aircraft. Also, they provide theoretical cost reductions in their examples based on assumptions related to overall propulsive energy requirements. Ribichini et al. indicate that their resulting cost savings are optimistic, and suggest the implementation of more realistic fuel burn parameters for both solo and formation flight segments. Additionally, their simulation could be extended to include more aircraft. This would increase the practical relevance of the research.

The next sections contain descriptions of those works from the discussed research area that were used as main sources for this thesis.

Chapter 3

Literature study: formation flight implementation in civil aviation

This section will elaborate on literature concerning the implementation of formation flight in civil aviation. From exploring the research area as discussed in Chapter 2, it was found that there are two distinct groups of approaches. One of those groups contains the works that apply a global optimization to estimate achievable fuel savings from the use of formation flight. What characterizes these approaches, is that every used formation is planned before any flight takes off. Using routes, departure and arrival times, as well as specific formation benefits as parameters, a set of flight plans is generated, through minimizing the overall fuel consumption or the total operational cost. The coordination of the formation flight implementation is done based on all the available information in the network at the time of the optimization. As this has to be centrally coordinated, approaches in the first group are hereby called *centralized approaches*.
The other group contains methods that do not plan on using a formation. A key property of the approaches in this group is that formation flight is an en-route option. Based on in-flight collectable data, such as speeds and flight plans of aircraft that happen to be relatively close by and are going in roughly the same direction, local optimizations are performed to determine if there are beneficial formation flight opportunities. If so, flight plans are updated with the actions required to take this opportunity. An opportunity is defined as a possibility to lower the overall fuel burn, by using formation flight, of the flights involved in these local communications. By adding up the savings of each used formation, an estimation of overall savings can be made when, for example, a large set of flights is considered. Since approaches such as these rely on local decisions on the use of formation flight, they are hereby called *decentralized approaches*.
The next sections summarize some of the leading examples of both approaches. Several considerations on fuel consumption modelling are given in Section 3.3. Section 3.4 presents a discussion, conclusions, and the resulting research objective.

3.1 Centralized Approaches

This section elaborates on two examples of centralized approaches found in recent literature. They can both be found the Research Area Map in Figure 2.2, in Chapter 2.

3.1.1 Aircraft Route Optimization for Formation Flight

A recent publication on a centralized approach to the implementation of formation flight in civil aviation, is the work of Xu et al. in 2014 [37]. With their paper, Xu et al. present one of the furthest developed centralized approach found in the literature. It contains a detailed fuel burn model (see Section 3.3), incorporating the work of Ning et al.[23] on rolled up wakes, allowing for, for example, the analysis of formations containing multiple aircraft types. It also includes, similar to other sources, an integer optimization to determine the optimal formation flight schedule. Two objectives are included and studied separately: minimize the overall fuel consumption and minimize the overall operational cost. As computational time was a limiting factor, formations could not contain more than three aircraft.

Xu et al. aim to simulate scenarios with many flights. The structure of their approach is illustrated in Figure 3.1. As their centralized approach considers all possible formation combinations, it also includes infeasible formation options, such as a formation of two aircraft going in opposite directions. Therefore, as a means to reduce computational time (more correctly, keep computational requirements workable), they perform a heuristic selection of possible formation candidates, before they initialize their optimization. They apply three heuristic rules. The first one considers the angle between two (or two out of three) great circle routes. If this angle is greater than a certain reference angle, formation flight for the corresponding flights will not be considered. Additionally, the total distance between the two origins, plus the distance between the two destinations, divided by the total to be flown distance, should be smaller than a certain factor to avoid consideration of routes that would require large detours. The third rule is related to departure times. Since two aircraft can not form a formation if one of them is still on the ground, the scheduled flight times of formation partners need to have a certain overlap, represented by a factor between overall and overlapping flight times.

The main design variables used by Xu et al. for each flight are altitude, weight and Mach number at the critical stages of each flight. These stages are the beginning and end of the cruise phase, either in a formation or while flying solo, as well as take-off and landing. The changes in weight represent consumed fuel and are constrained to ensure that each future segment can still be flown solo. Also, the two-dimensional locations (latitude and longitude) of the rendezvous and splitting points are included as design variables.
The most important constraints in the optimization problem originate from operational and regulatory limits. For example, an aircraft must have some excess thrust and enough fuel the complete the segment(s) ahead at any stage of its flight. The departure, arrival, as well as overall flight times, are allowed to vary a selected amount of minutes around the scheduled times in order to avoid aircraft flying at sub-optimal speeds to join their formation. Increasing the allowed variation in the schedule, increases possible overall savings. To ensure that a formation can be flown, the times of passing the rendezvous

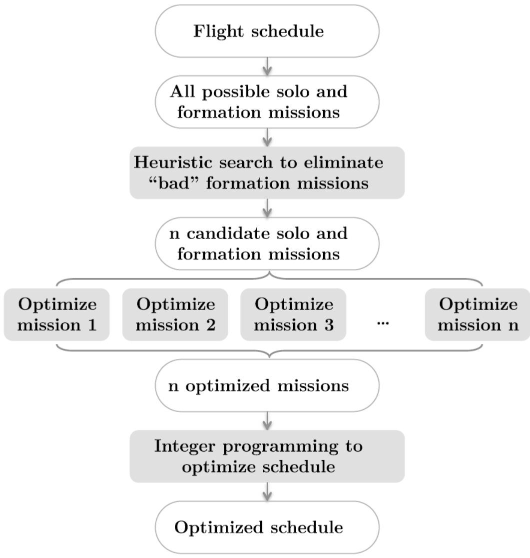


Figure 3.1: Stepwise illustration of the centralized approach by Xu et al.[37]

point and the splitting point are constrained to be equal for all the members of a specific formation.

If all the possible formations of either 1,2 or 3 aircraft are evaluated and their fuel and cost saving potential is determined, an integer program optimizes the combination of used formations in order to achieve either minimal fuel burn or minimal operational costs. The great majority of the evaluated options is not used, as an aircraft can only be in one formation at a time.

Xu et al. illustrate their method by showing two examples of route optimization after including formation flight. The largest of the two will be discussed here. Figure 3.2 shows the routes of 150 transatlantic flights from the Star alliance. Intuitively, these routes seem to be suitable for formation flight and since they all belong to one alliance, they are assumed to be willing to contribute to the overall objective by altering their route. Figure 3.3 depicts the formation flight routes resulting from the simulation performed by Xu et al.

This Star alliance example assumes an allowable variation in departure and arrival times of 6 minutes. The heuristics are set to exclude any combination of flights of which the intended headings differ more than 30 degrees. The overlap time is required to be at least 90% and the ratio of inter-airport distances may not be higher than 15%, in order to exclude relatively long detours. Minimizing the overall fuel consumption resulted in estimated overall fuel savings of 7.7%.

Centralized approaches are expected to have difficulties with adjusting to last-minute changes in schedules, mainly caused by delays. To demonstrate how their approach would handle delays, Xu et al. use a Monte Carlo simulation in which they randomly delay an arbitrary set of the 150 flights. They assume that a delayed flight can not take part in a formation. Figure 3.4 shows the means and standard deviations of achieved fuel savings for different amounts of delayed aircraft. A rough estimation yields a proportional relation between the percentage of delayed flights and the percentage of decrease in fuel savings

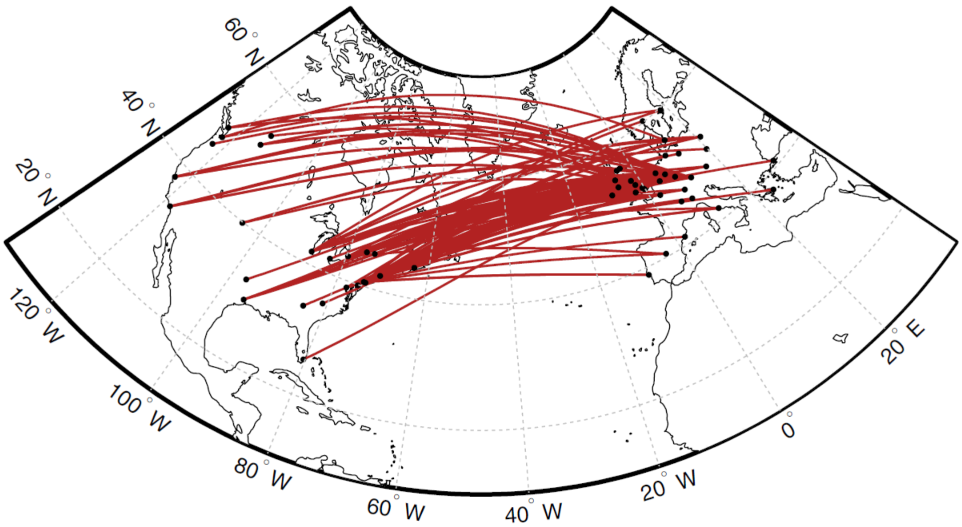


Figure 3.2: The great circle route of 150 flights of the Star alliance [37].

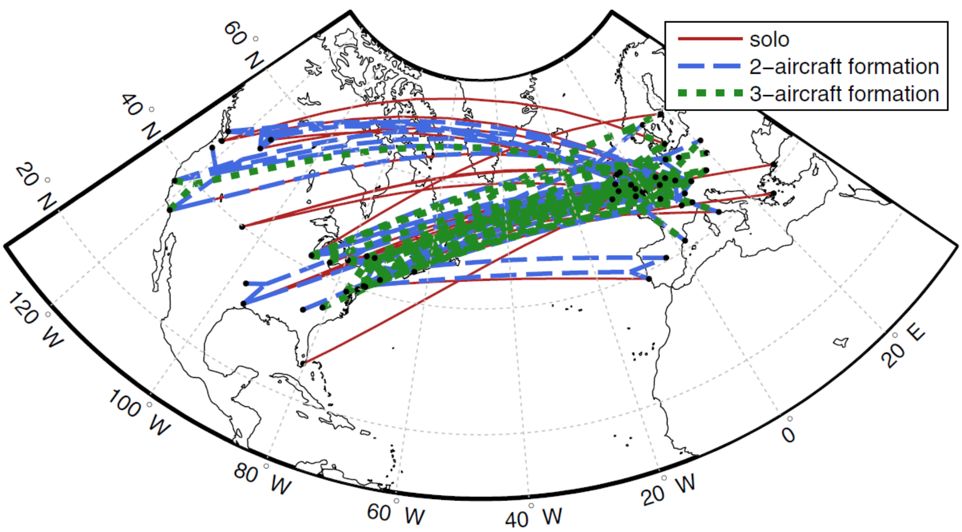


Figure 3.3: Formations selected by Xu et al., resulting from applying the minimum fuel objective. When operational costs are minimized, more solo missions are used, as the possibilities to save fuel are apparently outweighed by the cost originating from a longer flight or block time [37].

relative to the scenario without delays. Xu et al. indicate that more candidate formations as well as less restrictive heuristics would result in a less severe impact of delays on the overall achievable fuel savings.

Xu et al. faced significant challenges with respect to computational time. Especially the optimization of individual formation routes took a lot of computational effort. Even though their heuristics removed more than 99% of all flight combinations, evaluating the remaining options still was an elaborate task for their computers. Xu et al. used a gradient search method to reduce computational time on the optimization of individual formations. Once all the possible formations were optimized, selecting the set of formations that would

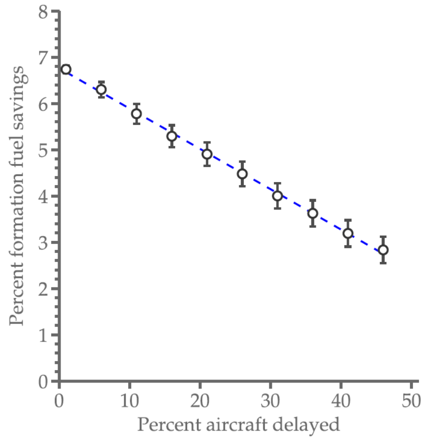


Figure 3.4: Estimated relation between the amount of delayed aircraft and the remaining achievable overall fuel savings [37].

minimize the selected objective was quickly done by integer optimization.
This vast computational requirement to optimize all possible formations within any used heuristics, is something that decentralized approaches do not require in general and as such defines a key characteristic to set the two approaches apart.

As one of their concluding statements, Xu et al. mention that they use conservative reserve fuel assumptions as well as the requirement for an aircraft to always be able to continue in solo flight. While they still can predict fuel savings, they can not let aircraft depart with less fuel than they use today. Actually, as formation flights usually require a larger overall distance to be flown by each aircraft, carried fuel may even have to increase. Xu et al. demonstrate in an example that this could lead to a 20% reduction in possible fuel savings. They mention the selection of contingency airports as a possible means to lower the requirements for carried fuel.

3.1.2 A Geometric Approach to Optimal Routing for Commercial Formation Flight

In 2012, the work of Kent et al. [17] was published, presenting their centralized approach to the implementation of formation flight in civil aviation. As can be seen in the map of the research area in Figure 2.2, the paper was inspired by several leading publications on the subject. Among others, the promising flight tests results of Ray et al. from 2002 [29] as well as the initial analysis of Ribichini et al. in 2003 [31] and the geometric method of Bower et al . [9] formed the state from which Kent et al. decided to continue. They assumed that all aircraft are of equal type and willing to strive for overall fuel savings, even if this means more fuel burn on their part, as well as complete departure and arrival time flexibility.

Similar to the work of Xu et al.[37] discussed in the previous section, Kent et al. start out with determining all the possible formations given a set of flights, with and upper limit (2 or 4) on the size of the formation. Accordingly, they minimize overall fuel burn trough optimization of the allocation of flights to possible formations. Chapter 6 discusses, implements, and extends the geometric method presented by Kent et al. to determine optimal formation routes defined by joining points and splitting points.

To increase the feasibility of their results, Kent et al. include a filter that does not allow aircraft to form formations before they have reached their cruise altitude. Additionally, they asses excessive computational requirements that arise when considered flight sets get larger. A simulated annealing approach is proven to be an adequate tool to arrive at good solutions in a reasonable amount of time. In a continuation of their work, published one year later, Kent et al. incorporate a differential fuel burn model (to include the fact that fuel burn per unit time changes while fuel is burned and with it the weight of the aircraft decreases) and an example involving wind patterns [18].

Kent et al. present a transatlantic case study that includes 210 flights. In this case study, they assume that a trailing aircraft, in a formation of two aircraft, is able to reduce its fuel flow by 20% on a formation flight segment. Accordingly, Kent et al. estimate the overall obtainable fuel savings to be 8.6%. As a means to study the fuel saving potential of using larger formations, they heuristically couple formations of size two into formations of size four. Assuming that a formation of four aircraft is able to reduce its cumulative fuel flow by 18% with respect to four solo flights along an identical segment, Kent et al. report that the estimation of the overall obtainable fuel savings in their case study exceeded 10%.

3.2 Decentralized Approaches

In this section, two decentralized approaches are discussed, along with an additional path planning method. For their place in the research area, one is again referred to the research area map in Figure 2.2, in Chapter 2.

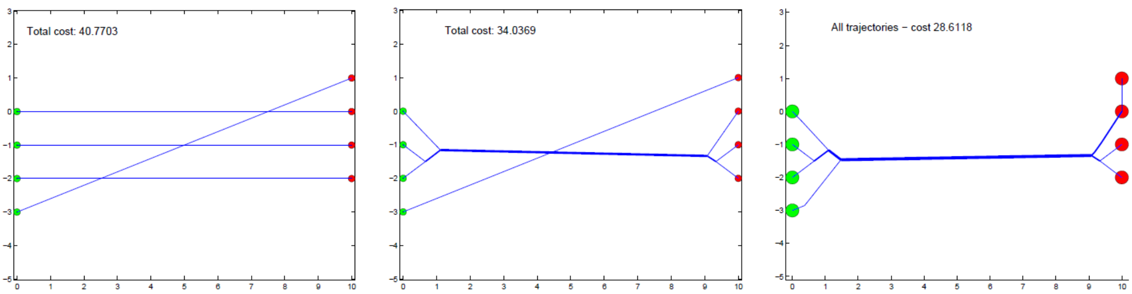
3.2.1 Energy-Efficient Coordination of Multiple-UAV systems

In 2003, Ribichini et al. published their analysis of possible energy savings resulting from having formation flight as an en-route option [31]. It is an initial work that considers UAV's and uses simplified units.
They start out by formulating a non-linear global optimization, aimed at minimizing the propulsive energy consumed by any set of flights. They conclude that this non-linear optimization is very difficult to solve, even though they use piece-wise linear trajectories, velocities, and power requirements. Thus, the global problem formulation is considered impractical, both for further study as well as for the practical implementation of formation flight. Accordingly, they approach the problem of formation flight implementation in a decentralized way.

They set up a method for two flying aircraft to determine if, and if so, how, they could join in formation in order to minimize their cumulative requirement for propulsive energy. The required power for a certain aircraft, on a certain segment, in a certain formation, gets lower as the number of aircraft flying in the formation increases. Knowing this, the optimal joining and splitting points are calculated with a two-dimensional geometrical model, based on the Weber problem that is treated by Drezner et al. in their book on facility location [11]. If this results in saving energy with respect to the individual minimum energy paths, which are straight lines in this work, the formation is formed. Ribichini et al. also include a so called 'rendezvous time'. This is the time at which the aircraft plan to meet at the determined joining point. The rendezvous time depends on the current locations and the speed ranges of the considered aircraft. It is likely that at least one of the aircraft has to speed up or slow down to ensure that the two aircraft meet at the joining point at the rendezvous time. At any time, it is assumed that all the aircraft fly at their minimum-drag velocity. Having to deviate from this, causes extra fuel burn, or the requirement for extra propulsive energy in this case. This consideration could lead to not using a formation option, even though geometrically it seems favourable.

Having set up this model for two aircraft, a method is required to enable the simulation of more flights. Ribichini et al. use a greedy algorithm that allows aircraft to communicate only with their direct neighbours and forms formations as soon as a reduction in required propulsive energy can be achieved by two communicating aircraft. When a formation option is chosen, the unique ID's (i.e. flight numbers) of the corresponding aircraft are set to be 'engaged' (otherwise aircraft are referred to as 'eligible'). When a formation is formed, the formation receives a new ID with which it is once again open for communication. This allows for formations to grow larger than two aircraft. If an aircraft has two or more neighbours, it first proposes a formation to the most promising partner.

Ribichini et al. provide an example which clearly illustrates how their methods works. Their results of a 4 aircraft simulation are given in Figure 3.5.



(a) (b) (c)

Figure 3.5: Example of Ribichini et al., showing how 4 aircraft would decide on their formation flight strategy [31].

The example contains 4 flights, with vertically aligned origins. All units are non-existent. Distances are indicated in units of distance.
In Figure 3.5(a), one finds the individual minimum energy routes for each flight. As mentioned before, these are just strait lines, as these represent the shortest distance between the origins and corresponding destinations. Numbering the aircraft from the top as 1 to 4, one finds in Figure 3.5(b) that aircraft 2 and 3 form a formation first, after which formation (2,3) is joined by aircraft number 1. This could have been formation (1,2) joining with aircraft three. However, the code is set up in such a way that a tie in the ranking of possible formation partners on profitability is settled on ID number, here resulting in aircraft number 2 favouring aircraft number 3 over aircraft number 1.
Figure 3.5(c) shows how aircraft number 4 joins the (1,2,3) formation for the major part of the to be flown distance. The total costs related to the three scenario's are indicated in the

top left corners of every plot in Figure 3.5. They drop significantly as more formations are formed. Since the cost parameters in the work of Ribichini et al. have a mainly illustrative purpose, the cost savings do not accurately represent achievable energy savings.
Next to the 4 aircraft example, Ribichini et al. present a 10 aircraft example, which further illustrates their method. It shows the development of larger formations and the possibility to apply the method to a larger group of aircraft.

Something that was not, but could have been, mentioned by Ribichini et al., is that in more complex examples, formation forming based on ID is very unlikely to occur, as two formation options would have to have exactly equal saving potential. Nonetheless, the example illustrates the significant commitment to a route, that has to be established based on local information. Also, it can be imagined that the second best formation option at a certain time, would result in superior saving potential at some later time. This notion touches upon the essential difference between centralized and decentralized approaches.
In their conclusion, Ribichini et al. mention the high level of cooperation between the aircraft in their example. In reality, not every aircraft will automatically change their route for the greater good, especially if it means that their own fuel consumption will (slightly) increase. For the time being, they assume full cooperation of all aircraft. As a conceptual work, this thesis will assume the same.

3.2.2 An Analysis of the Potential Savings from Using Formation Flight in the NAS

An other decentralized approach to the implementation of formation flight in civil aviation is published by Xue et al. in 2012 [39]. A key difference of this work with that of Ribichini et al. discussed in the previous section, is that Xue et al. do not include routing in the formation flight related communications between aircraft. They use a network of so called corridors in the sky along which flights are guided. They evaluate possible formations of aircraft that have, at some point in the corridor, a stream wise separation of less than 50 nautical miles.

Xue et al. use a genetic algorithm on current routes to determine corridors in the sky that could function as highways along which flights could be guided. This idea originated from attempts to reduce airspace complexity, described in the work of Alipio et al. in 2003 [2]. Since this routing structure clusters aircraft along a trajectory, it also creates a favorable environment for the introduction of formation flight. Figure 3.6 shows the 60 busiest corridors that Xue et al. identified.

