Math 104B: Homework 4

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1. (a)

```
function x = tridiagonal(d,e,f,b)
% Computer code for solving the nxn tridiagonal system for x
% Input:  d  --- vector of coefficients on the main diagonal of A
%         e  --- vector of coefficients on the lower diagonal of A
%         f  --- vector of coefficients on the upper diagonal of A
%         b  --- the vector in the solution Ax=b
% Output  x  --- solution vector to the system Ax=b
% Author: Raghav Thirumulu, Perm 3499720
% Date:   09/06/2018

% Set up appropriate vector lengths
n = length(b);
temp = zeros(n,1);
x = zeros(n,1);
temp2 = d(1);
x(1) = b(1)/temp2;

% Begin decomposition and forward substitution
for i=2:n
    temp(i-1) = f(i-1)/temp2;
    temp2 = d(i)-e(i)*temp(i-1);
    x(i) = (b(i)-e(i)*x(i-1))/temp2;
end

% Perform the back-substiution
for j=n-1:-1:1
   x(j) = x(j)-temp(j)*x(j+1);
end

end
```

(b) Let the matrix

$$A = \begin{bmatrix} 4 & 3 & 0 & 0 & 0 \\ 1 & 4 & 3 & 0 & 0 \\ 0 & 1 & 4 & 3 & 0 \\ 0 & 0 & 1 & 4 & 3 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix}, b = \begin{bmatrix} 0.1576 \\ 0.9706 \\ 0.9572 \\ 0.4584 \\ 0.8003 \end{bmatrix}$$

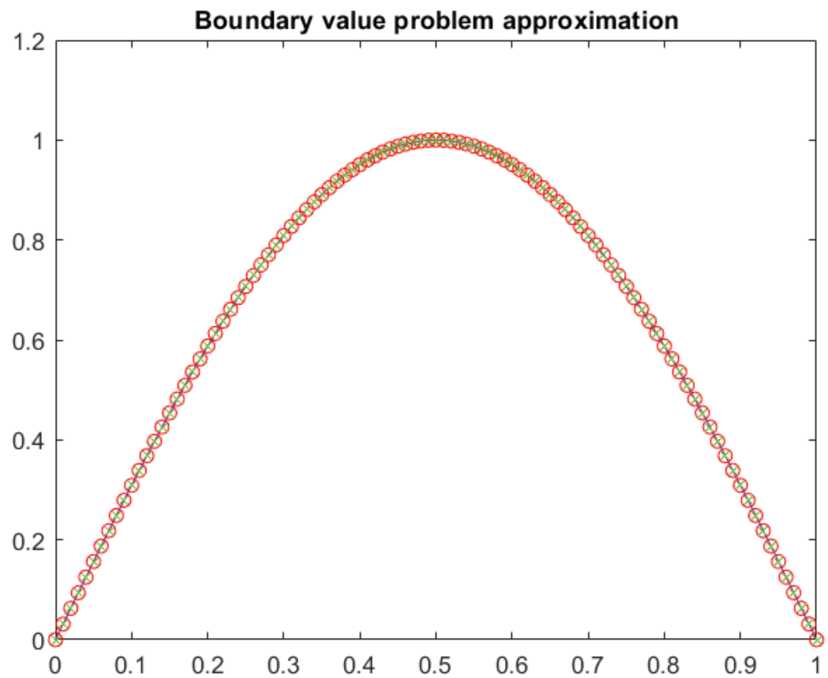
Using the tridiagonal solver from part (a) we obtain

$$x = \begin{bmatrix} 0.0742 \\ -0.0463 \\ 0.3606 \\ -0.1463 \\ 0.2366 \end{bmatrix}$$

We know that the code is correct because when we perform $A * x$ we obtain b.

2. (a) Using our tridiagonal solver, we obtain the following table of values:

x_j	v_j
0.000000	0.000000
0.020000	0.062801
0.040000	0.125354
0.060000	0.187412
0.080000	0.248731
0.100000	0.309068
0.120000	0.368185
0.140000	0.425849
0.160000	0.481833
0.180000	0.535915
0.200000	0.587882
0.220000	0.637529
0.240000	0.684660
0.260000	0.729089
0.280000	0.770640
0.300000	0.809150
0.320000	0.844467
0.340000	0.876451
0.360000	0.904976
0.380000	0.929929
0.400000	0.951213
0.420000	0.968742
0.440000	0.982449
0.460000	0.992278
0.480000	0.998191
0.500000	1.000164
0.520000	0.998191
0.540000	0.992278
0.560000	0.982449
0.580000	0.968742
0.600000	0.951213
0.620000	0.929929
0.640000	0.904976
0.660000	0.876451
0.680000	0.844467
0.700000	0.809150
0.720000	0.770640
0.740000	0.729089
0.760000	0.684660
0.780000	0.637529
0.800000	0.587882
0.820000	0.535915
0.840000	0.481833
0.860000	0.425849
0.880000	0.368185
0.900000	0.309068
0.920000	0.248731
0.940000	0.187412
0.960000	0.125354
0.980000	0.062801
1.000000	0.000000



(b) Using our table and plot from the previous question, we obtain a very low error value (0.00016450) relative to the exact equation. Therefore, it is safe to verify that the exact solution to the boundary problem is $u(x) = \sin \pi x$.

(c) I would expect the error to decrease by a factor of 4. For $N = 50$, we obtain an error of 0.00016450. For $N = 100$, we obtain an error of 0.000041124. Therefore, our rate of convergence is $O(N^2)$.

(d) We use the double mesh principle when the exact solution of an ODE is unknown. We can double the number of nodes and calculate the error using the formula

$$E_N = max|y_i^N - y_{2i}^{2N}|$$

y_{2i}^{2N} is the solution obtain containing the same number N nodes and N more nodes are aded by selecting the midpoints of the various x_i using the formula:

$$x_{i+1/2} = (x_i + x_{i+1})/2$$