

BAYESUVIUS QUANTICO

A VISUAL DICTIONARY OF
QUANTUM BAYESIAN NETWORKS



ROBERT R. TUCCI

Bayesuvious Quantico, a visual dictionary of Quantum Bayesian Networks

Robert R. Tucci
www.ar-tiste.xyz

August 14, 2025

This book is constantly being expanded and improved. To download
the latest version, go to

<https://github.com/rrtucci/bayes-quantico>

Bayesuvius Quantico

by Robert R. Tucci

Copyright ©2025, Robert R. Tucci.

This work is licensed under the Creative Commons Attribution-Noncommercial-No
Derivative Works 3.0 United States License. To view a copy of this license, visit the
link <https://creativecommons.org/licenses/by-nc-nd/3.0/> or send a letter to
Creative Commons, PO Box 1866, Mountain View, CA 94042.

Contents

Appendices	4
A Notational Conventions and Preliminaries	5
A.1 Set notation	5
A.2 Group	5
A.3 Group Representation	6
A.4 Group Theory References	6
A.5 Vector Space and Algebra over a field \mathbb{F}	7
A.6 Tensors	8
A.7 Permutations	10
B Birdtracks	12
B.1 Classical Bayesian Networks and their Instantiations	12
B.2 Quantum Bayesian Networks and their Instantiations	14
B.3 Birdtracks	15
1 Casimir Operators: COMING SOON	19
2 Clebsch-Gordan Coefficients	20
3 Determinants: COMING SOON	23
4 Dynkin Diagrams: COMING SOON	24
5 General Relativity Nets: COMING SOON	25
6 Group Integrals: COMING SOON	26
7 Invariants	27
8 Lie Algebras	31
8.1 Generators (infinitesimal transformations)	31
8.2 Clebsch-Gordan matrices	34
8.3 Structure Constants (3 gluon vertex)	35
8.4 Two types of gluon exchanges	38

9	Orthogonal Groups: COMING SOON	39
10	Quantum Shannon Information Theory: COMING SOON	40
11	Recoupling Equations: COMING SOON	41
12	Reducibility	42
12.1	Eigenvalue Projectors	42
12.2	$[P_i, M] = 0$ consequences	44
12.3	$[G, M] = 0$ consequences	45
13	Spinors: COMING SOON	47
14	Squashed Entanglement: COMING SOON	48
15	Symplectic Groups: COMING SOON	49
16	Symmetrization and Antisymmetrization	50
16.1	Symmetrizer	50
16.2	Antisymmetrizer	54
16.3	Levi-Civita Tensor	58
17	Unitary Groups: COMING SOON	60
17.1	$SU(n)$	60
18	Wigner Coefficients: COMING SOON	62
19	Wigner-Ekart Theorem: COMING SOON	63
20	Young Tableau: COMING SOON	64
	Bibliography	65

Appendices

Chapter 8

Lie Algebras

This chapter is based on Ref.[1].

8.1 Generators (infinitesimal transformations)

For some group \mathcal{G} , assume that any group element $G \in \mathcal{G}$ that is infinitesimal close to the identity 1 can be parametrized by

$$G = 1 + i \sum_i \epsilon_i T^i \quad (8.1)$$

where $T^i \in \mathbb{C}^{n \times n}$ for $i = 1, 2, \dots, N$, $\epsilon_i \in \mathbb{R}$ and $|\epsilon_i| \ll 1$.

Assume that the T^i matrices are Hermitian and that they satisfy

$$\text{tr}(T^i T^j) = K \delta(i, j) \quad (8.2)$$

We will call these matrices the **generators** of infinitesimal transformations for group \mathcal{G} .

It's customary to choose generators so that $K = \frac{1}{2}$.¹ However, we will often set $K = 1$ for intermediate calculations and restore $K \neq 1$ at the end by dimensional analysis. Just remember that each T^j scales as \sqrt{K} . For example, given the equation $\text{tr}(T^i T^j) = \delta(i, j)$, we know that when $K \neq 1$, $\text{tr}(T^i T^j) = K \delta(i, j)$ so both sides of the equation scale as K .

