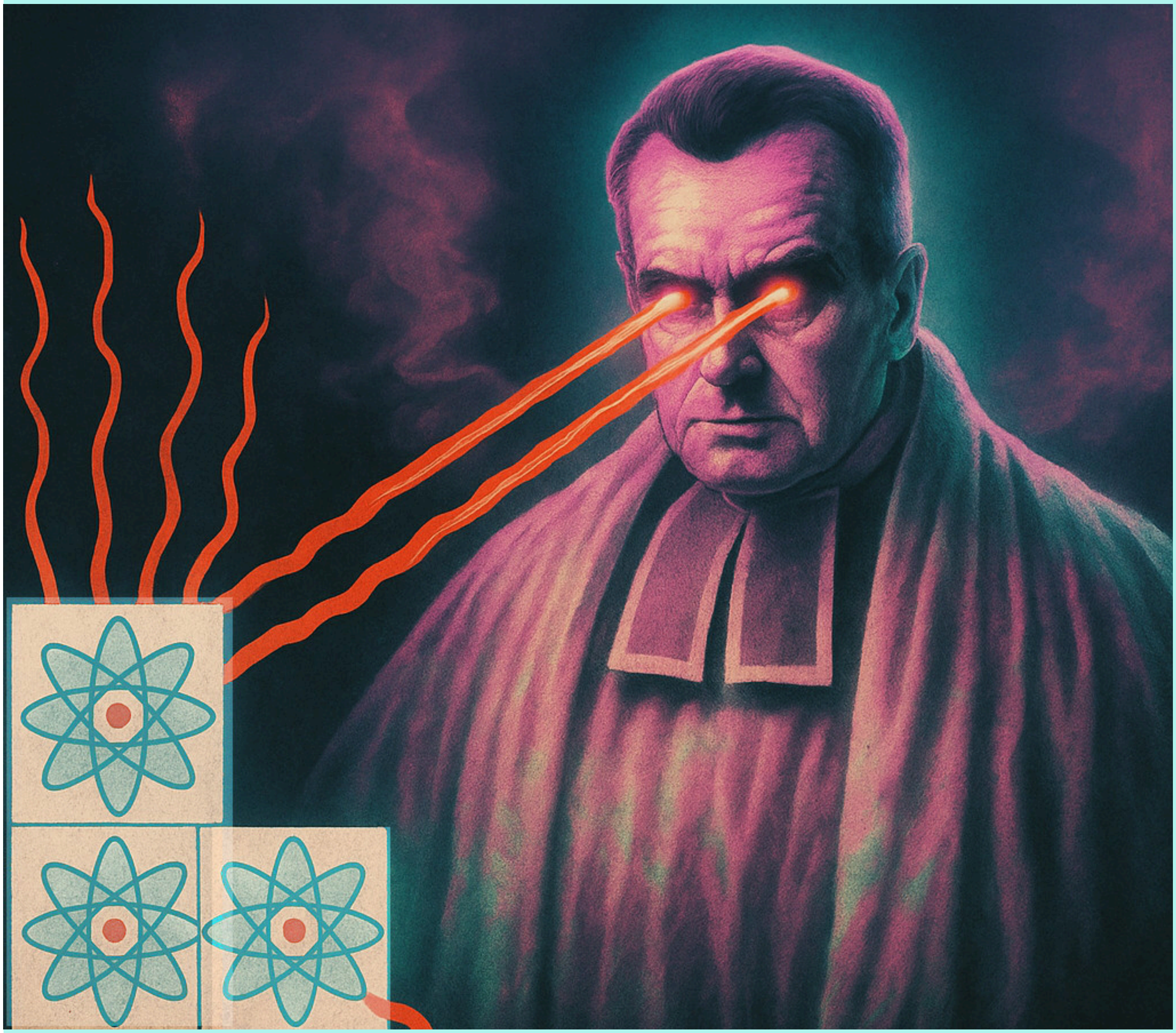
BAYESUVIUS QUANTICO

A VISUAL DICTIONARY OF QUANTUM BAYESIAN NETWORKS



ROBERT R. TUCCI

Bayesuvius Quantico,
a visual dictionary of Quantum Bayesian Networks

Robert R. Tucci
www.ar-tiste.xyz

July 19, 2025

This book is constantly being expanded and improved. To download the latest version, go to
`https://github.com/rrtucci/bayes-quantico`

