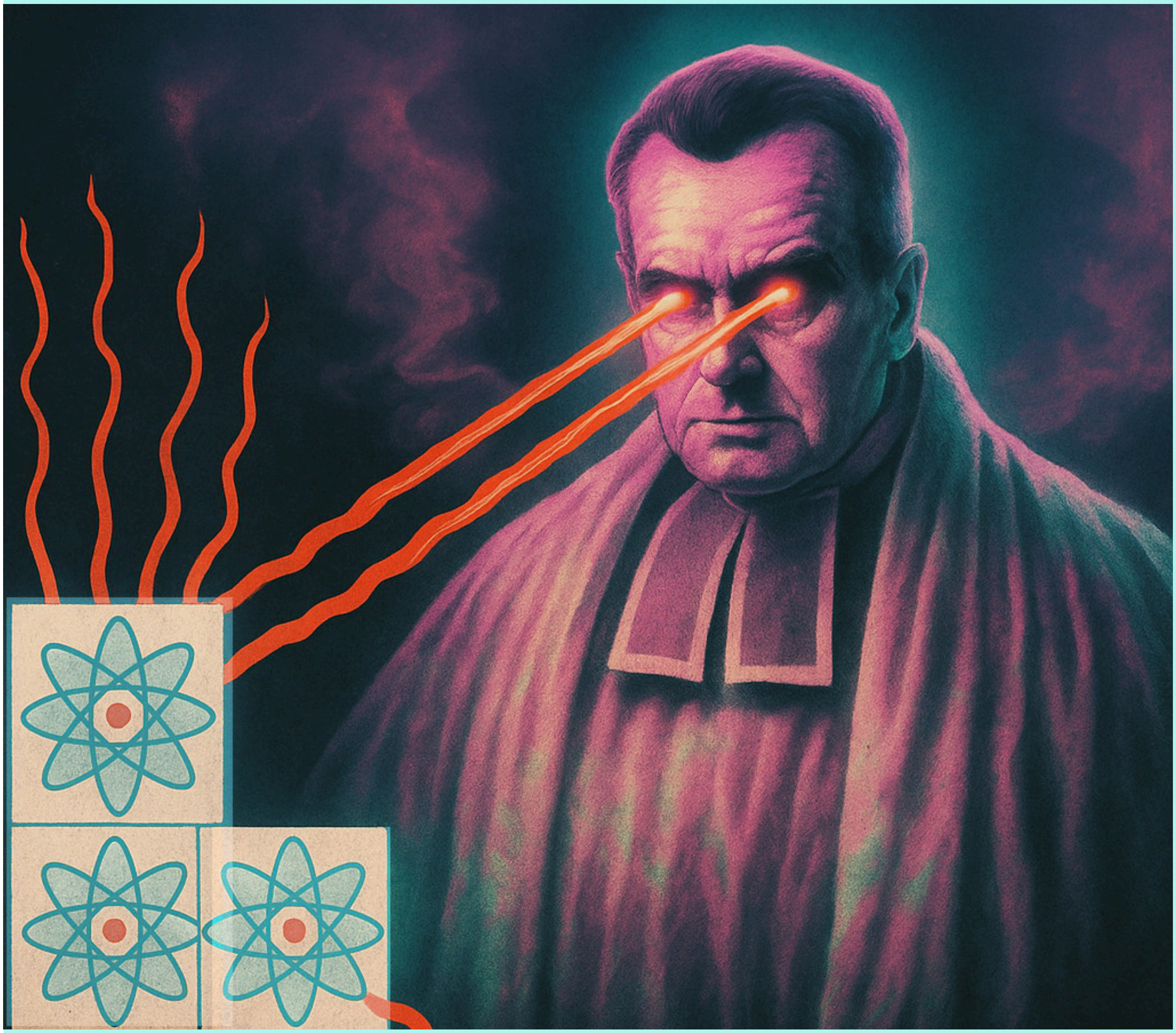
BAYESUVIUS QUANTICO

A VISUAL DICTIONARY OF QUANTUM BAYESIAN NETWORKS



ROBERT R. TUCCI

Bayesuvius Quantico,
a visual dictionary of Quantum Bayesian Networks

Robert R. Tucci
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This book is constantly being expanded and improved. To download the latest version, go to
`https://github.com/rrtucci/bayes-quantico`

