

BAYESUVIUS QUANTICO

A VISUAL DICTIONARY OF
QUANTUM BAYESIAN NETWORKS



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Bayesuvious Quantico, a visual dictionary of Quantum Bayesian Networks

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This book is constantly being expanded and improved. To download
the latest version, go to

<https://github.com/rrtucci/bayes-quantico>

Bayes Quantico

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Chapter 2

Birdtracks: COMING SOON

$$\delta(b, a) = \mathbb{1}(a = b) = \delta_a^b = a \leftarrow \bullet \longrightarrow b \quad (2.1)$$

$$X_{a, \bullet, b, \bullet}^{\bullet, c, \bullet, d} = \begin{array}{c} a \longleftarrow X_{a, \bullet, b, \bullet}^{\bullet, c, \bullet, d} \\ \nearrow \\ b \longleftarrow \\ \nearrow \\ c \longleftarrow \\ \nearrow \\ d \longleftarrow \end{array} \quad (2.2)$$

$$\langle a, b | X_{a, \bullet, b, \bullet}^{\bullet, c, \bullet, d} | c, d \rangle = \begin{array}{c} a \longleftarrow X_{a, \bullet, b, \bullet}^{\bullet, c, \bullet, d} \\ \nearrow \\ b \longleftarrow \\ \nearrow \\ c \longleftarrow \\ \nearrow \\ d \longleftarrow \end{array} \rightarrow \begin{array}{c} a, b \longleftarrow X_{a, \bullet, b, \bullet}^{\bullet, c, \bullet, d} \\ \nearrow \\ a, b \longleftarrow \\ \nearrow \\ c \longleftarrow \\ \nearrow \\ d \longleftarrow \end{array} \quad (2.3)$$

$X_{a, \bullet, b, \bullet}^{\bullet, c, \bullet, d} \in V^2 \otimes V_2$. Sometimes, we will omit denote this node simply by X . This is okay as long as we are not using, X to also denote a different version of $X_{\bullet, c, \bullet, d}^{a, \bullet, b, \bullet}$ with some of the indices raised or lowered or their order has been changed.

$$(X^\dagger)_{\bullet, c, \bullet, d}^{a, \bullet, b, \bullet} = \begin{array}{c} a \longrightarrow (X^\dagger)_{\bullet, c, \bullet, d}^{a, \bullet, b, \bullet} \\ \nearrow \\ b \longrightarrow \\ \nearrow \\ c \longrightarrow \\ \nearrow \\ d \longrightarrow \end{array} \quad (2.4)$$

$$(X^\dagger)_{\bullet, \underline{c}, \bullet, \underline{d}}^{a, \bullet, \underline{b}, \bullet} X_{a, \bullet, \underline{b}, \bullet}^{\bullet, \bullet, \underline{c}, \bullet, d} =$$
(2.5)

$$=$$
(2.6)

$$a^m \in \mathbb{Z}_+^m$$

$$R_{b_3^{n_3}, a_2^{m_2}}^{a_3^{m_3}, b_2^{n_2}} S_{b_2^{n_2}, a_1^{m_1}}^{a_2^{m_2}, b_1^{n_1}} =$$
(2.7)

$$\text{tr}_{\underline{b}} X_{a, \bullet, \underline{b}, \bullet}^{\bullet, \bullet, \underline{b}, \bullet, d} = \sum_b X_{a, \bullet, \underline{b}, \bullet}^{\bullet, \bullet, \underline{b}, \bullet, d} =$$
(2.8)

(2.9)

