Solutions for CCE : AIMIA Homework 1 Solutions

October 1, 2022

1   Q1

Use the given expert annotations (infection masks) and categorize the CT scans into three categories: Normal, Mild, and Severe. Report the dataset distribution (or count) of above mentioned classes. Also, display a sample for each category's infection mask and infection rate. [Hint : For a given slice, Normal : infection rate is 0, Mild : infection rate is (0%, 40%] and Severe : infection rate is > 40%].

Solution:-

The classification of the CT infection masks in to Normal, Mild, and Severe categories was done by calulating the ratio of total number of pixels occupied by the infected tissue and the total number of pixles covered by the segmented lung region (i.e, covered by both the healthy tissue and infected tissue) as shown in figure 1.

- The total number of infection masks presented in the dataset are equal to 3554

As per the technique discussed above(figure.1), the CT slices have been classified in to three categories i.e.,Severe, Mild, and Normal .

The distribution of classes are as follows:

- Severe infection was found in 159 slices.

- Mild infection was found in 1954 slices.

- There were 1441 normal slices.

    The infection rate of each class is as follows.

- The total Severe infection rate is 5%

- The total Mild infection rate is 55%
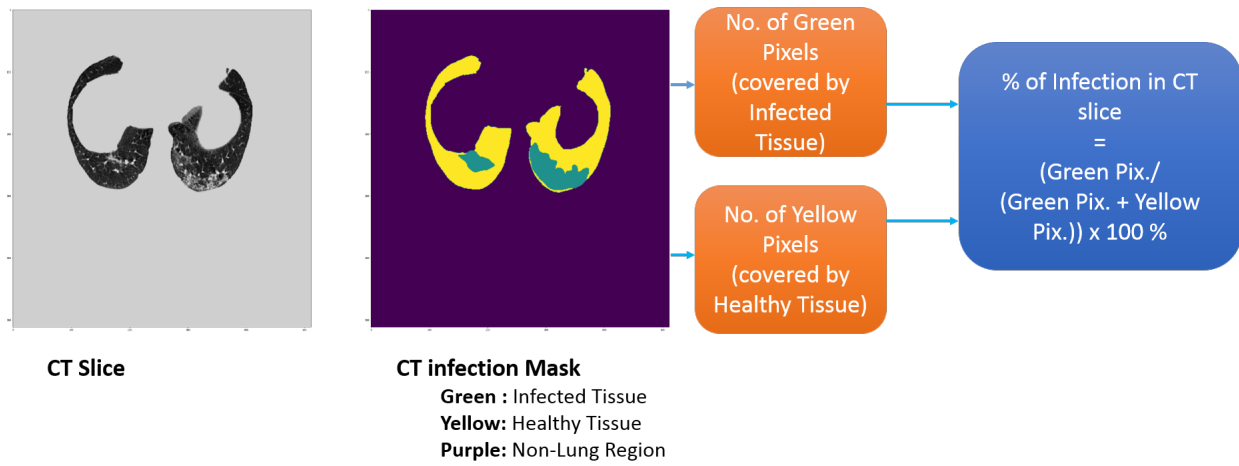
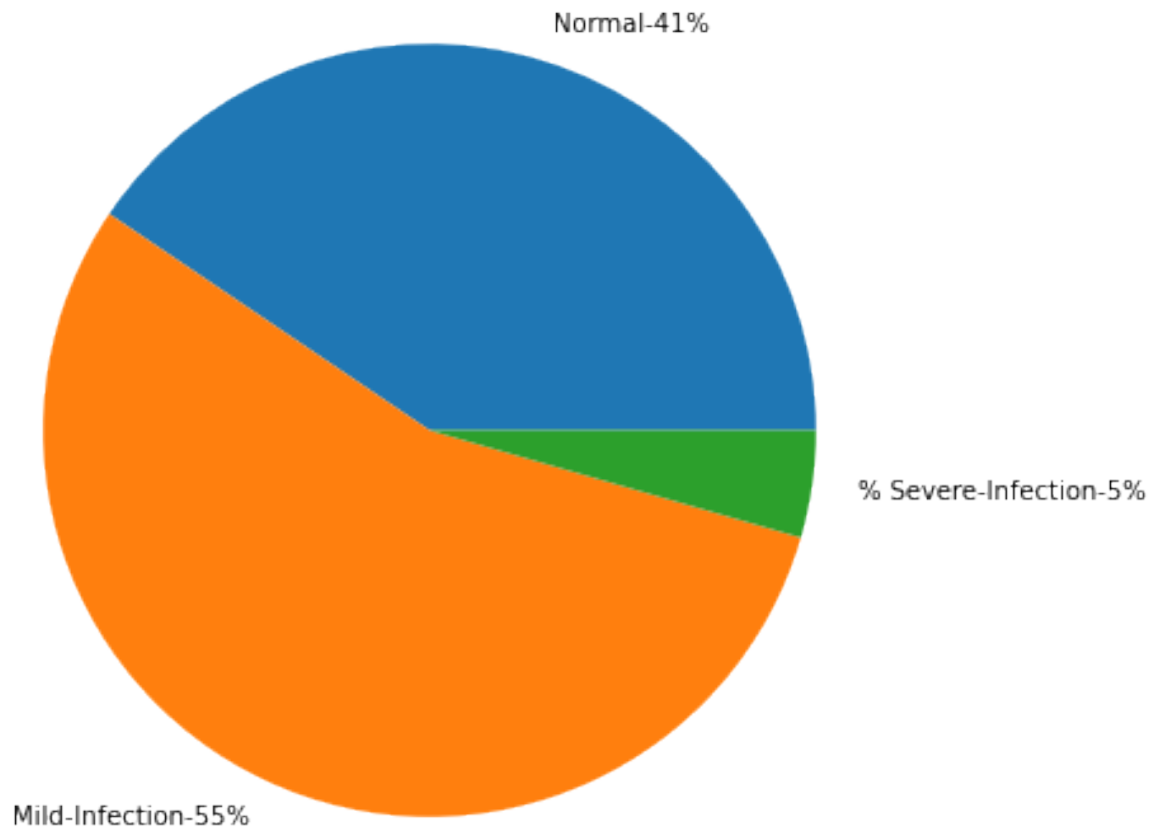- The total Normal slices rate is 41%

