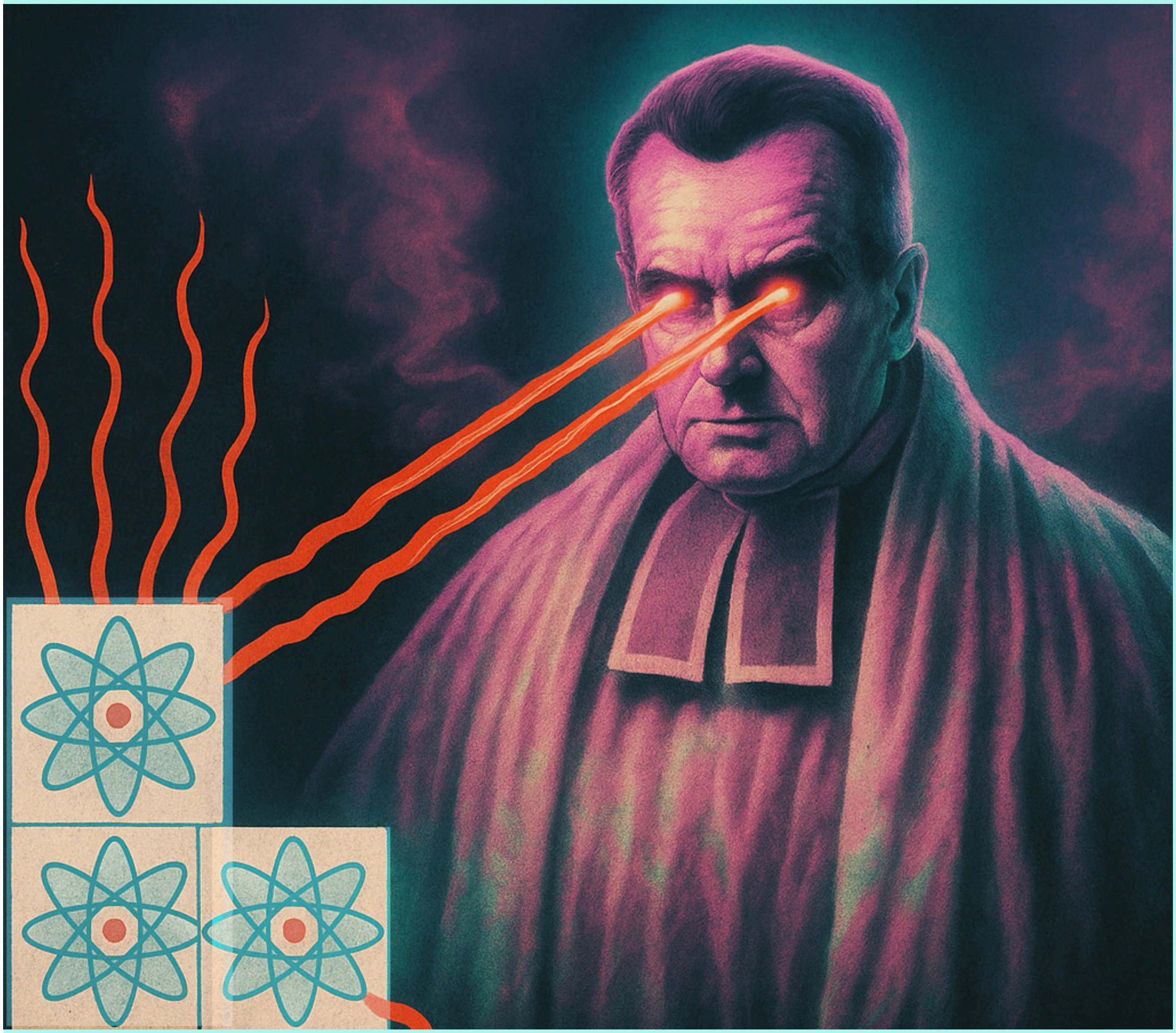
BAYESUVIUS QUANTICO

A VISUAL DICTIONARY OF QUANTUM BAYESIAN NETWORKS



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Bayesuvius Quantico,
a visual dictionary of Quantum Bayesian Networks

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This book is constantly being expanded and improved. To download the latest version, go to
`https://github.com/rrtucci/bayes-quantico`