We will use the following scaled version of T^j as a birdtrack. Define

¹For $SU(2)$, it is customary to choose $T^i = \frac{1}{2} \sigma_i$, where σ_i for $i = 1, 2, 3$ are the Pauli matrices. For $SU(3)$, it is customary to choose $T^i = \frac{1}{2} \lambda_i$ where λ_i for $i = 1, 2, \dots, 8$ are the Gell-Mann matrices. For both of these choices, $K = \frac{1}{2}$.

$$(C_{Adj}^i)_b^a = \frac{1}{\sqrt{K}}(T^i)_b^a = \frac{1}{\sqrt{K}} \quad i \sim T^i \begin{array}{c} a \\ \downarrow \\ b \end{array} \quad (8.3)$$

In the CC convention, we will always start reading the indices of this node at the wavy undirected leg.

Adj stands the Adjoint. In this node (vertex), an adjoint representation (adrep) particle (wavy line, gluon) is generated (released) by a fundamental representation (funrep) particle (straight solid line, arrow).

In terms of birdtracks, Eq.(8.2) becomes

$$\boxed{(T^i)_a^b (T^j)_b^a = \text{tr}(T^i T^j) = \delta(i, j)} \quad i \sim T^i \begin{array}{c} \xrightarrow{\Sigma^a} \\ \xleftarrow{\Sigma^b} \end{array} T^j \sim j = \leftarrow \bullet \quad (8.4)$$

We can now define the projection operator for the adrep (gluon exchange between 2 funrep particles)

$$\boxed{(P_{Adj})_b^a{}_d^c = \sum_i (T^i)_b^a (T^i)_d^c} \quad \begin{array}{c} b \swarrow \quad \nwarrow c \\ P_{Adj} \\ a \nearrow \quad \searrow d \end{array} = \begin{array}{c} b \quad c \\ \uparrow \quad \downarrow \\ a \sim \Sigma i \sim d \end{array} \quad (8.5)$$

The arrow that starts with a bar as in $\leftarrow \bullet$ indicates this is the first index in the CC convention.

Note that if $x \in V^n \otimes V^{\dagger n}$, then

$$(P_{Adj})_b^a{}_d^c x_c^d = \sum_i (T^i)_b^a \underbrace{[(T^i)_d^c x_c^d]}_{\epsilon_i \in \mathbb{R}} \quad (8.6)$$

Recall Eq.(A.24). If $x \in V^{n^p} \otimes V^{\dagger n^q}$, and $\mathbb{G} \in \mathcal{G} \subset GL(n^{p+q}, \mathbb{C})$,

$$(x')_{a:p}{}^{b:q} = \mathbb{G}_{a:p}{}^{b:q}{}_{rev(c:q)}{}^{rev(d:p)} x_{d:p}{}^{c:q}, \quad x'_\alpha = \mathbb{G}_\alpha^\beta x_\beta \quad (8.7)$$

where we define

$$\mathbb{G}_\alpha^\beta \stackrel{\text{def}}{=} \prod_{i=1}^p G_{a_i}^{d_i} \prod_{i=1}^q G^{\dagger b_i}_{c_i} \quad (8.8)$$

If \mathbb{G} is infinitesimally close to the identity, then we can parametrize it as

$$\mathbb{G}_\alpha^\beta = 1 + i \sum_j \epsilon_j (\mathbb{T}^j)_\alpha^\beta \quad (8.9)$$

$$G_{a_i}^{d_i} = 1 + i \sum_j \epsilon_j (T^j)_{a_i}^{d_i} \quad (8.10)$$

$$G^{\dagger b_i}_{c_i} = 1 - i \sum_j \epsilon_j (T^j)^{b_i}_{c_i} \quad (8.11)$$

Define

$$(\mathbb{T}^j)_\alpha^\beta = \left[(T^j)_{a_i}^{d_i} \frac{1}{\delta_{a_i}^{d_i}} - (T^j)^{b_i}_{c_i} \frac{1}{\delta_{c_i}^{b_i}} \right] \delta_{a:p}^{d:p} \delta_{c:q}^{b:q} \quad (8.12)$$

When $x'_\alpha = x_\alpha$, to first order in ϵ_i ,

$$0 = (\mathbb{T}^j)_\alpha^\beta x_\beta = \left[(T^j)_{a_i}^{d_i} \frac{1}{\delta_{a_i}^{d_i}} - (T^j)^{b_i}_{c_i} \frac{1}{\delta_{c_i}^{b_i}} \right] \delta_{a:p}^{d:p} \delta_{c:q}^{b:q} x_{d:p}^{c:q} \quad (8.13)$$