Bayes Quantico
by Robert R. Tucci

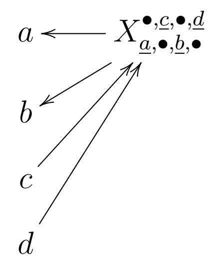
Contents

Chapter 2

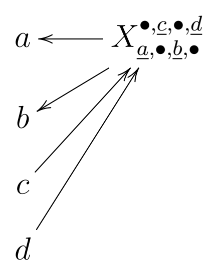
Birdtracks: COMING SOON

$$\delta(b,a) = \mathbb{1}(a=b) = \delta_a^b = \; a \leftarrow\!\bullet\!\!-\!\!- \; b \tag{2.1}$$

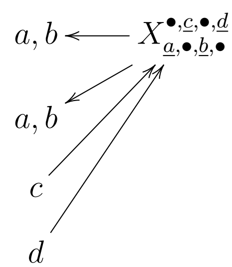
$$X^{\bullet,c,\bullet,d}_{a,\bullet,b,\bullet} = \tag{2.2}$$



$$\langle a,b|X^{\bullet,\underline{c},\bullet,\underline{d}}_{\underline{a},\bullet,\underline{b},\bullet}|c,d\rangle = \tag{2.3}$$

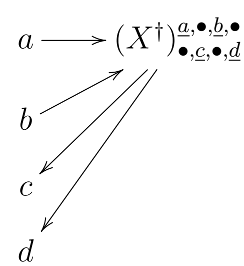


$\rightarrow$



$X^{\bullet,\underline{c},\bullet,\underline{d}}_{\underline{a},\bullet,\underline{b},\bullet} \in V^2 \otimes V_2$. Sometimes, we will omit denote this node simply b y X. This if okay as long as we are not using, X to also denote a different version of $X^{\underline{a},\bullet,\underline{b},\bullet}_{\bullet,\underline{c},\bullet,\underline{d}}$ with some of the indices raised or lowered or their order has been changed.

$$(X^\dagger)^{a,\bullet,b,\bullet}_{\bullet,c,\bullet,d} = \tag{2.4}$$



$$
(X^\dagger)^{a,\bullet,b,\bullet}_{\bullet,c,\bullet,d} X^{\bullet,c,\bullet,d}_{a,\bullet,b,\bullet} \quad = \quad
\begin{tikzcd}
(X^\dagger)^{\underline{a},\bullet,\underline{b},\bullet}_{\bullet,\underline{c},\bullet,\underline{d}} \arrow[dr] \arrow[ddr] \arrow[dddr] & \sum a \arrow[l] & X^{\bullet,\underline{c},\bullet,\underline{d}}_{\underline{a},\bullet,\underline{b},\bullet} \arrow[l] \arrow[dl] \\
 & \sum b \arrow[ul] & \\
 & \sum c \arrow[uur] & \\
 & \sum d \arrow[uuur] &
\end{tikzcd}
\tag{2.5}
$$

$$
= \quad
\begin{tikzcd}
X^\dagger \arrow[ddr] \arrow[dddr] & {} \arrow[l] & {} \arrow[l] & X \arrow[dll] \\
 & {} \arrow[ul] & & \\
 & {} \arrow[uurr] & & \\
 & {} \arrow[uuurr] & &
\end{tikzcd}
\tag{2.6}
$$

$a^m \in \mathbb{Z}^m_+$

$$
R^{a_3^{m_3},b_2^{n_2}}_{b_3^{n_3},a_2^{m_2}} S^{a_2^{m_2},b_1^{n_1}}_{b_2^{n_2},a_1^{m_1}} =
\begin{tikzcd}
b_3^{n_3} & R \arrow[l] \arrow[dr] & \sum b_2^{n_2} \arrow[l] & S \arrow[l] \arrow[dr] & b_1^{n_1} \arrow[l] \\
a_3^{m_3} \arrow[ur] & & \sum a_2^{m_2} \arrow[ur] & & a_1^{m_1}
\end{tikzcd}
\tag{2.7}
$$

