Math 104B Homework #4 *

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General Instructions: Please write your homework papers neatly. You need to turn in both your codes and descriptions on the appropriate runs you made by following TA's instructions. Write your own code, individually. Do not copy codes!

1. (a) Implement the tridiagonal solver seen in class. (b) Test your implementation.

2. Consider the boundary value problem:

$$\begin{aligned} -u'' + \pi^2 u &= 2\pi^2 \sin(\pi x), \quad 0 < x < 1, \\ u(0) &= u(1) = 0. \end{aligned} \tag{1}$$

We can find a numerical approximation to the solution of this problem by employing the finite difference method. Use a uniform grid with $N-1$ interior nodes to obtain, by replacing the second derivative with a second order finite difference and neglecting the (truncation) error, the linear system

$$\frac{-v_{j-1} + 2v_j - v_{j+1}}{h^2} + \pi^2 v_j = 2\pi^2 \sin(\pi x_j), \quad \text{for } j = 1, 2, \cdots, N-1, \tag{2}$$

where $h = 1/N$, v_j is the approximation to $u(x_j)$ for $j = 1, 2, \cdots, N-1$, and $v_0 = v_N = 0$.

   (a) Use your tridiagonal solver to solve (2) for $N = 50$ and plot your corresponding solution.

   (b) The exact solution to the boundary value problem (1) is $u(x) = \sin(\pi x)$. Check this.

   (c) For $N = 100$, by how much would you expect the error to decrease? Verify your answer by comparing the error for $N = 50$ and $N = 100$.

   (d) In real applications we do not know the exact solution. Describe a process to check the convergence and rate of convergence of your approximation if you don't know the exact solution.