Having the routes in place, formation flight has to be introduced. Xue et al. use several assumptions to do this. For example, they assume that when an aircraft enters a corridor, it continues to fly at his cruise speed. Also, they limit the maximum number of aircraft in a formation to 4, presumably to allow for the use of existing models on drag reduction in formation flight.
The method of Xue et al. contains a continuous time simulation. The properties of each aircraft, such as speed and location, are updated at every (small and equal) time step. As mentioned, an aircraft starts out in a corridor flying at his cruise speed. As soon as it gets within 50 nautical miles of an aircraft that entered the corridor earlier, an algorithm is triggered to determine if catching up and forming a formation with the (group of)

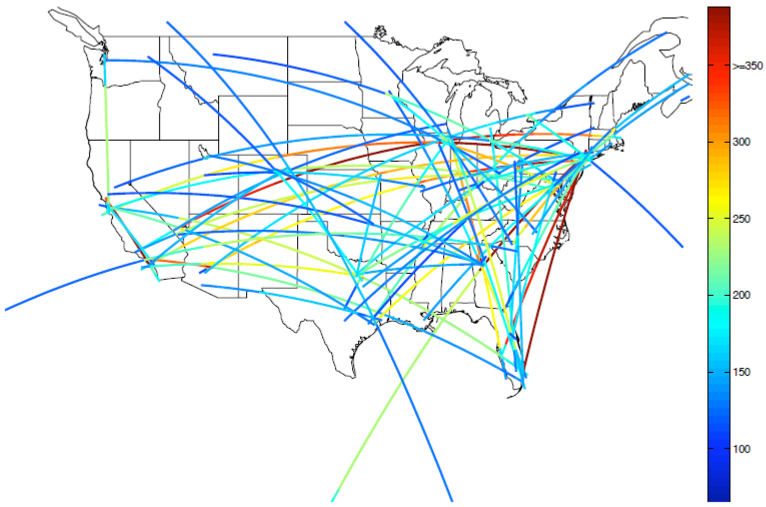


Figure 3.6: The 60 busiest corridors that Xue et al. identified. The color scale indicates the number of flights a day that is expected to use a corridor. [39].

aircraft ahead would result in an overall reduction of fuel burned (Note, slowing down of the leading aircraft is not mentioned as an option). If this is the case, the trailing aircraft speeds up and after some time the formation is formed. This process continues until aircraft have reached their exit points and leave the corridor and thus the formation.

To determine the drag reductions from formation flight, later translated to fuel savings, Xue et al use a drag reduction scheme based on the work of Nangia and Palmer [22] and Ning [23] for a two aircraft formation. Depending on the weight fraction of the two aircraft (Not specified by the paper, presumably (M)TOW fraction, as aircraft size is also mentioned), an achievable reduction in induced drag is assumed for the trailing aircraft. For equal weights, the assumed reduction in induced drag is 25%. As the induced drag causes approximately half of the total drag when an aircraft is in cruise, elaborated on by Henderson et al. in 1989 [14] and Smith in 1996 [32], the total drag reduction is deducted accordingly. The drag reduction scheme can be seen in Figure 3.7.

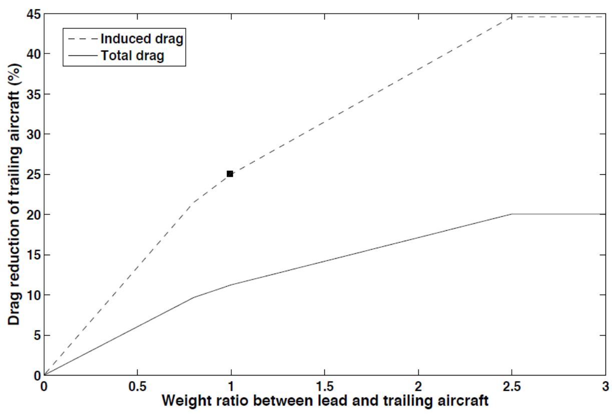


Figure 3.7: Drag reduction scheme used by Xue et al. to estimate drag reductions from formation flight [39].

Also visible from Figure 3.7, the scheme assumes that at weight ratio's larger than 2.5, the achievable total drag reduction for the trailing aircraft levels of at 20%.
For larger formations, it is assumed that an Echelon formation is used. Figure 3.8 gives three commonly considered formation configurations. The drag reduction for the added aircraft is calculated by the same drag reduction scheme, using the weight fraction with the aircraft at the back of the existing formation.
It should be noted that Xue et al. use the concept of extended formation flight, just as Xu et al [37] and Kent et al. [17].

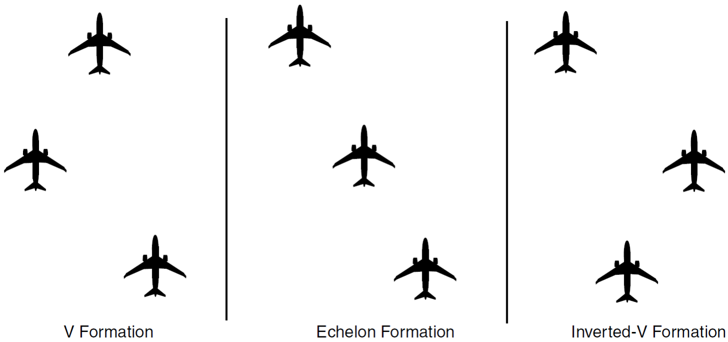


Figure 3.8: Three commonly considered formation geometries for three aircraft. [37]

The savings presented by Xue et al. are mostly given in dollars per year. In terms of fuel, their results are translated to being in the order of 50 million gallons of overall annual fuel savings. In their analysis, Xue et al. checked the effect of changing the allowed extra distance that may be flown to use the corridors. As the allowed distance was increased, fuel savings decreased. To evaluate the effect of increasing the traffic density, they doubled their number of flights. They found that this had a positive effect on the possible savings, which grew a little over proportionally.
As indicated in Chapter 2, a decentralized approach is expected to be able to cope with disruptions in a schedule. To test this, Xue et al. performed a Monte Carlo simulation with uncertain arrival and departure times on the busiest corridor. They used randomly generated departure times, taken from a gaussian distribution with a mean of zero minutes and a standard deviation of 15 minutes. This resulted in the histogram in Figure 3.9, showing the annual savings resulting from each of the 5000 performed runs. The results closely represent a gaussian distribution with a mean of 33 million dollars and a standard deviation of 2,5 million dollars. This illustrates that the model is capable of handling disruptions in the schedule, without losing significant amounts of achieved savings.

3.3 Considerations on modelling fuel consumption

The previous sections have summarized several of the leading works in the research area. These works, along with all the others treated in Chapter 2, have increasing fuel efficiency in aviation as a common objective. The reliability of an estimation of achievable fuel savings is dependant on the used fuel burn model, and how accurately this model represents reality. This chapter elaborates on several methods of fuel burn modelling that were encountered in the literature. Also, flight test results are discussed, in order to

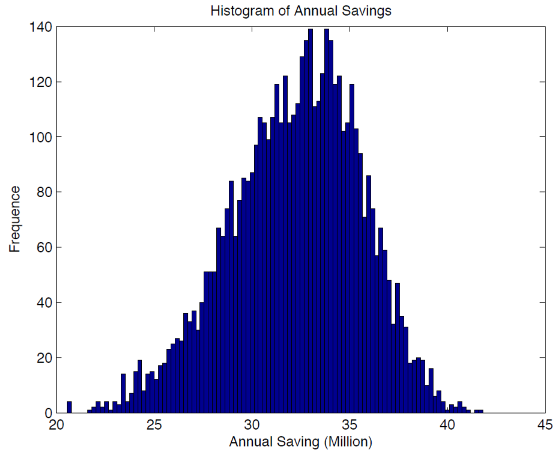


Figure 3.9: Histogram of annual savings, obtained from a Monte Carlo simulation on the busiest corridor with randomly generated, gaussian distributed, departure times [39].

put the theoretical models in perspective of what has been achieved regarding fuel flow reduction through formation flight.

3.3.1 Estimations of fuel flow reduction

In the past, numerous studies have elaborated on the aerodynamic characteristics of formation flight, in order to estimate the possible energy savings obtainable from formation flight. From the study of birds performed by Lissaman in 1969 [21], until the work of Ning et al. on extended formation flight in 2014 [26], the estimation of these savings have undergone continues refinement. When more aircraft are considered, the formation geometry affects the possible savings. In 2009, Bower et al. studied different formation geometries for three aircraft (see Section 3.2.2 for geometry illustrations) and concluded that the inverted-V formation was the most effective formation pattern with respect to drag reduction [9]. They estimated achievable overall fuel savings of 16% for a formation of three, against 10% for a formation of two in echelon formation. It is noted that Bower et al. used a stream wise separation of five wingspans. The extensive work of Ning et al. on extended formation flight, considered 10-50 wingspans of stream wise separation [23, 24, 25, 26]. As mentioned in Chapter 2, they predicted maximum induced drag reductions of 30$\pm$ 3% and 40$\pm$ 6% for a trailing aircraft in a two and three aircraft formation, respectively. Using the assumption that in the induced drag makes up around 50% of the total drag when an aircraft is in cruise (also assumed by Flanzer et al.[12]), these estimations are higher than those of Bower et al.[9].
Additionally, Veldhuis et al. estimated that a medium sized aircraft could obtain a 13% fuel flow reduction while following an identical aircraft [34]. This result, also based on extended formation flight, is in the same order of magnitude as earlier stated results, and as such increases the reliability of all the mentioned estimations.

To verify the theoretical predictions of achievable induced drag reductions, and accordingly, fuel savings, flight test are required. Two reports on formation flight tests were encountered in the literature. As mentioned in Chapter 2, Ray et al. reported in 2002

that a peak value of 18% fuel flow reduction for a trailing aircraft was achieved, in a close formation of two F/A-18 aircraft flying at Mach 0.56 at an altitude of 25,000 ft. In 2014, Flanzer et al. studied two military C-17 aircraft that flew in an extended formation, without any special formation flight equipment. At their best, these aircraft achieved a sustained fuel flow reduction of just over 10% for the trailing aircraft [12]. Note that these results were obtained with Mach number and altitude subject to optimization. The allowed range of the Mach number was 0.72 - 0.75 and the allowed altitudes ranged from 30,000 to 34,000 ft.
The savings obtained by Ray et al. were not sustained. The flight test results of Flanzer et al. are significantly less optimistic than theory would suggest. However, the theoretical estimations of obtainable fuel savings from formation flight may be realistic considering future developments. In the test performed by Ray et al., pilots were flying manually, whilst the trailing pilot received tracking guidance information. The flight test with C-17 aircraft relied on the auto pilot of that specific aircraft to keep the formation efficiently configured. Since wake tracking systems and formation flight auto pilot modules are still under development, the expectation that future flight tests will obtain higher fuel savings is widely shared in the literature. Without exception, the formation flight implementation studies treated earlier in this chapter include this expectation as an assumption.

To illustrate how the previously considered publications model the fuel consumption of aircraft, a concise overview is given.

3.3.2 Modelling fuel savings

Most literary sources use factors to model the fuel burn reduction that formation flight can achieve. Over a certain distance, an aircraft is said to experience a fixed percentage of fuel flow reduction with respect to a solo flight over the same distance. This percentage may depend on formation size and aircraft weight, depending on the source and it's assumptions.

The initial work by Ribichini et al. in 2003 uses unspecified energy reduction factors, dependent only on formation size, to model energy savings from formation flight. [31]. Inspired by the work of Ning [23] and Nangia and Palmer [22], Xue et al. define a percentage of possible drag savings, dependant on the relative sizes of the involved aircraft. This method is illustrated by Figure 3.7 in Section 3.2.2.
Kent et al. [17, 18] model the reduction of fuel burn with fixed percentages per unit of distance flown, depending on formation size. They later include a version of the Breguet range equation, in order to more accurately represent changes in aircraft weight over time.

The most advanced fuel burn model used in the considered literature on the implementation of formation flight, is used by Xu et al. in their work in 2012 and 2014 [38, 37]. It incorporates different aircraft types, with different engine efficiencies, wake propagation modelling and it determines the effect of a leader's wake on the induced drag of a specific trailing aircraft. The formation geometry, either V- or Echelon depending on which has the highest potential for the considered aircraft combination, is optimized by sorting the formation members by increasing thrust specific fuel consumption, with the most thrust efficient aircraft leading the formation. When optimizing a mission, the weight changes over a segment are obtained in accordance with the Breguet range equation, assuming that the range parameters change linearly.

Dekkers determined drag reductions of specific formations by the use of a Breguet range equation, an average aircraft to account for aircraft type diversity, and several assumptions on aerodynamic properties and state parameters of aircraft [10]. The assumption that all aircraft are of the same type, including a definition of this type, is common to the research area.

The use of a sophisticated fuel burn model has several advantages. The reliability of the estimated fuel savings becomes higher, as the fuel burn model becomes more realistic. Also, it quantifies some of the assumptions that most of the considered literary sources make on wake propagation and induced drag reducing effects of that wake. The inclusion of the Breguet range equation incorporates the significant effect that a change in weight has on the fuel consumption of an aircraft.
A simplified fuel born model, such as one that uses fuel flow reduction factors, is less representative for specific cases. However, it is suitable for the evaluation of scenarios in which more general questions are to be answered. There are two main reasons for this. Firstly, computing times are kept within workable limits as a Breguet range equation allows the evaluation of a weight change over an entire segment. Secondly, factors can be adjusted based on assumptions, to study any practical consideration such as speed changes or formation size. Especially the second property makes simplified fuel modelling attractive for conceptual work.

3.4 Discussion, Conclusions and Research Objective

The previous sections showed several leading examples of centralized as well as decentralized approaches to the implementation of formation flight in civil aviation. Both approaches have their strengths and opportunities for improvement. Centralized approaches provide solutions on a network level, indicating which flights could most beneficially be assigned to which formation. However, strong dependency on schedules as well as the computational requirements for larger problems are still limiting the potential of these methods. Decentralized approaches do not face these challenges. The opportunistic nature of decentralized approaches provides robust methods to form formations when schedule uncertainty is considered. However, it is very likely that the found optimum is a local optimum and it may occur that some beneficial formation flight opportunities go by unnoticed.

Xu et al. [37] mention that their centralized approach could only consider one rendezvous and splitting point for each formation due to the associated computational requirements. As these two sets of coordinates are treated as design variables by Xu et al., evaluating more of them significantly increases the amount of options, and as such the computational time. This limits the flexibility of introducing formation flight. As decentralized approaches only evaluate formation flight options when they are encountered, even without specifying how this is done, the computational requirements for decentralized approaches are expected to be significantly lower. Additionally, the amount of joining and splitting points is not necessarily limited.

Essentially, two main differences between centralized and decentralized approaches can be identified. The first one is the initial separation threshold between possible formation partners, which is essentially infinite for centralized approaches and finite for decentralized approaches. The second difference is found in the attitude towards collecting benefits. Where decentralized approaches use greedy methods to immediately collect and ensure benefits when possible, centralized approaches postpone the collection of benefits to the final steps of their calculations, in an attempt to avoid jeopardizing the optimality of their results.

Practically, a fuel related aspect of formation flight implementation is worth considering. Currently, every flight carries a certain amount of fuel, including reserve fuel. If formation flight is anticipated, and the reliability of the formation flight schedule is guarantied, flights could carry less fuel than required for a solo mission when departing from their origin. This would significantly increase the possible fuel savings, as less carried fuel means a lower weight and this again means less carried fuel. This reasoning favours centralized approaches, since a prediction of to be obtained fuel savings is now required in order to determine an aircrafts take-off weight. Unfortunately, as stated by Xu et al. [37], centralized approaches struggle with handling disruptions in the schedule and can not (yet) guaranty that a flight will be part of the planned formation, or any formation for that matter. This is due to the difficulty in finding and implementing a new optimum when a flight turns out to be delayed. Accordingly, flights should prepare for a possibly solo mission. They may even have to add fuel for when they fly a detour to form a formation, but their (set of) partner(s) does not show and they have to carry on flying solo. This means that, before any savings from a centralized approach can be collected, every involved flight has to increase his starting weight. Xu et al. [37] estimate, in a 31-aircraft example, that this may reduce overall obtainable fuel savings by 20%. They mention the selection of appropriate contingency airports as a possible measure. It is noted that this measure is usually used for emergencies only, considering it's substantial practical implications and costs. A measure that would increase the reliability of planned formation flight is preferred.
When applying a decentralized approach, there is no pre-flight knowledge on formation flight strategies. Therefore, flights would carry the same amount of fuel as in current operations. A formation flight segment is added to the flight plan when this is found, based on in-flight collected/available information, to result in overall fuel savings. As such, the probability of an unrewarded detour is reduced (assuming successful formation flight), and with it the requirement for extra contingency fuel. However, any pre-flight fuel savings, possibly resulting from planned-formation flight, are also eliminated.

One could argue that the intention of flying a longer distance than their solo route, requires aircraft to bring more reserve fuel, regardless of the formation flight implementation method. As this would severely reduce the potential of formation flight as a fuel saving mechanism for civil aviation, it is worthwhile to elaborate on how formation flight could be executed, such that it becomes a reliable fuel saving strategy.

Any disruptions in flight plans, which are local events, should be solved locally as well, as re-optimizing unaffected parts of a set of flight plans is likely to be inefficient. In other words, when a disruption in the implementation of formation flight occurs, a decentralized element or module is probably most suitable for resolving the disruption.

In general, if only operational measures are considered in order to obtain fuel savings from formation flight, one could consider to use a centralized approach to maximize the likelihood with which formation flight options occur, as well as to maximize the potential overall profitability of formation flight. Accordingly, a decentralized approach could be used to turn the generated potential into actual savings.

From this discussion, and the findings in Chapter 2, it is concluded that there are many challenges left in defining an operationally attractive method to implement formation flight in civil aviation. Considering how various centralized and decentralized approaches are described in the literature, it was found that the development of a completely decentralized approach would have significant scientific and operational potential. The word "completely" emphasises the complete absence of any pre-flight planning related to formation flight. Accordingly, the objective of this thesis project is formulated as follows:

The objective of this research is to evaluate the fuel saving potential for civil aviation of a completely decentralized approach to formation flight routing.

Chapter 4

Operational concept

After the performed literature study, two main requirements for an operationally attractive implementation method were identified. The first one is related to the calculation time that is required to evaluate a scenario in which formation flight is considered. The critical information can be summarized as follows: When, should which aircraft, be flying where, and how fast should it be going? The current literature often notes the substantial amount of required calculations to obtain this information. One of the inherent advantages of a decentralized approach is that the number of calculations is limited and therefore the required time to obtain a solution is limited. To what extend they are limited, depends on the efficiency of the model and methods that it contains. Therefore, throughout the development of this work, efforts were made to keep the model as quick as possible, whilst conserving the value of the model output. The second requirement for an operationally attractive implementation method is flexibility. From the literature, this flexibility is primarily desired in determining which aircraft will end up in a formation together. One can imagine that taking away any restrictions on possible formation partners will increase the amount of formation flight options and with it the potential fuel savings. The flexibility requirement applies to formation size as well. Formations may get more efficient as they contain more aircraft [15]. Current centralized approaches limit themselves to formations of size 3 [37] or 4 (solely by combining two formations of size 2) [18], in order to keep the required calculations within workable limits. In a decentralized approach, one would want formations to be able to grow at will, to any size that may be found appropriate. In order to investigate which formation sizes would be used given a certain network of flights, the model itself should not impose any limits on formation size.

This chapter describes the model that has been developed as a means to demonstrate the fuel saving potential for civil aviation of a decentralized approach to formation flight routing. Inspired by the initial work of Ribichini et al. [31], a simulation was developed from the ground up. From the start, realistic units of distance were used as well as representative aircraft parameters such as range, empty weight and cruise speed. Before details of the simulation are discussed, the design philosophy and the validity of the obtained model will be addressed briefly.

4.1 Model definition

Before going into the actual applied methods and developed functions, the modelling context is laid out and the simulated scenario is specified. The simulated operational concept is clarified, followed by an elaboration on the formulation of the simulation objective and the validity of the developed model and its results.

4.1.1 Simulated scenario

Intuitively, formation flight has the highest fuel saving potential when considering long-haul flights with substantial route similarities such as heading and geographic origin and destination location, but also departure time. One can name several regions in the world where many flights a day fit into this picture. For example, the flights connecting Europe, through for instance Dubai, to South-East-Asia. In the literature, flights over the Atlantic Ocean are often considered. To be able to cope with the high traffic density in this region of the world, without having radar coverage in the area, a structure of highways in the sky has been developed: the North Atlantic Tracks. The requirement for such an air traffic control instrument to guarantee flight safety, partially explains the popularity of the region in formation flight studies. To be able to compare the findings in this work with those of the leading literary sources, the here discussed model was developed to eventually simulate flights over the Atlantic Ocean. Section 7.2 contains a case study on 347 eastbound transatlantic flights. The model simulates aircraft that are flying their great circle route in a two dimensional space. Solo routes are therefore represented by straight lines between origin and destination. These flights depart at pre-defined departure times, just like they would do in current operations.

4.1.2 Simulated operational concept

If the formation flight functionalities of the developed model were to be turned off, all aircraft would fly their great circle routes and consequentially no fuel will be saved. When turning on the formation flight modules, formation flight becomes an *in-flight option* for each flight. This in-flight options is the key concept in a decentralized approach. Aircraft are now allowed to communicate with each other, if they are within their combined communication range. This communication range is defined as a circle around each aircraft with radius CR (short for Communication Range). As soon as two circles, belonging to two different aircraft, touch or overlap, the corresponding aircraft are allowed to communicate. In this work, the communication process consists of two parts: determining a formation flight strategy and deciding whether or not the aircraft will commit to it. This commitment consists of updating the two flight plans to include the newly determined formation flight segment. This commitment is indisputable. As soon as a commitment is made, it is always honored in order to secure the savings that were predicted. Effectively, this means that the developed model does not adjust formation flight strategies that have been accepted.

As soon as an option to save fuel is encountered, the savings are secured by committing to the formation flight strategy. This greedy way of collecting fuel savings is based on

the work of Ribichini et al.[31]. A greedy algorithm, such as the one used by Ribichini et al., is designed for efficient evaluation of complex problems. It cuts a large problem into smaller, preferably straight forward problems. After obtaining a local optimum for one of these smaller problems, this optimum is never revised. Hence, these algorithms are referred to as being greedy. The solutions for the smaller problems are combined in a solution for the original problem [33].
Note that greedy algorithms usually do not return a global optimum. However, since less options have to be considered, these algorithms are favourable for calculation speed. This work limits itself to a greedy way of collecting savings because of this computational advantage and because initial results in this work were judged promising enough to spend the remaining project time on developing other aspects of the model, such as the amount of included flights and the size to which formations can grow. In general, if the developed model can prove the potential of a decentralized approach while using a greedy savings collection algorithm, the potential of a decentralized approach in general is increased, given the known sub-optimality of a greedy algorithm. From a practical point of view, a decentralized approach requires quick decisions. Any time that is spent on either evaluating a formation flight option or waiting for a better one, reduces the overall fuel saving potential through the fact that during this time, each aircraft is already flying towards his destination. As the distance that a flight has to cover reduces, the average time that it is likely to spent in a formation reduces, and with it the overall obtainable fuel savings that may be expected.