$$
\mathrm{tr}_{\underline{b}} X^{\bullet,\underline{b},\bullet,d}_{a,\bullet,\underline{b},\bullet} = \sum_b X^{\bullet,b,\bullet,d}_{a,\bullet,b,\bullet} =
\begin{tikzcd}
a & X^{\bullet,\underline{c},\bullet,\underline{d}}_{\underline{a},\bullet,\underline{b},\bullet} \arrow[l] \arrow[dl] \\
{} & \\
{} \arrow[uur] & \\
d \arrow[uuur] &
\end{tikzcd}
\tag{2.8}
$$

$$
\begin{tikzcd}
 & {} \arrow[d] \arrow[rr, dash, red] & & {} & \\
{} & R \arrow[l] \arrow[dr] & & S \arrow[u] \arrow[dr] & {} \arrow[l] \\
{} \arrow[ur] & & {} \arrow[ur] & & {}
\end{tikzcd}
\tag{2.9}
$$

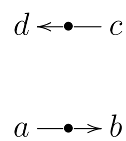
Chapter 19

Unitary Groups: COMING SOON

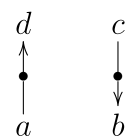
19.1 SU(n)

$$m(p,q) = \delta^a_b \sum_{a=1}^{n} (p_a)^* q_a \tag{19.1}$$

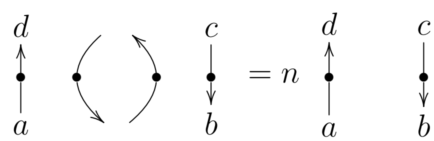
$$\mathbb{1}^{a,c}_{d,b} = \delta^a_b \delta^c_d = \tag{19.2}$$



$$\mathbb{X}^{a,c}_{d,b} = \delta^a_d \delta^c_b = \tag{19.3}$$



$$\mathbb{X}^2 = n\mathbb{X} \tag{19.4}$$

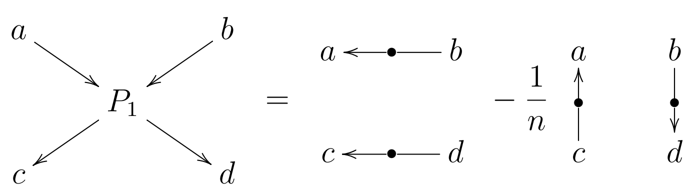


(19.5)

$$P_i = \sum_{j \neq i} \frac{M - \lambda_j}{\lambda_i - \lambda_j} \tag{19.6}$$

$\lambda_1 = n$

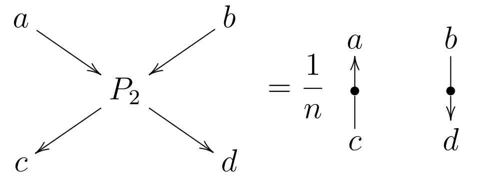
$$P_1 = \frac{\mathbb{X} - n}{0 - n} = 1 - \frac{1}{n}\mathbb{X} \tag{19.7}$$



(19.8)

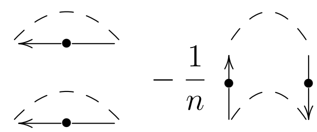
$\lambda_2 = 0$

$$P_2 = \frac{\mathbb{X} - 0}{n - 0} = \frac{1}{n}\mathbb{X} \tag{19.9}$$



(19.10)

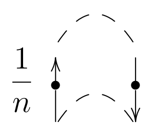
$\mathrm{tr}P_1 \;=$



(19.11)

$$= \; n^2 - 1 \tag{19.12}$$

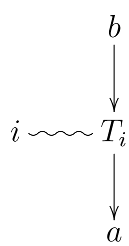
$\mathrm{tr}P_2 \;=$



(19.13)

$$= \; 1 \tag{19.14}$$

$(T_i)^b_a =$



(19.15)

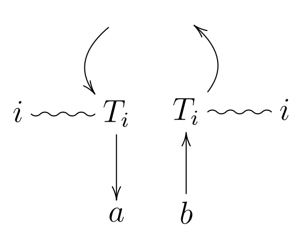
$$T_i^\dagger = T_i \tag{19.16}$$

Claim 1

$$C_F \delta^b_a = (T_i T_i)^b_a = \frac{n^2 - 1}{n}\delta^b_a \tag{19.17}$$

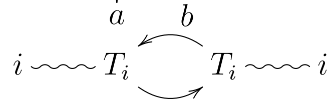
proof:

$(T_i T_i)^b_a \;=\; \sum_i$



(19.18)

$= \; \sum_i$



(19.19)

QED

Bibliography