For example, if we define

$(\mathbb{T}^j)_{a_1 a_2}^{b_1 c_1 d_2 d_1} = (T^j)_{a_1}^{d_1} \delta_{a_2}^{d_2} \delta_{c_1}^{b_1} + \delta_{a_1}^{d_1} (T^j)_{a_2}^{d_2} \delta_{c_1}^{b_1} - \delta_{a_1}^{d_1} \delta_{a_2}^{d_2} (T^j)^{b_1}_{c_1}$

$$\begin{array}{c} \begin{array}{c} a_1 \swarrow \quad \nwarrow d_1 \\ a_2 \leftarrow \mathbb{T}^j \rightarrow d_2 \\ b_1 \nearrow \quad \searrow c_1 \end{array} = \begin{array}{c} \begin{array}{c} a_1 \leftarrow T^j \leftarrow d_1 \\ a_2 \leftarrow \quad d_2 \\ b_1 \longrightarrow c_1 \end{array} + \begin{array}{c} \begin{array}{c} a_1 \leftarrow \quad d_1 \\ a_2 \leftarrow T^j \leftarrow d_2 \\ b_1 \longrightarrow c_1 \end{array} - \begin{array}{c} \begin{array}{c} a_1 \leftarrow \quad d_1 \\ a_2 \leftarrow \quad d_2 \\ b_1 \rightarrow T^j \rightarrow c_1 \end{array} \end{array} \end{array} \quad (8.14)$$

then

$0 = (\mathbb{T}^j x)_{a_1 a_2}^{b_1} = \left[(T^j)_{a_1}^{d_1} \delta_{a_2}^{d_2} \delta_{c_1}^{b_1} + \delta_{a_1}^{d_1} (T^j)_{a_2}^{d_2} \delta_{c_1}^{b_1} - \delta_{a_1}^{d_1} \delta_{a_2}^{d_2} (T^j)^{b_1}_{c_1} \right] x_{d_1 d_2}^{c_1}$

$$0 = \begin{array}{c} \begin{array}{c} a_1 \swarrow \quad \nwarrow x \\ a_2 \leftarrow \mathbb{T}^j \rightarrow \parallel \\ b_1 \nearrow \quad \searrow \parallel \end{array} = \begin{array}{c} \begin{array}{c} a_1 \leftarrow T^j \leftarrow x \\ a_2 \leftarrow \quad \parallel \\ b_1 \longrightarrow \parallel \end{array} + \begin{array}{c} \begin{array}{c} a_1 \leftarrow \quad x \\ a_2 \leftarrow T^j \leftarrow \parallel \\ b_1 \longrightarrow \parallel \end{array} - \begin{array}{c} \begin{array}{c} a_1 \leftarrow \quad x \\ a_2 \leftarrow \quad \parallel \\ b_1 \rightarrow T^j \rightarrow \parallel \end{array} \end{array} \end{array} \quad (8.15)$$

8.2 Clebsch-Gordan matrices

The Clebsch Gordan (CG) matrices are introduced in Ch.2. Note that the generators $(T^i)_a^b$ are a simple kind of CG matrix, one with

- a gluon (adrep) particle instead of a general λ rep particle emanating from the i index,
- a particle of the funrep entering and another leaving the node, instead of any number of funrep particles entering and leaving.

Since $\mathbb{G} = 1 + i \sum_j \epsilon_j \mathbb{T}^j$, generators decompose in the same way as the group elements

$$\boxed{\mathbb{T}^j = \sum_{\lambda} C_{\lambda}^{\dagger} T_{\lambda}^j C_{\lambda}}$$

$$\begin{array}{c} j \\ \vdots \\ \swarrow \mathbb{T}^j \nwarrow \\ \leftarrow \mathbb{T}^j \rightarrow \\ \swarrow \searrow \end{array} = \sum_{\lambda} \begin{array}{c} j \\ \vdots \\ \swarrow C_{\lambda}^{\dagger} \nwarrow T_{\lambda}^j \nwarrow C_{\lambda} \leftarrow \\ \leftarrow C_{\lambda}^{\dagger} \rightarrow T_{\lambda}^j \rightarrow C_{\lambda} \rightarrow \\ \swarrow \searrow \end{array} \quad (8.16)$$

The CG matrices are matrix invariants.

$$C_\lambda = G_\lambda^\dagger C_\lambda G \quad (8.17)$$

Hence,

$$0 = -T_\lambda^j C_\lambda + C_\lambda T^j \quad (8.18)$$

$$0 = \left\{ \begin{array}{l} - \text{Diagram 1} + \text{Diagram 2} \\ + \text{Diagram 3} - \text{Diagram 4} \end{array} \right\} \quad (8.19)$$