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CT Slice

CT infection Mask
Green : Infected Tissue
Yellow: Healthy Tissue
Purple: Non-Lung Region

No. of Green Pixels (covered by Infected Tissue)

No. of Yellow Pixels (covered by Healthy Tissue)

% of Infection in CT slice = (Green Pix./ (Green Pix. + Yellow Pix.)) x 100 %

Figure 1: CT infection Rate calculation

Normal-41%

% Severe-Infection-5%

Mild-Infection-55%
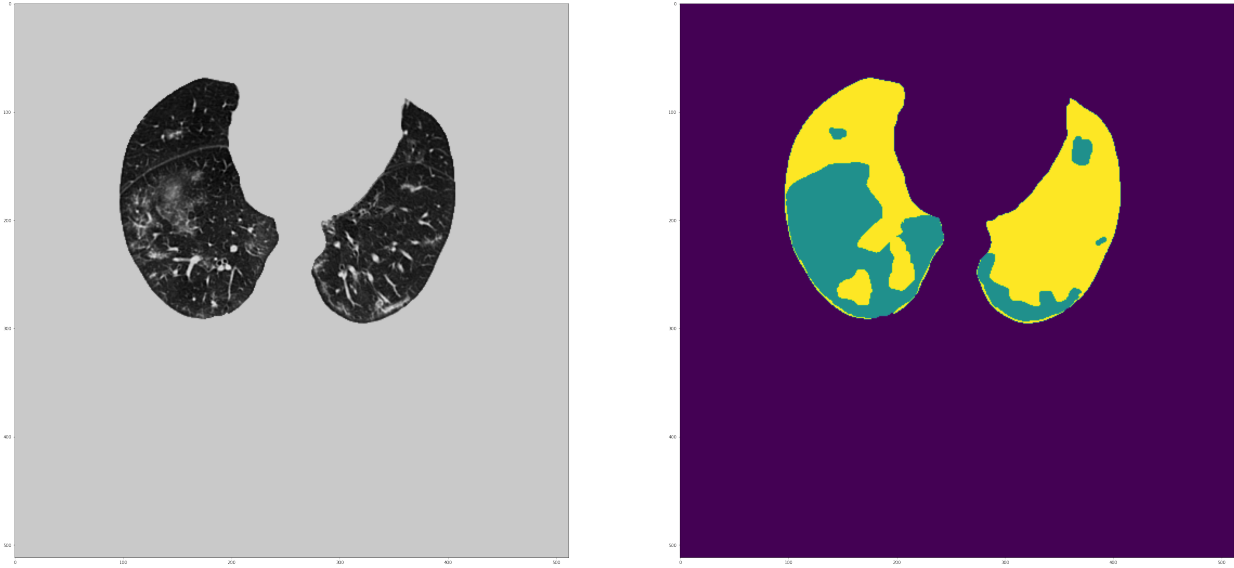
Figure 2: Distribution of infection rate in (%)

Figure 3: The Infection rate is 74.50122230108036 percent, it is a severly infected CT slice!!!!
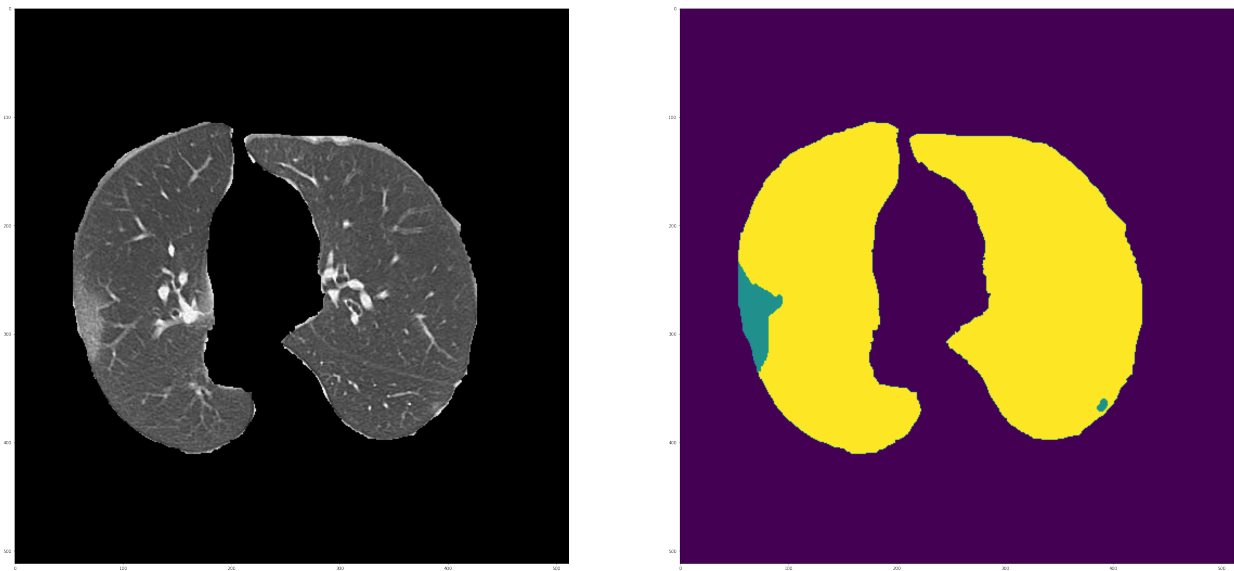

