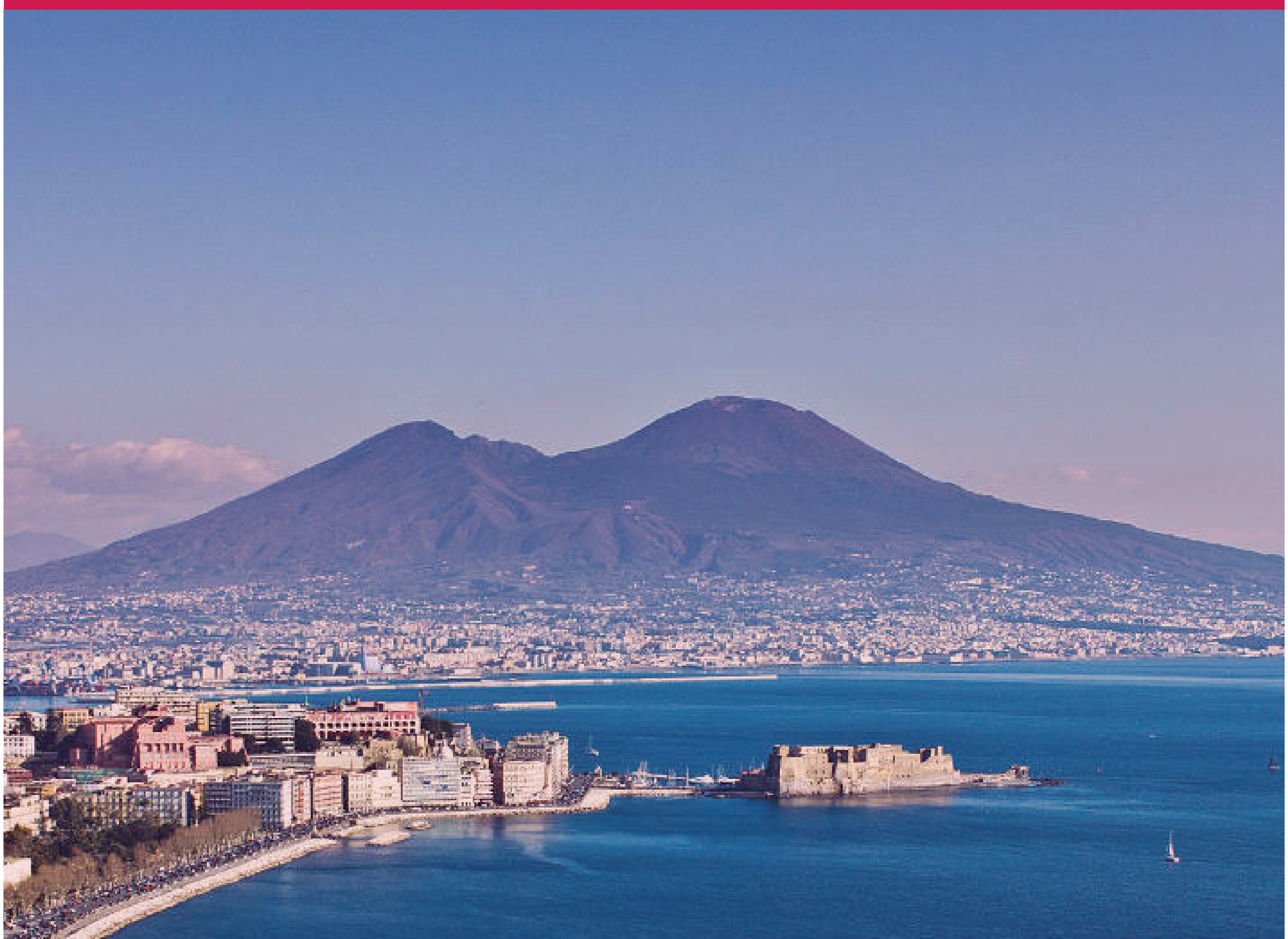
BAYESUVIUS

A visual dictionary of Bayesian Networks and Causal Inference



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Bayesuvius,
a visual dictionary of Bayesian Networks and Causal Inference

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This book is constantly being expanded and improved. To download the latest version, go to `https://github.com/rrtucci/Bayesuvius`

Bayesuvius
by Robert R. Tucci

Figure 1: View of Mount Vesuvius from Pompeii

Figure 2: Mount Vesuvius and Bay of Naples

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Chapter 84

Sentence Splitting with SentenceAx

The Openie6 (O6) software (at github repo Ref.[2]) splits complex or compound sentences into simple ones.[1] Sentence splitting is a necessary step when doing DAG extraction from text (DEFT) (See Chapter 14).