Bayes Quantico
by Robert R. Tucci

Contents

Appendices

Chapter 6

Levi-Civita

$$\epsilon^{123\ldots p} = \epsilon_{123\ldots p} = 1 \tag{6.1}$$

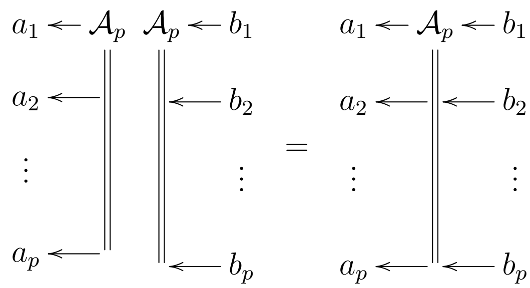
$$\epsilon_{rev(a^p)} = (-1)^{\binom{p}{2}} \epsilon_{a^p} \tag{6.2}$$

where $rev(a^p)$ is the reverse of a^p. $rev(a_1, a_2, \ldots, a_p) = (a_p, a_{p-1}, \ldots, a_1)$

$$(C_{\mathcal{A}_p})_1^{a^p} = e^{i\phi}\frac{\epsilon^{a^p}}{\sqrt{p!}} = \begin{array}{l} \mathcal{A}_p \leftarrow a_1 \\ \Vert \longleftarrow a_2 \\ \Vert \quad\;\; \vdots \\ \Vert \longleftarrow a_p \end{array} \tag{6.3}$$

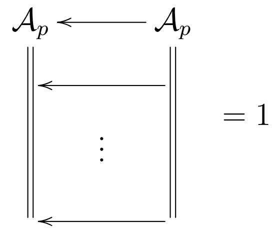
$$(C^\dagger_{\mathcal{A}_p})^1_{a^p} = e^{-i\phi}\frac{\epsilon_{a^p}}{\sqrt{p!}} = \begin{array}{r} a_1 \leftarrow \mathcal{A}_p \\ a_2 \longleftarrow \Vert \\ \vdots \quad\;\; \Vert \\ a_p \longleftarrow \Vert \end{array} \tag{6.4}$$

$$\frac{1}{p!}\epsilon_{a^p}\epsilon^{b^p} = \mathcal{A}_{p}{}_{a^p}^{b^p} \tag{6.5}$$



(6.6)

$$e^{i2\phi}\frac{1}{p!}\epsilon^{a^n}\epsilon_{a^n} = \delta_1^1 = 1 \tag{6.7}$$



(6.8)

For Convention 1, we will use $\phi = 0$.
For Convention 2, we must choose

$$e^{i2\phi} = (-1)^{\binom{p}{2}} = e^{i\pi\frac{p(p-1)}{2}} \tag{6.9}$$

so

$$\phi = \frac{\pi}{4}p(p-1) \tag{6.10}$$

Chapter 16

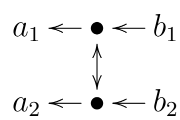
Symmetrization and Antisymmetrization: COMING SOON

$(1,2)$ transposition, swaps 1 and 2, $1 \to 2 \to 1$. $(3,2,1)$ means $3 \to 2 \to 1 \to 3$. A reordering of $(1,2,3,\ldots,p)$ is a permutation on p letters. A permutation can be expressed as a product of transpositions $(3,2,1) = (3,2)(2,1)$ is an even permutation because it can be expressed as a product of an even number of transpositions. An odd permutation can be expressed as a product of an odd number of permutations.

