Math 104B: Homework 5

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Method: I first find the matrices A and b for the equation $Ax = b$, for $N = 50$ and $N = 100$. I then plug these values into the Jacobi and Gauss-Seidel iterations.

1. (a)

```
% Computer code for finding matrices A and B for Jacobi/Gauss-Seidel iteration
% Input:  none
% Output  vectors needed for iteration
% Author: Raghav Thirumulu, Perm 3499720
% Date:   09/12/2018

N=50;
h=1/N;
x = 0:h:1;

% Set up vector sizes for iteration later
A=zeros(N-1,N-1);
b=zeros(N-1,1);
x2=zeros(N+1,1);

% Various diagonals
temp1=-1/(h^2);
temp2=2/(h^2) + (pi)^2;
temp3=-1/(h^2);

% Iterate through, solving for A
for i=1:N-1
    if(i ~= 1 && i ~= N-1)
            A(i,i-1)=temp1;
            A(i,i)=temp2;
            A(i,i+1)=temp3;
    end
    if(i ==1 )
            A(i,i)=temp2;
            A(i,i+1)=temp3;
    end
    if(i == N-1)
            A(i,i-1)=temp1;
            A(i,i)=temp2;
    end
end

% Iterate through, solving for b
for j=1:N-1
    if(j~=1 && j~=N-1)
        b(j)=2*pi^2*sin(pi*x(j+1));
    end
    if(j==1)
        b(j)=2*pi^2*sin(pi*x(j+1))-temp1*x2(1);
    end
end
```

```
% Computer code for evaluating the Jacobi iteration
% Author: Raghav Thirumulu, Perm 3499720
% Date:   09/12/2018

n=50;
x=zeros(n);
```

```
stop=Inf;

crit = 0.1 * (1/n); itr=0;

while stop>crit
    xold=x;
    for i=1:n
        sigma=0;
        for j=1:n
            if j~=i
                sigma=sigma+A(i,j)*x(j);
            end
        end
        x(i)=(1/A(i,i))*(d(i)-sigma);
    end
    itr=itr+1;
    stop=abs(xold-x);
end
```

For N=50, we need 1575 iterations. For N=100, we need 7000 iterations.

Here is the solution vector for N=50:

$$\begin{bmatrix} 0 \\ 0.0627 \\ 0.1251 \\ 0.1870 \\ 0.2482 \\ 0.3085 \\ 0.3675 \\ 0.4250 \\ 0.4809 \\ 0.5348 \\ 0.5867 \\ 0.6363 \\ 0.6833 \\ 0.7276 \\ 0.7691 \\ 0.8075 \\ 0.8428 \\ 0.8747 \\ 0.9032 \\ 0.9281 \\ 0.9493 \\ 0.9668 \\ 0.9805 \\ 0.9903 \\ 0.9962 \\ 0.9982 \\ 0.9962 \\ 0.9903 \\ 0.9805 \\ 0.9668 \\ 0.9493 \\ 0.9281 \\ 0.9032 \\ 0.8747 \\ 0.8428 \\ 0.8075 \\ 0.7691 \\ 0.7276 \\ 0.6833 \\ 0.6363 \\ 0.5867 \\ 0.5348 \\ 0.4809 \\ 0.4250 \\ 0.3675 \\ 0.3085 \\ 0.2482 \\ 0.1870 \\ 0.1251 \\ 0.0627 \\ 0 \end{bmatrix}$$

Here is the solution vector for N=100:

