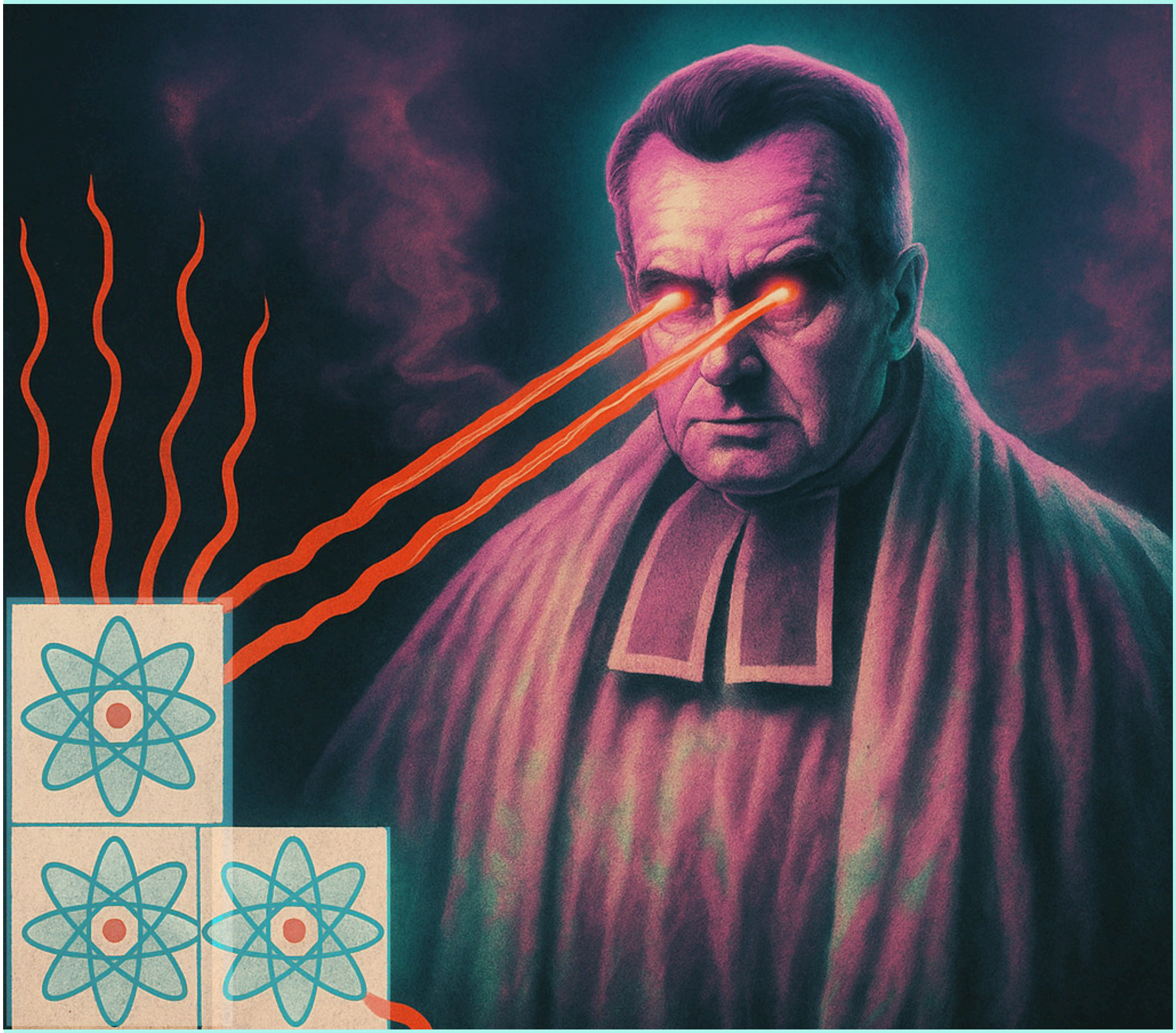
BAYESUVIUS QUANTICO

A VISUAL DICTIONARY OF QUANTUM BAYESIAN NETWORKS



ROBERT R. TUCCI

Bayesuvius Quantico,
a visual dictionary of Quantum Bayesian Networks

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This book is constantly being expanded and improved. To download the latest version, go to
`https://github.com/rrtucci/bayes-quantico`