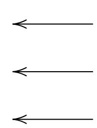
16.1 Symmetrization

$$\mathbb{1}_{a_1,a_2}^{b_2,b_1} = \delta_{a_1}^{b_1}\delta_{a_2}^{b_2} = \begin{array}{c} a_1 \leftarrow b_1 \\ \\ a_2 \leftarrow b_2 \end{array} \tag{16.1}$$

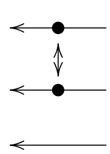
$$(\sigma_{(1,2)})_{a_1,a_2}^{b_2,b_1} = \delta_{a_1}^{b_2}\delta_{a_2}^{b_1} = \tag{16.2}$$

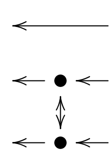


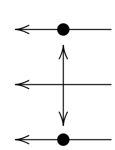
$$\mathbb{1} = \tag{16.3}$$



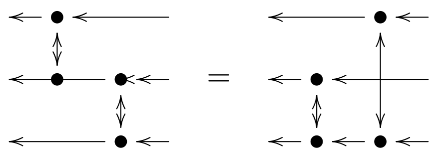
$$\sigma_{(1,2)} = \quad \sigma_{(2,3)} = \quad \sigma_{(1,3)} = \tag{16.4}$$





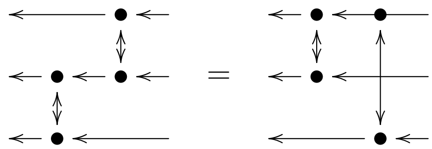


$\sigma_{(1,2,3)} =$

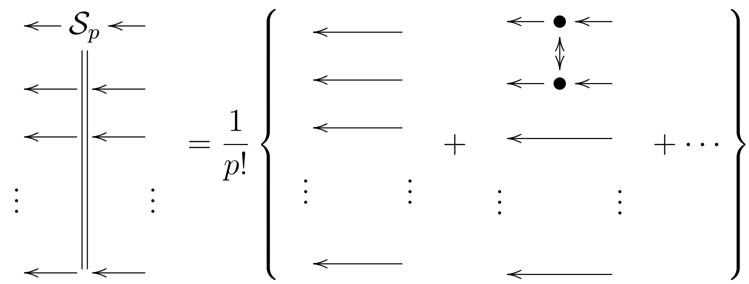


(16.5)

$\sigma_{(1,3,2)} =$

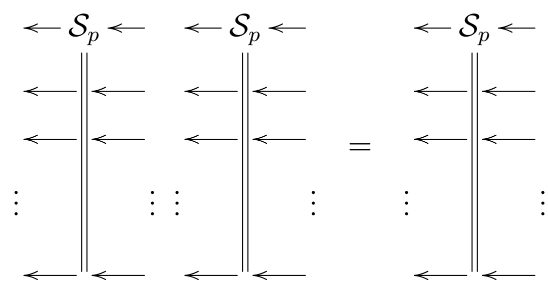


(16.6)

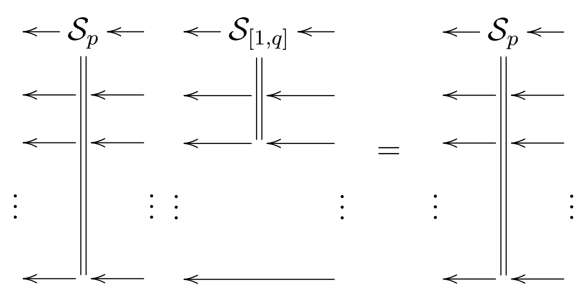


(16.7)

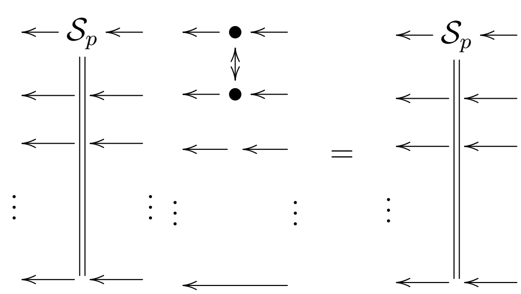
$$\mathcal{S}_p^2 = \mathcal{S}_p \tag{16.8}$$



(16.9)