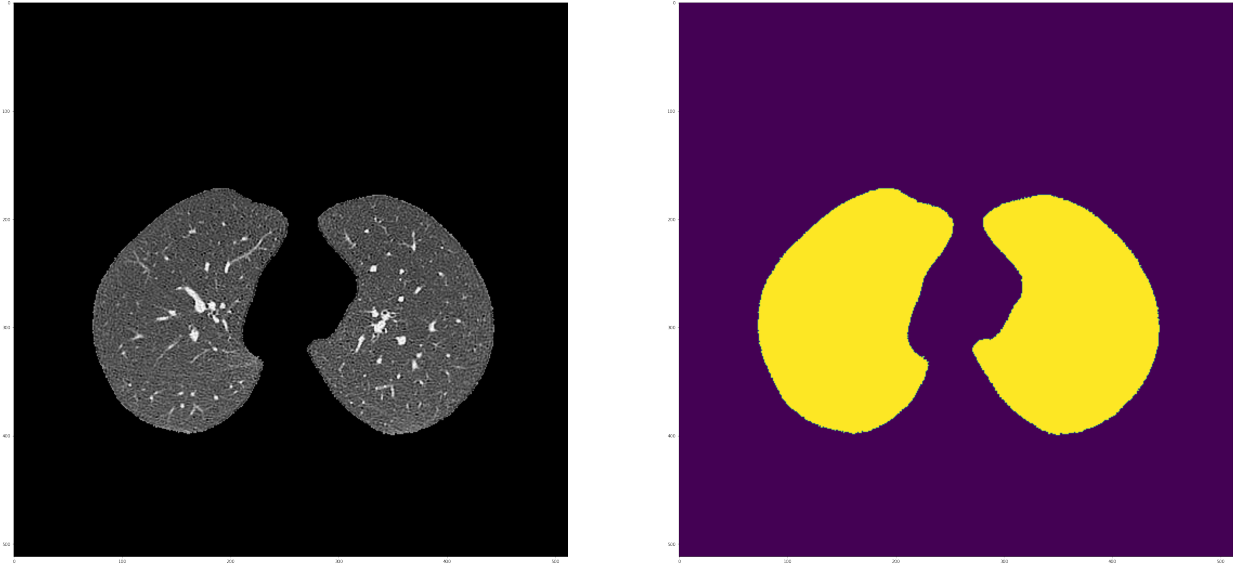Figure 4: The Infection rate is 2.1793374997138475 percent, it is a mildly infected CT slice!!

Figure 5: The Infection rate is 0.0 percent, it is a normal CT slice

2 Q2

Apply the K-Means algorithm on the given CT Scans for segmenting the infected and healthy regions of the lung tissue. Compare the predicted mask from K-Means with expert annotation and report the averaged dice score, sensitivity, specificity, and accuracy for the normal and infected regions. Also, display two sample slices along with the expert infection mask and predicted mask.

Solution:-

The k-means is an unsupervised algorithm and it is used to segment the region of interest from the background. It groups the given data into K-clusters. Mostly the algorithm is used with unlabeled data. The K-means iteratively assigns the data points to one of the K clusters based on how near the point is to the cluster centroid. The sequence of steps in k-means algorithms are as follows.

Assume that, we have input data points

$$x_1, x_2, x_3, ..., x_n$$

and the number of clusters needed denoted as K.

Step1 : Pick K points as the initial centroids, either randomly or the first K.

Step2 : Calculate the Euclidean distance of each point in the dataset with the identified cluster centroids

Step3: Assign each data point to the closest centroid based on the distance obtained from step 2.

Step4 : Find the new centroid by taking the average of the points in each cluster.

Step5: Repeat the steps 2, 3 and 4 till the centroids don't change or for a fixed number of iterations.

The Euclidean distance between two points in space

$$x(x_1, x_2)$$

and

$$y(y_1, y_2)$$

is given by

$$d(x, y) = \sqrt{(y_1 - x_1)^2 + (y_2 - x_2)^2}$$

Assigning each point to the nearest cluster:
If each cluster centroid is denoted by ci, then each data point x is assigned to a cluster based on

$$arg \min_{c_i \subset C} dist(c_i, x)^2$$

Here dist() is the euclidean distance
Finding the new centroid from the clustered group of points:

$$c_i = |\frac{1}{S_i}| \sum_{x_i \subset S_i} x_i$$

[Refer 8] where Si is the set of all points assigned to the ith cluster
Evaluation Metrics:

The performance of the k-means segmentation can be evaluated by calculating averaged dice score, sensitivity, specificity, and accuracy for the normal and infected regions.

Sensitivity :
Sensitivity of a classifier is the ratio between how much were correctly identified as positive to how much were actually positive.

$$Sensitivity = \frac{TP}{(FN + TP)}$$

Specificity :
Specificity of a classifier is the ratio between how much were correctly classified as negative to how much was actually negative.

$$Specificity = \frac{TN}{(FP + TN)}$$

Precision:
How much were correctly classified as positive out of all positives.

$$Precision = \frac{TP}{(TP + FP)}$$

Recall:
Recall and sensitivity are one and the same.

$$Recall = \frac{TP}{(FN + TP)}$$

The harmonic mean of precision and recall gives a score call f1 score which is a measure of performance of the model's classification ability.

$$F1score = \frac{2 * (precision * recall)}{(precision + recall)}$$

F1 score is considered a better indicator of the classifier's performance than the regular accuracy measure.

The k-means segmented and the expert infection masks have been depicted in Figure 6 and Figure 7.

The Average Accuracy, Sensitivity, Specificity, and Avg Dice Socre obtained for the 3554 CT slices is as given below

Average Sensitivity is: 0.4917266328796307

Average Specificity is: 0.4707865949677128

Average Accuracy(%): 47.322597631898546

Average Dice score: 0.11304791992172793

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3 Q3

Reconstruct the given CT Scans from limited angle Sinograms (4x and 8x) and report the averaged quality metrics PSNR and SSIM for the limited angle reconstructions. Also, display a sample slice along with 4x and 8x reconstruction. Repeat Q2 on the limited angle CT reconstructed data.