Bayes Quantico
by Robert R. Tucci

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Appendices

Chapter 18

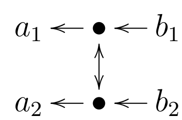
Symmetrization and Antisymmetrization: COMING SOON

$(1,2)$ transposition, swaps 1 and 2, $1 \to 2 \to 1$. $(3,2,1)$ means $3 \to 2 \to 1 \to 3$. A reordering of $(1,2,3,\ldots,p)$ is a permutation on p letters. A permutation can be expressed as a product of transpositions $(3,2,1) = (3,2)(2,1)$ is an even permutation because it can be expressed as a product of an even number of transpositions. An odd permutation can be expressed as a product of an odd number of permutations.

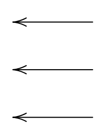
18.1 Symmetrization

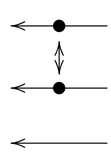
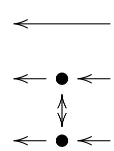
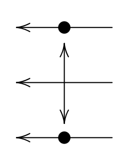
$$\mathbb{1}^{b_2,b_1}_{a_1,a_2} = \delta^{b_1}_{a_1}\delta^{b_2}_{a_2} = \begin{array}{c} a_1 \leftarrow b_1 \\ \\ a_2 \leftarrow b_2 \end{array} \tag{18.1}$$

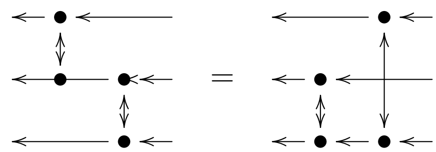
$$(\sigma_{(1,2)})^{b_2,b_1}_{a_1,a_2} = \delta^{b_2}_{a_1}\delta^{b_1}_{a_2} = \tag{18.2}$$

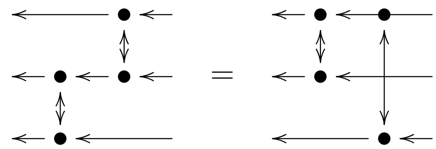


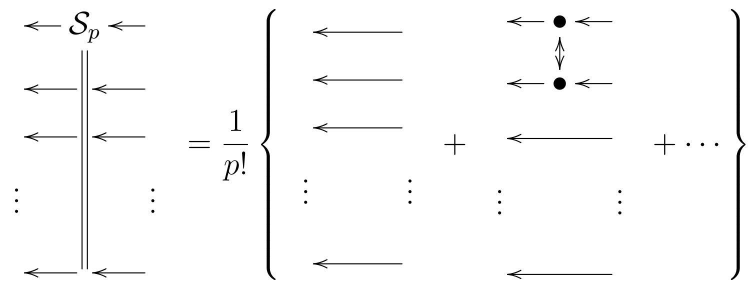
$$\mathbb{1} = \tag{18.3}$$



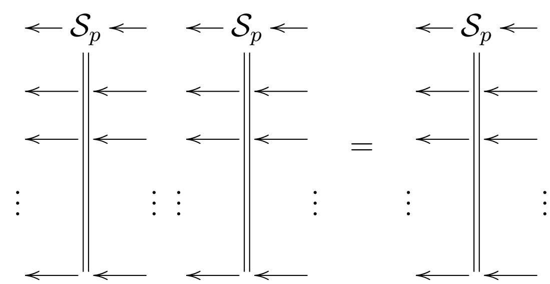
$\sigma_{(1,2)} =$  $\sigma_{(2,3)} =$  $\sigma_{(1,3)} =$  (18.4)

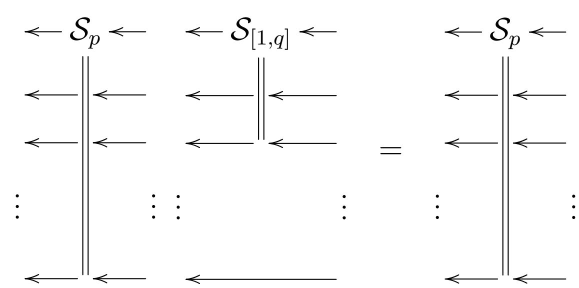
$\sigma_{(1,2,3)} =$  (18.5)

