Math 104B Homework #1 *

Instructor: Lihui Chai

General Instructions: Please write your homework papers neatly. You need to turn in both your codes and descriptions on the appropriate runs you made by following TA's instructions. Write your own code, individually. Do not copy codes!

1. Derive directly the elementary Simpson rule on $[-1,1]$ by approximating the integrand f with the Hermite-interpolation polynomial, which interpolates $f(-1)$, $f(0)$, $f'(0)$, and $f(1)$.

2. Let

$$f(x) = \begin{cases} 1+x, & -1 \leq x \leq 0, \\ 1-x, & 0 \leq x \leq 1. \end{cases}$$

   Find an approximation of the integral of f over $[-1,1]$ by using

   (a) the elementary Trapzoidal rule over $[-1,1]$;

   (b) the elementary Simpson rule over $[-1,1]$;

   (c) the elementary Trapezoidal rule over $[-1,0]$ and then elementary Trapzoidal rule over $[0,1]$.

   Compute the error in each case and explain the differences in the accuracy of the results.

3. Construct a quadrature rule of the form

$$\int_{-1}^{1} f(x)dx \approx A_0 f(x_0) + A_1 f(x_1) + A_2 f(x_2),$$

   which is exact for polynomials of degree ≤ 2.

   (a) Derive the 3-point (Legendre) Gaussian quadrature to approximate $\int_{-1}^{1} f(x)dx$ (i.e. you need to obtain the nodes x_0, x_1, x_2 and the corresponding weights A_0, A_1, A_2).

   (b) Verify its degree of precision.

   (c) Compare the accuracy of this 3-point Gaussian quadrature with that of the elementary Simpson rule for approximating $\int_{-1}^{1} e^x dx$.

(d) Show that the 3-point Gaussian quadrature can be used for approximating $\int_a^b f(x)dx$ by doing a simple change of variables and apply this to approximate

$$\int_0^4 \frac{\sin x}{x} dx.$$