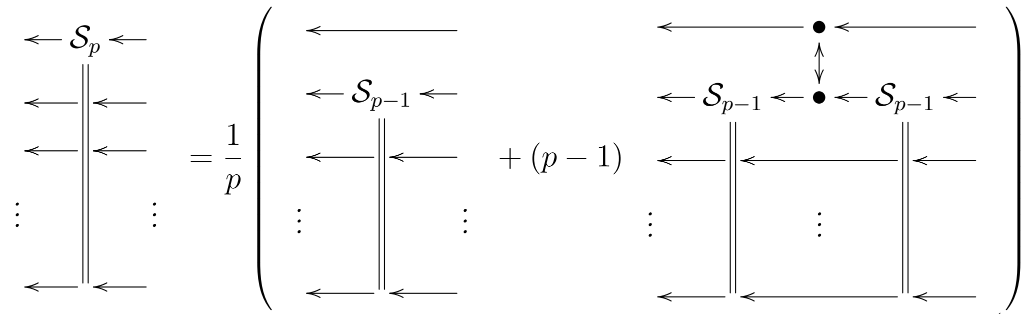


(16.10)



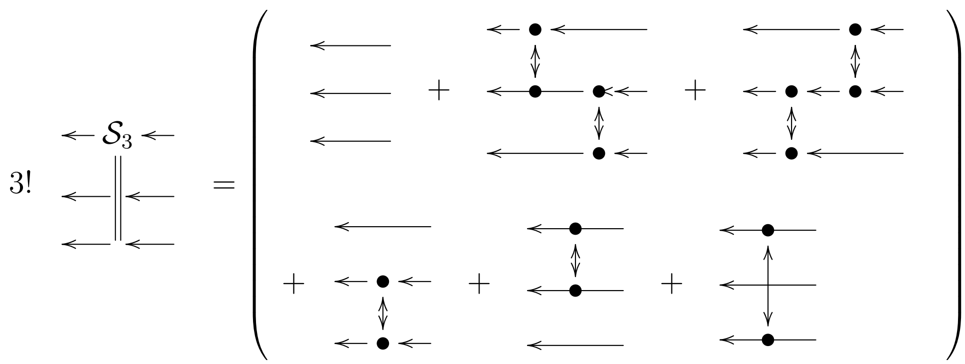
(16.11)

Claim 1

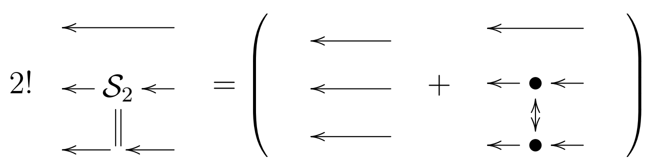


(16.12)

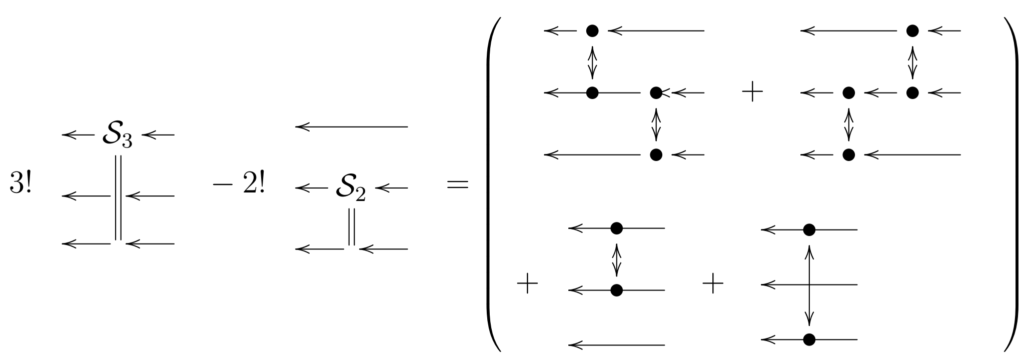
proof: We only prove it for $p = 3$.