When two flights commit to a formation flight strategy, they are referred to as engaged (named by Ribichini et al. [31]) and they are no longer available for communication. They fly to their joining point and continue afterwards as a formation of size two. Formation flight is treated as a black-box in this work. The two aircraft are simulated to fly at the same speed, heading, location etc. The only difference between them is the required fuel to cover a certain distance. The trailing aircraft now saves 10% of fuel per unit of distance flown, compared to a solo flight over the same segment. After a formation has been formed, it becomes available for communication once again. The formation leader, one of the formation members, may decide to add additional formation flight segments to its existing formation flight mission. This feature of the model allows formations to grow as time progresses.

Figure 4.1 presents a flow diagram of the operational concept that has been described above. This diagram closely represents the structure of the developed model. The decisions that may have to be taken with respect to formation flight are given. Note that the decision scheme in Figure 4.1 is written from the perspective of a single flight. Any airborne flight in this work is continuously using this decision scheme.

There are several loops in the decision scheme in Figure 4.1. These illustrate the continuous effort of flights to find potential formation flight partners with whom they may achieve additional cumulative fuel savings. As soon as flights are prepared, meaning that they have determined their starting weight and their initial heading, they depart at their assigned departure time. Note how each flight starts out on a solo flight segment that may extend itself all the way to the its destination. This is a key feature of a decentralized approach; individual flights do not anticipate the use of formation flight. At some point in time, two flights may commit to a formation flight strategy and adjust their flight plans accordingly. They alter their heading and speed in order to meet each other at the

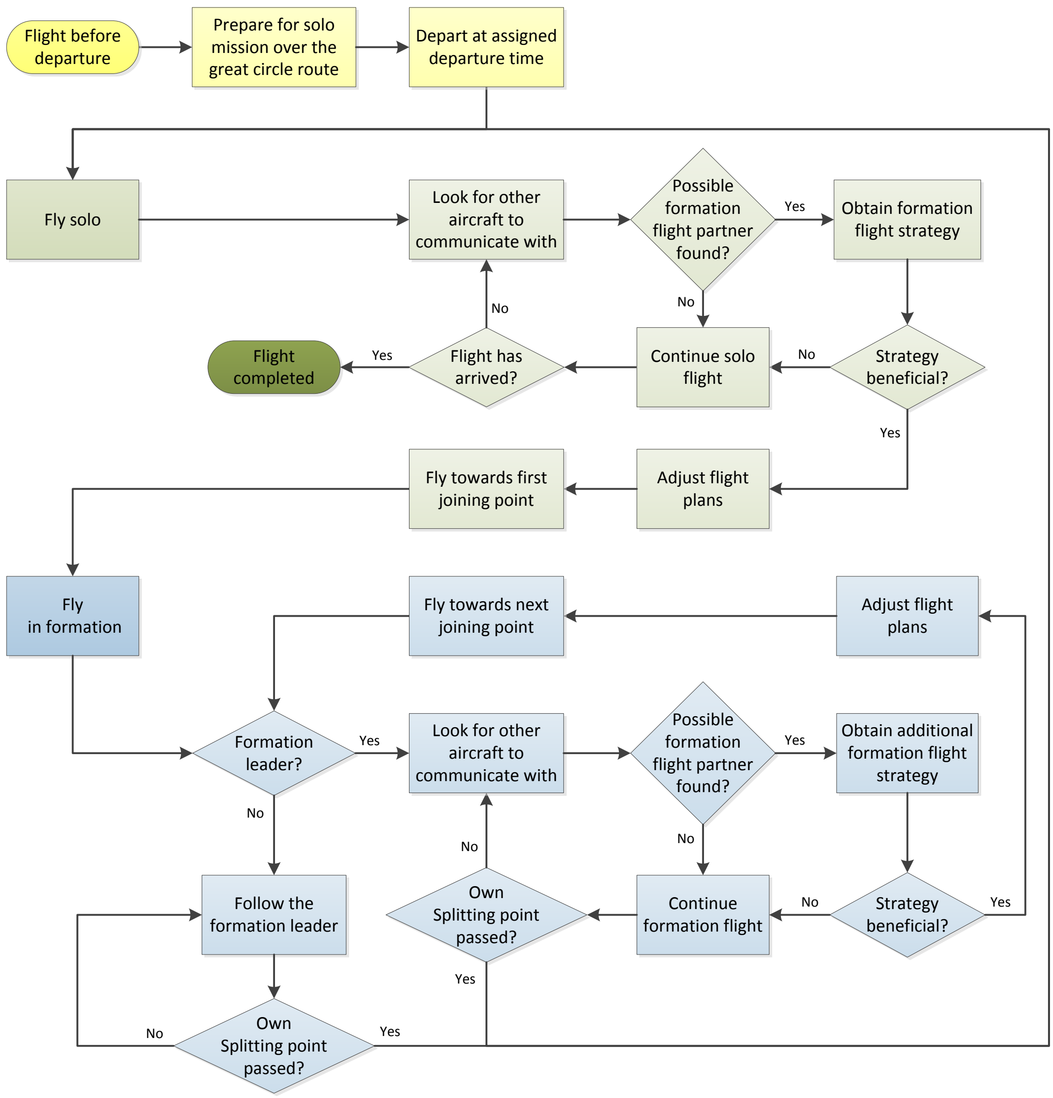


Figure 4.1: Diagram of the operational concept that is simulated in this work. The illustrated concept applies to each considered flight.

agreed time and location, the latter of which is called the joining point. The process in which aircraft spend time and fuel to join in formation is, in this work, referred to as synchronization.

When two, or more, flights have successfully joined in a formation, one of the aircraft has to be assigned as the formation leader. Within the scope of this work, the formation leader does not experience any benefits from formation flight. To be able to track the fuel requirements of each flight, a leader must therefore be selected in each formation. In the developed model, that formation member with the lowest flight number is elected as the formation leader.

The formation leader may, similarly to a solo flight, attempt to communicate with other aircraft as a means to find additional formation flight partners. Note how the formation leader follows the same decision scheme as a solo flight, be it on behalf of the entire formation. The formation continues to exist up until the splitting point, defined as a point were a flight leaves a formation. After flying over a splitting point, the two flights that committed to that splitting point, head off to their individual destinations. Note that these destinations may correspond to the splitting points of smaller formations that together, until recently, formed a larger formation. A flight that does not lead a formation, is referred to as a follower. Followers have no control over their own flight plan anymore, until they reach their splitting point. The formation leaders ensure that they only commit their formation to formation flight strategies that result in additional overall benefits.

Figure 4.1 suggests that each last segment of a mission is a solo flight segment. In both reality and within this work, this is most likely to occur, hence the representation of the process as such. However, it is noted that the developed model allows communicating flights with common destinations to place their splitting point on this common destination.

4.1.3 Modelling technique

To realize a simulation in which aircraft can, at any random time, choose to fly in a formation, one needs to guaranty the availability of the data that is required for that decision at any random time. The most intuitive way to achieve this, is through continuous time simulation. However, continuous time simulations can quickly become time consuming. Since one of the goals of this research is to obtain results as fast as possible, continuous time simulation is not the preferred choice. Alternatively, one could use a form of discrete-event simulation. Time becomes a later derived parameter and changes in model parameters are evaluated from event to event. An event could be an encounter between aircraft, or the joining of two aircraft in a formation. The challenge here is to define all the events and to determine when they will occur. Especially when aircraft may alter their flight plan at any time, it may become an elaborate task to evaluate all the events and act on them in such a way that a realistic flight scenario is obtained.

During the development of the model, it became clear that for most modelling steps, such as the evaluation of encounters or the re-routing of aircraft, it was required to know where each involved aircraft was at that specific time. Also, for development, validation, and illustration purposes, the availability of an aircraft's location continuously through time was desired.

After considering the arguments above, it was decided to combine the two mentioned simulation methods. Essentially, the developed model is a continuous time simulation over small and equal time steps. However, the only parameter that is continuously simulated is the location (x- and y-coordinates) of all aircraft. All other aircraft parameters that are incorporated in the model are only revisited when this is required for a formation flight related decision. In this way, one limits the required calculation time, by only calculating what is required, when it is required.

4.1.4 Schematic model representation

With the operational concept defined and the modelling technique selected, this section outlines the developed simulation. Figure 4.2 provides a diagram that illustrates how the operational concept (Section 4.1.2) was translated into a simulation that used the selected modelling technique (Section 4.1.3).

In the top row of Figure 4.2, the flights that will be used in the simulation are defined. A flight is defined by a flight number, an origin, a destination, and a departure time. This data can be generated by the model, allowing for the evaluation of a broad spectrum of scenarios. For the case study of this work, presented in Section 7.2, routes and departure times were imported from a data set of real transatlantic flights. Flight numbers run from 1 to N, where N is the total number of flights in the simulation. Each flight prepares for a solo flight to their destination. In their flight plan, they determine their initial heading and the amount of fuel they require.

When all the flight plans are in order, a continues time simulation is started. As soon as the departure time of a flight has passed, it takes off and immediately reaches its cruise altitude. All airborne flights fly for a pre-defined amount of time, up to their next time step location. All flights are simulated to be steady and level. At this point in the simulation, the model only determines the position of each aircraft at the end of the current time step. After moving all flying aircraft to their next location, a check is performed that makes sure that those flights that have arrived are no longer considered in the remainder of the simulation. When all the flight have arrived, the simulation is terminated.

After each time step, the communication algorithm is called upon. This algorithm determines which flights, of all flying flights, are allowed to communicate. In this work, a flight is allowed to communicate when it is either a solo flight heading towards its destination, or a formation leader heading towards a splitting point. When flights are flying towards a joining point or following another flight, they are not allowed to communicate. From those flights that are allowed to communicate, so called communication sets are formed. A communication set is defined as a pair of flights of which the communication range circles, see Section 4.1.2, touch or overlap; they are close enough to each other to communicate. The communication algorithm finds all possible communication sets. Note that a single flight can be part of multiple communication sets.

The list of communication sets is loaded into the routing algorithm. This list is sorted by increasing flight number of the first flight in the communication set. The routing algorithm works down the list of communication sets. This is where the model shows the greedy nature that was mentioned in Section 4.1.2. Starting with the first communication set in the list, the routing algorithm determines a possible formation flight route for this set of flights. To do this, a routing module is used that produces initial locations for the joining point and splitting point. Accordingly, the synchronization module may adjust the joining point in order to make sure that the flights are physically able to fly the suggested formation flight route. Next, the routing algorithm determines the cumulative obtainable fuel savings of the considered formation flight route with respect to using solo flights for both members of the communication set. In a two aircraft scenario, this would come down to the obtainable fuel savings with respect to two solo flights. In parallel, the additional flight time for each flight that will be affected by the suggested formation flight

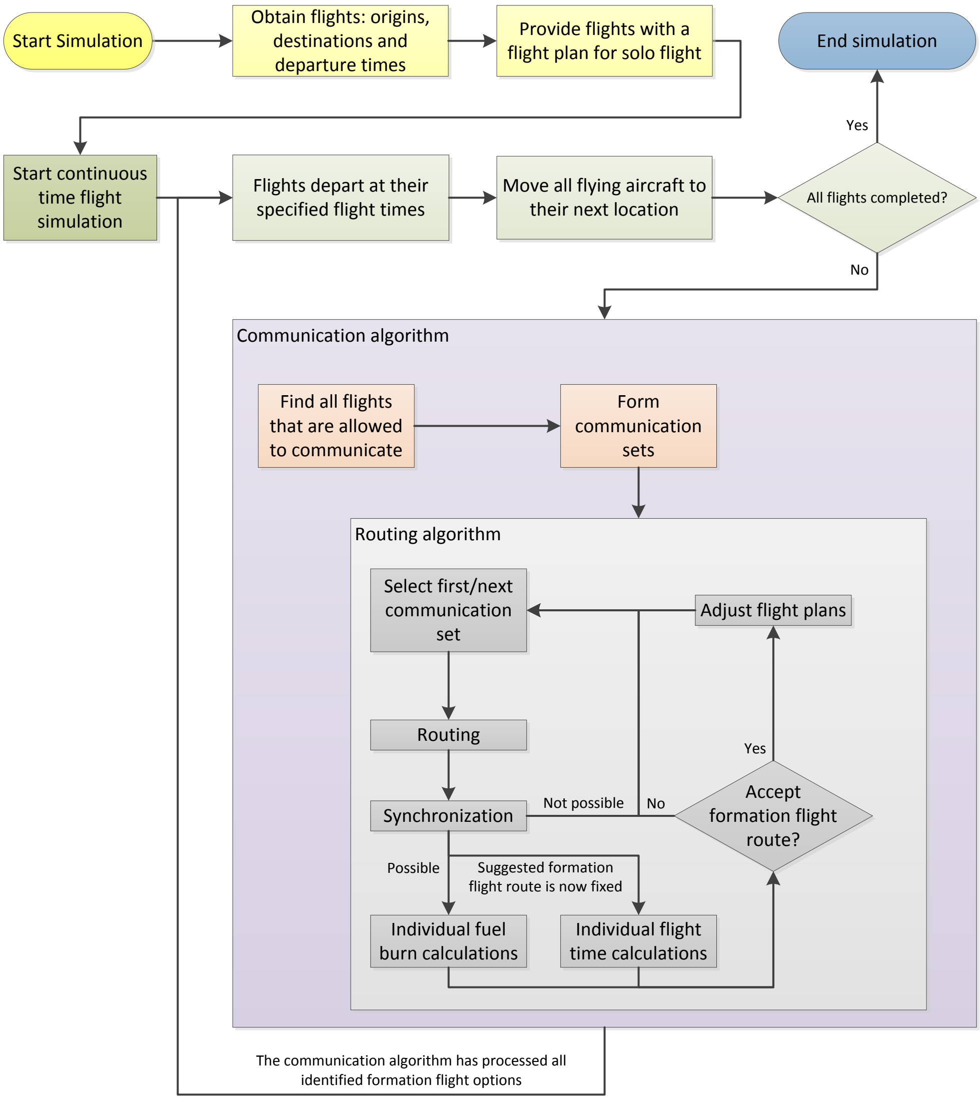


Figure 4.2: This diagram shows how the operational concept from Figure 4.1 is simulated in this work.

strategy is determined. Based on these two figures, kilograms of saved fuel and minutes of individually incurred additional flight time, the algorithm decides if it wants to use the suggested formation flight route. If so, the two flights in the currently considered communication set commit to the determined route and include it in their flight plans. Additionally, these flights will also turn their status to engaged. Since these flights may be part of another communication set that is yet to be considered, the algorithm needs to know that these two flight are no longer available for communication. As Figure 4.2 indicates, the routing algorithm will now move on to the next communication set and repeat the described process until all communication sets have been evaluated. Several routes that the routing module provides, will be rejected by the synchronization module. If this occurs, the location of the joining point, that is required to ensure synchronization, renders the suggested route obsolete. For example, it is theoretically possible that the synchronization module requires the joining point to be located beyond the suggested splitting point.

After re-routing all the flights that found a formation flight partner, the next time step is taken and all airborne flights are moved to their next location, determined by their, possibly updated, speed and heading. With the new location of each aircraft determined, the communication algorithm is called upon once again. The described process continuous until all flights have arrived.

4.2 Validity of the model: governing assumptions

The model that is developed in this work is based on a number of assumptions. As these assumptions directly affect the value of the results, the most significant assumptions are presented here to define the scope of this work. They are listed below, in no particular order, and are clarified when appropriate.

- All aircraft are of the same type. In Section 5.3, a standard aircraft is defined for this work
- Any effect of wind is not included in the model
- The simulation space is two dimensional and uses cartesian coordinates
- All considered routes are great circle routes in reality. In the two dimensional simulation space, this is translated as flying from one point to the other in a straight line.
- Free routing is allowed. All aircraft may change their flight plan at any time and there are no separation related routing restrictions
- Cooperation of aircraft in any formation flight strategy is guaranteed
- At the end of a solo mission, an aircraft would be out of fuel. This is used to determine the starting weight of an aircraft
- Flying in a formation will generate a 10% reduction in fuel flow for any aircraft that is not leading a formation
- Every aircraft flies at an equal speed, 251 m/s, unless it is directed to do otherwise
- The minimum flight speed is 207 m/s
- The fuel flow to the engines increases linearly with decreasing flight speed, with a maximum penalty of 10% for flying at V_{min} (deduced from [36])
- When allowed, inter-aircraft communications are always possible within the set communication range

- All flight takes place at 11 km altitude. This defines the free stream density
- The overall obtainable savings are independent from which aircraft leads a formation
- The flight with the lowest flight number leads the formation
- Any attempt to fly in formation is successful and saves the predicted amount of fuel
- All flights are steady, level flights, such that T = D and L = W on all segments. This requires instantaneous heading and speed changes
- An aircraft is able to fly in formation as soon as it has taken off
- All weight changes of aircraft that are considered in this work are due to fuel consumption

From these assumptions, one finds that the reality of flight operations has been simplified in many ways. In doing so, this work was able to arrive at several statements on the value of a decentralized approach to formation flight routing within the given time frame. Most assumptions are not new to the research area. Therefore, this work is not restrained by its assumptions to contribute to the current knowledge on the research topic.

4.3 Formulation of the objective

The primary objective of this work is to save fuel through the implementation of formation flight. For now, it is of no particular importance who receives these savings. Adding to this the assumption that the obtained savings are independent of who leads a formation, which was found to be accurate within the scope of this work, allows for selecting the flight with the lowest flight number as the formation leader by default. Additionally to the fuel saving objective, one may select a maximum amount of flight time that a flight receives from committing to a formation flight strategy. This objective was added in the final phase of the project, as flight times tended to increase significantly, sometimes up to an hour of additional flight time for an eight hour flight.

It is noted that this study aims to evaluate the fuel saving potential of formation flight with respect to not using formation flight. While the developed model will estimate the amount of saved fuel in a certain scenario, this will be an estimation with high uncertainty. The percentage of saved fuel with respect to only using solo flights is more informative. In this case, the solo flights and the formation flights are evaluated in the same (simplified) conditions, using exactly the same fuel consumption determination method; the one that is included in this work. This relative result is more representative for the potential fuel savings in the real world, than the absolute fuel savings obtained from the developed model.

The objective of saving fuel is indirectly pursued in this work. Within the scope of this research, the fuel consumption during a flight depends on the starting weight of, and the distance flown by, the corresponding aircraft. The starting weight is determined using the solo route distance and the zero-fuel weight of the defined standard aircraft. Since, in a decentralized approach, formation flight is not anticipated, the starting weight is now fixed. This leaves the to be flown distance to be minimized in order for the fuel consumption to be minimized. The next chapter elaborates on the fuel consumption calculations performed in this work.

Chapter 5

Fuel consumption modelling

This chapter elaborates on the method that this work uses to model the fuel consumption of aircraft. First, the origin and context of the method will be introduced. Accordingly, the implemented Breguet range equation is discussed. At the end of this chapter, a standard aircraft is defined that will be used to perform each flight that is considered in this work.

5.1 Introduction of fuel consumption modelling method

From the literature that was treated in Section 3.3, it is found that earlier works have used a variety of methods to model the fuel consumption of an aircraft that had to cover a certain distance. Methods of different complexity were encountered, ranging from straight forward estimations to full aerodynamic analyses that included multiple aircraft types. The majority of publications that were encountered in the research area, use a version of the Breguet range equation to evaluate weight changes of an aircraft due to fuel consumption. The differences between encountered fuel consumption modelling methods, often arise from the assumptions used to derive and implement a version of the Breguet range equation. Of these assumptions, the drag reduction, or fuel flow reduction, that a trailing aircraft is able to achieve when flying in a formation, is the most important. Referring back to Section 3.1.2, one finds from Table 6.1 that Kent et al. assume that a trailing aircraft in a formation of size two is able to achieve a fuel flow reduction of 20% [17]. Figure 3.7 in Section 3.2.2 shows that Xue et al. assume a total drag reduction of about 12% for the trailing aircraft in a formation of size two, provided that the two aircraft have equal gross weights [39]. The assumption used by Xue et al. has been proven to be realistic in 2014. In this year, Flanzer et al. reported that two military C-17 aircraft had achieved a sustained fuel flow reduction of just over 10% for the trailing aircraft [12].

This work aims to evaluate the fuel saving potential of a decentralized approach to formation flight routing. Therefore, a fuel consumption model is required that can process flights over many different segments. Since this work does not intend to refine earlier estimations of overall achievable fuel savings, a highly complex model is not required. The fuel consumption model that is used must guaranty that the results of this work may be compared with the results from the literature. This is required to establish the value of a decentralized approach. The work of Kent et al. [17] [18] has been a valuable source to this work. It contains a version of the Breguet range equation that allows for the calculation of weight changes of aircraft over a segment of any realistic length. Because of this property, and since this research does not intend to innovate on fuel consumption modelling, the same equation has been implemented in this work. Section 5.2 will show this equation and discuss the associated assumptions. After selecting an equation that would be used to evaluate weight change, the parameters in this equation have to be quantified. Most of these are included in the properties of the standard aircraft that will be presented in the next section. What remains to be quantified is the obtainable benefit from formation flight. As introduced in the previous paragraph, different formulations have been used for this. Considering the conceptual nature of this work, comparability of results is essential to determine the value of these results. Therefore, it was chosen to determine fuel savings from formation flight with the method that Kent et al. presented. However, different values are assumed for the achievable fuel flow reduction. This work conservatively assumes that a trailing aircraft in a formation of size two, containing equal aircraft, is able to maintain a fuel flow reduction of 10%. With this assumption, this work adheres to the earlier mentioned flight test results, mainly motivated by the level of realism that this provides. As mentioned in Section 3.1.2, Kent et al. assume that in a formation of three aircraft, the third member is able to save more fuel than the second member. This work neglects any additional benefits that formation members may have in larger formations. By choosing a conservative fuel flow reduction percentage, and by using this percentage for all formation members except the formation leader, this work intends to avoid overestimating the benefits of using formation flight in civil aviation.

For synchronization purposes, aircraft are often required to fly at a reduced speed. The developed model includes a variable fuel consumption penalty for flying at a sub-optimal speed. From the literature, it is known that, assuming cruise conditions, a reduction in airspeed is likely to result in a higher fuel requirement per unit of flown distance [36]. By raising the fuel flow of an aircraft that reduces its speed, this work accounts for additional fuel that an aircraft may have to invest to become part of a formation. Note how, in this work, the fuel investment that is required of an aircraft in order to achieve formation flight, is thus determined by two parameters: the speed related increase in fuel consumption and the detour that must be flown.