The O6 software is described by its creators in the paper Ref.[30], which we will henceforth refer to as the O6 paper.

The SentenceAx (Sax) software (at github repo Ref.[78]) is a complete rewrite of the O6 software. Sax is 95% identical algorithmically to O6, but it has been rewritten in what we hope is a friendlier form.

Sax is a fine-tuning of the BERT model. What this means in the language of Bayesian Networks is that we use BERT as a prior probability. The BERT model is the encoder part of the vanilla Transformer network which we discuss in Chapter 95.

This chapter describes the technical aspects of Sax. Although this chapter can be read without reading the O6 paper, we highly recommend to our readers that they read the O6 paper also. Some parts of this chapter are taken almost verbatim from the O6 paper. Other parts try to fill-in gaps in the explanations provided by the O6 paper or to improve those explanations. Yet others parts describe small changes that we made to the O6 software, in an effort to improve its clarity.

In this chapter, we will use the Numpy-like tensor notation discussed in Section C.48. In particular, note that $[n] = [0 : n] = \{0, 1, \ldots, n-1\}$ and that $T^{[n],[m]}$ is an $n \times m$ matrix.

84.1 Preliminary Conventions

84.1.1 Tensor Notation

Our tensor notation is discussed in Section C.48. Here is a quick review of some of the more salient facts in that section on tensors. Below, we will often accompany

[1]Simple sentences are essentially the same as the triples (subject, relationship, object) which, when visualized as a directed or undirected graph, is called a “knowledge graph".

an equation in tensor component notation with the equivalent matrix equation, in parenthesis.

We use Greek letters for tensor indices.

Let $\alpha \in [a]$, $\beta \in [b]$, $\gamma \in [c]$, $\delta \in [d]$, $\nu \in [n]$, $\Delta \in [D]$.

- **reshaping**

$$T^{\nu,\delta} \rightarrow T^{\Delta} \quad \left(T^{[n_{\underline{h}}],[d]} \rightarrow T^{[D]}\right) \tag{84.1}$$

$$T^{\Delta} \rightarrow T^{\nu,\delta} \quad \left(T^{[D]} \rightarrow T^{[n_{\underline{h}}],[d]}\right) \tag{84.2}$$

- **concatenation**

$$T^{[n]} = (T^0, T^1, \ldots, T^{n-1}) = (T^\nu)_{\nu\in[n]} \tag{84.3}$$

- **Hadamard product (element-wise, entry-wise multiplication)**

$$T^{[n]} * S^{[n]} = (T^\nu S^\nu)_{\nu\in[n]} \tag{84.4}$$

- **Matrix multiplication**

  $T^{[n]} = T^{[n],[1]}$ is a column vector.

$$(T^{[n]})^T S^{[n]} = \text{scalar} \tag{84.5}$$

$$T^{[a],[b]} S^{[b],[c]} = \left[\sum_{\beta\in[b]} T^{\alpha,\beta} S^{\beta,\gamma}\right]_{\alpha\in[a],\gamma\in[c]} \tag{84.6}$$

Most treatments of Transformer Networks (tranets), including the the O6 paper and PyTorch, order the operations chronologically from left to right (L2R). So if A occurs before B, they write AB. This is contrary to what is done in Linear Algebra, where one orders the operations chronologically from right to left (R2L), and one writes BA. In Chapter 95 on tranets, we followed the Linear Algebra convention. In this chapter, we will follow the PyTorch convention, because Sax is written with PyTorch so it uses the PyTorch convention.