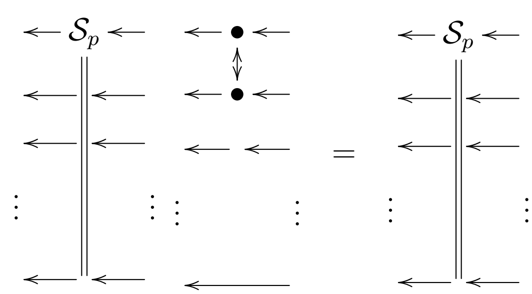
$\sigma_{(1,3,2)} =$  (18.6)

 (18.7)

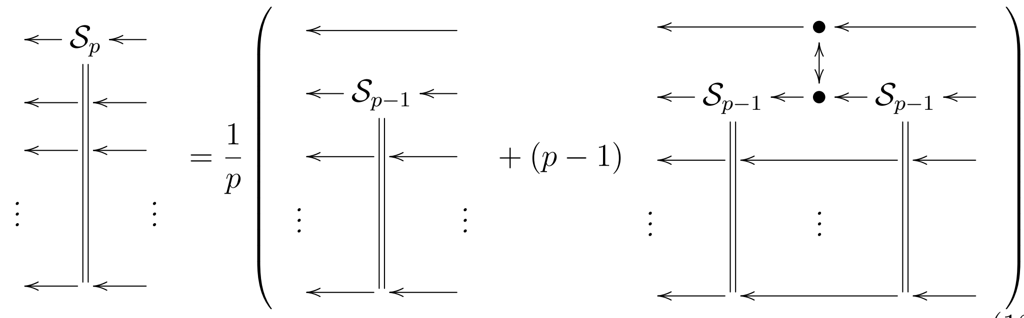
$$\mathcal{S}_p^2 = \mathcal{S}_p \tag{18.8}$$

 (18.9)

 (18.10)

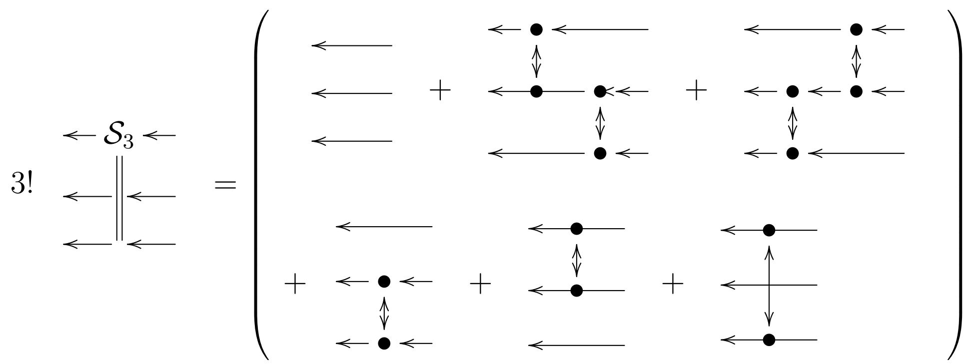
 (18.11)

Claim 1



(18.12)

proof: We only prove it for $p = 3$.



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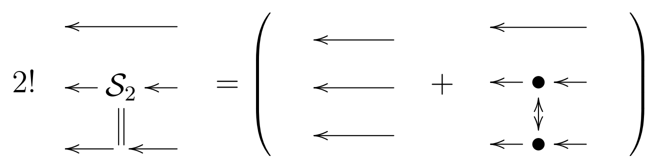