In this work, fuel consumption calculations are performed for each aircraft individually, on any segment. While a formation flight segment is identical for all aircraft that have included it in their flight plan, each aircraft will enter this formation flight segment at a different gross weight. Therefore, this chapter presents a fuel consumption modelling method that applies to a single aircraft. From a simulation perspective, this allows for careful tracking of fuel requirements for each flight and it allows for making decisions based on this information. If an aircraft is to receive a benefit from flying in a formation, this will be included in its individual fuel consumption calculation.

5.2 Breguet range equation

A well known concept on fuel calculations is the Breguet range equation. Multiple versions exist, all adjusted to be used in slightly different circumstances. They all describe a relation between three parameters of an aircraft: a flown distance, the starting weight, and the end weight. This section presents the formulation of the Breguet range equation that is implemented in this work. As mentioned in the previous section, it has been adopted from Kent et al. [18]. The implemented range equation has two main advantages over possible other formulations. Section 5.1 stated that it returns usable results for any realistic segment length. This is an advantage, since this work must at least be able to evaluate weight changes of aircraft over segments of about 100 to 10.000 km. A second advantage originates from the assumption made by Kent et al., stating that the airspeed does not have to be constant for the range equation to be valid. Accordingly, the speed at which aircraft are flying can be altered by the developed model, without degrading the results of the fuel consumption calculations.

5.2.1 Assumptions

The quality of the results that are provided by any Breguet range equation, depends on the correlation between the simulated scenario and the assumptions through which the used equation was derived. The assumptions that are used in this work further define, or confine, the simulated operational concept that was discussed in Chapter 4.

The following is assumed in the derivation of the Breguet range equation that is used in this work:

- The free stream density is constant throughout the simulation. All flight takes place at 11 km altitude in ISA
- The coefficients of lift, C_L, and drag, C_D, the thrust specific fuel consumption, C_t, and the wing surface area S are constant for a specific flight on a specific segment
- All flight is steady and level: the equalities T = D and L = W are valid during all flight stages. Aircraft are assumed to be able to change their heading, speed, and formation membership instantaneously

In the performed simulations, these assumptions are slightly adjusted. For all flights, the free stream velocity is constant over a segment. This is in violation with the combination of the definition $W = L = \frac{1}{2}\rho V^2 S C_L$ and the other assumptions; the free stream velocity must change if the weight is to change. Since the optimum flight conditions for formation flight are still a current research topic, and since this method for fuel consumption calculations already presents a severely simplified reality, this work assumes that slight variations in the coefficients C_t, C_L, C_D along a segment allow for the velocity to be simulated as being constant on a segment.

5.2.2 Implemented range equation

With the assumptions in place, a formulation of the Breguet range equation can be obtained. The full derivation, taken from Kent et al.[18], can be found in appendix A. Starting from equation 5.1, which relates a change in weight over a certain amount of time to the applied thrust specific fuel consumption and duration, one can derive the relation given in equation 5.2. In equation 5.2, W_0 represents the starting weight, W_1 the end weight, and R the length of a segment. Given the assumptions in the previous section, all other parameters can be combined in one constant parameter M, specified by equation 5.3. Substituting equation 5.3 into equation 5.2 allows for re-writing equation 5.2 into the expression given by equation 5.4.

$$dW = -C_t T_A dt \tag{5.1}$$

$$R = \sqrt{\frac{2}{\rho_\infty S}} \frac{C_L{}^{\frac{1}{2}}}{C_t C_D} \left(2W_0^{\frac{1}{2}} - 2W_1^{\frac{1}{2}} \right) \tag{5.2}$$

$$M = \sqrt{\frac{2}{\rho_\infty S}} \frac{C_L{}^{\frac{1}{2}}}{C_t C_D} \tag{5.3}$$

$$W_1 = \left(\frac{M\sqrt{W_0} - R}{M} \right)^2 = \left(\sqrt{W_0} - \frac{R}{M} \right)^2 \tag{5.4}$$

Equation 5.4 provides the equality that is used throughout this work to evaluate weight changes of aircraft as a consequence of flying a certain segment length. The next section elaborates on how this work uses the parameter M to adjust the fuel flow of aircraft. Since M may adopt a range of different values, representing the relative fuel efficiency at which an aircraft is operating, this work will refer to the parameter M as the M-value.

5.2.3 The M-value

Besides modelling the fuel consumption of an aircraft that flies a certain mission, this work intends to evaluate the effect on a flight's fuel consumption of adding one or more formation flight segments to the flight plan. From the literature, it was found that the effect of formation flight on fuel consumption can be interpreted as a reduced fuel flow for the trailing flights. From inspection of equation 5.4, one can see that at a constant segment length R, and for a given starting weight W_0, the end weight, W_1, is directly determined by the value of M. This work assumes that any change in the weight of an aircraft is due to fuel consumption. Accordingly, by adjusting the value of M, one can control the amount of fuel that is consumed by a certain flight on a certain segment. It is found that increasing the M-value will amount to reducing the fuel consumption.

Section 5.1 mentioned that this work intends to model the benefits of formation flight as a fuel flow reduction of 10% for any trailing aircraft. Therefore, values for M need to be obtained that represent this. Note how the collection of parameters that are represented by M in equation 5.3, is varied as being one parameter. In order to further quantify the different M-values, one must know more about the type of aircraft that will be used on a mission and the operational restrictions that are imposed on the considered flights.

5.3 Standard aircraft

Currently, many different aircraft types are operated by airlines on a daily basis. Considering the conceptual nature of this work, as well as the fact that the following assumption is not unusual to the research area, it was chosen to use a single aircraft type for every flight that is part of this research.

5.3.1 Properties of the standard aircraft

For the definition of the standard aircraft, the Boeing 777 was used as a reference aircraft family [8]. This family of aircraft is frequently used on the transatlantic routes that will be studied in Chapter 7. The derived properties of the standard aircraft are given in Table 5.1. It is noted that for this work, the use of realistic aircraft properties is adequate. With the simplifications and assumptions of this work in mind, accurate representation of a specific real aircraft would not add significant value to this research.

The properties in Table 5.1 are used extensively in this work. The maximum endurance speed is defined as the speed at which an aircraft burns the lowest possible amount of fuel per unit of time. In this work, the maximum endurance speed is used as the minimum speed at which an aircraft may fly.

Table 5.1: Properties of the standard aircraft used in this study

Aircraft Property	Value
Zero fuel weight	171 $tons$
Maximum speed/cruise speed	251 m/s (M0.85 @ 11 km altitude in ISA)
Maximum endurance speed = V_{min}	207 m/s (M0.70 @ 11 km altitude in ISA)
Design range	7500 km
Fuel consumption on design mission	47.3 $tons$
Starting weight on design mission	$171 + 47.3 = 218.3$ $tons$

The design range of the standard aircraft has been set to 7500 km. For design ranges in this order of magnitude, the reference data provided estimations of the fuel consumption over an entire mission [8]. Accordingly, the design mission was used as a calibration point for the fuel consumption calculations in this research. As mentioned in Section 4.3, this work determines the starting weight of an aircraft by adding up its zero fuel weight and the required fuel weight to complete a solo mission. The zero fuel weight includes all contributors to the take off weight, except all the required fuel. Thus, a starting weight of 218.3 $tons$ is found for the design mission of the standard aircraft. This starting weight falls well within the reference data on the maximum take off weight of various members of the Boeing 777 family [8].

5.3.2 Fuel consumption of the standard aircraft

In Table 5.1, one finds the zero fuel weight, assumed to be equal to the end weight W_1, and the fuel consumption for a design mission of the standard aircraft. Accordingly, assuming that there is no contingency fuel, equation 5.4 can be used to derive the value of M that must be used to retrieve the starting weight. A value for M of 140 was found. Accordingly, the fuel consumption on the design mission was lowered by 10%, in order to obtain the value for M that would account for an entire design mission performed as a trailing aircraft in formation flight. A value for M of 158 was determined. For segment lengths from 100 to 10.000 km, it was experimentally verified that changing the M-value from 140 to 158 resulted in a fuel consumption reduction of approximately 10% over the entire segment.

In the performed research, it is assumed that flying at the maximum endurance speed increases the fuel flow by 10% with respect to flying at the cruise speed, V_{max}. This value has been deduced from a study on the fuel consumption of a C-141 Starlifter aircraft done in a paper by Visser in 1991 [36]. To account for the increased fuel flow, simulations were performed to find M-values that would represent this increase. It is assumed that the M-value that corresponds to flying at some speed in between the minimum and the maximum speed, may be obtained through linear interpolation of the two determined extremes. Note how the M-value of 143.8, corresponding to a trailing aircraft flying at V_{min}, does not intuitively reflect the fact that the aircraft increases and decreases its fuel flow by 10% respectively. This is due to the nonlinearity of equation 5.4.

Table 5.2 presents an overview of the M-values that this work uses to model the effect of formation flight on the fuel consumption of aircraft. In the developed model, the M-value of an aircraft is revisited after each adjustment to its speed or formation flight status. In this way, the model specifies a new fuel flow setting for an aircraft, for each consecutive segment that is flown by the corresponding aircraft.

Table 5.2: During flight, the fuel consumption of an aircraft is governed by these M-values. Speed variations and formation flight status are taken into account.

Aircraft Property	Value
M-value solo/leader, V_{max}	140
M-value trailing, V_{max}	158
M-value, solo/leader, V_{min} to V_{max}	linear interpolation between 127.4 and 140
M-value, trailing, V_{min} to V_{max}	linear interpolation between 143.8 and 158

It is assumed that the standard aircraft is able to carry the required fuel for any mission it is assigned to in this work. It is noted that this work aims for a relative study with and without formation flight. In general, slight deviations in the properties given in Table 5.1 are not expected to lead to significant changes in the final result.

In Section 4.1.2 it was mentioned that this work assigns the flight with the lowest ID number to be the formation leader. This decision was made with the idea of a standard aircraft in mind. Later, during the development of this work, it was experimentally verified that the overall obtainable fuel savings could safely be assumed to be independent of which aircraft was assigned to be the formation leader. This finding can be explained by the fact that all aircraft in the performed simulations are very similar, both in gross-weight and in fuel requirements.

Chapter 6

Elements of the developed model

The research presented in this work aims to address both the desire to more efficiently evaluate possible formation flight routes in large scenarios and the handling of delayed flights. Accordingly, methods have been developed that enable this work to face these challenges. This chapter describes the key elements of the developed model. It focusses on obtaining a new route for two parties that are considering to engage in formation flight. While these parties may consist of multiple aircraft, the communication algorithm that was presented in Chapter 4 forces them to act as a single entity.

After describing the routing method that has been developed by Kent et al. [17], this chapter elaborates on how this method has been adjusted and extended. Accordingly, the obtained routing method is verified and the criteria for accepting a formation flight strategy are discussed. Section 6.4 elaborates on how formations are enabled to grow in size.

6.1 Routing of formation flight missions

This section provides the geometric routing method that was developed by Kent et al. [17]. Several adjustments and extensions are presented and verified. Through minimizing the length of each route, the method aims to minimize overall fuel consumption.

6.1.1 Geometric routing method by Kent et al.

The geometric routing method that this section elaborates on, is based on the Fermat point problem [13, 35]. Considering the left part of Figure 6.1, the method intends to determine the location of point P that minimizes the overall route length from origins A and B, through a joining point P towards a common destination C.

On the right in Figure 6.1, the geometrical method to find the point P that minimizes the cumulative length of the vectors AP, BP, and PC is depicted. By drawing circles through

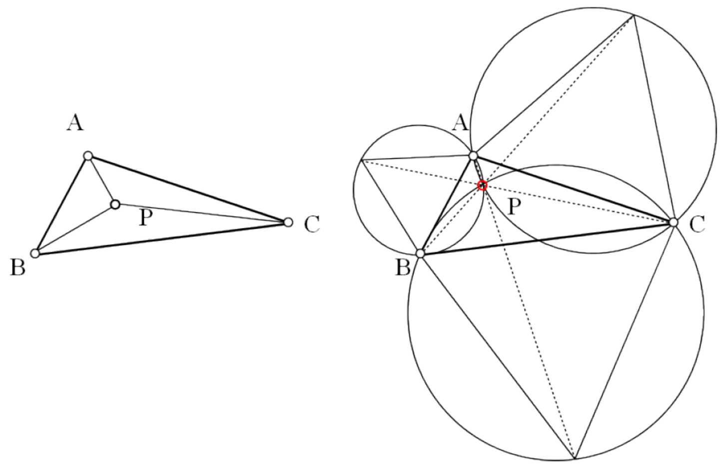


Figure 6.1: The Fermat point problem aims to minimize the distance AP+BP+PC, by adjusting the location of point P [17].

the corners of the equilateral triangles that can be formed on segments AB, AC and BC respectively, the desired point P is found.
Having obtained a location for point P, one can imaging two flights. Originating from A and B, these flights join in formation at point P and fly in formation to point C. The smallest angle between the segments AP and BP is defined as the formation angle, θ_f. Figure 6.2 illustrates the definition of the formation angle.

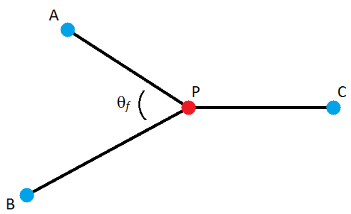


Figure 6.2: The formation angle is defined as the smallest angle between two flight paths towards a joining point [17].

To favour the use of a formation flight segment, Kent et al. assign weights to every segment length. As the formation size on a segment increases, it's cumulative weight per unit length decreases, thereby favouring this segment in the geometrical optimization. The used weights, given in Table 6.1, are based on several aerodynamic studies published in 1970-2005 [6, 7, 19]. When the assumed fuel saving weights in Table 6.1 are compared to the flight test results of Flanzer et al. in 2014 [12], it is found that these weights strongly anticipate future improvements in formation flight efficiency. They imply a 20% fuel flow reduction for the trailing aircraft in a formation of two identical aircraft. Flanzer et al. present test results, obtained with C-17 aircraft, indicating sustainable fuel savings for the trailing aircraft just over 10%.

The segment weights are introduced in an equation that updates the formation angle. Equation 6.1, obtained from formulating a minimization of the cumulative weighted length of segments AP, BP and CP, contains the relation between the formation angle and the

Table 6.1: The fuel burn metric used by Kent et al.[17]

Formation size (number of aircraft)	1	2	3	4
Fuel burn fraction (per member)	1	0.9	0.85	0.82

weights of each segment.

$$\theta_f = \cos^{-1}\left(\frac{-{w_A}^2 - {w_B}^2 + {w_f}^2}{2w_A w_B}\right) \tag{6.1}$$

In equation 6.1, w_A, w_B and w_C correspond to the weights of segment AP, BP and PC respectively. Referring to Table 6.1, Kent et al. use the value 1 for w_A and w_B, and $2 \cdot 0.9 = 1.8$ for w_C. Now that the weight factors for different segments have been introduced, the effective overall route length will be referred to as the weighted distance.

The formation angle forms the basis for the construction of a formation flight route. Consider Figure 6.3, which depicts routes AC and BD for which a formation flight route is to be found. The updated formation angle is used to define arcs of possible joining points. Along the bold printed arcs that run from A to B, the formation angle is constant. These arcs are extended to form full circles. On the right in Figure 6.1, a circle was drawn through the corners of the equilateral triangle that contained segment AB. The same has been done in Figure 6.3, generating the point X_1. Inspection of Figure 6.1 shows that point P is determined by the intersection of the arc from A to B and the line from the point X_1 to C. This concept is, up to a certain extend, independent of the location of point C. If the weights of segments AP and BP are not equal, the triangle ABX_1 is to be constructed whilst respecting the relation $\mid AB \mid$:$\mid BX_1 \mid$:$\mid X_1A \mid = w_C$:w_B:w_A. By doing so, one will, eventually, optimally redistribute segments lengths. Note how in Figure 6.3, w_A is larger than w_B, leading to $\mid BX_1 \mid$ being larger than $\mid X_1 \mid$). This notion serves as an illustration only, Figure 6.3 is not to scale.

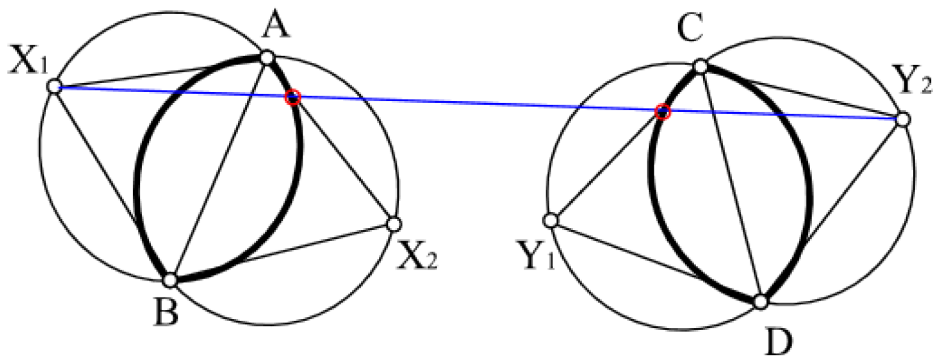


Figure 6.3: Illustration of the geometric method of Kent et al. to determine the shortest formation routes by determining joining and splitting points [17].

Similarly to what has been done for origins A and B, one can construct arcs of constant formation angle between the destinations C and D. Obtaining point Y_2 in similar fashion as point X_1, allows for the points X_1 and Y_2 to be connected by a line. This line, given in blue in Figure 6.3, intersects with the arcs of constant formation angle that have been drawn between origins A and B and destinations C and D respectively. The joining point and splitting point that provide a route with minimal weighted distance, are given by the two intersections that are marked with red circles in Figure 6.3. The formation flight route, with joining point P and splitting point Q is obtained and presented in Figure 6.4.

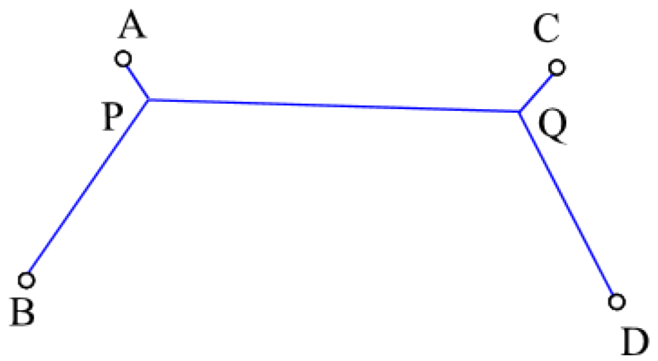


Figure 6.4: Shortest routes with a common segment on route AC with route BD [17].

6.1.2 Extension of the geometric routing method

As suggested in the previous section, the geometric routing method of Kent et al. has restrictions to its validity. The method as described above, will not return the minimum weighted distance route for any set of routes AC and BD. This means that there are restrictions to the route sets for which formation flight can be evaluated. This property of the geometric method is undesirable, since this work plans to evaluate large sets of diverse routes using automated procedures.

While the geometric method can be applied to any set of unequal origins and destinations, it may return a sub-optimal route in certain cases. As an example, consider Figure 6.5. Note that a joining point and splitting point are now indicated by J and S, respectively. The depicted formation flight route may be the result from applying the geometric method as reported by Kent et al. [17]. The two solo routes that are to be joined in formation flight are straight lines between AC and BD.

Due to the particular locations of the origins A and B in Figure 6.5, the joining point is located to the left of origin B. Intuitively, this route does not seem to minimize the weighted distance. By noting that one can reduce the length of segments JS and AJ+BJ simultaneously, by moving J over the circle towards B, the sub-optimality is identified.
Provided that the formation flight segment is sufficiently long, having one of the aircraft fly in the wrong direction to join a formation may still result in overall fuel savings. However, the distance that must be covered will never be minimal, regardless of the used routing weights. Hence, it is expected that the route will not be optimal from a fuel consumption perspective.

Based on these considerations, the developed routing algorithm will never suggest an alternative route that requires a heading change of over 90 degrees of one or more of the involved aircraft. Accordingly, the result of the geometric method has to be occasionally overruled when this does occur. In this work, two possible measures are defined that may

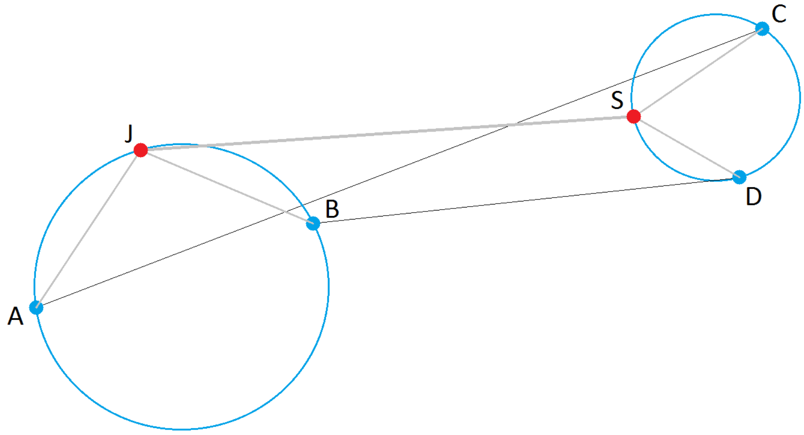


Figure 6.5: Illustration of a route that may result from applying the geometric routing method as it is described in the paper of Kent et al [17].

be used simultaneously. One can move the joining point to the most right origin, and one can move the splitting point to the most left destination. When either the joining point or the splitting point is moved, the respective other has to be revisited. Looking at Figure 6.3, Y_2 will remain stationary if the joining point is moved. Therefore, the optimal splitting point will move due to the re-location of the joining point. The same analogy holds for initially moving the splitting point. Moreover, cases have been encountered in which the joining was initially usable, but the splitting point had to be moved, which in turn rendered the joining point sub-optimal, requiring revision of the joining point after all.

To be able to obtain the minimum weighted distance route for any random combination of flights, all the possible exceptional cases are incorporated in the routing algorithm. To illustrate, Figure 6.6 shows the formation flight route that will be returned by the developed routing algorithm for the solo flights from A to C and from B to D given in Figure 6.5.