84.1.2 PyTorch conventions

- **Linear**

  Some pseudo-code

```
lin = nn.Linear(b, a)
y^{[n],[b]} = lin(x^{[n],[a]})
```

In the L2R (left to right) convention followed by PyTorch

$$x^{\nu,[a]} \to y^{\nu,[b]} = x^{\nu,[a]} W^{[a],[b]} \tag{84.7}$$

for all $\nu \in [n]$. Alternatively, in the R2L convention followed in Linear Algebra,

$$x^{[a],\nu} \to y^{[b],\nu} = W^{[b],[a]} x^{[a],\nu} \tag{84.8}$$

Note that in PyTorch, the rightmost index of the input is the one that is summed over.

The weights matrix $W^{[b],[a]}$ is learned by training.

- **Dropout**

  Some pseudo-code

```
dropo = nn.Dropout(p_{drop})
y^{[n],[a]} = dropo(x^{[n],[a]})
```

$$x^{\nu,[a]} \to y^{\nu,[a]} = x^{\nu,[a]} \mathcal{W}_R^{[a],[a]} \quad \text{(in L2R)} \tag{84.9}$$

$$x^{\nu,[a]} \to y^{\nu,[a]} = \mathcal{W}_L^{[a],[a]} x^{\nu,[a]} \quad \text{(in R2L)} \tag{84.10}$$

for all $\nu \in [n]$.

`Dropout` learns a weight matrix W just like `Linear`. But at the end of the training, `Dropout` flips a coin for each row of $W_L^{[a],[a]}$ (resp., column of $W_R^{[a],[a]}$), with

$$P(Heads) = p_{drop}, \text{ and } P(Tails) = 1 - p_{drop} = p_{keep}.$$

If the coin lands on T, it keeps that row of $W_L^{[a],[a]}$ (resp., column of $W_R^{[a],[a]}$), whereas if lands on H, it sets that row (resp., column) to zero. Then the matrix is divided by p_{keep}. The final matrix after all these operations is denoted by $\mathcal{W}_L$ (resp., $\mathcal{W}_R$).

- **Embedding**

  Some pseudo-code

```
emb = nn.Embedding(num_embeddings=L, embedding_dim=d)
Y^{[n1],[n2],[d]} = emb(λ^{[n1],[n2]})
```

In Sax, we use $L = 100$ and $d = 768$ (for BERT base). The d is the "hidden dimension" of BERT. The L could be as large as the vocab size of BERT, but since we consider only sentences with 100 words at most, we may set $L = 100$. L doesn't appear in the final answer because we sum over $\lambda \in [L]$.

Next, we explain in more detail the meaning of the tensors λ and Y.

Let

$L =$ number of embeddings

$d =$ embedding dimension

$\lambda \in [L]$, $\alpha \in [\ell]$, $\nu_1 \in [n_1]$, $\nu_2 \in [n_2]$

$\ell = \nu_1 \nu_2$.

Consider matrices Y, E, X such that

$$Y^{\delta,\alpha} = \sum_\lambda E^{\delta,\lambda} X^{\lambda,\alpha} \quad \left(Y^{[d],[\ell]} = E^{[d],[L]} X^{[L],[\ell]}\right) \tag{84.11}$$

Assume that matrix X has 1-hot columns

$$X^{\lambda,\alpha} = \delta(\lambda, \lambda(\alpha)) \tag{84.12}$$

where $\lambda() : [\ell] \to [L]$.

Hence,

$$Y^{\delta,\alpha} = E^{\delta,\lambda(\alpha)} \tag{84.13}$$

If we define

$$\Lambda^\alpha = \lambda(\alpha) \tag{84.14}$$

then `emb()` maps

$$\Lambda^\alpha \to Y^{\delta,\alpha} = E^{\delta,\lambda(\alpha)} \ \ (\Lambda^{[\ell]} \to Y^{[d],[\ell]}) \tag{84.15}$$

Now replace α by (ν_1, ν_2). `emb()` also maps

$$\Lambda^{\nu_1,\nu_2} \to Y^{\delta,\nu_1,\nu_2} = E^{\delta,\lambda(\nu_1,\nu_2)} \ \ (\Lambda^{[n_1],[n_2]} \to Y^{[d],[n_1],[n_2]}) \tag{84.16}$$