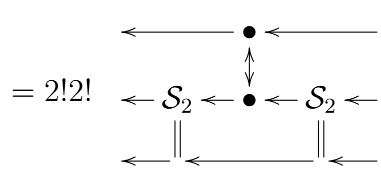
(16.13)



(16.14)

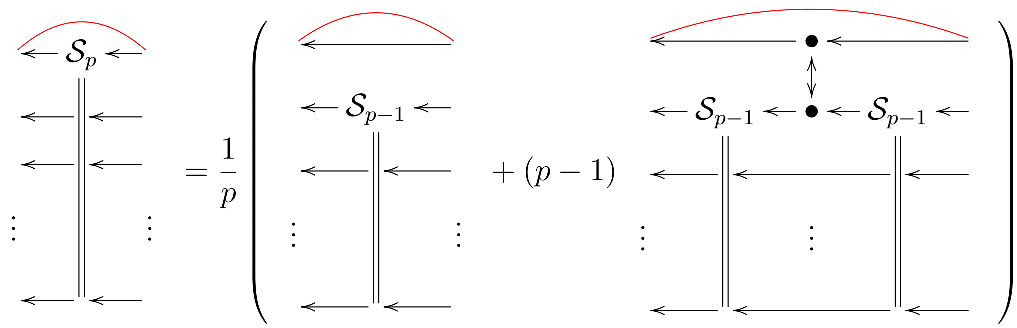


(16.15)

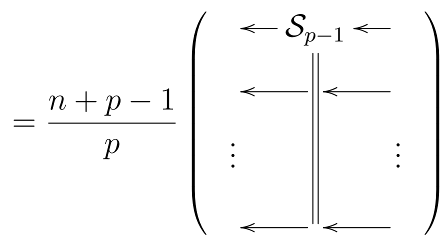


(16.16)

QED



(16.17)



(16.18)

$$\mathrm{tr}_{\underline{a}_1}\mathcal{S}_p = \frac{n+p-1}{p}\mathcal{S}_{p-1} \tag{16.19}$$

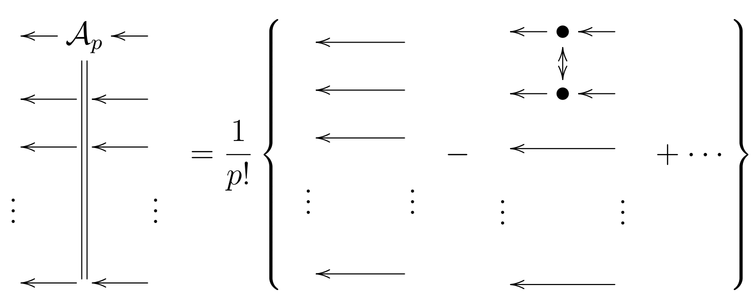
$$\mathrm{tr}_{\underline{a}_1,\underline{a}_2,\ldots,\underline{a}_k}\mathcal{S}_p = \frac{(n+p-1)(n+p-2)\ldots(n=p-k)}{p(p-1)\ldots(p-k+1)}\mathcal{S}_{p-k} \tag{16.20}$$

$$d_{\mathcal{S}_p} = \mathrm{tr}_{\underline{a}^p}\mathcal{S}_p = \frac{(n+p-1)!}{p!(n-1)!} = \binom{n+p-1}{p} \tag{16.21}$$

For $p = 2$,

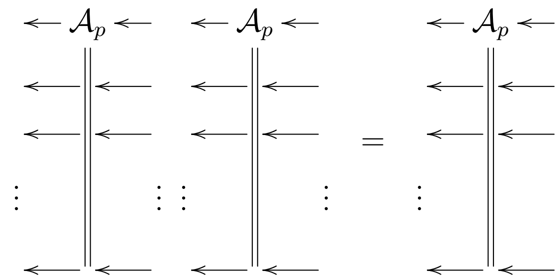
$$d_{\mathcal{S}_2} = \frac{(n+1)n}{2} \tag{16.22}$$

16.2 Antisymmetrization

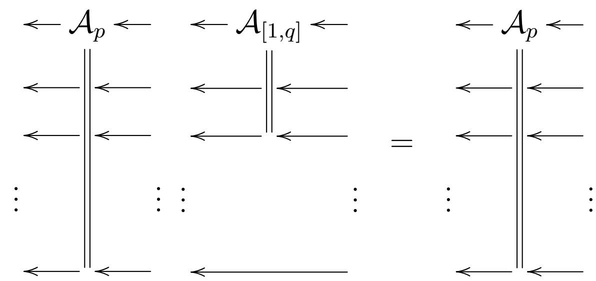


(16.23)