$$\begin{bmatrix} 00.0314 \\ 0.0627 \\ 0.0940 \\ 0.1252 \\ 0.1563 \\ 0.1872 \\ 0.2179 \\ 0.2485 \\ 0.2787 \\ 0.3087 \\ 0.3384 \\ 0.3678 \\ 0.3968 \\ 0.4254 \\ 0.4536 \\ 0.4813 \\ 0.5086 \\ 0.5353 \\ 0.5615 \\ 0.5872 \\ 0.6123 \\ 0.6368 \\ 0.6607 \\ 0.6839 \\ 0.7064 \\ 0.7283 \\ 0.7494 \\ 0.7698 \\ 0.7894 \\ 0.8082 \\ 0.8263 \\ 0.8435 \\ 0.8599 \\ 0.8755 \\ 0.8902 \\ 0.9040 \\ 0.9169 \\ 0.9289 \\ 0.9400 \\ 0.9501 \\ 0.9594 \\ 0.9677 \\ 0.9750 \\ 0.9813 \\ 0.9867 \\ 0.9912 \\ 0.9946 \\ 0.9971 \\ 0.9985 \\ 0.9990 \\ 0.9985 \\ 0.9971 \\ 0.9946 \\ 0.9912 \\ 0.9867 \\ 0.9813 \\ 0.9750 \\ 0.9677 \\ 0.9594 \\ 0.9501 \end{bmatrix}$$

(b)

```
% Computer code for evaluating the Gauss-Seidel iteration
% Author: Raghav Thirumulu, Perm 3499720
% Date:   09/12/2018

x=zeros(n,1)
stop=Inf;
crit=0.1 * (1/n);
itr=0;

while stop>crit
    x_temp=x;
    for i=1:n
        temp=0;
        for j=1:i-1
                temp=temp+A(i,j)*x(j);
        end
        for j=i+1:n
                temp=temp+A(i,j)*x_temp(j);
        end
        x(i)=(1/A(i,i))*(b(i)-temp);
    end
    itr=itr+1;
    stop=norm(x_temp-x);
end
```

For N=50, we need 789 iterations. For N=100, we need 3501 iterations.

Here is the solution vector for N=50:

$$\begin{bmatrix} 0 \\ 0.0627 \\ 0.1251 \\ 0.1870 \\ 0.2482 \\ 0.3084 \\ 0.3674 \\ 0.4249 \\ 0.4808 \\ 0.5348 \\ 0.5866 \\ 0.6362 \\ 0.6832 \\ 0.7276 \\ 0.7690 \\ 0.8075 \\ 0.8427 \\ 0.8747 \\ 0.9031 \\ 0.9280 \\ 0.9493 \\ 0.9668 \\ 0.9805 \\ 0.9903 \\ 0.9962 \\ 0.9982 \\ 0.9962 \\ 0.9903 \\ 0.9805 \\ 0.9669 \\ 0.9494 \\ 0.9281 \\ 0.9032 \\ 0.8748 \\ 0.8429 \\ 0.8076 \\ 0.7692 \\ 0.7277 \\ 0.6834 \\ 0.6363 \\ 0.5868 \\ 0.5349 \\ 0.4809 \\ 0.4251 \\ 0.3675 \\ 0.3085 \\ 0.2483 \\ 0.1871 \\ 0.1251 \\ 0.0627 \\ 0 \end{bmatrix}$$

Here is the solution vector for N=100:

$$
\begin{bmatrix}
0 \\
0.0314 \\
0.0627 \\
0.0940 \\
0.1252 \\
0.1563 \\
0.1872 \\
0.2179 \\
0.2484 \\
0.2787 \\
0.3087 \\
0.3384 \\
0.3678 \\
0.3968 \\
0.4254 \\
0.4535 \\
0.4813 \\
0.5085 \\
0.5353 \\
0.5615 \\
0.5872 \\
0.6123 \\
0.6368 \\
0.6607 \\
0.6839 \\
0.7064 \\
0.7283 \\
0.7494 \\
0.7698 \\
0.7894 \\
0.8082 \\
0.8263 \\
0.8435 \\
0.8599 \\
0.8755 \\
0.8901 \\
0.9039 \\
0.9169 \\
0.9289 \\
0.9400 \\
0.9501 \\
0.9594 \\
0.9676 \\
0.9750 \\
0.9813 \\
0.9867 \\
0.9912 \\
0.9946 \\
0.9971 \\
0.9985 \\
0.9990 \\
0.9986 \\
0.9971 \\
0.9946 \\
0.9912 \\
0.9867 \\
0.9814 \\
0.9750 \\
0.9677 \\
0.9594
\end{bmatrix}^T
$$