Actually, `emb()` orders the tensor indices of the output so that the δ index is on the right side rather than the left side of the input indices. Thus,

$$Y^{[n_1],[n_2],[d]} = \texttt{emb}(\Lambda^{[n_1],[n_2]}) \tag{84.17}$$

- **Cross Entropy Loss**

  Some pseudo-code

  ```
  loss = nn.CrossEntropyLoss()
  output = loss(input=x^{[n_c],[n_s]}, target=t^{[n_s]})
  ```

  Cross Entropy in Information Theory:

  $$H(P^\sigma_{tar}, P^\sigma_{in}) = -\sum_{\gamma\in[n_{\underline{c}}]} P_{tar}(\gamma|\sigma)\ln P_{in}(\gamma|\sigma) \tag{84.18}$$
  $$= -\sum_{\gamma\in[n_{\underline{c}}]} P_{tar}(\gamma|\sigma)\ln\left[\frac{P_{in}(\gamma|\sigma)}{P_{tar}(\gamma|\sigma)}P_{tar}(\gamma|\sigma)\right] \tag{84.19}$$
  $$= H(P^\sigma_{tar}) + D_{KL}(P^\sigma_{in} \parallel P^\sigma_{tar}) \tag{84.20}$$

  Cross Entropy Loss in PyTorch:

  Let

  $n_{\underline{s}}$ = total number of samples being considered (usually batch size). $\sigma \in [n_{\underline{s}}]$

  $n_{\underline{c}}$ = number of classes in classification. $\gamma \in [n_{\underline{c}}]$

  $x^{[n_{\underline{c}}],[n_{\underline{s}}]}$ = input samples

  $t^{[n_{\underline{s}}]}$ = target samples

  Define

  $$P_{in}(\gamma|\sigma) = \frac{\exp(x^{\gamma,\sigma})}{\sum_{\gamma'\in[n_{\underline{c}}]}\exp(x^{\gamma',\sigma})} \tag{84.21}$$
  $$= \text{softmax}(x^{[n_{\underline{c}}],\sigma})(\gamma|\sigma) \tag{84.22}$$

  Suppose $W^\gamma : values(\underline{t}) \to [0,1]$ for all $\gamma \in [n_{\underline{c}}]$.

  Define

  $$P_{tar}(\gamma|\sigma) = \frac{W^\gamma(t^\sigma)\mathbb{1}(t^\sigma \neq -100)}{\sum_{\gamma\in[n_{\underline{c}}]} numerator} \tag{84.23}$$

  The -100 on the right side of the last equation can be replaced by any other integer in $values(\underline{t})$ for which we want the loss to be zero (for example, it could be an integer used for padding)

  Now define the cross entropy loss $\mathcal{L}_{CE}$ by

$$\mathcal{L}^\sigma_{CE}(t^\sigma, x^{[n_{\underline{c}}],\sigma}) = H(P_{tar}(\cdot|\sigma), P_{in}(\cdot|\sigma)) \tag{84.24}$$

$$\mathcal{L}_{CE} = \frac{1}{n_{\underline{s}}} \sum_{\sigma\in[n_{\underline{s}}]} \mathcal{L}^\sigma_{CE} \tag{84.25}$$

For example, if $W^\gamma = 1$, and $n_{\underline{c}} = 2$,

$$\mathcal{L}_{CE} = \frac{1}{n_{\underline{s}}} \sum_{\sigma\in[n_{\underline{s}}]} [P_{tar}(0|\sigma)\ln P_{in}(0|\sigma) + P_{tar}(1|\sigma)\ln P_{in}(1|\sigma)] \tag{84.26}$$

- **unsqueeze-repeat-gather**

  Some pseudo-code

```
lll_loc = ll_loc0.unsqueeze(2).\
    repeat(1, 1, lll_state.shape[2])
lll_out = torch.gather(
    input=lll_state, dim=1, index=lll_loc)
```

  Sax uses the trio of operations unsqueeze-repeat-gather in the manner of the above pseudo-code. We have already discussed in Section C.48 how each of these 3 operations acts individually. Here we will discuss how they act jointly, when used as in the above pseudo-code.