The CT reconstruction was carried out by employing Radon transform and Filtered Back-Projection.
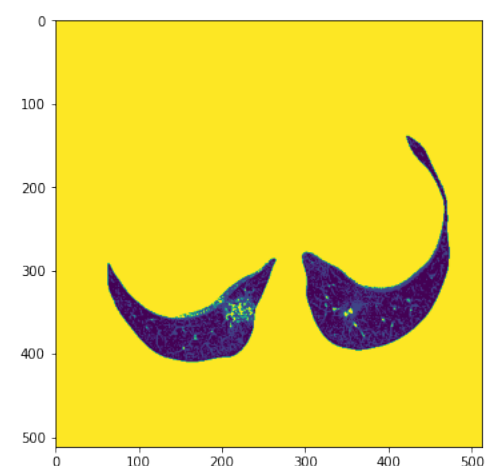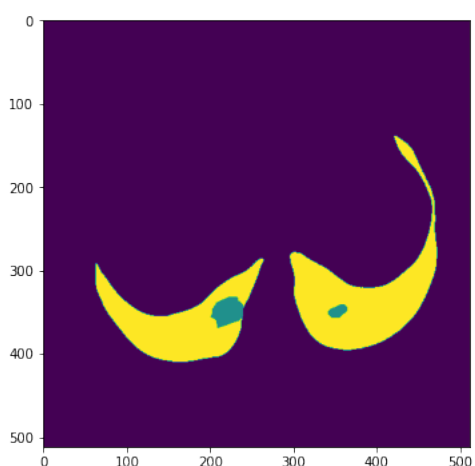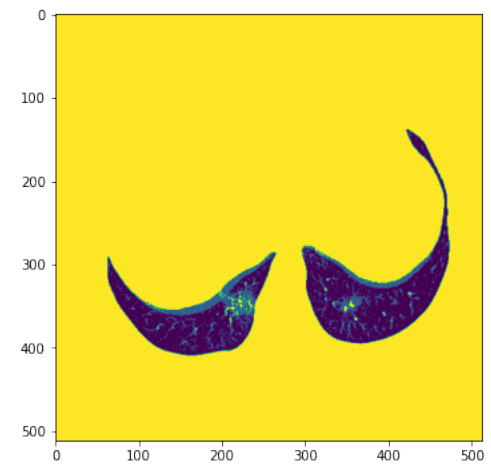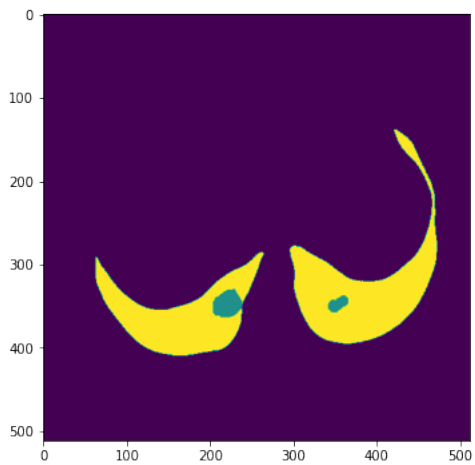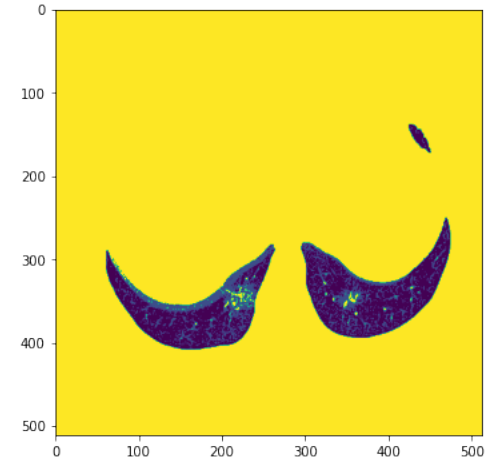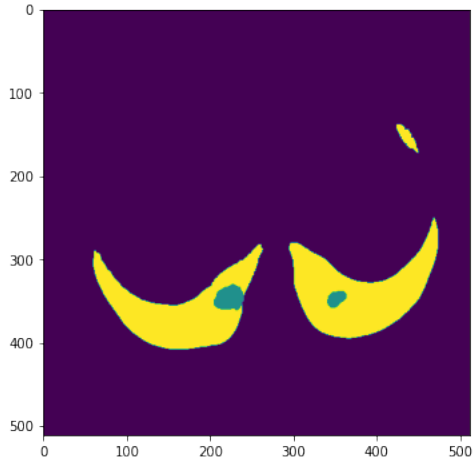
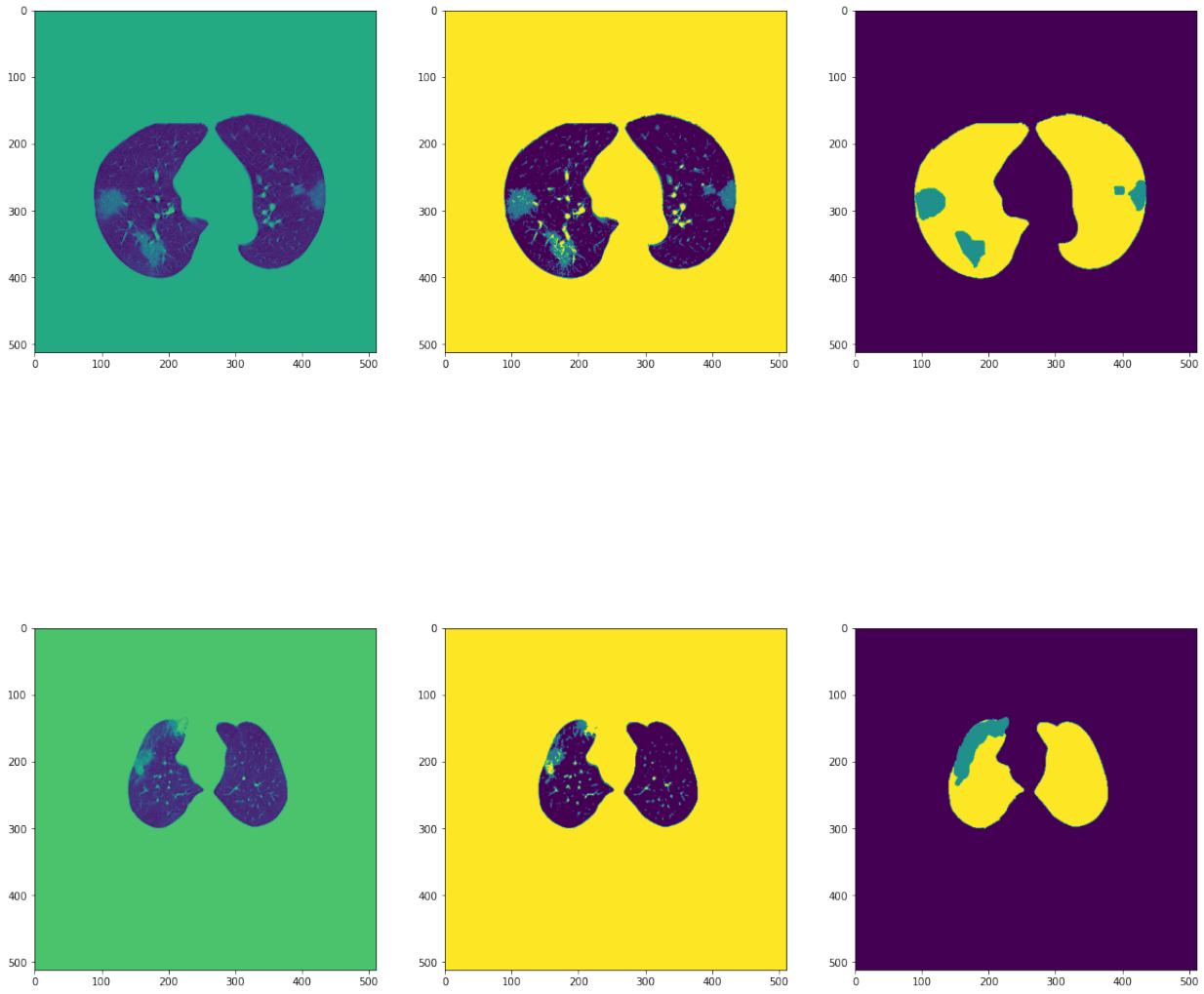Figure 6: The expert infection mask vs. the k-means segmentation mask