Multiplying on the left by C_λ^\dagger , we obtain an expression for the generator T_λ^i in term the generators T^j (and C_λ CG matrices).

$$\begin{aligned}
& \begin{array}{c} j \\ \textcolor{red}{\text{~}} \\ a \leftarrow T_\lambda^j \leftarrow a' \end{array} = \\
& \begin{array}{c} j \\ \textcolor{red}{\text{~}} \\ T^j \\ \swarrow \quad \searrow \\ a \leftarrow C_\lambda \quad C_\lambda^\dagger \leftarrow a' \end{array} + \begin{array}{c} j \\ \textcolor{red}{\text{~}} \\ \swarrow \quad \searrow \\ a \leftarrow C_\lambda \quad T^j \quad C_\lambda^\dagger \leftarrow a' \end{array} - \begin{array}{c} j \\ \textcolor{red}{\text{~}} \\ \swarrow \quad \searrow \\ a \leftarrow C_\lambda \quad C_\lambda^\dagger \leftarrow a' \\ \quad \quad \quad \swarrow \quad \searrow \\ \quad \quad \quad T^j \end{array} \quad (8.20)
\end{aligned}$$

8.3 Structure Constants (3 gluon vertex)

See Sec.A.5 for the definition of an algebra over a field.

$$\underbrace{T^i T^j - T^j T^i}_{[T^i, T^j]} = i f_{ijk} T^k \quad (\text{Lie Algebra commutation relations})$$

$$\begin{array}{c} i \quad j \\ \textcolor{red}{\text{~}} \quad \textcolor{red}{\text{~}} \\ a \leftarrow T^i \leftarrow T^j \leftarrow c \end{array} - \begin{array}{c} i \quad j \\ \textcolor{red}{\text{~}} \quad \textcolor{red}{\text{~}} \\ a \leftarrow T^j \leftarrow T^i \leftarrow c \end{array} = i \begin{array}{c} i \quad j \\ \textcolor{red}{\text{~}} \quad \textcolor{red}{\text{~}} \\ f_{ijk} \\ \textcolor{red}{\text{~}} \\ a \leftarrow T^k \leftarrow c \end{array} \quad (8.21)$$

The f_{ijk} tensors are called the **structure constants** of the Lie Algebra. They define a 3 gluon vertex in term of the generators T^i .²

If $(T^j)_a^b$ are the matrix rep (the funrep) of the generators of a group \mathcal{G} , then Eq.(8.21) shows that the matrices $(M^k)_{ij} = -i f_{ijk}$ are also a matrix rep (the adrep) of the generators of \mathcal{G} .

Since $\text{tr}(T^k T^{k'}) = \delta(k, k')$, Eq.(8.21) implies

$$\text{tr}([T^i, T^j] T^k) = i f_{ijk} \quad (8.22)$$

²It's possible to distinguish between upper and lower gluon indices (i.e., to give the gluon arrows a direction. In that case, the Lie Algebra commutation relations would be $[T^i, T^j] = f^{ij}_k T^k$ and the gluon indices could be lowered and raised using the metric $g_{ij} = \text{tr}(T^i T^j)$. But since we are assuming $g_{ij} = K \delta_i^j$, there is no need to do this.

$$\boxed{if_{ijk} = \text{tr}([T^i, T^j]T^k) = (T^i)_a{}^c (T^j)_b{}^a (T^k)_c{}^b - (T^j)_c{}^b (T^i)_a{}^c (T^k)_b{}^a}$$

(8.23)

Note that

$$\boxed{f_{ijk} = -f_{jik}}$$

(8.24)

In fact, the tensor f_{ijk} is **totally antisymmetric** (i.e., it changes sign under a transposition of any two indices).

Claim 1 f_{ijk} is a real number.

proof:

$$\left[i \text{tr}([T^i, T^j]T^k) \right]^\dagger = (-i) \text{tr}(T^k [T^j, T^i]) \quad (8.25)$$

$$= (-i) \text{tr}([T^j, T^i]T^k) \quad (8.26)$$

$$= i \text{tr}([T^j, T^k]T^i) \quad (8.27)$$

QED

Note that the birdtrack for the Lie Algebra commutation relations Eq.(8.21) can be understood as the statement that the generators T^j are matrix invariants. Below we restate Eq.(8.21) to make that obvious

$$0 = \begin{array}{c} i \quad j \\ \text{red wavy} \quad \text{wavy} \\ a \leftarrow T^i \leftarrow T^j \leftarrow c \end{array} - \begin{array}{c} i \quad j \\ \text{red wavy} \quad \text{wavy} \\ a \leftarrow T^j \leftarrow T^i \leftarrow c \end{array} - i \begin{array}{c} i \quad j \\ \text{red wavy} \quad \text{wavy} \\ f_{ijk} \\ \text{wavy} \\ a \leftarrow T^k \leftarrow c \end{array} \quad (8.28)$$