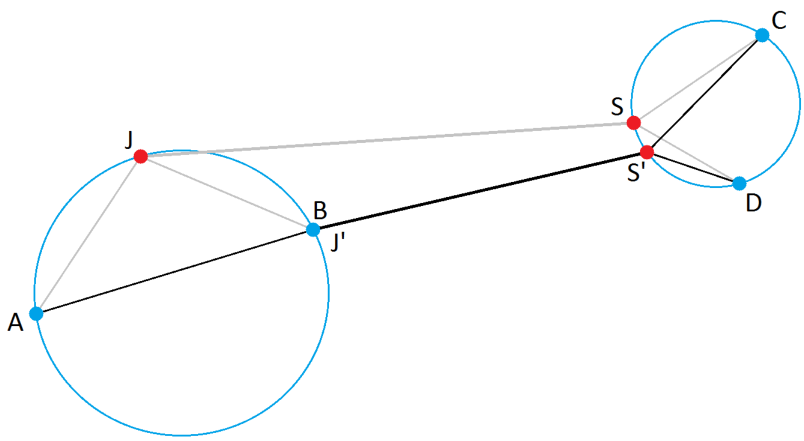


Figure 6.6: The sub-optimal route from Figure 6.5 is repeated in grey. The black route is returned by the developed routing algorithm.

The black route represents a route as it would be returned by the routing algorithm. One can see that the new joining point J' has been moved onto origin B. Accordingly, the splitting point S has been relocated to the point S'. The cumulative weighted distance from A to C and from B to D is now minimized.

In this work, communicating flights may find that they have a common destination. If this occurs, this destination is selected as the splitting point for any formation flight route that they may consider.

With the extensions mentioned in this section, the obtained routing algorithm is able to find the minimum weighted distance formation flight route for any two flights that may be communicating at some point in this work. This will be verified in Section 6.2.

6.1.3 Extension of the weighting scheme

Equation 6.1 provides an expression for the formation angle. As this work intends to use the routing algorithm for the re-routing of formations as well, a different and more elaborate weighting scheme is developed. Note that the weights discussed in this section have a purely geometric function. There is no direct relation between the weights and the expected fuel savings from formation flight.

For routing two solo aircraft, the values for w_A and w_B are set equal to 1, in accordance with Kent et al. It was experimentally found that $w_C = 1.87$ would lead to the highest overall fuel savings for a generic route. The fuel consumption calculations were performed as described in Chapter 5.

In this work, a formation of two may communicate with other flights to acquire additional formation members. Accordingly, the routing algorithm must be able to generate a route for a communication set that contains a solo flight and a formation of size 2. Intuitively, it seems to be undesirable to assign the formation of size two with a detour that is comparable to the detour that has to be flown by the solo flight in order for the three aircraft to join in formation together. As stated in Section 5.3.1, all considered aircraft are of the same standard type. Because of this, the distance that has to be covered by the formation is relatively more expensive than the distance that has to be covered by the solo flight. Since the weights in the geometric routing method are designed to represent exactly this cost of distance, the routing weight of the formation of size 2 in increased. The developed model uses the number of aircraft in a formation as the routing weight corresponding to a solo segment of that specific formation. Denoting the formation of size 2 as flight A and the solo flight as flight B, one obtains $w_A = 2$, $w_B = 1$. The routing weight of the formation flight segment remains to be determined. The explanation for the used method to obtain this last routing weight is aided by the extension of the example above to a scenario in which the formation leader of N flights communicates with the formation leader of n flights, where N is larger than n. Accordingly, w_N should be larger than w_n, while w_N and w_n represent the respective routing weights of the formations. As N becomes significantly larger than n, one wants w_N and w_n to reflect this difference in cost of flown distance. The expression in equation 6.2 has been formulated to determine the routing weight of the formation flight segment, w_C.

$$w_C = min\left(\left(w_A + w_B - 0.01\right), 0.87\left(w_A + w_B - 1\right)\left(1 + 0.035\left(w_A + w_B - 2\right)\right) + 1\right) \quad (6.2)$$

It was experimentally shown that the use of the expression in equation 6.2 positively influenced the overall obtainable fuel savings in the simulation of large scenarios. Note that when $w_A = 1$ and $w_B = 1$, equation 6.2 returns the previously mentioned value of 1.87. For the previously used example in which a formation of size 2 is routed with a solo flight, where $w_A = 2$, $w_B = 1$, one obtains from equation 6.2 that $w_C = 2.8$.

As an example, consider that $w_A = 6$ and $w_B = 3$. From equation 6.2, it is found that $w_C = 8.99$. If one calculates the formation angle with equation 6.1, the relatively small angle of 5.7 degrees is the result. Equation 6.2 has been designed to perform in this way, because of the fact that re-routing formations becomes less and less attractive as formations increase in size. The benefits from forming a formation is that one additional flight can enjoy a reduced fuel flow. This is true for the merger of any two formations. Therefore, it is relatively less rewarding to re-route larger formations. Equation 6.2 aims to incorporate this knowledge in the routing method. By doing so, the obtained formation flight route suppresses detours of larger formations. A low formation angle provides the flight with the highest routing weight with the smallest detour.

In equation 6.2, the value of w_C can not be raised above the sum of w_A and w_B, minus 0.01. This factor of 0.01 has been added to avoid a situation in which $w_A + w_B = w_C$. Including this equality would be the same as stating that formation flight does not come with any benefits or additional costs. The routing algorithm needs to be provided with a reason to include a formation flight segment. Later in the simulation, a route can still be rejected on fuel considerations if this turns out to be required.

6.1.4 Synchronization

After the minimum weighted distance formation flight route has been obtained, it must be evaluated if it can actually be used. The involved aircraft must be able to arrive at the joining point at the same time. As introduced in Section 4.1.2, the efforts of aircraft to simultaneously arrive at their joining point are referred to as synchronization efforts. This section presents the synchronization method that has been developed for this work. Since the formation flight route that is suggested by the routing algorithm can be altered by the synchronization algorithm, synchronization is part of the routing process.

Synchronization strategy

The synchronization process makes sure that flights are able to fly the formulated formation flight route. In this work, it is assumed that formation flight is to be realised as fast as possible, since getting flights into formation is the objective of this research. Accordingly, a formation flight route that is accepted, should contain those synchronization segments that get the involved flights in formation as soon as possible.

The synchronization algorithm is allowed to change the speeds of the two (groups of) aircraft as well as the location of the joining point, in order to realize synchronization. As mentioned in Section 4.2, all aircraft cruise at their maximum allowed speed. For synchronization purposes, one (group of) aircraft is slowed down while the other continues to fly at its maximum speed, in order to catch up with the other.

First, one must verify if synchronization is at all possible on the suggested formation flight route. Therefore, it is verified if synchronization would be possible at the suggested splitting point. If this is not the case, the suggested formation flight route must be rejected, as it is certainly not beneficial. This decision is depicted in Figure 4.2. If synchronization is possible at the splitting point, it is evaluated if synchronization is possible at the suggested joining point. If this is the case, the algorithm derives which (group of) aircraft is to be slowed down to which speed. The suggested formation flight route is marked as flyable and the synchronization process is complete.

It may occur that synchronization is possible at the suggested splitting point, but not possible at the suggested joining point. In these cases, the synchronization algorithm relocates the joining point to a usable location. It has been decided that in this work, the new joining point has to be chosen on the original formation flight segment. Intuitively, the joining point should be moved towards the splitting point, since moving it further away from the splitting point will only increase the synchronization problem. The major advantage of moving J over the formation flight segment towards S, is that an exact method allows for direct determination of the new joining point. This method will be presented in the next section. It is compact and does not require any optimisation. These properties are desirable for the synchronization function, as it will be used to evaluate thousands of optional formation flight routes. Secondly, moving the joining point towards S is the only direction for which both the detours that have to be flown by both (groups of) aircraft reduce in length. This is a positive side effect of postponing formation flight. Also, having minimal additional flight distance is assumed to be a general operational preference for each flight individually, both from a fuel and a flight planning perspective. Note that moving the joining point over the formation flight segment towards the splitting point may render the new formation route sub-optimal with respect to weighted distance. This is actually proven later in Section 6.2. However, the loss of optimality is considered insignificant and unharmful at this stage of the project, especially considering the impact of the governing assumptions on synchronization.

Re-locating the joining point

This section describes the method that has been developed to re-locate the joining point in accordance with the synchronization strategy from the previous section. The method will be illustrated by means of an example.

Consider Figure 6.7. If synchronization is not possible at the proposed joining point J, one needs to bring the ratio of the segments lengths AJ and BJ closer to 1. In fact, the largest ratio of segments lengths AJ and BJ that will make synchronization possible is the ratio of the minimum and maximum speed. For this work, this ratio is equal to 251/207=1.21.

By using the law of cosines, a method has been developed that exactly determines the new location of the joining point, given the synchronization strategy in Section 6.1.4. Figure 6.8 contains the left part of Figure 6.7 as well as several definitions that are used to relocate the joining point.

In Figure 6.8, point X is defined on the intersection of the line segments $J_{old}X_1$ and AB. This point X creates the two triangles AXJ_{new} and BXJ_{new}. Since the locations of the

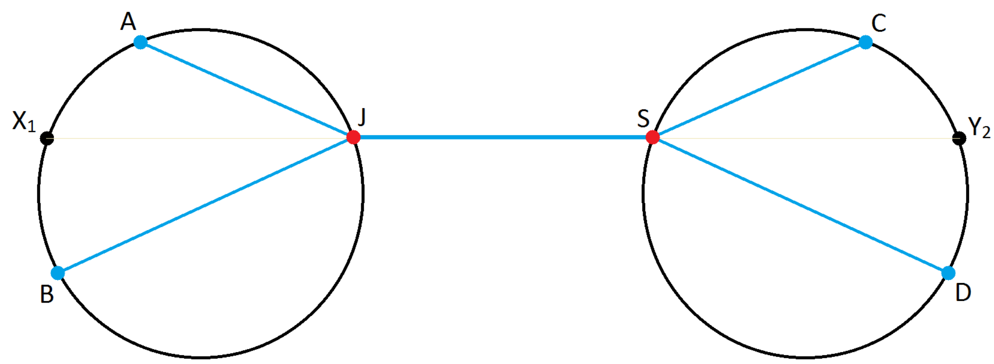


Figure 6.7: Example route based on solo routes AC and BD. The layout of this image has been adopted from Kent et al.[17].

points A, B, X_1 and J_{old} are known, the angles α and β can be found. Solving for the side length XJ_{new} would provide the new location of the joining point.

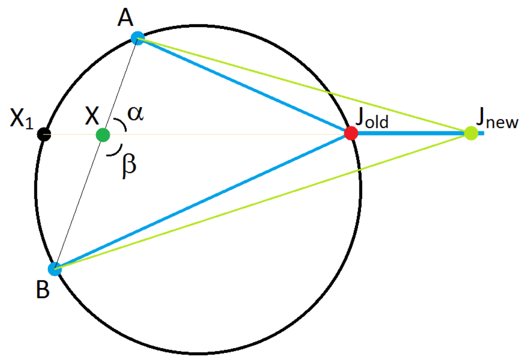


Figure 6.8: From the triangles AXJ_{new} and BXJ_{new} and the preferred ratio AJ_{new}/BJ_{new}, point J_{new} can be located.

The missing angles and side lengths of a triangle can be determined through the law of cosines, if at least one angle and two side lengths are known. In the triangles AXJ_{new} and BXJ_{new}, one angle and one side length, AX and BX respectively, are known. The law of cosines can therefore not be applied directly.

By making use of the assumption that it is desired to obtain the location of J_{new} that ensures $AJ_{new}/BJ_{new} = 1.21$, and by using the fact that side XJ_{new} is shared by triangles AXJ_{new} and BXJ_{new}, it is possible to obtain a solvable quadratic expression for the segment length XJ_{new}. Note that in approximately half of all cases, one would be considering $BJ_{new}/AJ_{new} = 1.21$, depending on the locations of points A and B.

The first step to obtaining the expression for XJ_{new} is to write the law of cosines for both triangles AXJ_{new} and BXJ_{new}. For triangle AXJ_{new}, one finds the expression for AJ_{new} as given in equation 6.3.

$$(AJ_{new})^2 = (XJ_{new})^2 + (XA)^2 - 2XJ_{new}XA \cdot \cos\alpha \tag{6.3}$$

Similarly, now considering triangle BXJ_{new}, one finds the expression for BJ_{new} as given in equation 6.4.

$$(BJ_{new})^2 = (XJ_{new})^2 + (XB)^2 - 2XJ_{new}XB \cdot \cos\beta \tag{6.4}$$

Using the relation $BJ_{new} = 1.21 \cdot AJ_{new}$, allows for combining equations 6.3 and 6.4 into equation 6.5

$$(1.21 \cdot AJ_{new})^2 = (XJ_{new})^2 + (XB)^2 - 2XJ_{new}XB \cdot \cos\beta \tag{6.5}$$

Inserting equation 6.3 into equation 6.5 yields

$$1.21^2\left((XJ_{new})^2 + (XA)^2 - 2XJ_{new}XA \cdot \cos\alpha\right) = (XJ_{new})^2 + (XB)^2 - 2XJ_{new}XB \cdot \cos\beta \tag{6.6}$$

which rearranges to

$$1.21^2\left((XJ_{new})^2 + (XA)^2 - 2XJ_{new}XA \cdot \cos\alpha\right) - (XJ_{new})^2 - (XB)^2 + 2XJ_{new}XB \cdot \cos\beta = 0 \tag{6.7}$$

From equation 6.7, the quadratic expression in equation 6.8 is obtained.

$$\left(1.21^2 - 1\right)(XJ_{new})^2 + \left(2XB \cdot \cos\beta - 1.21^2 \cdot 2XA \cdot \cos\alpha\right) XJ_{new} + 1.21^2\,(XA)^2 - (XB)^2 = 0 \tag{6.8}$$

Equation 6.8 can be solved for side length XJ_{new}. After choosing the appropriate solution, the one that is larger than side length XJ_{old}, one can construct the new joining point, J_{new}.

As mentioned in Section 6.1.2, it may happen that the geometric routing algorithm generates a route for which the joining point is located on one of the origins. For these cases, the location of a joining point that allows for synchronization can be derived be means of a method that is similar to the one presented in this section. One can construct a single triangle ABJ_{new}, for which one angle, the side length AB and the desired ratio $AJ_{new}/BJ_{new} = 1.21$ are known. The law of cosines can be applied again to obtain a quadratic expression for XJ_{new}.

The formation flight route that has been obtained at this point, will no longer be adjusted in this work. Referring back to Figure 4.2, the current route is either accepted or rejected.

6.2 Verification of routing method

To verify that the developed routing method always returns the minimum weighted distance formation flight route, two verification steps have been taken. Note that during these verification steps, the segments weights w_A, w_B and w_C are kept constant.

First, the formation flight route in Figure 6.9 is generated. This route has both common locations for the origins and rare locations for the destinations. It can be seen that the splitting point has been placed on destination D.

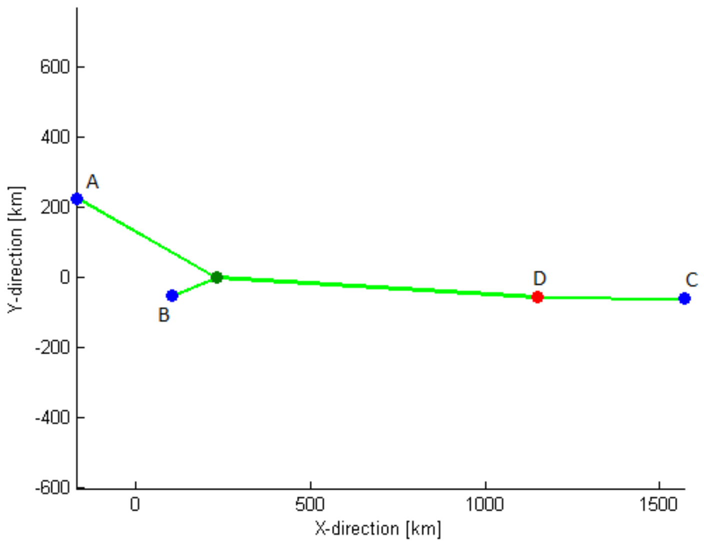


Figure 6.9: Example route that is generated by the developed routing algorithm from the solo routes AC and BD.

In an effort to demonstrate that this route is indeed the minimum weighted distance route, Figure 6.10 is presented. On the route in Figure 6.9, the joining point is moved to every integer location in a 200 km by 200 km square around the joining point that is generated by the routing algorithm. After each relocation, the resulting weighted distance is calculated. The color scale in Figure 6.9 indicates the value of the weighted distance savings for each joining point location. Afterwards, the same exercise is repeated for the splitting point.

From the surface plots in Figure 6.10, one finds that the route that is returned by the routing algorithm is indeed the minimum weighted distance route. Also, in the right plot one finds a steep decrease in weighted distance savings for moving the splitting point to the right. Moving it to the right would mean moving it past destination D. According to the discussion in Section 6.1.2, this would lead to a sub-optimal route. The right plot in Figure 6.10 confirms that moving de splitting point to the right will significantly decrease the obtained weighted distance savings.

In order to demonstrate that the routing algorithm can handle any random route set that it receives as input, the verification process above is applied to 10.000 randomly generated flight pairs. The x- and y-coordinates of the origins and destinations were

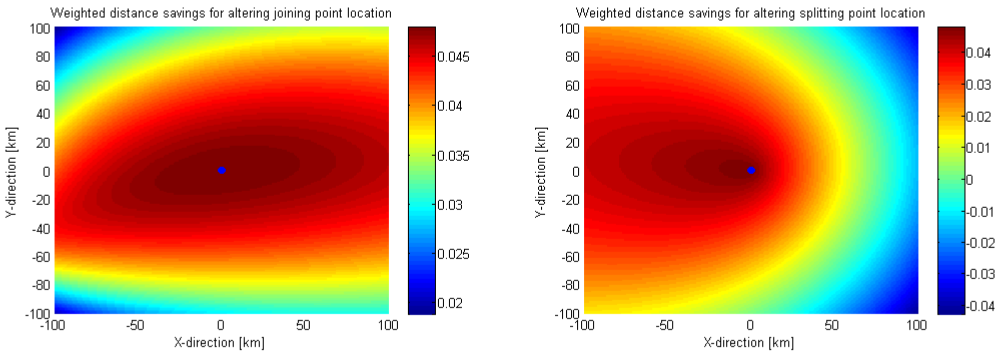


Figure 6.10: These surface plots show the effect of moving either the joining point or the splitting point on the total weighted distance savings

obtained randomly from a normal distribution with a standard deviation of 500 km. The means for the origins and destinations were set to the points [0,0] and [7000km,0] respectively. Figure 6.10 leads to the conclusion that, given a splitting point, there is only one optimal location for the joining point. Accordingly, the square in which the joining point and splitting point are moved is shrunk to 10 km by 10 km, in order to increase the computational efficiency of the verification simulation. Figure 6.11 shows the weighted distance savings that the routing algorithm had returned versus the maximum encountered weighted distance savings from moving the joining point and splitting point around their original locations. It is concluded that the routing algorithm is capable of finding the minimum weighted distance route for two random flights.

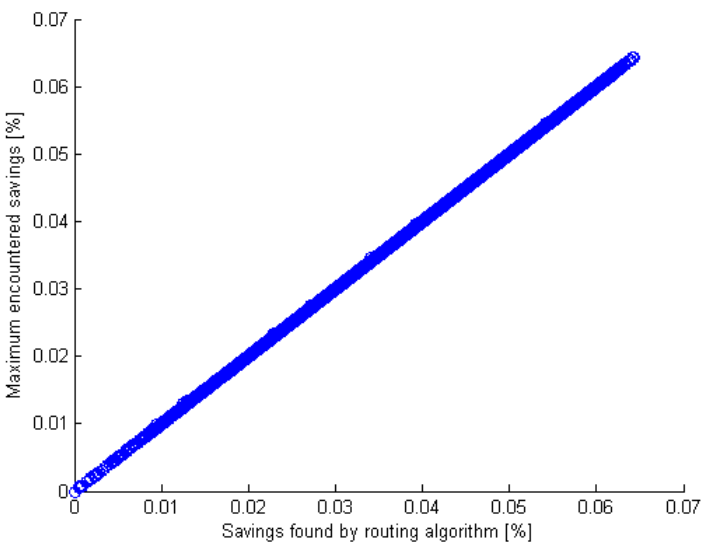


Figure 6.11: The weighted distance savings that the routing algorithm selects, correspond in all 10.000 cases to the maximum possible weighted distance savings.

Section 6.1.4 mentioned that moving the joining point along the original formation flight segment, for synchronization purposes, may result in a sub-optimal formation flight route. This is confirmed by inspection of Figure 6.10. For the formation flight route in Figure

6.9, the synchronization algorithm will move the joining point to the right and slightly down, due to the location of the splitting point. From Figure 6.10, it follows that moving the joining point to the right and slightly upward would have resulted in slightly higher weighted distance savings. The slight upward movement of the joining point would partially relieve the synchronization difficulties. Considering the conceptual nature of this work, as well as the fact that this paragraph addresses exceptional routing scenarios, the synchronization algorithm is not further developed after this notion.

It is noted that the developed routing algorithm is not able to cover *any* random pair of flights. For example, flights in opposite directions can not be routed. In fact, the algorithm requires all the flights to be from left to right in the two dimensional set up of the simulation. For the study of formation flight implementation, these model properties do not pose any restrictions. If a pair of flights is to benefit from formation flight, they must have certain similarities, considering for example solo heading and the geographic location of their origins and destinations.

6.3 Accepting a formation flight strategy

When optimizing a certain part of a solo flight trajectory, one must accept a solution at some point, otherwise the flight can not be performed. In this work, the use of formation flight is treated as an in-flight option. Accordingly, one requires some sort of strategy or objective formulation in order to make formation flight related decisions. One may include all kinds of cost parameters in this additional decision moment. In general, a suggested formation flight route would have consequences for the overall fuel consumption, flight time, risk profile and operational cost. More specifically, it requires the involved aircraft to be willing to cooperate and update their flight paths. Any flight may reach different conclusions on the use of a suggested formation flight route when evaluating these decision variables. For example, a flight may be time-critical due to unforseen circumstances and as such it may choose to reject any formation flight route.