Figure 7: The CT slice vs. the k-means segmentation Slice Vs.Infection Mask

The Radon transform of an image represented by the function f(x,y) can be defined as a series of line integrals through f(x,y) at different offsets from the origin. This is mathematically as follows

$$R(p, \tau) = \int_{-\infty}^{\infty} f(x, px + \tau) dx$$

, where where p and tau are the slope and intercepts of the line. (Refer link 9)

A more directly applicable form of the transform can be defined by using a delta function:

$$R(r, \theta) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \delta(x cos\theta + y sin\theta - r) dx dy$$

(Refer link 9)

where theta is the angle of the line, and r is the perpendicular offset of the line.

calculation of Inverse Radon Transform using Filtered Back Projection (FBP) method
To reconstruct the image from the sinogram, the inverse Radon transform is applied.
The projection phase is very similar to the Radon transform.

$$f(x, y) = \int_{0}^{\pi} f'(x cos\theta + y sin\theta, \theta)$$

, where f' is the filtered data. (Refer link 9)

A common discrete approximation of above equation is given below.

$$f(x_m, y_m) \approx \triangle\theta \sum_{t=0}^{T-1} f'(x_m cos\theta_t + y_n sin\theta_t, \theta_t)$$

(Refer link 9)

This equation can be used to determine the pixel values at a given point. As more projections are added, the quality of the reconstruction will increase accordingly.

Evaluation Metrics : PSNR, SSIM

PSNR (Peak Signal-to-Noise Ratio)

PSNR is the ratio between the maximum possible power of a signal and the power of corrupting noise that affects the accuracy of its representation.

$$PSNR = 20 log_{10}(\frac{MAX_f}{\sqrt{(MSE)}})$$

[Refer 4, 5]

where

$$MSE = \frac{1}{mn} \sum_0^{m-1} \sum_0^{n-1} ||f(i,j) - g(i,j)||^2$$

[Refer 4, 5]
SSIM: Structural similarity

SSIM is a perception-based model that considers image degradation as perceived change in structural information, while also incorporating important perceptual phenomena, including both luminance masking and contrast masking.
Structural information is the idea that the pixels have strong inter-dependencies particualrly when they are spatially close.

These dependencies carry important information about the structure of the objects in the visiblel scene.

The mathematical represention of the SSIM is given by the below equation

$$SSIM(x,y) = \frac{((2\mu_x\mu_y) + c_1)(2\sigma + c_2)}{(\mu_x^2 + \mu_y^2 + c_1)(\sigma_x^2 + \sigma_y^2 + c_2)}$$

[Refer 4, 5]
The PSNR and SSIM for the full angle Reconstruction of the CT slice (Figure 8) is as given below

PSNR = 12.42797968024357 dB
SSIM = 0.6532645623728417

The average PSNR and SSIM for the 4X Reconstruction of the CT slices (ex. Figure 9) is as given below

PSNR = 6.0560631 dB.
SSIM = 0.16157094

The average PSNR and SSIM for the 8X Reconstruction of the CT slices (ex. Figure 10) is as given below

PSNR = 6.05618027 dB.
SSIM = 0.16150934

The kmeans algorithm has been applied on the 8X Reconstructed slice(ex. Figure 11).The PSNR and SSIM are as follows:
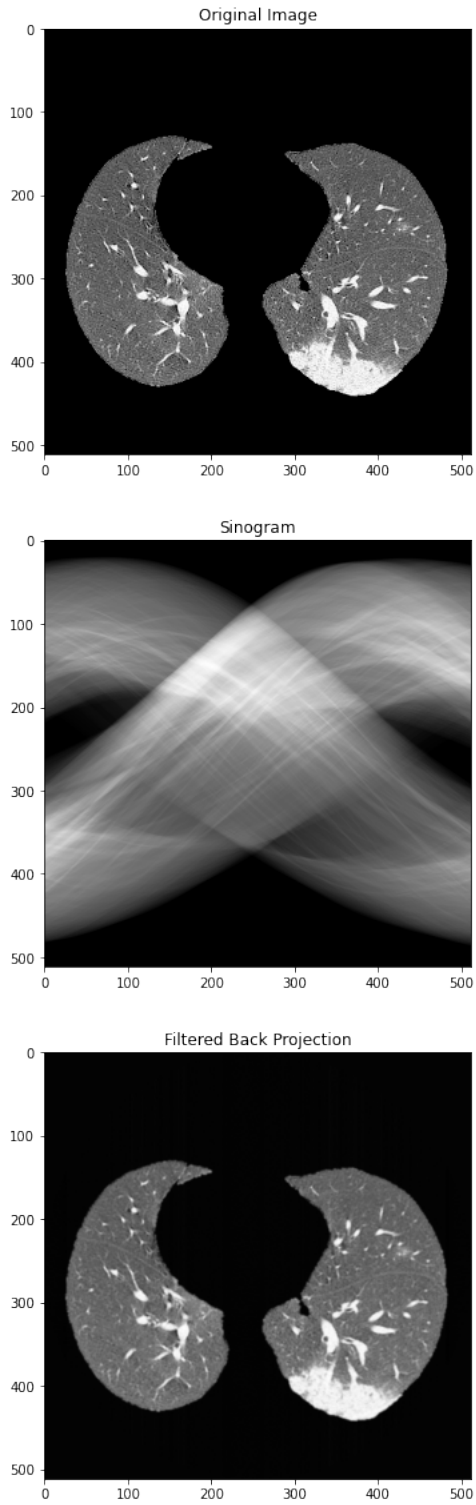
PSNR = 2.56 dB
SSIM = 0.0018