Claim 2

$$\boxed{f_{ijm}f_{mkl} - f_{ljm}f_{mki} = f_{iml}f_{jkm} \quad (\text{Jacobi identity})}$$

$$\begin{array}{c} i \quad l \\ \text{wavy} \quad \text{wavy} \\ f_{ijm} \sim \sum m \sim f_{mkl} \\ \text{wavy} \quad \text{wavy} \\ j \quad k \end{array} - \begin{array}{c} i \quad l \\ \text{wavy} \quad \text{wavy} \\ f_{ijm} \sim \sum m \sim f_{mkl} \\ \text{wavy} \quad \text{wavy} \\ j \quad k \end{array} = \begin{array}{c} i \quad l \\ \text{wavy} \quad \text{wavy} \\ f_{iml} \\ \sum m \\ f_{jkm} \\ \text{wavy} \quad \text{wavy} \\ j \quad k \end{array} \quad (8.29)$$

proof:

Note that

$$\text{tr} \left([[T^i, T^j], T^k] T^l \right) = \text{tr} \left(f_{ijm} [T^m, T^k] \right) \quad (8.30)$$

$$= \text{tr} \left(f_{ijm} f_{mkl'} T^{l'} T^l \right) \quad (8.31)$$

$$= f_{ijm} f_{mkl} \quad (8.32)$$

so the Jacobi identity can be restated as

$$\text{tr} \left(\left\{ [[T^i, T^j], T^k] + [[T^j, T^k], T^i] + [[T^k, T^i], T^j] \right\} T^l \right) = 0 \quad (8.33)$$

Hence, the claim follows if we can prove that

$$\underbrace{[[T^i, T^j], T^k] + [[T^j, T^k], T^i] + [[T^k, T^i], T^j]}_{\text{cyclic permutations of } ijk} = 0 \quad (8.34)$$

If we expand the left hand side on Eq.(8.34), we find 6 terms that cancel in pairs.

QED

Note Claim 2 can be understood as the Lie Algebra commutation relations Eq.(8.21), but stated in the adrep instead of the funrep. Indeed, if

$$\mathbb{T}_{jk}^i = -iC_{ijk} \quad (8.35)$$

then Claim 2 becomes

$$(\mathbb{T}^i \mathbb{T}^l - \mathbb{T}^l \mathbb{T}^i)_{jk} = iC_{ilm}(\mathbb{T}^m)_{jk} \quad (8.36)$$

Note that Claim 2 can be understood as a statement of the fact that f_{ijk} is a tensor invariant.

$0 = f_{ijm}f_{mkl} - f_{ljm}f_{mki} - f_{iml}f_{jkm}$

(8.37)

8.4 Two types of gluon exchanges

$(P_{Adj})_a^b{}_c^d = \sum_i (T^i)_a^b (T^i)_c^d$

(8.38)

$\mathbb{P}_a^b{}_\beta^\gamma = \sum_i (T^i)_a^b (T_\lambda^i)_\beta^\gamma$

(8.39)

$$0 = \left\{ \begin{array}{c} \begin{array}{ccc} a & & b \\ & \searrow & \nearrow \\ i & \text{wavy red line} & \\ \beta \leftarrow T_\lambda^i \leftarrow \mathbb{P} \leftarrow \gamma & & \\ a \rightarrow T^i \rightarrow \mathbb{P} \rightarrow b & & \\ & \nearrow & \searrow \\ \beta & & \gamma \end{array} & - & \begin{array}{ccc} a & & b \\ & \searrow & \nearrow \\ i & \text{wavy red line} & \\ \beta \leftarrow \mathbb{P} \leftarrow T_\lambda^i \leftarrow \gamma & & \\ a \rightarrow \mathbb{P} \rightarrow T^i \rightarrow b & & \\ & \nearrow & \searrow \\ \beta & & \gamma \end{array} \\ - i & & + i \\ \begin{array}{ccc} \beta & & \gamma \end{array} & & \begin{array}{ccc} \beta & & \gamma \end{array} \end{array} \right\} \quad (8.40)$$