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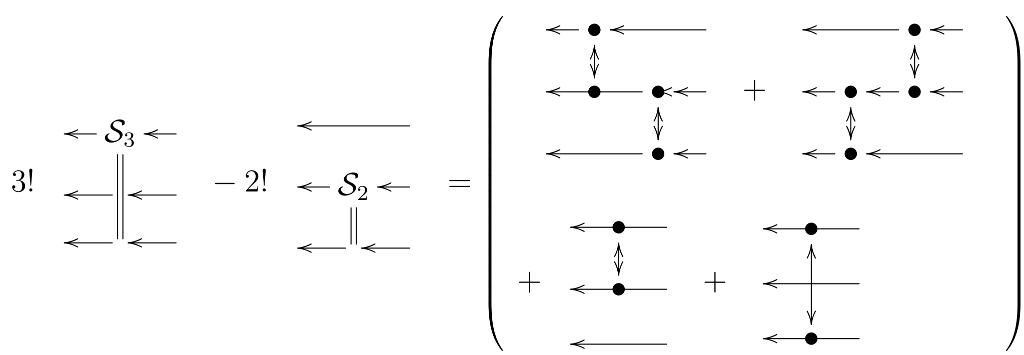
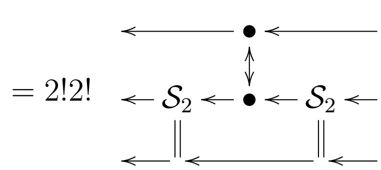
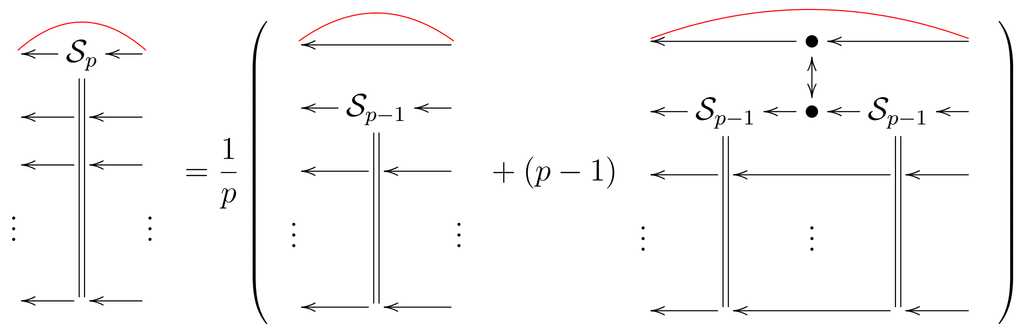


(18.15)

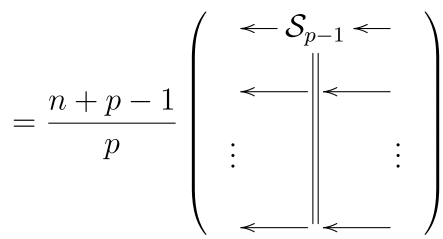


(18.16)

QED



(18.17)



(18.18)

$$\mathrm{tr}_{\underline{a}_1}\mathcal{S}_p = \frac{n+p-1}{p}\mathcal{S}_{p-1} \tag{18.19}$$

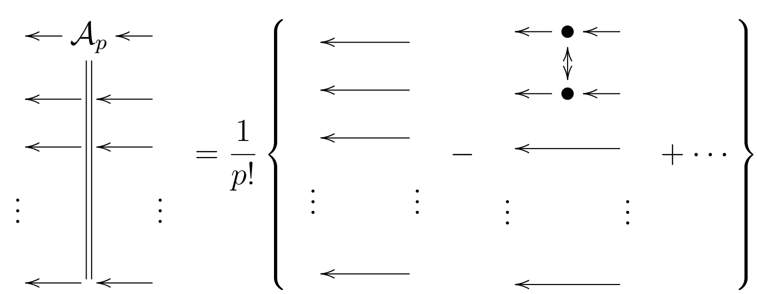
$$\mathrm{tr}_{\underline{a}_1,\underline{a}_2,\ldots,\underline{a}_k}\mathcal{S}_p = \frac{(n+p-1)(n+p-2)\ldots(n=p-k)}{p(p-1)\ldots(p-k+1)}\mathcal{S}_{p-k} \tag{18.20}$$

$$d_{\mathcal{S}_p} = \mathrm{tr}_{\underline{a}^p}\mathcal{S}_p = \frac{(n+p-1)!}{p!(n-1)!} = \binom{n+p-1}{p} \tag{18.21}$$