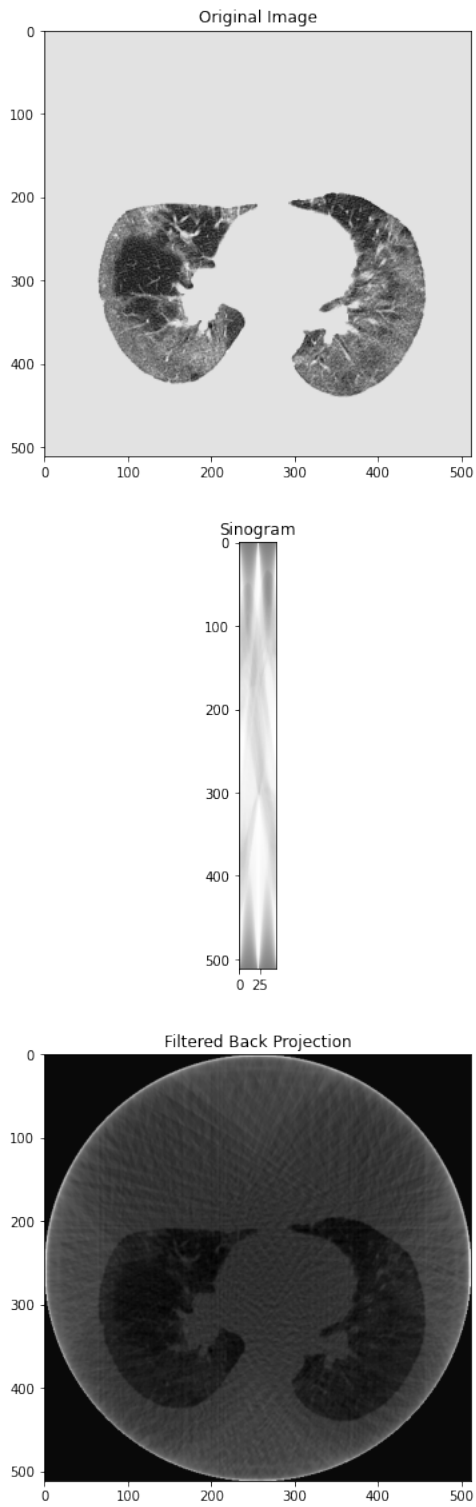Figure 8: The full angle reconstructed CT slice with PSNR = 12.43 dB. and SSIM = 0.65
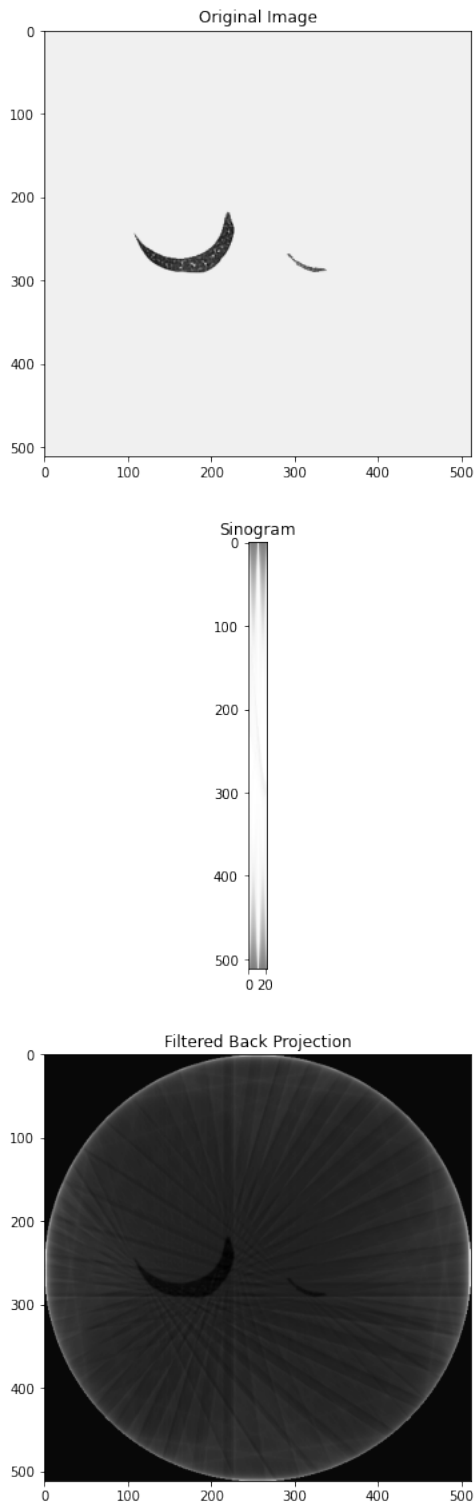
Figure 9: The 4X reconstructed CT slice with PSNR = 6.07 dB. and SSIM = 0.16