This work aims to determine, on a conceptual level, the value of a decentralized approach to formation flight routing. To do this in an adequate manner, it must be assumed that all flights have a positive attitude towards formation flight. As mentioned in Section 4.2, this work assumes that all flights are willing to cooperate in any formation flight strategy that contributes to the objective of reducing the cumulative fuel consumption. Because of this assumption, any formation flight strategy with a positive result in cumulative fuel savings may be accepted. A second decision variable considers the additional flight time for each individual flight that would be involved in a suggested formation flight route. From an operational point of view, excessive additional flight time is unwanted. It will cause difficulties on connecting flights, increase the operational costs related to delayed arrival and decrease passenger comfort. Also, when a flight commits to a formation flight strategy that requires high synchronization efforts in terms of time and fuel, the pressure is increased on the formation flight segment, which has to provide sufficient return on these investments.

In this work, the determination and acceptance of a formation flight route is not an iterative process. For any communication set, it is attempted to determine a formation flight route. If a route is found, it is either accepted or rejected. This design choice enables efficient evaluation of large scenarios, such as the case study that will be presented in Chapter 7.

6.4 Facilitate growing formations: Dummy-flight system

This work differentiates itself from the present literature, by letting formations grow to any size whenever this is ruled to be beneficial in terms of overall fuel burn. Practically, this is quite comprehendible; smaller formations will join together in larger formations. To be able to simulate formation growth, one must define how this may occur.

Inspired by Ribichini et al. [31], a formation growth coordination system has been developed for this work. As can be found is Section 3.2.1, Ribichini et al. use unique communication ID codes for formations. One of the aircraft in a formation is made responsible for the routing of the formation. Considering that this thesis intends to simulate scenarios with hundreds of flights, the reliability and programmability of this concept was questioned. The challenge to keep track of formation hierarchy and navigational responsibility was identified. Considering also the desire to record flight data for analysis and visualisation, it was decided to extend the concept of using a unique ID for formations to communicate with. Instead of using just a separate ID to route a formation, a complete virtual flight, or dummy-flight, is used to account for the navigation of a formation. Figure 6.12 illustrates how a dummy-flight route is generated when a formation is formed. The virtual route has origin E and destination F, which are placed on the joining point and splitting point respectively.

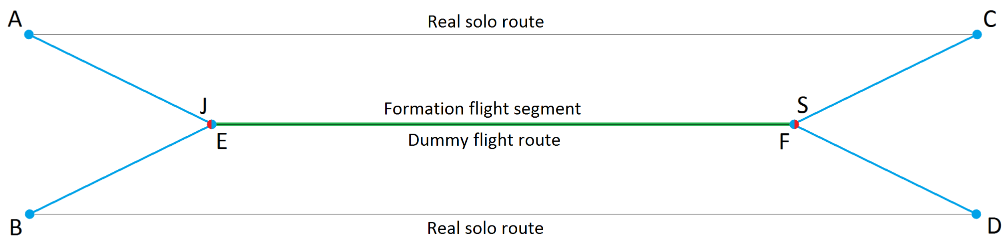


Figure 6.12: When two flights meet at their joining point, a dummy-flight is created to navigate the formation. It uses the joining point and splitting points of the real flights as its origin and destination.

When generated, a dummy-flight immediately departs from its destination. A dummy-flight and a real flight have identical properties and capabilities, with the exception that a dummy-flight does not consume any fuel. The two real flights are instructed to follow the dummy-flight, until this dummy-flight has reached its destination. When the dummy-flight has arrived, the two real flights are released and they will fly towards their individual destinations.

While flying, the dummy-flight follows the operational concept as described in Figure 4.1. As such, it may communicate with other aircraft and commit to additional formation flight strategies. When a dummy-flight decides on using an additional formation flight segment, it takes into account the consequences for its two followers. Therefore, the followers can blindly follow their dummy-flight, until they have reached their determined splitting point. In fact, within the simulated operational concept as presented by Section 4.1.2, followers are not allowed to leave their dummy-flight. This design choice ensures that all expected fuel savings are actually obtained.

A dummy-flight is created at each joining point that is used. Accordingly, dummy flights can become followers of yet an other dummy-flight. Through this process, formation can grow to essentially any size. Chapter 7 will illustrate how the possibility of growing formations manifests itself in a case study of 347 transatlantic flights.

In reality, the navigational role of a dummy-flight is to be performed by a formation leader. This does not pose additional operational challenges, if it is assumed that real aircraft are able to alternate between acting as a leader and acting as a follower.

Chapter 7

Results

The developed model has been used to evaluate numerous formation flight scenarios on different networks with varying model configurations. This chapter mainly elaborates on a case study of 347 transatlantic flights. Before its results are presented, additional examples are provided with the purpose of introducing the model on a smaller scale.

7.1 Illustration of the developed model

This section will provide examples that illustrate the workings of the developed decentralized approach to formation flight routing. Several formation flight decisions are illustrated and explained. Additionally, some model characteristics, that are known to be sub-optimal, are clarified. These characteristics contribute to the conclusions and recommendations provided in Chapter 8. Throughout this section, the communication range is set to 250 km, all departure times are equal and all routes are orientated to be flown from left to right in the readers perspective.

Figure 7.1 shows four fictional solo routes. Figure 7.2 contains the formation flight routes that result from applying the developed model to the routes in Figure 7.1.

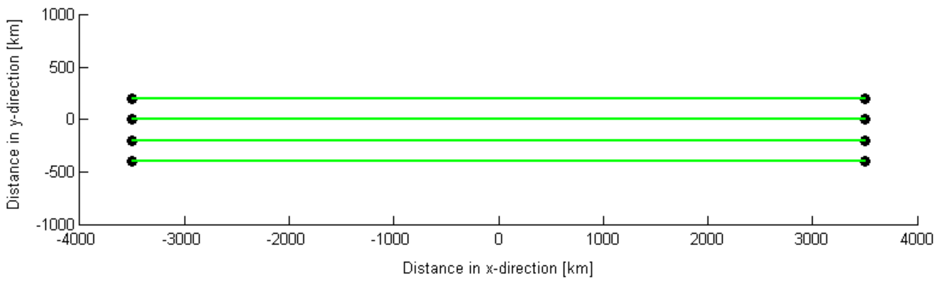


Figure 7.1: Four similar solo routes

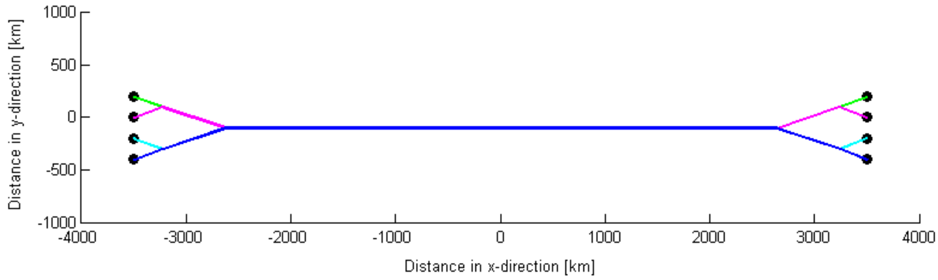


Figure 7.2: Formation flight routes for the four similar solo routes from Figure 7.1

From Figure 7.2 one can observe a symmetric route pattern. After departure, two formations of size 2 are formed, that consecutively join together in a formation for size 4. Due to the symmetry in the orientation of the origins and destinations, the routes up to the joining point are similar to those after the splitting point. Note how the formations of size 2 act as a single flight from the joining point to the splitting point of two individual aircraft. Once any two flights have committed to a formation flight strategy, it is always honoured to secure the predicted fuel savings. However, if the individual flights would have been aware of their future membership of the formation of size four, they may have wanted to relocate their first joining points in order to optimize for fuel consumption.

Figure 7.3 shows four different solo routes, now somewhat more randomized in orientation with respect to Figure 7.1 in order to stimulate the model to show more of it's capabilities. Figure 7.4 contains the formation flight routes that were obtained from the developed model.

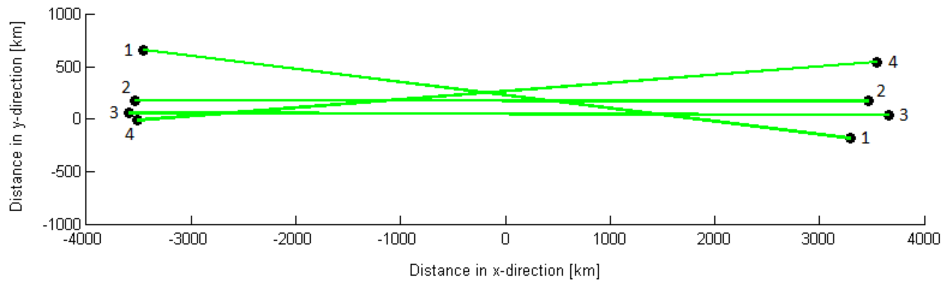


Figure 7.3: Four random solo routes

As expected, the flights that leave from the three closely spaced origins 2, 3 and 4 progress to form a formation of size 3. When flights 2 and 3 cross their joining point, they are once again open for communication and commit to a formation flight strategy with flight 4. Flight 1 started its mission with a solo segment. As soon as the formation of size 3 is formed, it may communicate again and establishes contact with flight 1. A formation flight strategy is found and committed to. Note how flight 1 performs a more significant heading change than the formation of size 3. This is an example of the models efforts to reduce the detours of larger formations, as discussed in Section 6.1.3. In the right part of Figure 7.4, it can be observed that the aircraft leave the formation of size 4 one by one. The remaining aircraft stick together in their previously determined formation flight

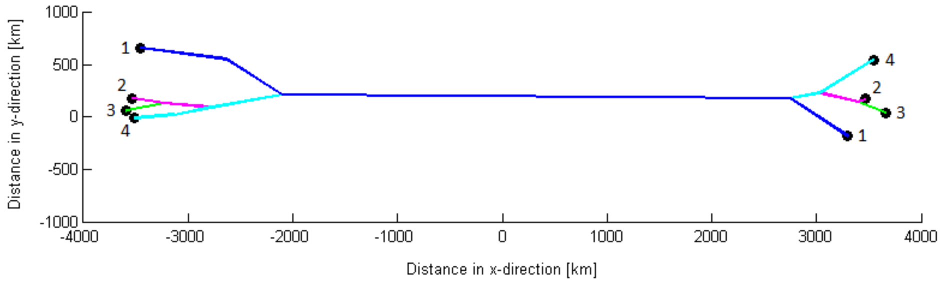


Figure 7.4: Formation flight routes for the four random solo routes from Figure 7.3

strategies.

Figure 7.5 contains four random solo routes that are generated to have a larger vertical spread in their origin and destination locations. Figure 7.6 provides the found formation flight routes.

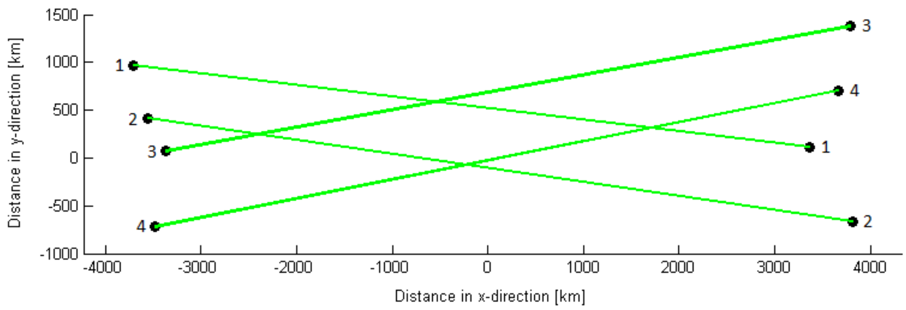


Figure 7.5: Four random solo routes with a higher spread in origin and destination location

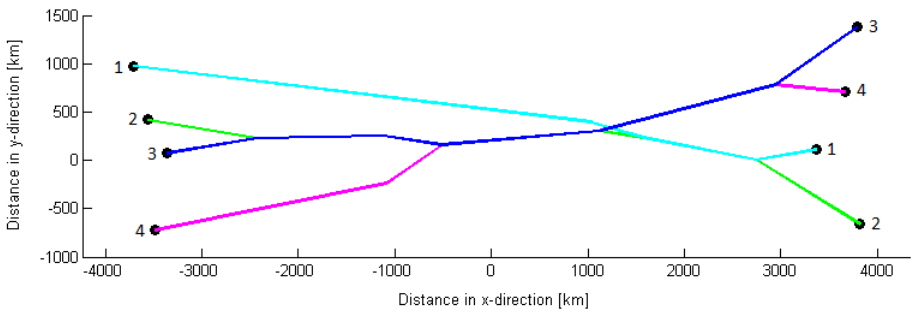


Figure 7.6: Formation flight routes for the four solo routes from Figure 7.5

As a result of the wider spread in the origins and destinations of the flights, one can observe a more fragmented use of formation flight. From examining Figure 7.6, it is found that flight 2 and 3 commit to a formation flight strategy right after take off. Flight 1 does not encounter any profitable formation flight options in the first half of its mission, first limited by communication range and later by fuel considerations in the communications with the formation of size 2. Flight 4 is out of range for any communications during the first part of its mission. When the flights are about to reach the -1000 km marker, flight 4 and the formation of size two enter each others communication circles. A formation flight strategy is found and implemented. As the formation of size 3 flies past the 0 km marker, no profitable formation flight strategies have been encountered that include flight 1. After passing the 1000 km marker, flight 2 splits off from the formation of size 3. It now becomes available for communication and finds a consecutive formation flight strategy together with flight 1. The two formations of two fly on towards their splitting points and all flights perform a concluding solo segment.

Figure 7.6 demonstrates the flexibility of the developed model. Formation flight strategies are accepted throughout the planned flight time and flight 2 is part of two consecutive formation flights. In the case study on 347 flights, discussed in the next section, these two capabilities of the developed model are used frequently.

Figure 7.7 elaborates on a specific example of a consecutive formation flight segment. A formation of size 4 enters the figure at the 2500 km marker. First, it splits in two formations of size 2. Shortly after this, flight 2 splits away from flight 4. Flight 2 continues to fly solo and sets course for its destination. After the splitting point of flights 1 and 3, a beneficial formation flight strategy is encountered by flights 1 and 2. This specific outcome of the model indicates what may occur if flights are not allowed or able to communicate with multiple others at the same time. If flight 2 had been aware of the flight plan of flight 1, it is considered likely that flight 2 would have continued in a formation of size 3 with flights 1 and 3, right after flight 4 had left the formation of size 4. Even though the model is currently operating sub-optimally in this specific area, it remains a novelty to the research area that aircraft are capable of consecutive formation flight segments.

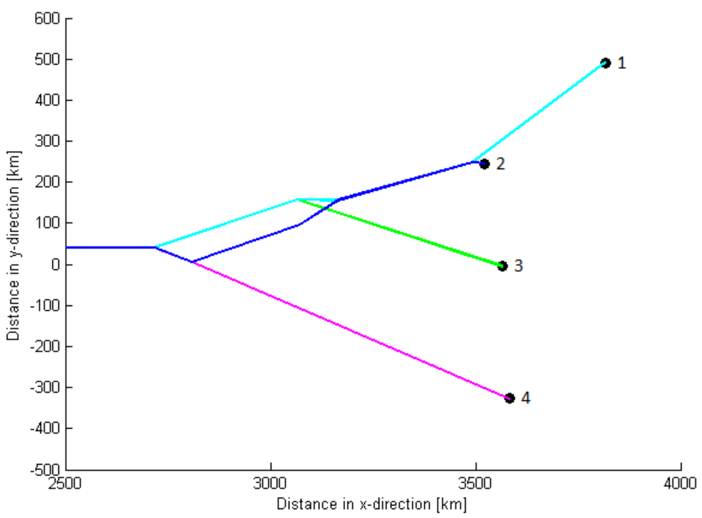


Figure 7.7: Illustration of a sub-optimal consecutive formation flight segment

Figure 7.8 contains a specific example of how the developed model may construct a formation of size 3. Due to the formulation of the communication algorithm, flight 1 engages to flight 2 right after departure. Flight 3 sets out on a solo mission, unaware of the formation flight strategy of the other two flights. When the formation of size two is formed, flight 3 is well within the required communication range and commits to a formation flight strategy with the formation of size 2. One can safely assume that it would have been more fuel efficient if flights 1 and 3 had engaged in formation flight communication first. Not only would this have reduced the detours of flights 1 and 3, it would also have increased the overall usage rate of formation flight.

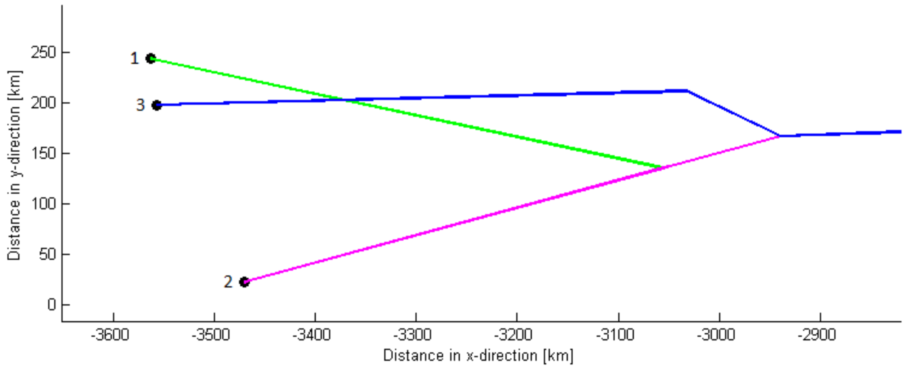


Figure 7.8: Illustration of a sub-optimality in how the developed model may form larger formations

7.2 Case study: 347 transatlantic flights

This section will elaborate on the application of the developed model to a set of real flights that took place on the 14th of march, 2010. Intuitively, formation flight implementation has a higher probability of success when the considered routes are similar in both their range and the geographical locations of their origins and destinations. Also, the implementation of formation flight on transatlantic routes has been studied in the literature before. From the available data set, the route set given in Figure 7.9 has been obtained. It contains eastbound transatlantic flights that departed between 17:00h and 23:00h. The origins and destinations were filtered on minimum and maximum longitude and latitude, to avoid inaccurate representation of routes by the projection method that will be presented in the next section. The flights that remain are all used in this case study, making no distinction in operating airline or route specifics.

This section presents the main case study of this chapter. The following sections elaborate on additional considerations with respect to the same case.

7.2.1 Projection of routes

The developed model assumes a two dimensional world with cartesian coordinates. Since the data on the route set is available in longitude-latitude coordinates, a projection method is required. The projection method used in this work is a so called azimuthal-equidistant projection method. This means that two parameters from the data are preserved during the projection process.

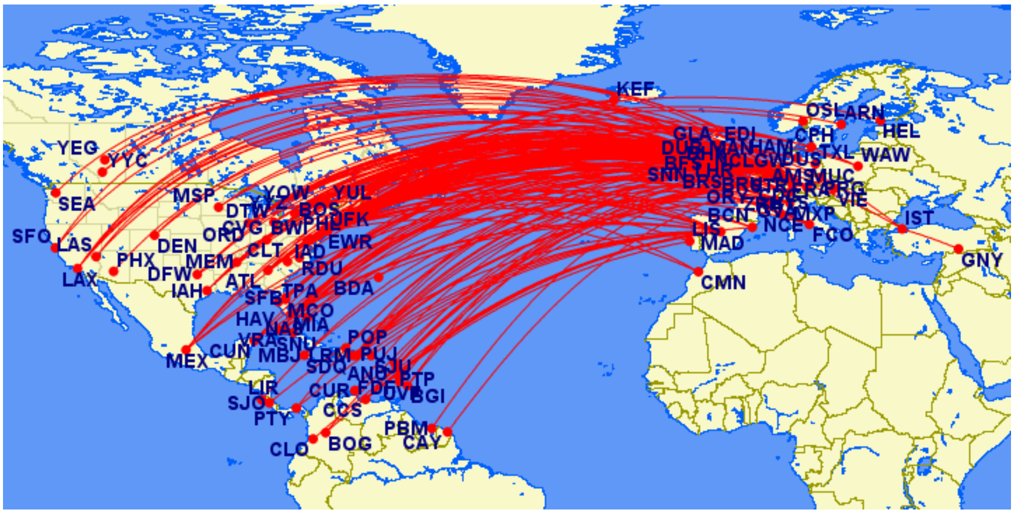


Figure 7.9: Rectangular projection of the 347 great circle routes used in this case study.[1]

First, a sphere is defined with its radius equal to the average radius of the earth. Next, a center point is defined by the average longitude and average latitude of each considered origin and destination, counted once for each flight that uses them. The first parameter that is preserved during projection, is the great circle distance from each origin/destination to the center point. The second preserved parameter is the angle between the horizontal and the great circle route from the origin/destination to the center point.

With this projection method, the relative locations of the origins and destinations, as well as the route lengths and relative flight headings are reasonably well translated into cartesian coordinates. Since these route characteristics are most significant for the success of formation flight implementation, the projection method is considered suitable for this thesis.

The blue dots in figures 7.11 to 7.13 show the projected origins and destinations that were obtained from the available data set, by means of the projection method. To verify that the projection method functions correctly, Figure 7.10 gives the azimuthal-equidistant projection as generated with a well known online route mapping tool [1]. Noting a slight tilt between Figure 7.10 and Figure 7.11 to 7.13, which does not affect the results of this case study, it is concluded that the projection method performs to an adequate standard for this work.

7.2.2 Simulation of formation flight implementation

With the routes imported and projected, the model needs to be configured before it can be run. Table 7.1 provides the values of the most important configuration parameters that were used in this case study.

During this research, it was found that a time step of 5 minutes was sufficiently small for the simulation to perform up to its potential. While the user of the developed model is free to apply any time step size, all simulations that are discussed in this thesis used a

Figure 7.10: Azimuthal equidistant projection of the 347 great circle routes used in this case study.[1]

Table 7.1: Model configuration parameters

Parameter	Value
dt	5 min
Communication Range	250 km
Allowed additional flight time/decision/aircraft	∞ min, no limit
Departure times	According to schedule
Accept formation flight route if	It saves any amount of fuel

time step of 5 minutes, in order to maintain computational efficiency. The communication range for the simulation in this section has been set to 250 km. Section 7.5 elaborates on using different values for the communication range. To demonstrate the results of using a purely fuel orientated objective, the allowed additional flight time per formation flight decision per aircraft is set to infinity. Section 7.4 elaborates on using a combined objective of fuel consumption reduction and additional flight time limitation.

