Graphs 4

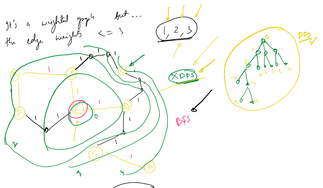
Wednesday, 16 January 2023 7:03 PM

Q Given a graph (unweighted). Find the min dist from a source node to all other nodes

BFS

TC : O(V+E)

→ It's a weighted graph but... the edge weights <= 3



G (V, E)

V' = V + 2E

E' = 3E

BFS → O(V'+E')

O(V+I+2+3E)

O(V+5E)

O(V+E)!

Q No limit on weights

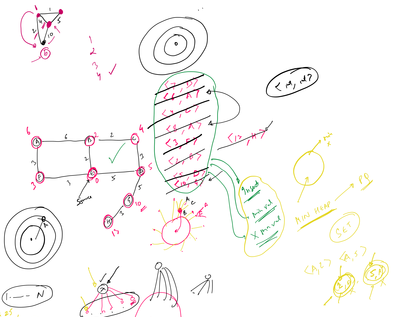
1 ≤ U ≤ 10^9

Dijkstra's Algorithm → [Weighted graph (+ve)]

(Single source Shortest path algorithm).

Input: Weighted graph, Source Node

Output:



TC:

1 → N

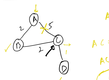
TC: O(V+E log E)

SC: O(V+E)

$V, E \leftarrow 10^5$

$10^5 + 10^5 \cdot 17$

$\sim 1.7 \times 10^6$



AC = 5

AC = min(AC, AB+BC)

AC = min(5, 4+2)

AC = 5

AD = AC + CD

4 + 1

AD = 5

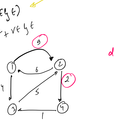
Floyd Warshall Algorithm → [-ve weights]

[All pair shortest path]

E >> V

V(V+E log E)

$V^2 + VE \log E$



d(1, 4) via 3

= d(1,3) + d(3,4)

= -4 + 7

= 3

d(1,4) via 2

= d(1,2) + d(2,4)

= 2 + 9 = 11

A_0

	1	2	3	4
1	0	8	-4	∞
2	4	0	∞	2
3	∞	5	0	∞
4	∞	∞	1	0

A_1

	1	2	3	4
1	0	8	-4	∞
2	4	0	∞	2
3	∞	5	0	∞
4	∞	∞	1	0

A_2

	1	2	3	4
1	0	8	-4	10
2	4	0	∞	2
3	9	5	0	7
4	∞	∞	1	0

A_3

A_4

All pair shortest path Problem!

$d_1(1,3) = d_0(1,1) + d_0(1,3)$

min($d_0(1,3)$)

= (∞ + -4)

= -4

$d_2(1,3) = \min(d_1(1,3), d_1(1,2) + d_1(2,3))$

min(8 + 2, -4)

10, -4

$d^k[i][j] = \min(d^{k-1}[i][j], d^{k-1}[i][k] + d^{k-1}[k][j])$

TC: O(V³)

SC: O(V²)

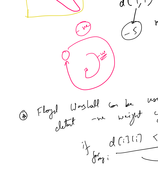
f(k: 1 → n)

f(i: 1 → n)

f(j: 1 → n)

dp[i][j] =

-ve cycle

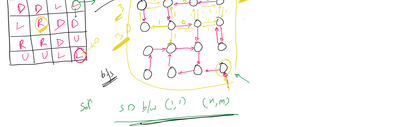


d(1,1) = (d(1,2) + d(2,1)) min

d(1,1)

Floyd Warshall can be used to detect -ve weight cycles

if d_{ii}: d[i][i] < 0



Sol: SD b/w (1,1) (n,m)