  Let

  `lll_state`= $S^{[s_{ba}],[a],[d]}$

  `ll_loc0`= $L_0^{[s_{ba}],[a]}$

  `lll_loc`= $L^{[s_{ba}],[b],[d]}$

  `lll_out`= $O^{[s_{ba}],[b],[d]}$

  $\sigma \in s_{ba}, \alpha \in [a], \beta \in [b], \delta \in [d]$

  `unsqueeze(2)` takes

$$L_0^{[s_{ba}],[a]} \to L_0^{[s_{ba}],[a],0} \tag{84.27}$$

  `lll_state.shape[2]` equals d, and `repeat(1, 1, d)` takes

$$L_0^{[s_{ba}],[a],0} \to L^{[s_{ba}],[a],[d]} = (\underbrace{L_0^{[s_{ba}],[a],0}, \ldots, L_0^{[s_{ba}],[a],0}}_{d \text{ times}}) \tag{84.28}$$

  Now define

$$\lambda(\sigma, \alpha) = L^{\sigma,\alpha,\delta} = L_0^{\sigma,\alpha} \tag{84.29}$$

Then the `gather()` with `dim=1` outputs.

$$O^{\sigma,\alpha,\delta} = S^{\sigma,\lambda(\sigma,\alpha),\delta} \tag{84.30}$$

84.2 Bayesian Network for this model

Let

$\ell_{pad} = 86$, padding length, for this batch

$\ell_{enc} = 121$, encoded length, for this batch, $\ell_{enc} \geq \ell_{pad}$

$n_{dep} = 5$, number of copies of plain box connected in series, number of depths

$n_{att} = 2$, number of copies of dashed box connected in series, number of iterative (attention) layers.

$d = 768$, hidden dimension per head

$n_{\underline{h}} = 12$, number of heads (BERT base)

$D = dn_{\underline{h}}$, hidden dimension for all heads

$s_{ba} = 24$, batch size

$n_{il} = 6$, number of ilabels

$d_{me} = 300$, merge dimension

Fig.84.1 shows the bnet for Sax.[2]. The structural equations, printed in blue, for that bnet, are as follows.

$\underline{a}^{[86]}$: `ll_greedy_ilabel`
$\underline{B}^{[121],[768]}$: `lll_hidstate`
$\underline{d}^{[121],[768]}$: `lll_hidstate`
$\underline{E}^{[86],[768]}$: `lll_pred_code`
$\underline{G}^{[86],[768]}$: `lll_word_hidstate`
$\underline{I}^{[121],[768]}$: `lll_hidstate`
$\underline{L}^{[86],[6]}$: `lll_word_score`
$\underline{M}^{[86],[300]}$: `lll_word_hidstate`
$\underline{S}^{[86],[768]}$: `lll_word_hidstate`
$\underline{X}^{[86],[6]}$: `lll_word_score`

$$\begin{aligned} a^{[86]} &= \text{argmax}(X^{[86],[6]}; dim = -1) \\ &: \texttt{ll\_greedy\_ilabel} \end{aligned} \tag{84.31a}$$

$$\begin{aligned} B^{[121],[768]} &= \text{BERT}() \\ &: \texttt{lll\_hidstate} \end{aligned} \tag{84.31b}$$

[2]The bnet of Fig.84.1 and its structural equations printed in blue, were produced via the texnn software (Ref.[80])