Figure 7.11 shows a snapshot of the simulation at the 41st time step. Referring to the legend in Table 7.2, several observations can be made from Figure 7.11. It is noted that an engaged formation of flights will still be shown in purple.

Table 7.2: Legend of figures 7.11 to 7.13

Dot property	This represents:
Blue	Origin/Destination location
Green	Solo flight
Cyan	Engaged flight
Purple	Formation
Dot size	Proportional to formation size

From the origins in the left half of Figure 7.11, indicated by blue dots, flights are departing. As they fly towards their destination, they start out as a green dot, as they are still flying solo. After some time, most flights engage to another flight, causing them to change their flight plan and to turn cyan in the figure. Some flights are still flying solo, even though they have been in the air for a while. Note most of these flights are not in the vicinity of other aircraft that are allowed to communicate. Later, the majority of these flights will be part of a formation, illustrating the sub-optimality of the greedy communication algorithm, discussed in Section 4.1.2. The purple dots show the formed formations and their size. In the middle, two formations of size 5 have already been established. In the top region, many flights are still in the process of merging their routes by flying synchronization segments.

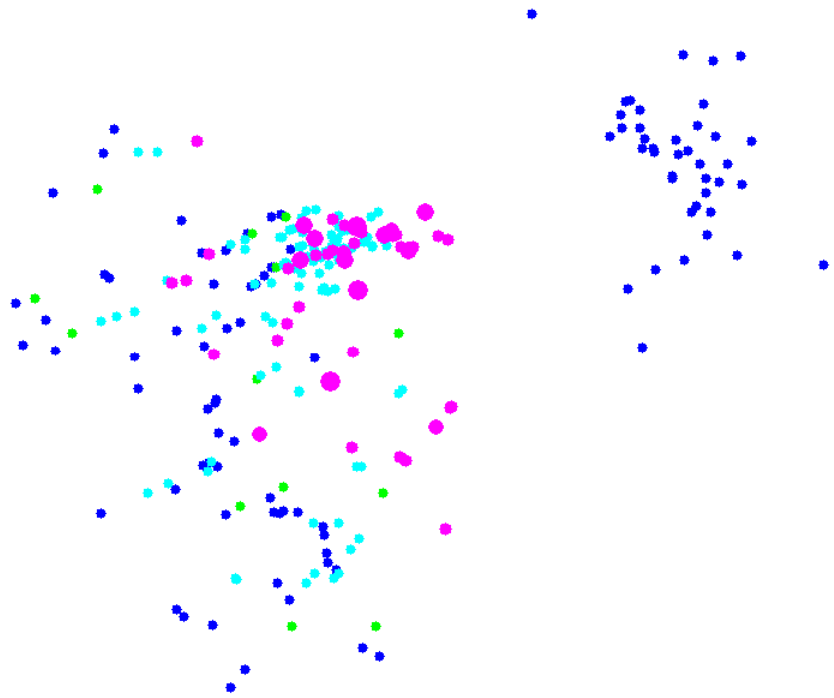


Figure 7.11: Snapshot from the simulation at time step 41

Moving on to Figure 7.12, showing the same flights 200 real minutes later, it can be seen that the supply to the eastbound stream of aircraft has reduced. In fact, all flights have departed at this stage. Many formations of different sizes have emerged in the top half of the figure. The largest formation in use at this point, and at any point, comprises 15 aircraft. Analysis showed that a formation of size 8 and a formation of size 7 had earlier accepted a formation flight strategy. The green dots at the front indicate flights that have separated from their formations; they are heading towards their destination by means of a solo flight segment.

Figure 7.12: Snapshot from the simulation at time step 81

Figure 7.13 provides the situation another 200 real minutes later, at which point quite some flights have arrived. Many solo flights are completing their final segment towards their destination. While only three cyan dots are distinguishable at this specific time step, it was found that many more flights and formations re-engage to a next formation flight strategy in the second half of their respective missions. It is not surprising that a few of the flights that were last in line to depart, do not manage to become a formation member. The combination of the location of their origin and their departure time prevents them from encountering any beneficial formation flight options.

Table 7.3 gives the general results of the simulation performed in this section. Note that the objective was solely to collect fuel savings and that fuel savings are modelled as a fuel flow reduction of 10% for any trailing aircraft, as discussed in Section 4.2.

The overall fuel savings are calculated with respect to the cumulative solo fuel requirement for all included flights. This means that a flight that never joins a formation, and as such does not even have to be aware of the fact that other flights did, weighs down the found overall fuel savings percentage. Since this may very well happen in reality, this consideration is included in the presentation of the results. Note how the obtained fuel

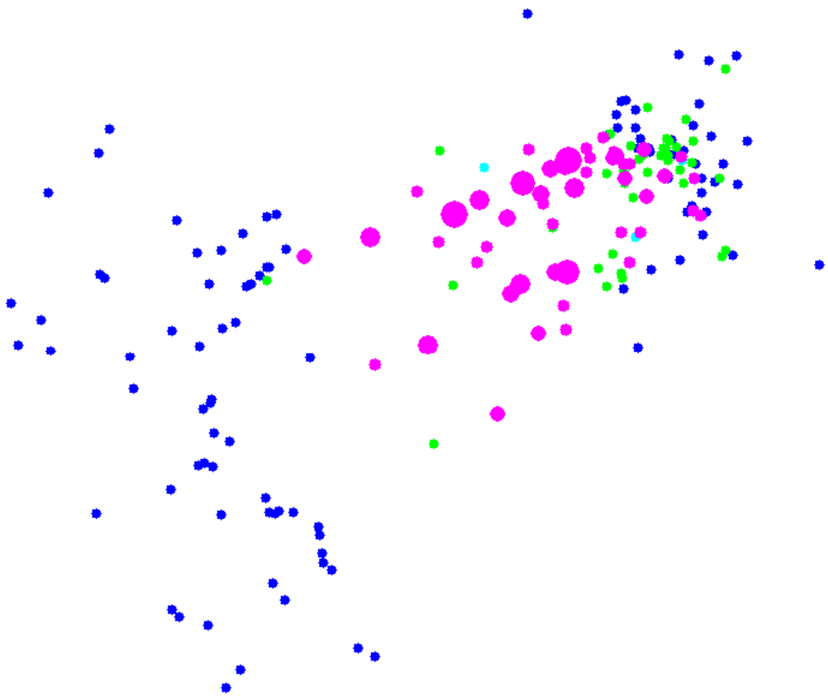


Figure 7.13: Snapshot from the simulation at time step 121

Table 7.3: Results for 347 transatlantic flights

Parameter	Value
Overall fuel saved	$5.6 \cdot 10^5\ kg$
Overall fuel savings	3.6%
Average added flight time	17.6 min
Maximum added flight time	64.2 min
Formation flight usage rate	72%

savings require a significant time investment. Section 7.4 addresses the use of limited additional flight time as an objective in combination with the fuel consumption reduction objective.

Figures 7.14 and 7.15 show distributions of added flight times, due to formation flight, and frequency of use of each formation size, respectively. It is concluded that the additional flight times due to formation flight are significant, often lengthening the total flight time of an aircraft by 5% or more (the average solo flight time is around 8 hours). The used formation sizes indicate that formations larger than size 4 are not uncommon. In the

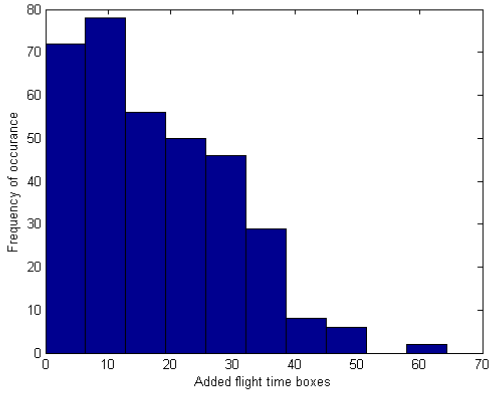


Figure 7.14: Added flight times for each flight

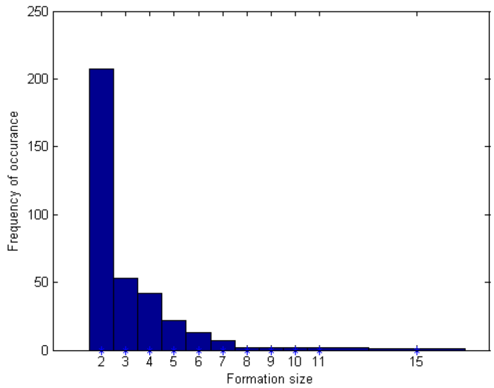


Figure 7.15: Frequency of use of each formation size

encountered literature, formations up to size 4 were considered. Note that in this work, any formation starts by a formation of size 2 due to the setup of the communication algorithm. Since 207 formations of size two are used, and 347 flights are simulated, it is known that a consecutive formation flight strategy is chosen at least 67 times, counting flights individually. Considering that there are flights that never fly in formation underlines the frequent use of consecutive formation flight strategies. On top of this, larger formations tend to split up and re-group with other formations in the second half the their missions, increasing the usage rates of formations larger than size 2.

Next to the frequency with which a certain formation size is used, it was studied for how long each formation size was used. Inspired by the relatively short segment on which the formation of size 15 was used in the performed simulation, the histogram in Figure 7.16 was created. It shows the distribution of overall flight time over each formation size. In the literature, formation flight usage rates are often based on covered distance of flight time. For comparability reasons, it is desired to use similar indicators for the formation flight usage rate. If this work would calculate the formation flight usage rate based on covered distance, the formation flight usage rate may be overestimated, as lower speeds on synchronization segments would be excluded. Since this is undesirable, as it would weigh the more expensive synchronization segments equal to segments at cruise speed, this work determines the formation flight usage rate based on total flight time.

In Figure 7.16, one can observe that indeed a relatively small amount of the total flight time is spent in the formation of size 15. What stands out, is that that 72% of all flight time is spent whilst flying in a formation. This result may indirectly be compared to a number found by Xu et al. in their transatlantic example that was discussed in Section 3.1.1. Xu et al. report that 67.5% of the total flight distance is spent in formation flight [37]. Due to the use of formation flight, their overall flight time is increased by 7.4%. As they also take variable formation flight speeds into account, where aircraft in this study only deviate from their default speed for synchronization purposes, a direct comparison can not be made. However, the two usage rate figures are very similar in meaning, as they are both indicators of the participation rate of aircraft in formation flights. The fact that they are of similar magnitude brings the potential of a centralized and a decentralized approach closer together.

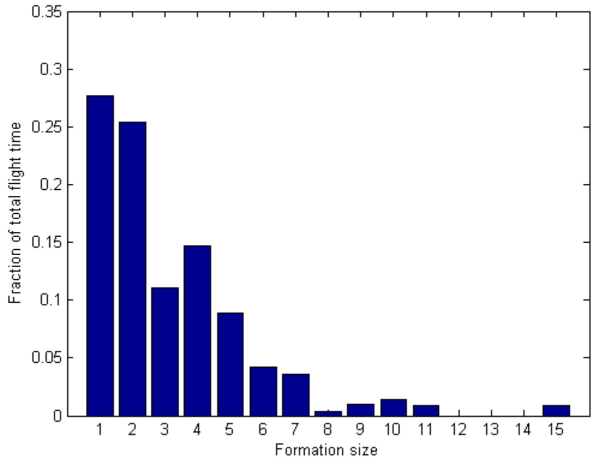


Figure 7.16: Formation size usage rate for the 347 flights in this case study

A novelty of this work is the elaborate consideration of formations that comprise more than four aircraft. In Figure 7.16, it can be seen that significant amounts of flight time are spent in formations of sizes 4 to 7. Larger formations are used for considerably shorter durations.

Note that the modelled fuel savings of formations larger than size 3 is purely theoretical and aimed to be conservative. Therefore, the fact that formations larger than size 3 are frequently used by the model, may inspire research into the actually obtainable fuel savings from the use of larger formations.

7.3 Delayed flights

From Section 3.4, it was found that current centralized approaches experience difficulties with handling delayed flights in an operationally attractive amount of time. Since flights are delayed every day, it would speak in favour of a decentralized approach if it is able to handle delayed flights. The nature of the developed decentralized approach suggests that it can, since any in-flight encounter between aircraft may lead to formation flight. The simulation that was described in the previous section has been repeated with an alternative departure time schedule. Figure 7.17 gives an overview of the delay times that were randomly assigned to the flights. These delay times were randomly generated from a normal distribution, such that the maximum delay time would approximately equal 10% of the average solo mission time.

Figure 7.18, considering the added flight time due to formation flight, shows a similar picture as Figure 7.14. Note that the delays are not included in the added flight time, as they are unrelated to formation flight. Figure 7.19 and 7.20 show that the largest used formation becomes one of size 13 and that, relative to the case without delays, more flight time is spent in formations of sizes 2 and 3. The formation flight usage rate has increased to 74%.

The relatively similar results in the figures 7.14 to 7.16 and figures 7.18 to 7.20 show that

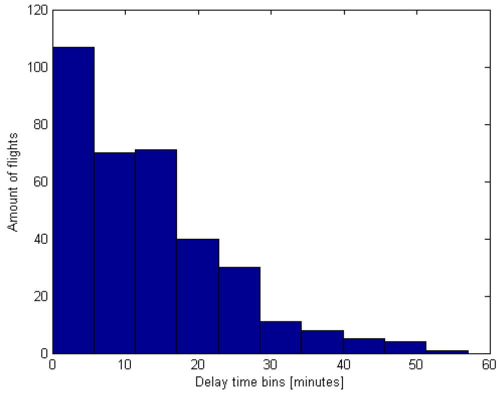


Figure 7.17: Distribution of the random delays that have been added to the departure times of the 347 flights

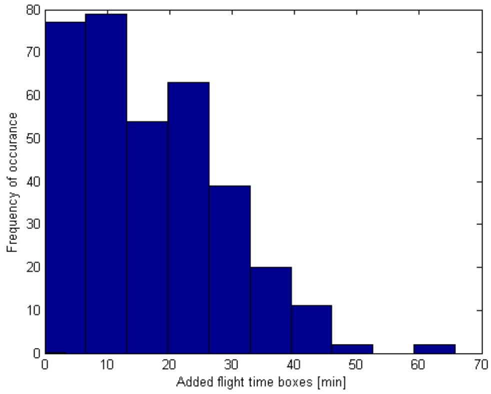


Figure 7.18: Added flight time when the 347 flights are delayed according to Figure 7.17. Delays are not included.

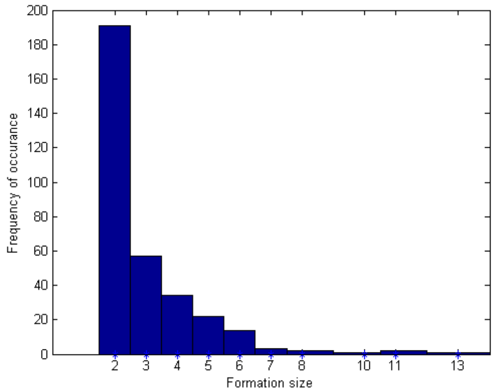


Figure 7.19: Frequency of use of each formation size when the 347 flights are delayed according to Figure 7.17

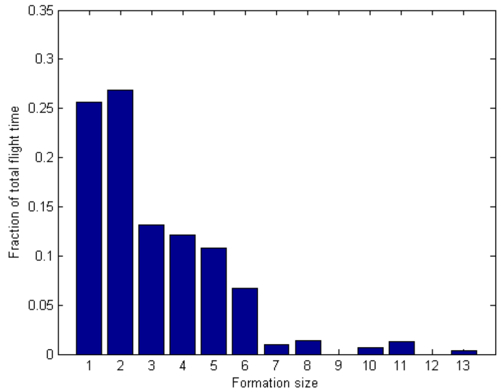


Figure 7.20: Formation size usage rate when the 347 flights are delayed according to Figure 7.17

in both cases, the model is capable of introducing formation flights. To quantify this result, Table 7.4 shows that the obtained fuel savings are equal. Note that they were not found to be exactly equal, but the differences are negligible and removed by the rounding of the numbers. Comparison of tables 7.3 and 7.4 gives that the average and maximum added flight times are not significantly affected when delayed flights are considered.

Any other distribution of delays is expected to generate similar results, mainly due to the flexibility of the developed model with respect to forming formations and the density of the considered traffic. Extreme delays may certainly cause aircraft to have to fly solo, which will somewhat reduce the obtained fuel savings. As this research aims to evaluate the potential of a decentralized approach, it is concluded that, within the scope of this work, the developed model is capable of letting delayed flights contribute to the overall

Table 7.4: Results for 347 transatlantic flights, including delayed flights

Parameter	Value
Overall fuel saved	$5.6 \cdot 10^5$ kg
Overall fuel savings	3.6%
Average added flight time	18.3 min
Maximum added flight time	59.2 min
Formation flight usage rate	74%

objective of reducing fuel consumption through the use of formation flight.

7.4 Limiting additional flight time

It was found that when the objective formulation of the simulation only contained the overall reduction of fuel consumption, the average added flight time is 17.6 minutes and the maximum added flight time is 64.2 minutes (see Table 7.3). These findings inspired a study that limited the acceptable added flight time per formation flight decision, as a means to reduce the overall additional flight time. From experiments, it was found that limiting the additional flight time for each aircraft to 10 minutes per formation flight decision, generated significant improvements on both the additional flight time as well as the fuel consumption reduction objective. As further optimization of the used limit would only provide case-specific results, the 10 minute limit is applied in a new simulation that is otherwise identical to the one described in Section 7.2.

The results in Table 7.5 reveal that the maximum added flight time reduced to 33.7 minutes and the average added flight time reduced to 9.9 minutes. The overall obtained fuel savings went up significantly, from 3.6% to 4.2%. Apparently, the formation flight strategies that are removed by the limit on additional flight time are less beneficial than the alternatives that the model is able to find consecutively. Again, the sub-optimal nature of the method that is used to collect fuel savings becomes apparent.

Table 7.5: Results for 347 transatlantic flights, including a limit on the added flight time due to a formation flight decision

Parameter	Value
Overall fuel saved	$6.5 \cdot 10^5$ kg
Overall fuel savings	4.2%
Average added flight time	9.9 min
Maximum added flight time	33.7 min
Formation flight usage rate	72%

Figure 7.21 shows the distribution of added flight times, obtained from using the limit of 10 minutes on the added flight time per aircraft per formation flight decision.

Compared to Figure 7.14, Figure 7.21 shows a significant decrease in additional flight times. Figure 7.16 and 7.22 illustrate a similar use of formations. Since the overall

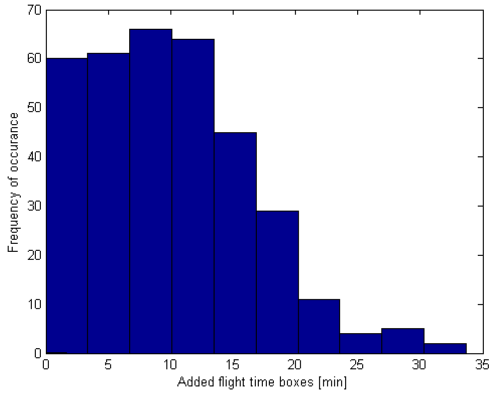


Figure 7.21: Added flight times when the 347 flights have a limited additional flight time per formation flight decision of 10 minutes.

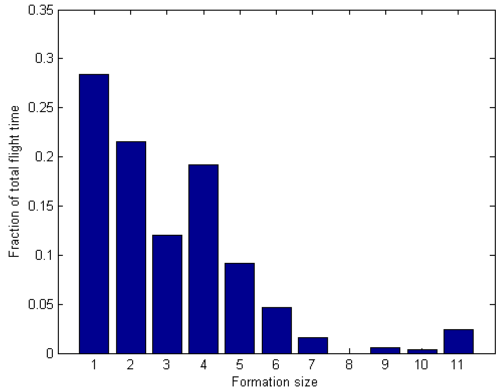


Figure 7.22: Formation size usage rate when the 347 flights have a limited additional flight time per formation flight decision of 10 minutes.

fuel savings have increased significantly, it is concluded that extending the objective formulation of the simulation to include a limit on the added flight time, has prevented the model from selecting formation flight options that are relatively less profitable.

7.5 Varying the communication range

In Section 3.4, it was noted that one of the main differences between a centralized and a decentralized approach is the maximum distance between aircraft at the moment of determining a possible formation flight strategy. In the core of a centralized approach, this distance is essentially unlimited. In the developed decentralized approach, the communication range is directly defined by the model. Choosing a certain communication range has direct operational consequences when considering encountered formation flight options and inter-aircraft communication. Accordingly, a study was performed on the impact of the communication range on the obtainable fuel savings. The communication range will be varied in the scenario of Section 7.2. Note that the communication range is herein defined as the radius of a circle around each aircraft. As soon as two circles touch of overlap, the aircraft may communicate if they are not otherwise restricted to do so.

Figure 7.23 shows the overall fuel savings that were obtained in each of the sixty simulations that were performed. From one simulation to the next, the communication range was increased by 10 km. At a communication range of 0 km, no fuel will be saved, since the communication algorithm is effectively deactivated by this setting.