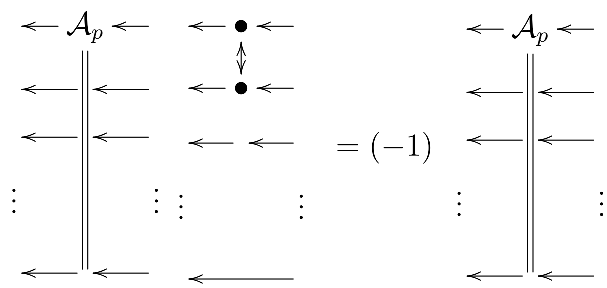
$$\mathcal{A}_p^2 = \mathcal{A}_p \tag{16.24}$$



(16.25)

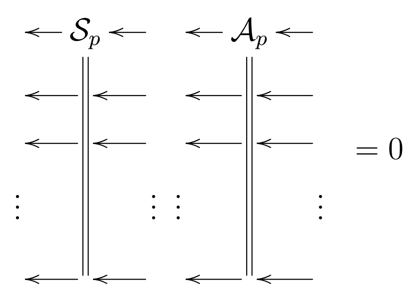


(16.26)

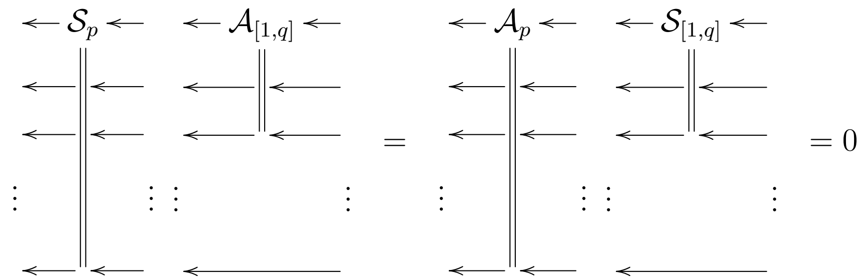


(16.27)

$$\mathcal{S}_p \mathcal{A}_q = \mathcal{A}_p \mathcal{S}_q = 0 \tag{16.28}$$

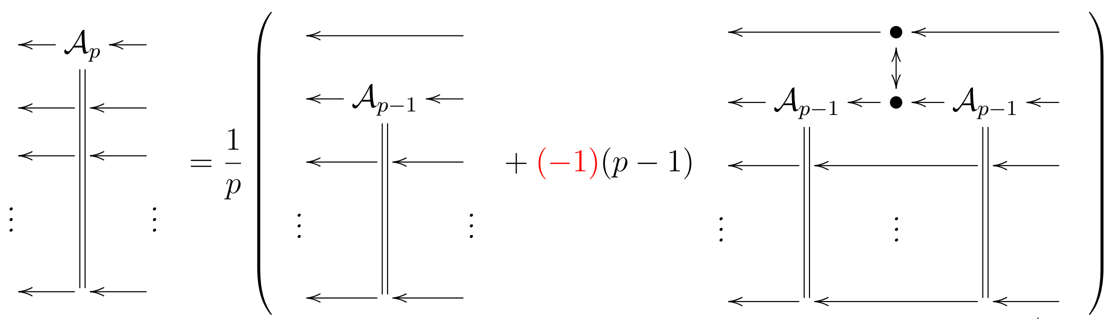


(16.29)



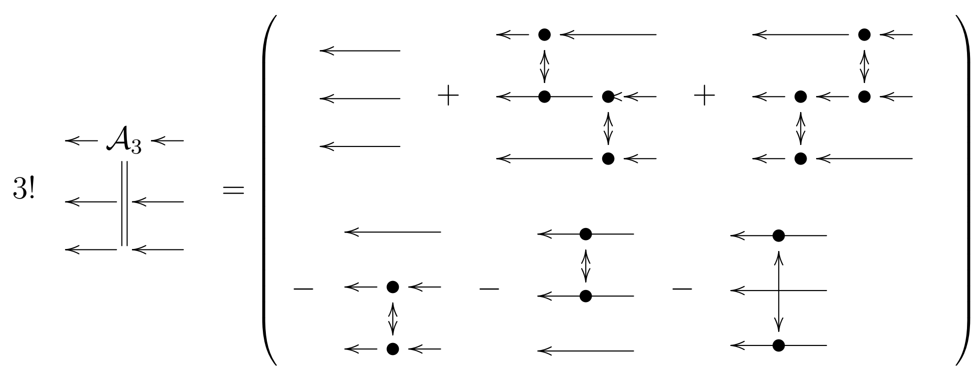
(16.30)