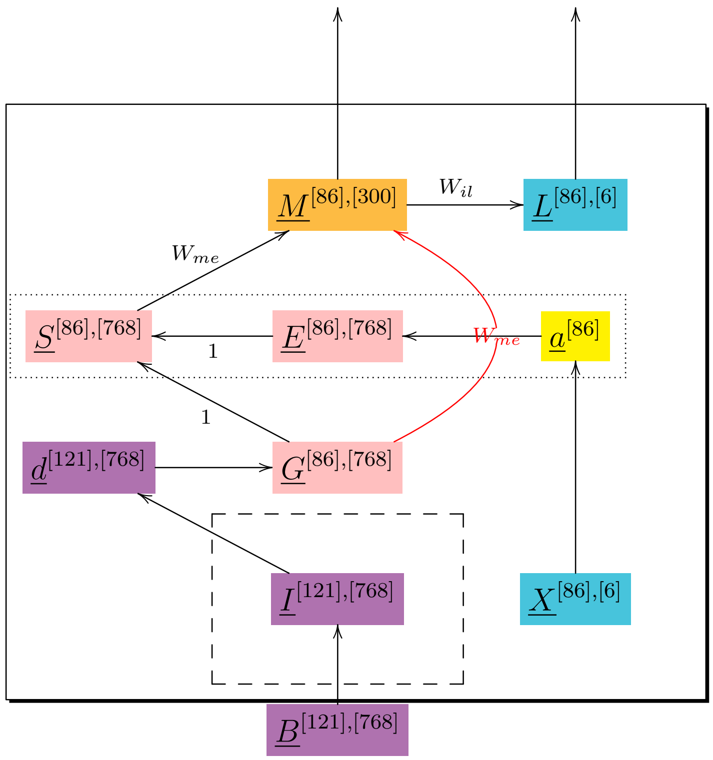

Figure 84.1: Sax bnet. 2 copies of dashed box are connected in series. 5 copies (5 depths) of plain box are connected in series. However, in the first of those 5 plain box copies, the dotted box is omitted and node $\underline{G}$ feeds directly into node $\underline{M}$ (indicated by red arrow). We display the tensor shape superscripts in the PyTorch L2R order. All tensor shape superscripts have been simplified by omitting a $[s_{ba}]$ from their left side, where $s_{ba} = 24$ is the batch size.

$$d^{[121],[768]} = \text{dropout}(I^{[121],[768]}) \quad : \texttt{lll\_hidstate} \tag{84.31c}$$

$$E^{[86],[768]} = \text{embedding}(a^{[86]}) \quad : \texttt{lll\_pred\_code} \tag{84.31d}$$

$$G^{[86],[768]} = \text{gather}(d^{[121],[768]}; dim = -2) \quad : \texttt{lll\_word\_hidstate} \tag{84.31e}$$

$$I^{[121],[768]} = \left[B^{[121],[768]} \mathbb{1}(depth = 0) + M^{[86],[300]} \mathbb{1}(depth > 0)\right] \tag{84.31f}$$
$$: \texttt{lll\_hidstate}$$

$$L^{[86],[6]} = M^{[86],[300]} W_{il}^{[300],[6]} \tag{84.31g}$$
$$: \texttt{lll\_word\_score}$$

$$M^{[86],[300]} = \left[G^{[86],[768]} \mathbb{1}(depth = 0) + S^{[86],[768]} \mathbb{1}(depth > 0)\right] W_{me}^{[768],[300]} \tag{84.31h}$$
$$: \texttt{lll\_word\_hidstate}$$

$$S^{[86],[768]} = E^{[86],[768]} + G^{[86],[768]} \tag{84.31i}$$
$$: \texttt{lll\_word\_hidstate}$$

$$X^{[86],[6]} = L^{[86],[6]} \mathbb{1}(depth > 0) \tag{84.31j}$$
$$: \texttt{lll\_word\_score}$$

84.3 Loss for this model

The Loss $\mathcal{L}$ is the sum of the Cross Entropy Loss $\mathcal{L}_{CE}$ and 4 penalty losses $\mathcal{L}_i$ for $i \in PL$ where $PL = \{POSC, HVC, HVE, EC\}$.

$$\mathcal{L} = \mathcal{L}_{CE} + \sum_{i \in PL} \lambda_i \mathcal{L}_i \tag{84.32}$$

where the λ_i are hyper-parameters to be determined heuristically.

In an earlier section, we discussed the Cross Entropy Loss at length. In this section, we will discuss the 4 penalty losses.

Below, we will use the standard notation for the **positive-part function** (a.k.a. the **reLU** function)

$$(x)_+ = \begin{cases} x & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases} \tag{84.33}$$
$$= \max(0, x) \tag{84.34}$$

Since loss is supposed to be bounded below (usually it is defined to be greater or equal to zero), the positive-part function comes in handy when defining a loss.