$$\boxed{\mathbb{P}_{d\beta}^c \nu \mathbb{P}_{b\nu}^a \gamma - \mathbb{P}_{b\beta}^a \nu \mathbb{P}_{d\nu}^c \gamma = (P_{Adj})_x^a d^c \mathbb{P}_{b\beta}^x \gamma - \mathbb{P}_{x\beta}^a \gamma (P_{Adj})_b^x d^c}$$

$$\begin{array}{ccc} \begin{array}{ccc} a & & b \\ & \searrow & \nearrow \\ c & & d \\ \beta \leftarrow \mathbb{P} \leftarrow \mathbb{P} \leftarrow \gamma & & \\ a \rightarrow T^i \rightarrow \mathbb{P} \rightarrow b & & \\ c \rightarrow T^i \rightarrow d & & \\ & \nearrow & \searrow \\ \beta & & \gamma \end{array} & - & \begin{array}{ccc} a & & b \\ & \searrow & \nearrow \\ c & & d \\ \beta \leftarrow \mathbb{P} \leftarrow \mathbb{P} \leftarrow \gamma & & \\ a \rightarrow \mathbb{P} \rightarrow T^j \rightarrow b & & \\ c \rightarrow T^j \rightarrow d & & \\ & \nearrow & \searrow \\ \beta & & \gamma \end{array} = \end{array} \quad (8.41)$$

$$\sum_{i,j} (T_\lambda^i)_\beta \nu (T_\lambda^j)_\nu \gamma \left[(T^i)_d^c (T^j)_b^a - (T^i)_b^a (T^j)_d^c \right] = \sum_k (T_\lambda^k)_\beta \gamma \left[\begin{array}{c} (T^k)_b^x (P_{Adj})_x^a d^c \\ -(P_{Adj})_b^x d^c (T^k)_x^a \end{array} \right] \quad (8.42)$$

$$= \sum_{k,i} (T^i)_d^c (T_\lambda^k)_\beta \gamma \left[\begin{array}{c} (T^k)_b^x (T^i)_x^a \\ -(T^i)_b^x (T^k)_x^a \end{array} \right] \quad (8.43)$$

$$\sum_{ijk} \frac{1}{2} i C_{ijk} (T_\lambda^k)_\beta \gamma \left[(T^i)_d^c (T^j)_b^a - (T^i)_b^a (T^j)_d^c \right] = \sum_k (T_\lambda^k)_\beta \gamma \left[\begin{array}{c} (T^k)_b^x (P_{Adj})_x^a d^c \\ -(P_{Adj})_b^x d^c (T^k)_x^a \end{array} \right] \quad (8.44)$$

$$\sum_{ij} \frac{1}{2} i C_{ijk} \left[(T^i)_d{}^c (T^j)_b{}^a - (T^i)_b{}^a (T^j)_d{}^c \right] = \left[\begin{array}{c} (T^k)_b{}^x (P_{Adj})_x{}^a{}_d{}^c \\ -(P_{Adj})_b{}^x{}_d{}^c (T^k)_x{}^a \end{array} \right] \quad (8.45)$$

$$T_d{}^c T_b{}^a - T_b{}^a T_d{}^c = (P_{Adj})_b{}^a{}_x{}^c T_d{}^x - T_x{}^c (P_{Adj})_b{}^a{}_x{}^c \quad (8.46)$$

$$T_d{}^c T_b{}^a - T_b{}^a T_d{}^c = T_d{}^a \delta_b{}^c - T_b{}^c \delta_d{}^a \quad (8.47)$$

For $SU(N)$,

$$(P_{Adj})_b{}^x{}_d{}^c = \delta_{bd}^{xc} - \delta_{db}^{xc} \quad (8.48)$$

$$T_d{}^c T_b{}^a - T_b{}^a T_d{}^c = T_d{}^a \delta_b{}^c - T_b{}^c \delta_d{}^a \quad (8.49)$$

Bibliography

- [1] Predrag Cvitanovic. *Group theory: birdtracks, Lie's, and exceptional groups*. Princeton University Press, 2008. <https://birdtracks.eu/course3/notes.pdf>.
- [2] JP Elliott and PG Dawber. *Symmetry in Physics, vols. 1, 2*. Springer, 1979.
- [3] Robert R. Tucci. Bayesuvius (free book). <https://github.com/rrtucci/Bayesuvius>.
- [4] Robert R. Tucci. Quantum Bayesian nets. *International Journal of Modern Physics B*, 09(03):295–337, January 1995.