As the communication range is increased, flights are allowed to communicate with other flight that are further away. Up to a communication range of 50 km, a steep increase in obtained overall fuel savings is recorded. This can be explained by the fact that the amount of encounters between aircraft increases significantly. This leads to more formation flight options being evaluated and accepted. For communication ranges of 50 to 120 km, the increase in obtained fuel savings continues with a smaller gradient. The model

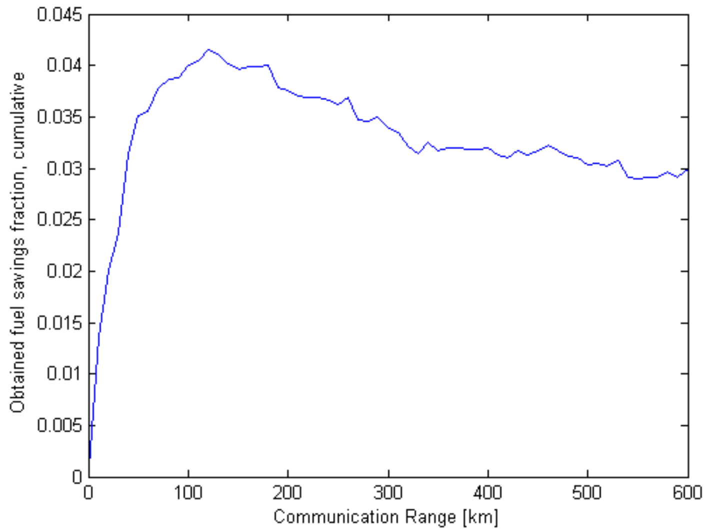


Figure 7.23: This plot shows the overall obtained fuel savings at different values of the communication range. Additional flight time was not limited when this data was produced.

still finds additional/more beneficial formation flight options. At a communication range of 120 km, the maximum obtainable fuel savings are found. These savings round up to 4.2% with respect to solo flights over all simulated routes. Increasing the communication range from 120 to 600 km reveals a gradual decrease in obtained fuel savings that appears to level off around 3.0%.

The gradual decrease in obtained savings can be explained by what happens when the communication range is increased. As the communication range is increased above 120 km, formation flight options that require relatively larger detours are encountered. Some of these will be accepted by the greedy communication algorithm, if they will result in cumulative fuel savings. Apparently, these decisions are sub-optimal, as the overall obtained fuel savings decrease when the communication range is increased above 120 km.

Section 7.4 showed that the overall obtainable fuel savings may be increased by introducing a limit on additional flight time. This inspired a simulation in which the communication range was varied in the scenario of Section 7.4, where the additional flight time per aircraft per formation flight decision was limited to 10 minutes. Figure 7.24 shows the obtained results with a green line. The blue line from Figure 7.23 is repeated to illustrate the difference in results.

Consider the green line in Figure 7.24, for increasing values of the communication range. Note how, after arriving in the 4.2% region, overall fuel savings are maintained as the communication range increases further. It is concluded that the limit on additional flight time counteracts the negative effect that encountering and accepting sub-optimal formation flight strategies has on the overall obtainable fuel savings.

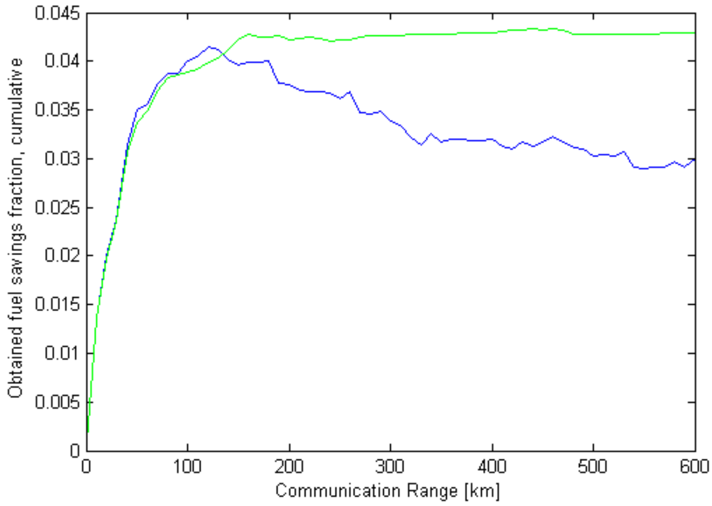


Figure 7.24: Plot of the overall obtained fuel savings at different communication ranges. The blue line shows the results of using only a fuel saving objective. The green line represents the results obtained from using a fuel saving objective in combination with a limit on the additional flight time.

7.6 Optimum model configuration

Referring to Figure 7.24, the optimum in obtained fuel savings, within this work, occurs at a communication range of 440 km. Including the limit on additional flight time of 10 minutes per aircraft per formation flight decision, one obtains the results given in Table 7.6. Note that, for a communication range between 200 and 600 km, the green line in Figure 7.24 is nearly horizontal. This indicates that the optimum at a communication range of 440 km is case specific.

Table 7.6: Results for 347 transatlantic flights, when the maximum allowed additional flight time per aircraft per formation flight decision was limited to 10 minutes. The communication range was set to 440km.

Parameter	Value
Overall fuel saved	$6.7 \cdot 10^5$ kg
Overall fuel savings	4.3%
Average added flight time	10.3 min
Maximum added flight time	38.6 min
Formation flight usage rate	73%

From Table 7.6, the additional flight time due to the use of formation flight has an average value of 10.3 min. With this extra ten minutes of flight time, each flight is predicted to save, on average, $6.7 \cdot 10^5/347 = 1.93$ tons of fuel.

In this work, the highest estimation of the total amount of saved fuel, which was not elaborated on as a result before now, amounts to 6.7 10^5 kg. As this number is heavily dependent on the assumptions in Section 4.2, the obtained fuel savings, with respect to solo flights that were simulated in equal conditions, are more valuable. This leads to the overall savings percentage of 4.3%. This value is conservative when it is compared to estimations from the literature. However, the assumptions in this work are also conservative.

To approach the assumptions used by Kent et al. [18], the developed model was configured to use a 20% fuel flow reduction for any trailing aircraft. The communication range and the limit on additional flight time remain unchanged with respect to the simulation that generated Table 7.6. The obtained estimation of overall fuel savings in the case study on 347 transatlantic flights increased to 9.7%. This work then still neglects any additional benefits for third and higher formation members, which is a significant factor that i.e. Kent et al. do include to arrive at their highest indicated fuel savings of just over 10% for 210 transatlantic flights. Xu et al. report a 7.7% reduction in overall fuel consumption in their case study on 150 transatlantic flights.
From Figure 7.25, it is noted that after doubling the assumed profitability of formation flight, no significant differences were observed in how the model formed formations. With respect to figures 7.16 and 7.22, the usage rate of formations of size 3 to 6 has slightly increased. This accounts for part of the additional overall fuel savings. However, it is concluded that the increase in obtained fuel savings mainly originates from the fact that similar formation flight strategies have become more profitable by assumption. This conclusion has been confirmed by the similarities in aircraft behaviour that have been encountered through visualisation of the flight data from the respective simulations.

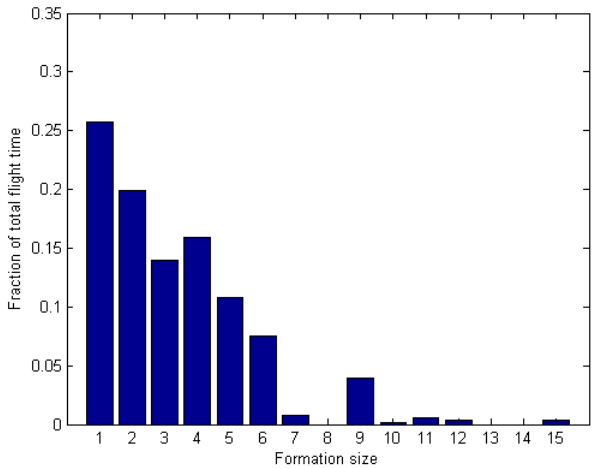


Figure 7.25: Formation size usage rate of the 347 flights in this case study, while assuming a 20% reduction in fuel flow for any trailing aircraft.

7.7 Interpretation of results

A model has been developed that illustrates a possible use of a decentralized approach to formation flight routing. In order to determine the added value of the performed research and the developed model, this section will reflect on the meaning of the obtained results with respect to the literature that was presented in Chapter 3.

Up to now, the literature provided several extensive studies on the possible fuel savings that could be obtained through the implementation of formation flight in civil aviation. However, many challenges remain considering the operational implementation of formation flight in civil aviation. Fuel savings are shown to be possible, but many risks and uncertainties are recognized, such as low model flexibility due to high computational requirements, sustaining fuel savings when flights are delayed, and the consideration of formations that comprise more than 3 aircraft.
The developed model addresses all of these challenges, while introducing only one new challenge that is not common to the research field: aircraft are assumed to be able to communicate with each other within their communication radius.

Considering the results of the case study that were presented in Section 7.2, this research has successfully demonstrated the value of a decentralized approach to formation flight routing. A model has been developed that, within the scope of this research, shows the following properties:

- Flexibility: Results can be obtained for many different route sets and the method is independent of network properties, such as origin and destination pairs. Also, within the defined communication scheme, flights are not restricted in their choice of formation flight partners
- Reliability: Formation flight is repeatedly and extensively opted for in various scenarios without performing any pre-flight planning
- Robustness: Delayed flights can still contribute to the overall objective of saving fuel through the use of formation flight
- Operational suitability: computational requirements are not expected to pose challenges in operation, as they can be performed on both a local scale and on local computing units, i.e. onboard of each aircraft

The formation flight route sets that result from the developed model are comparable to those generated by the centralized approaches in the literature. However, they are more efficiently obtained and there is no limit on formation size. From an operational point of view, it is valuable to compare the formation flight routes that result from different approaches. Many routes obtained in this research are very similar to those in the literature. Additionally, the developed model includes more routing options. Flights may decide to form a formation at any time and it has been shown that, given the right conditions, they tend to exploit this possibility. Multiple consecutive formation flights are also possible, which has not been seen in the literature that was consulted during this thesis.

Since the actual obtainable fuel savings from formation flight are still a topic of current research, significant assumptions are required to model reductions in fuel flow. Because of this, findings on actual fuel savings are not the primary result of this work.

The developed model conservatively estimates the obtainable fuel savings from formation flight. As this number is heavily reliant on the governing assumptions, one should keep this in mind when considering its actual value. With respect to results from the literature, an in-the-order-of comparison is more appropriate. When the obtained estimations of overall fuel savings are compared to estimations from the literature, the following is noted:

In the developed model:

- Fuel consumption reductions are based on flight tests that were performed without specially developed equipment
- Departure times are not flexible
- A variable fuel consumption penalty is included for flying at a sub-optimal speed (for synchronization purposes)
- Additional benefits from larger formations are not considered (i.e. formation member 3 profiting more than formation member 2)
- Flights that never join a formation are included in the calculation of overall fuel savings

While not all of these model properties are exclusive to this work, they do form a conservative set relative to the literature. It is noted that, while synchronization through departure time variation will be more fuel efficient, other costs will be incurred when departure times are required to be flexible.

In the transatlantic case study, the overall additional flight time was reduced significantly by using an upper limit on the additional flight time, for an individual aircraft, resulting from a single formation flight decision. It was demonstrated that the same measure enabled the model to maintain obtainable fuel savings for increasing values of the communication range. By restricting the greedy communication algorithm with respect to acceptable additional flight time, it was forced to find more time efficient formation flight routes. Since additional flight time is strongly related to additional fuel burn, the use of more time efficient routes led to an increase in fuel efficiency as well. This example demonstrates the ability of a decentralized approach to increase its performance by extending its local objective formulation. Note that the second objective of limited additional flight time barely affected the required calculation time.

The transatlantic case study illustrates that a significant amount of the total flight time may be spent in formations that comprise 2 to 6 aircraft. Evaluating a similar scenario in a centralized way would result in a tremendous amount of required calculations, since the number of formation flight options increases exponentially with an increase in the size of the largest formation that may be used. The decentralized approach in this research was able to generate a possible set of formation flight routes within minutes, on a standard university computer.

The highest obtained estimation of overall fuel savings is 4.3% in the transatlantic case study. Comparison with leading sources by Kent et al.[18] and Xu et al.[37], shows that this percentage is about half of what has been reported for centralized approaches. However, the assumptions of this work have been, on purpose, formulated conservatively, in order to increase the credibility of any positive findings. The comparison with findings of Xu et al. can only be made indirectly, as they used both aerodynamic models and variable airspeeds to obtain estimations for formation specific induced drag reductions.

Comparison with findings of Kent et al. is possible to a greater extent, since the work of Kent et al. was the leading sources for several elements of this thesis. Referring to Section 3.1.2, Kent et al. assume that in a formation of two aircraft, the fuel flow reduction for the trailing aircraft is twice as high as the value that is used in this work. Additionally, they assume increasing fuel flow reductions for additional formation members.
Accordingly, the developed model was configured to better represent the assumptions of Kent et al. with respect to formation flight benefits. The obtained estimation of overall fuel savings increased to 9.7%. This figure is in the same order of magnitude as the fuel consumption reductions that are estimated by centralized approaches in the literature.

The synchronization segments that are required to get aircraft in a formation, are a burden on the overall fuel savings obtainable with a decentralized approach. The penalties on the fuel consumption can be interpreted as the price to pay for not planning the formation flight up front. Changing the routing system from a distance minimization problem to a fuel minimization problem may help to reduce these synchronization penalties. However, elaborate flight plan adjustments will always be necessary to establish formations, if formation flight is not anticipated.

To get the most out of formation flight, one may consider a combination of a centralized and a decentralized approach. For example, as was suggested in Section 3.4, one could use a centralized approach to optimize departure schedules for formation flight potential, i.e. by maximizing the predicted amount of encounters between flights. Accordingly, a decentralized approach, like the one developed in this work, could be used to turn the generated potential into actual fuel savings.

Given the assumptions on which this work is based, the presented results are still theoretical. However, this research has shown that a decentralized approach has significant potential when it comes to introducing formation flight in current operations. Formation flight usage rates up to 74% of overall flight time were recorded and flexible use was made of formations comprising up to 15 aircraft.

Chapter 8

Conclusions and recommendations

Based on the work that has been done, several statements can be formulated considering the value of a decentralized approach to formation flight routing. These statements are presented in this chapter as part of the conclusions of this work. During the development of the model, possible additions and extensions to the work were discovered. These are given as recommendations at the end of this chapter.

8.1 Conclusions

The governing objective of the research area is to save fuel through the use of formation flight in civil aviation. This thesis is devoted to evaluating the potential of one particular approach to formation flight routing in civil aviation: a fully decentralized approach.

The performed simulations contain elements that have not been used in the research area. The research area is here confined to the consulted literature presented in Chapter 3. From the interpretation of the obtained results, see Section 7.7, and the process of developing a decentralized approach, a number of conclusions can be presented. These conclusions contain properties of the developed decentralized approach that could contribute to the governing objective of saving fuel in civil aviation through the use of formation flight.

- The developed model shows that a decentralized approach can obtain results, among which are route sets and fuel savings estimations, that are comparable to centralized approaches from the literature
- The developed model shows that a decentralized approach has many operational advantages with respect to a centralized approach, such as real-time flight path optimization, low computing requirements and resilience
- The route sets that have been obtained show many similarities to those in the literature. However, the developed model:

  - obtains these route sets more efficiently
  - allows formations to grow to any appropriate size
  - allows aircraft to fly multiple consecutive formation segments
  - enables randomly delayed flights to join a formation, thereby increasing the reliability of formation flight usage rate estimations
  - makes formation flight decisions based only on information that can, within reason, be assumed to be available in flight

- Compared to the literature, a high formation flight usage rate was found whilst using conservative assumptions on formation flight benefits as well as a greedy savings collection algorithm
- The model implies that any findings on the benefits of larger formations (> 3 aircraft) would have a significant effect on the overall achievable fuel burn reduction through the use of formation flight in civil aviation
- The number of considered flights is significantly higher in this work than in the considered literature. Given the sustained calculational efficiency of the developed model, the large-scale potential of a decentralized approach is underlined

The fuel savings in this work are collected through a greedy communication algorithm. It has been demonstrated that this leads to sub-optimality in several areas. Still, the results of this work indicate that a decentralized approach to formation flight routing has significant potential. This potential is bound to increase with the development of the communication algorithm.

In the transatlantic case study, the highest estimated overall fuel savings percentage was 4.3%. If the assumptions in the different works are considered, this number is in the same order of magnitude as the the overall savings presented by leading literary sources. However, the impact of the assumptions of both this work and the literature, make this 4.3% a rough indication. The more general results of this thesis are more valuable. While using conservative assumptions on the benefits of formation flight, the developed decentralized approach achieved a 73% usage rate of formation flight in the presented transatlantic case study. The use of consecutive formation flight segments contributes significantly to this formation flight usage rate.

This work has shown that a decentralized approach to introducing formation flight in civil aviation is operationally attractive properties. It has been demonstrated that a high formation flight usage rate can be obtained without any pre-flight planning. The in-flight calculations that the developed method requires are expected to be manageable in real-time. By eliminating the need for pre-flight planning, delayed flights do not pose any challenges to the fuel saving objective. In fact, their delay may even lead to additional fuel savings, since current flight schedules are not optimized for formation flight implementation.

Since the flexibility and diversity with which this work introduces formation flight has not been encountered in the research area, the fuel saving potential for civil aviation of a decentralized approach to formation flight routing is considered to be significant.

8.2 Recommendations

During this project, many findings suggested that the highest fuel saving potential of the use of formation flight in civil aviation is not to be found by using just a centralized or a decentralized approach. Instead, it is expected that a combination of the two allows for compensating the weaknesses of the one with the strengths of the other. For example, the difficulty in handling delayed flights, experienced by centralized approaches, can be dealt with in a decentralized way. Accordingly, the speed related penalties on synchronization segments, incurred by decentralized approaches, can be remedied by introducing a form of pre-flight planning, or more complex communication methods that could be developed as local centralized approaches. After conducting the literature study, it was already expected that centralized and decentralized approaches could benefit from each other, provided that a decentralized approach could be proven to have significant potential for implementing formation flight. This thesis shows the potential of a decentralized approach. Therefore, it is recommended to study a combination of a centralized and a decentralized approach to the implementation of formation flight in civil aviation.

Additionally, several more specific considerations are recommended for further research:

- Expand the model such that aircraft can communicate with multiple potential formation flight partners at the same time. Accordingly, they could find more beneficial formation flight strategies. Essentially, local centralized approaches, using only locally available information, are suggested
- Use the current gross weights of aircraft in the selection of formation flight strategies. Heavier aircraft are expected to benefit more from formation flight, in terms of absolute fuel savings
- The current model uses a routing method based on minimizing the distance that has to be flown. Incorporating a routing method that obtains formation flight routes through directly minimizing the predicted overall fuel consumption may bring additional benefits
- Study the effect of variable airline cooperation in formation flight implementation
- Investigate fuel optimal synchronization
- Investigate ways to optimize formation organization; which aircraft flies in which formation position and why
- Elaborate on the formulation of the simulation objective and additional information that could be used to decide if a formation flight strategy is accepted or rejected
- Study the effects of the increased uncertainty in overall flight time
- Study the effects of considering a non-homogeneous fleet composition
- Study the effect of uncertain and varying traffic density

These recommendations will contribute to finding formation flight routes that are more efficient and more realistic.

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Appendix A

Breguet range equation

In this appendix, the derivation of the variation on the Breguet range equation used by Kent et al.[18] is given. The presented derivation is literally taken from Kent et al.[18], since the resulting variation of the Breguet range equation is used throughout this thesis. Note that two presumed typing errors were removed. Kent et al. retrieved the derivation from Anderson [3].

Let dW denote a change in weight of an aircraft due to fuel consumption over an increment of time dt, then given a thrust specific fuel consumption factor, C_t, and the thrust available TA, the following relation holds

$$dW = -C_t T_A dt \tag{A.1}$$

which rearranged with respect to dt is

$$dt = \frac{-dW}{C_t T_A.} \tag{A.2}$$

For the incremental distance dr travelled by the aircraft, over an increment of time dt, equation A.2 is multiplied by a stream-free velocity V_∞ so

$$dr = V_\infty dt = \frac{-V_\infty dW}{C_t T_A}. \tag{A.3}$$

Rearranging equation A.3 leads to the rate of fuel burnt per unit of distance

$$\frac{dW}{dr} = -\frac{C_t T_A}{V_\infty} \tag{A.4}$$

Assuming steady level flight, then thrust available, T_A, should equal thrust required, T_R, and for a given coefficient of lift, C_L, and drag, C_D, then $T_A = T_R = \mathrm{W}/(C_L/C_D)$ (which depends on W). The W required to evaluate this equation is determined after a certain flight distance, by following through with this derivation enables its calculation. First integrate equation A.3 between the limits s = 0 (when W = W_0, the initial weight) and s = R (when W = W_1, the final weight),

$$R = \int_0^R dr = \int_{W_0}^{W_1} \frac{V_\infty dW}{C_t T_A}, \tag{A.5}$$

$$R = \int_{W_0}^{W_1} \frac{V_\infty C_L dW}{C_t C_D W} \tag{A.6}$$

Using the definition that for a given density ρ_∞, $V_\infty = \sqrt{\frac{2W}{\rho_\infty S C_L}}$ results in

$$R = \int_{W_1}^{W_0} \sqrt{\frac{2}{\rho_\infty S}} \frac{{C_L}^{\frac{1}{2}} dW}{C_t C_D W^{\frac{1}{2}}}. \tag{A.7}$$

Assuming constant C_t, C_L, C_D and density ρ_∞ (at a constant altitude) then

$$R = \sqrt{\frac{2}{\rho_\infty S}} \frac{{C_L}^{\frac{1}{2}}}{C_t C_D} \int_{W_1}^{W_0} \frac{dW}{W^{\frac{1}{2}}}, \tag{A.8}$$

$$R = \sqrt{\frac{2}{\rho_\infty S}} \frac{{C_L}^{\frac{1}{2}}}{C_t C_D} \left(2W_0^{\frac{1}{2}} - 2W_1^{\frac{1}{2}}\right), \tag{A.9}$$

completing the derivation of the Breguet range equation. Setting $M = \sqrt{\frac{2}{\rho_\infty S}} \frac{{C_L}^{\frac{1}{2}}}{C_t C_D}$ to be the contribution of the constant terms then

$$R = M \left(2W_0^{\frac{1}{2}} - 2W_1^{\frac{1}{2}}\right), \tag{A.10}$$

$$W_1 = \left(\frac{M\sqrt{W_0} - R}{M}\right)^2 = \left(\sqrt{W_0} - \frac{R}{M}\right)^2. \tag{A.11}$$

M is assumed to be a non-zero constant, so given a distance R and initial weight W_0 an aircrafts weight can be calculated at that point. Equations A.4 and A.11 enable an estimation of fuel burn rates after any distance (up to the range of the aircraft). Equation A.11 requires knowledge of an initial weight in order to estimate any en-route weights, therefore it is necessary to also calculate the required fuel for the entire journey.

Section 4.3 elaborates on how the starting weight of an aircraft is determined in this thesis. Note that this is done differently than in the work of Kent et al.[18], as a different approach towards the implementation of formation flight is used.