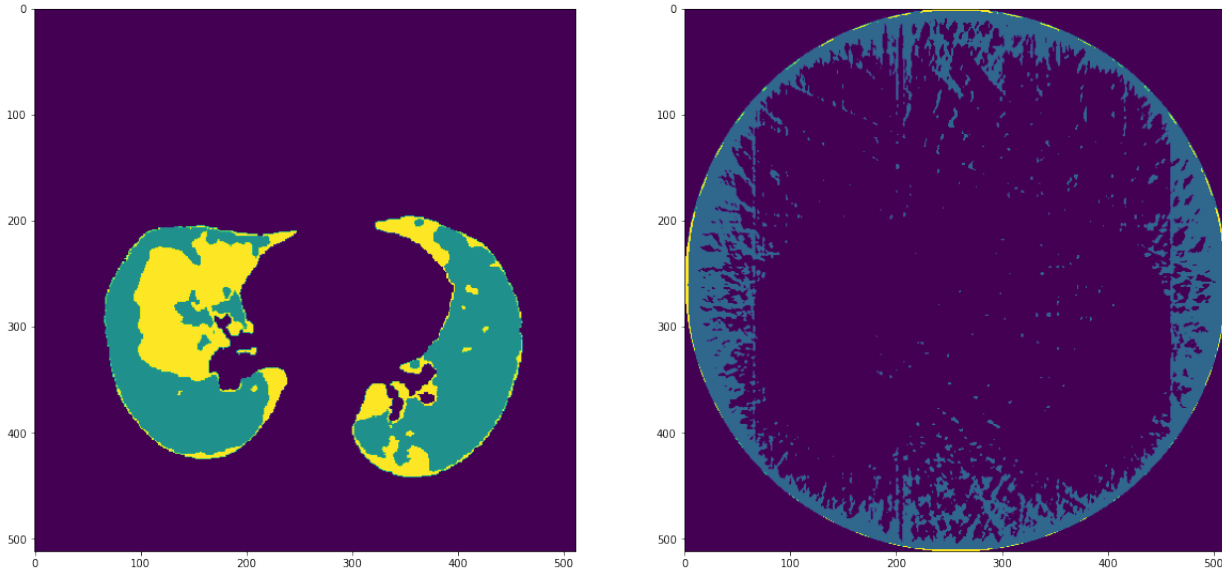Figure 10: The 8X reconstructed CT slice with PSNR = 6.07 dB. and SSIM = 0.16

Figure 11: The K-means Segmentation of 8X reconstructed CT slice with PSNR = 2.56 dB. and SSIM = 0.0018

Both the metrics confirms the poor performance of the reconstruction and K-means segmentation

For 4x Reconstructed K-means Segmentation , the Average Accuracy, Sensitivity, Specificity, Avg Dice Socre for a batch of 20 slices is as given below.
The corresponding plots have been highligted in Figure 12

Average Sensitivity is: 0.22868437199093297

Average Specificity is: 0.5751187464387966

Average Accuracy(%): 56.54590622319074

Average Dice score: 0.0376779274170505

For 8x Reconstructed K-means Segmentation , the Average Accuracy, Sensitivity, Specificity, Avg Dice Socre for a batch of 20 slices is as given below.
The corresponding plots have been highligted in Figure 13