Let
ℓ = number of words, length of sentence. $\alpha \in [\ell]$
M = number of depths. $\mu \in [M]$
w^α = word at position α
$T_{pos} = \{N, V, JJ, RB\}$, POS tags, POS=Part Of Speech, N=Noun, V=Verb, JJ=Adjective, RB=Adverb
$T_{ex} = \{S, R, O, N\}$[3]. extraction tags (extags), S=Subject, R=Relation, O=Object, N=None
$T_{ex} \backslash N = T_{ex} - \{N\}$
$POS^\alpha \in T_{pos}$, Part Of Speech of w^α.
Importance indicator function.

$$IMP^\alpha = \mathbb{1}(POS^\alpha \in T_{pos}) \tag{84.35}$$

Head verb indicator function. A **head verb** is any verb that isn't a **light verb** (do, be, is, has, etc.).

$$HV^\alpha = \mathbb{1}(w^\alpha \text{ is a head verb}) \tag{84.36}$$

Let $P(\underline{t}^{\mu,\alpha} = t)$ denote an empirical probability for a table element $\underline{t}^{\mu,\alpha} \in T_{ex}$, for all $\mu \in [M]$ and $\alpha \in [\ell]$.

The O6 paper uses the following sentence to exemplify the 4 types of penalty losses.

Obama gained popularity after Oprah endorsed him for the presidency.

Henceforth, we will refer to this sentence as the red-sent.

For the red-sent, the head verbs are gained, endorsed

Two valid extractions from red-sent are: (Obama; gained; popularity) and (Oprah; endorsed him for; the presidency).

1. **Part of Speech Coverage (POSC)**

   Penalize if some important words do not belong to at least one extraction.

   In red-sent: all the words Obama, gained, popularity, Oprah, endorsed, presidency must be covered in the set of extractions.

$$\mathcal{L}_{POSC} = \sum_{\alpha \in [\ell]} IMP^\alpha P_{POSC}(\alpha) \tag{84.37}$$

[3]The Sax software uses a different set for T_{ex} than $T_{ex} = \{S, R, O, N\}$. In Sax, we use for T_{ex} the list `BASE_EXTAGS` (defined globally in the file `sax_globals`.) In `BASE_EXTAGS`, N becomes NONE (or 0) and R becomes REL (or 3). Also note that 2 tranets are trained by Sax, one for extraction (task=ex), and one for splitting (task=cc). For task=cc, T_{ex} is replaced by T_{cc}. In Sax, we use for T_{cc} the list `BASE_CCTAGS` (defined globally in the file `sax_globals`.) In `BASE_CCTAGS`, N becomes NONE (or 0) and R becomes CC (or 3).

$$P_{POSC}(\alpha) = 1 - \max_{\mu\in[M]}\max_{t\in T_{ex}\setminus N} P(\underline{t}^{\mu,\alpha} = t) \tag{84.38}$$

2. **Head Verb Coverage (HVC)**

   Penalize if a head verb is not present in the relation (R) of any extraction.

   In red-sent: (Obama; gained; popularity), (Obama; gained; presidency) is not a comprehensive set of extractions.

$$\mathcal{L}_{HVC} = \sum_{\alpha\in[\ell]} HV^{\alpha} P_{HVC}(\alpha) \tag{84.39}$$

$$P_{HVC}(\alpha) = \left| 1 - \sum_{\mu\in[M]} P(\underline{t}^{\mu,\alpha} = R) \right| \tag{84.40}$$

3. **Head Verb Exclusivity (HVE)**

   Penalize extractions that contain more than one head verb in their relation (R).

   In red-sent: gained popularity after Oprah endorsed is not a good relation as it contains two head verbs

$$\mathcal{L}_{HVE} = \sum_{\mu\in[M]} \left(\sum_{\alpha\in[\ell]} HV^{\alpha} P(\underline{t}^{\mu,\alpha} = R) - 1 \right)_{+} \tag{84.41}$$

4. **Extraction Count (EC)**

   Penalize if the total number of extractions with head verbs in the relation (R) is too small; i.e., it is smaller than the number of head verbs in the sentence.

$$\mathcal{L}_{EC} = \left(\sum_{\alpha\in[\ell]} HV^{\alpha} - \sum_{\mu\in[M]} EC^{\mu} \right)_{+} \tag{84.42}$$

$$EC^{\mu} = \max_{\alpha\in[\ell]} HV^{\alpha} P(\underline{t}^{\mu,\alpha} = R) \tag{84.43}$$

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