For $p = 2$,

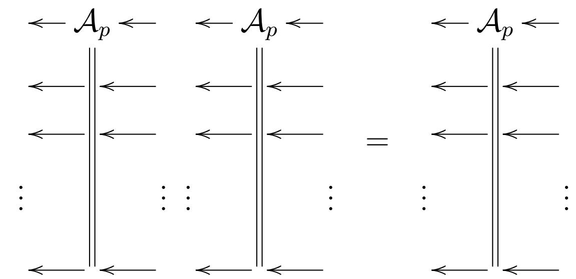
$$d_{\mathcal{S}_2} = \frac{(n+1)n}{2} \tag{18.22}$$

18.2 Antisymmetrization

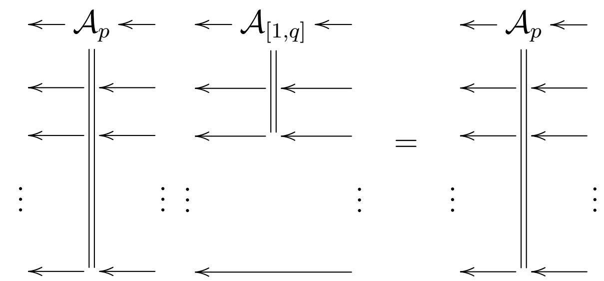


(18.23)

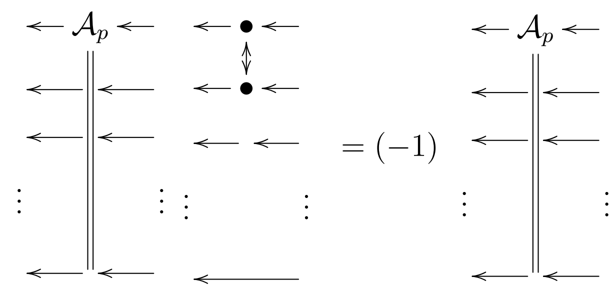
$$\mathcal{A}_p^2 = \mathcal{A}_p \tag{18.24}$$



(18.25)

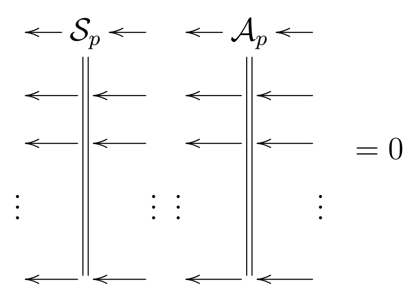


(18.26)

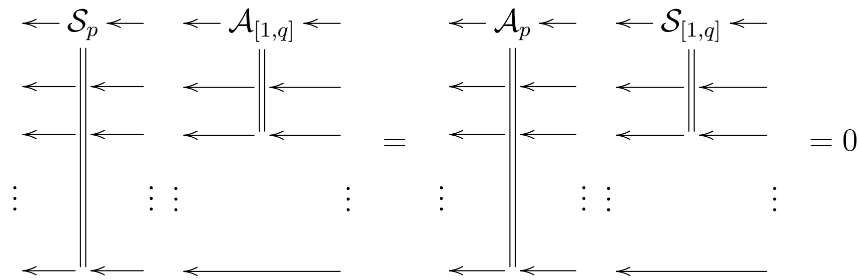


(18.27)

$$\mathcal{S}_p \mathcal{A}_q = \mathcal{A}_p \mathcal{S}_q = 0 \tag{18.28}$$

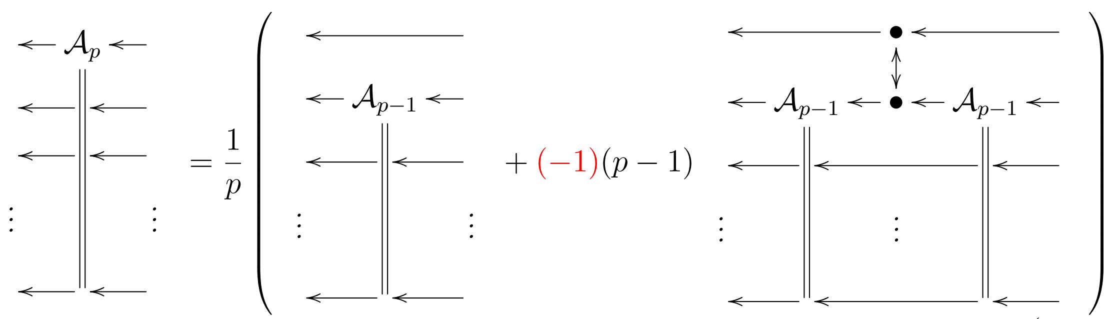


(18.29)