Average Sensitivity is: 0.2679735396720146

Average Specificity is: 0.6555515364059492

Average Accuracy(%): 64.09300681599676

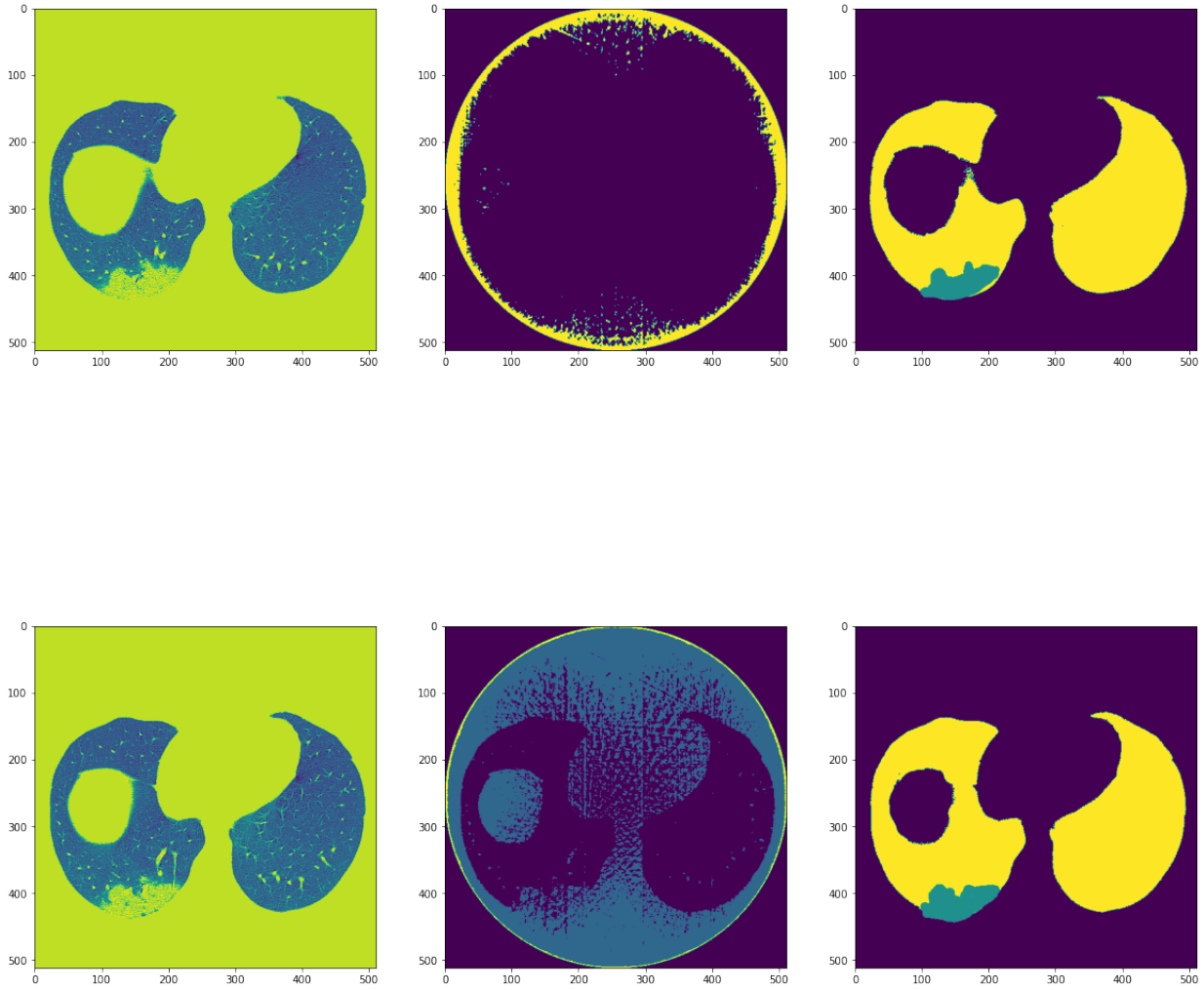Average Dice score: 0.06877999729029094

Figure 12: The K-means Segmentation of 4X reconstructedCT slices along with original image and expert infection mask
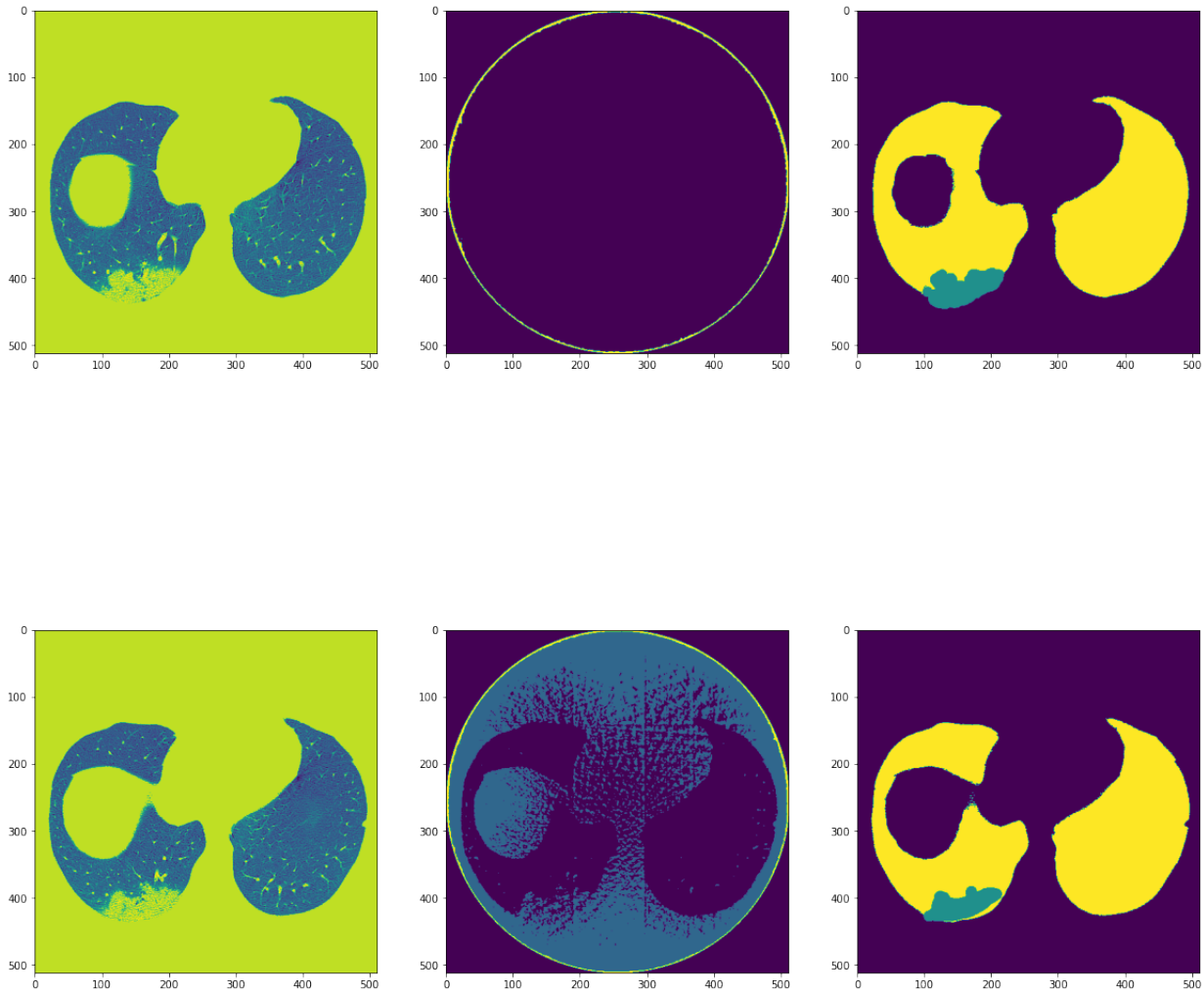
Figure 13: The K-means Segmentation of 8X reconstructed CT slices along with original image and expert infection mask

4 References

1.Qiblawey Y et.all., Detection and Severity Classification of COVID-19 in CT Images Using Deep Learning. Diagnostics (Basel). 2021 May 17;11(5):893. doi: 10.3390/diagnostics11050893. PMID: 34067937; PMCID: PMC8155971.

2. Link https://github.com/SanketD92/CT-Image-Reconstruction/

3. Link https://github.com/adh182/ct-reconstruction/

4. Link https://pythonmana.com/2021/08/20210810150803572p.html/

5. Link https://cvnote.ddlee.cc/2019/09/12/psnr-ssim-python/

6. Link https://github.com/suhas-nithyanand/Image-Segmentation-using-K-Means/

7. Link https://github.com/DSGeek24/Image-segmentation-KMeans/

8. Linl https://muthu.co/mathematics-behind-k-mean-clustering-algorithm/

9. Link `https://homepages.inf.ed.ac.uk/rbf/CVonline/LOCAL_COPIES/AV0405/HAYDEN/` `Slice_Reconstruction.html/`

All the links accessed on 29-09-2022