Bayes Quantico
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$$
\begin{aligned}
P(y) &= \sum_x P(y|x)P(x) \qquad (7.1)\\
\langle y|\psi\rangle &= \sum_x \underbrace{\langle y|A|x\rangle}_{A(y|x)} \langle x|\psi\rangle \qquad (7.2)
\end{aligned}
$$

$$
\longleftarrow \;=\; \xleftarrow[\sum a]{} \;=\; \sum_a |a\rangle\langle a| \qquad (7.3)
$$

$$
\begin{aligned}
\langle a|q\rangle &= \sum_b \langle a|G|b\rangle \langle b|q\rangle \qquad (7.4)\\
q_a &= \sum_b G_a^b q_b \qquad (7.5)\\
\xleftarrow[a]{} q &= \xleftarrow[a]{} G \xleftarrow[\Sigma b]{} q \qquad (7.6)
\end{aligned}
$$

$$
\begin{aligned}
\langle q|a\rangle &= \sum_b \left\langle b|G^\dagger|a\right\rangle \langle q|b\rangle \qquad (7.7)\\
q^a &= \sum_b (G^\dagger)_b^a q^b \qquad (7.8)\\
q \xleftarrow[a]{} &= q \xleftarrow[\sum b]{} G^\dagger \xleftarrow[a]{} \qquad (7.9)
\end{aligned}
$$

$$
\xleftarrow[a]{} q \;=\; a \longleftarrow q \qquad (7.10)
$$

$$
q \xleftarrow[a]{} \;=\; q \longleftarrow a \qquad (7.11)
$$

$$G^{d,e}_{a,b,c} = \langle a,b,c|G|d,e\rangle = \begin{tikzcd} a & G \arrow[l] \arrow[dl] \arrow[ddl] & d \arrow[l] \\ b & & e \arrow[ul] \\ c & & \end{tikzcd} \tag{7.12}$$

$$\langle b_1, b_2|h|a_1, a_2\rangle = \left\langle G^\dagger b_1, G^\dagger b_2|h|Ga_1, Ga_2\right\rangle \tag{7.13}$$

$$\begin{tikzcd} a_1 \arrow[dr] & & a_2 \arrow[dl] \\ & h \arrow[dl] \arrow[dr] & \\ b_1 & & b_2 \end{tikzcd} = \begin{tikzcd} a_1 \arrow[d] & & a_2 \arrow[d] \\ G \arrow[dr] & & G \arrow[dl] \\ & h \arrow[dl] \arrow[dr] & \\ G^\dagger \arrow[d] & & G^\dagger \arrow[d] \\ b_1 & & b_2 \end{tikzcd} \tag{7.14}$$

$$G^a_b = \delta^a_b + i\sum_j \epsilon_j (T_j)^a_b \tag{7.15}$$

$$\xleftarrow[b]{} G \xleftarrow[a]{} \;=\; \xleftarrow[b]{} \delta \xleftarrow[a]{} \; + i\sum_j \epsilon_j \; \xleftarrow[b]{} T_j \xleftarrow[a]{} \tag{7.16}$$

Assume $T_j^\dagger = T_j$. To first order in ϵ_j,

$$0 = i\sum_j \epsilon_j \left(\begin{tikzcd}[column sep=small] a_1 \arrow[d] & & a_2 \arrow[d] \\ T_j \arrow[dr] & & \delta \arrow[dl] \\ & h \arrow[dl] \arrow[dr] & \\ \delta \arrow[d] & & \delta \arrow[d] \\ b_1 & & b_2 \end{tikzcd} + \begin{tikzcd}[column sep=small] a_1 \arrow[d] & & a_2 \arrow[d] \\ \delta \arrow[dr] & & T_j \arrow[dl] \\ & h \arrow[dl] \arrow[dr] & \\ \delta \arrow[d] & & \delta \arrow[d] \\ b_1 & & b_2 \end{tikzcd} \right) \tag{7.17}$$

$$- i\sum_j \epsilon_j \left(\begin{tikzcd}[column sep=small] a_1 \arrow[d] & & a_2 \arrow[d] \\ \delta \arrow[dr] & & \delta \arrow[dl] \\ & h \arrow[dl] \arrow[dr] & \\ T_j \arrow[d] & & \delta \arrow[d] \\ b_1 & & b_2 \end{tikzcd} + \begin{tikzcd}[column sep=small] a_1 \arrow[d] & & a_2 \arrow[d] \\ \delta \arrow[dr] & & \delta \arrow[dl] \\ & h \arrow[dl] \arrow[dr] & \\ \delta \arrow[d] & & T_j \arrow[d] \\ b_1 & & b_2 \end{tikzcd} \right) \tag{7.18}$$

from which we get one equation for each ϵ_j.

Bibliography