Claim 2

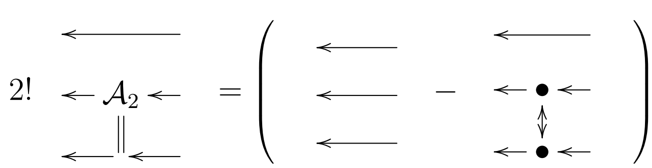


(16.31)

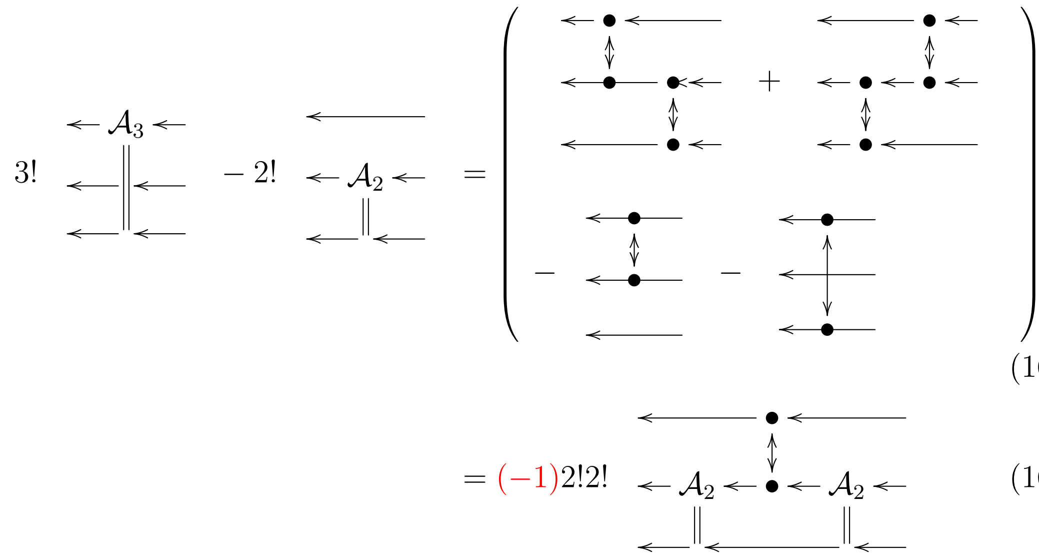
proof: We only prove it for $p = 3$.



(16.32)



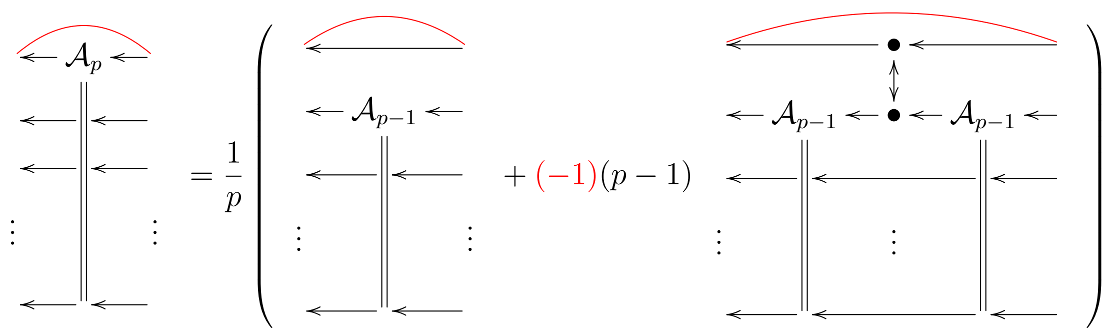
(16.33)