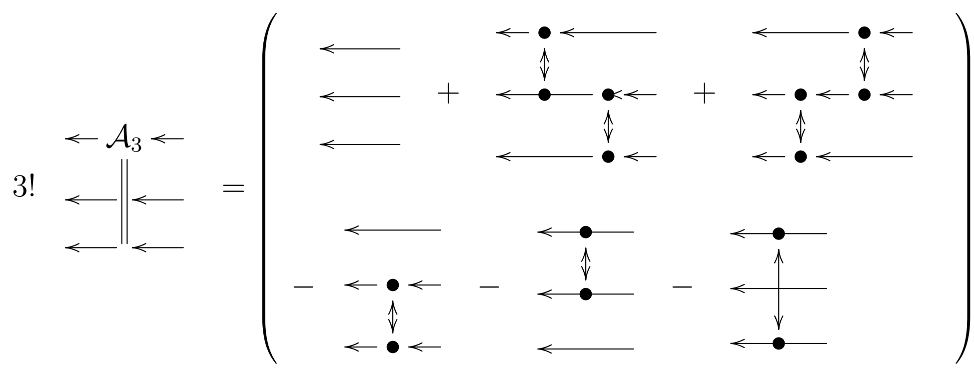
(18.30)

Claim 2

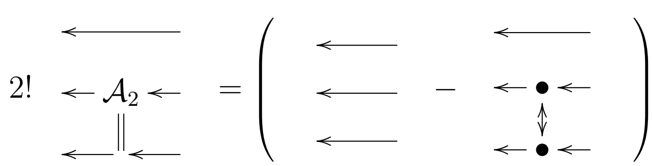


(18.31)

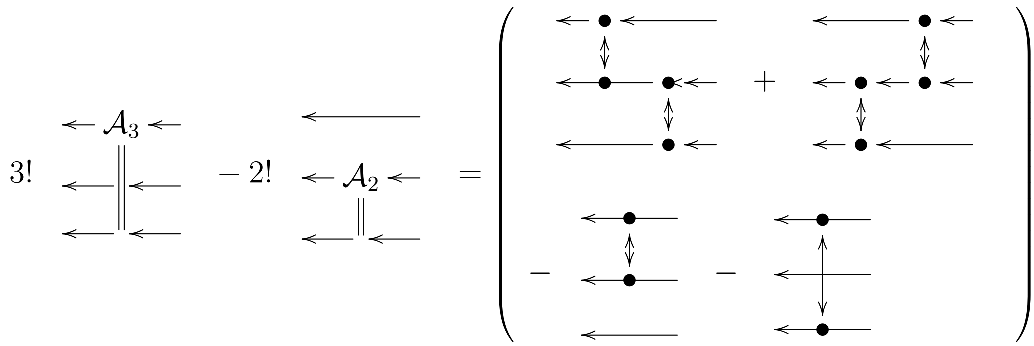
proof: We only prove it for $p = 3$.



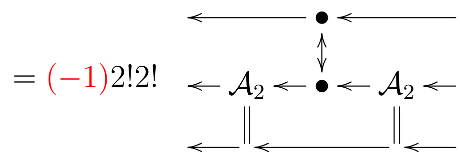
(18.32)



(18.33)