Chapter 19

Unitary Groups: COMING SOON

19.1 SU(n)

$$m(p, q) = \delta_b^a \sum_{a=1}^n (p_a)^* q_a \quad (19.1)$$

$$\mathbb{I}_{d,b}^{a,c} = \delta_b^a \delta_d^c = \begin{array}{c} d \leftarrow \bullet \longrightarrow c \\ a \longrightarrow \bullet \longrightarrow b \end{array} \quad (19.2)$$

$$\mathbb{X}_{d,b}^{a,c} = \delta_d^a \delta_b^c = \begin{array}{cc} d & c \\ \uparrow & \downarrow \\ \bullet & \bullet \\ \downarrow & \uparrow \\ a & b \end{array} \quad (19.3)$$

$$\mathbb{X}^2 = n\mathbb{X} \quad (19.4)$$

$$\begin{array}{c} d \\ \uparrow \\ \bullet \\ \downarrow \\ a \end{array} \begin{array}{c} \curvearrowright \\ \curvearrowleft \end{array} \begin{array}{c} c \\ \downarrow \\ \bullet \\ \uparrow \\ b \end{array} = n \begin{array}{c} d \\ \uparrow \\ \bullet \\ \downarrow \\ a \end{array} \begin{array}{c} c \\ \downarrow \\ \bullet \\ \uparrow \\ b \end{array} \quad (19.5)$$

$$P_i = \sum_{j \neq i} \frac{M - \lambda_j}{\lambda_i - \lambda_j} \quad (19.6)$$

$$\lambda_1 = n$$

$$P_1 = \frac{\mathbb{X} - n}{0 - n} = 1 - \frac{1}{n}\mathbb{X} \quad (19.7)$$

$$\begin{array}{ccc} a & & b \\ & \searrow \quad \swarrow & \\ & P_1 & \\ & \swarrow \quad \searrow & \\ c & & d \end{array} = \begin{array}{ccc} a \leftarrow \bullet \longrightarrow b \\ c \leftarrow \bullet \longrightarrow d \end{array} - \frac{1}{n} \begin{array}{cc} a & b \\ \uparrow & \downarrow \\ \bullet & \bullet \\ \downarrow & \uparrow \\ c & d \end{array} \quad (19.8)$$

$$\lambda_2 = 0$$

$$P_2 = \frac{\mathbb{X} - 0}{n - 0} = \frac{1}{n} \mathbb{X} \quad (19.9)$$

$$\begin{array}{ccc} a & & b \\ & \searrow \quad \swarrow & \\ & P_2 & \\ & \swarrow \quad \searrow & \\ c & & d \end{array} = \frac{1}{n} \begin{array}{cc} a & b \\ \uparrow & \downarrow \\ \bullet & \bullet \\ \downarrow & \downarrow \\ c & d \end{array} \quad (19.10)$$

$$\text{tr} P_1 = \begin{array}{c} \text{---} \bullet \text{---} \\ \text{---} \bullet \text{---} \end{array} - \frac{1}{n} \begin{array}{c} \text{---} \bullet \text{---} \\ \uparrow \quad \downarrow \\ \bullet \quad \bullet \\ \downarrow \quad \downarrow \end{array} \quad (19.11)$$

$$= n^2 - 1 \quad (19.12)$$

$$\text{tr} P_2 = \frac{1}{n} \begin{array}{c} \text{---} \bullet \text{---} \\ \uparrow \quad \downarrow \\ \bullet \quad \bullet \\ \downarrow \quad \downarrow \end{array} \quad (19.13)$$

$$= 1 \quad (19.14)$$

$$(T_i)_a^b = \begin{array}{c} b \\ \downarrow \\ i \text{ --- } T_i \\ \downarrow \\ a \end{array} \quad (19.15)$$

$$T_i^\dagger = T_i \quad (19.16)$$

Claim 1

$$C_F \delta_a^b = (T_i T_i)_a^b = \frac{n^2 - 1}{n} \delta_a^b \quad (19.17)$$

proof:

$$(T_i T_i)_a^b = \sum_i \begin{array}{ccc} & \curvearrowright & \curvearrowleft \\ i \text{ --- } T_i & & T_i \text{ --- } i \\ \downarrow & & \uparrow \\ a & & b \end{array} \quad (19.18)$$

$$= \sum_i \begin{array}{ccc} & \curvearrowright & \\ i \text{ --- } T_i & & T_i \text{ --- } i \\ & \curvearrowleft & \end{array} \quad (19.19)$$

QED

Bibliography