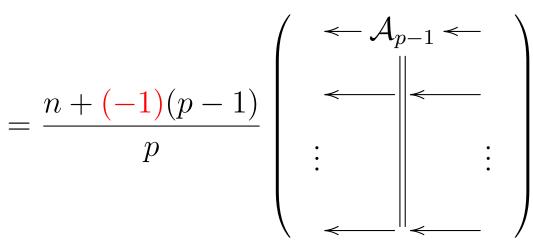
(16.34)

(16.35)

QED



(16.36)



(16.37)

$$\mathrm{tr}_{\underline{a}_1}\mathcal{A}_p = \frac{n-p+1}{p}\mathcal{A}_{p-1} \tag{16.38}$$

$$\mathrm{tr}_{\underline{a}_1,\underline{a}_2,\ldots,\underline{a}_k}\mathcal{A}_p = \frac{(n-p+1)(n-p+2)\ldots(n-p+k)}{p(p-1)\ldots(p-k+1)}\mathcal{A}_{p-k} \tag{16.39}$$

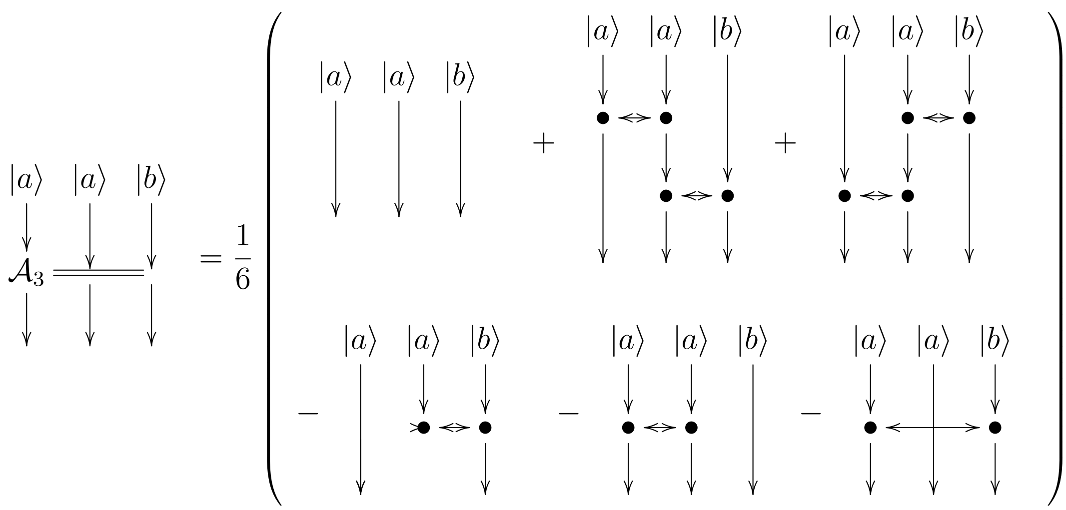
$$
\begin{aligned}
d_{\mathcal{A}_p} = \mathrm{tr}_{\underline{a}^p}\mathcal{A}_p &= \frac{\prod_{i=n-p+1}^{n} i}{p!} && (16.40)\\
&= \frac{\prod_{i=n}^{n-p+1} i}{p!} && (16.41)\\
&= \begin{cases} \frac{n!}{p!(n-p)!} = \binom{n}{p} & \text{if } p \leq n \\ 0 & \text{otherwise} \end{cases} && (16.42)
\end{aligned}
$$

For $p = 2 \leq n$,

$$d_{\mathcal{A}_2} = \binom{n}{2} \tag{16.43}$$

$$\mathcal{A}_p = 0 \text{ if } n < p \tag{16.44}$$

For example, for $n = 2$ and $p = 3$



(16.45)

$$
\begin{aligned}
\mathcal{A}_3|a,a,b\rangle &= \frac{1}{6}\begin{pmatrix} |a,a,b\rangle + |a,b,a\rangle + |b,a,a\rangle \\ -|a,b,a\rangle - |a,a,b\rangle - |b,a,a\rangle \end{pmatrix} && (16.46)\\
&= 0 && (16.47)
\end{aligned}
$$

Bibliography

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