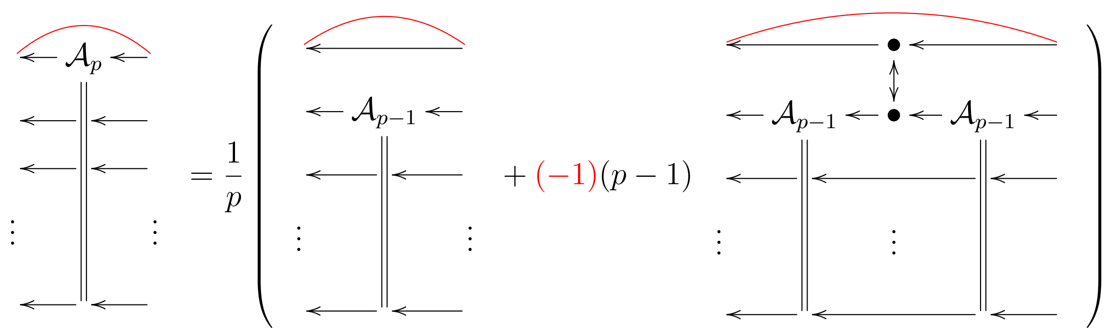


(18.34)

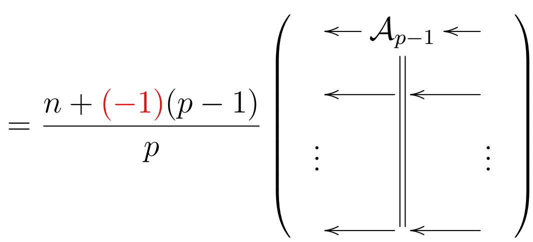


(18.35)

QED



(18.36)



(18.37)

$$\mathrm{tr}_{\underline{a}_1}\mathcal{A}_p = \frac{n-p+1}{p}\mathcal{A}_{p-1} \tag{18.38}$$

$$\mathrm{tr}_{\underline{a}_1,\underline{a}_2,\ldots,\underline{a}_k}\mathcal{A}_p = \frac{(n-p+1)(n-p+2)\ldots(n-p+k)}{p(p-1)\ldots(p-k+1)}\mathcal{A}_{p-k} \tag{18.39}$$

$$d_{\mathcal{A}_p} = \mathrm{tr}_{\underline{a}^p}\mathcal{A}_p = \frac{\prod_{i=n-p+1}^{n} i}{p!} \tag{18.40}$$

$$= \frac{\prod_{i=n}^{n-p+1} i}{p!} \tag{18.41}$$

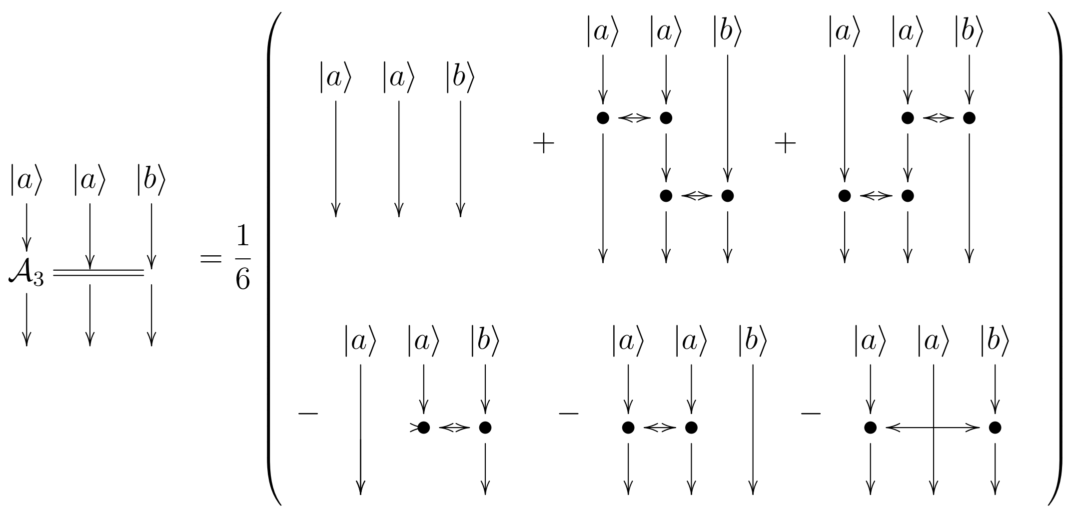
$$= \begin{cases} \frac{n!}{p!(n-p)!} = \binom{n}{p} & \text{if } p \le n \\ 0 & \text{otherwise} \end{cases} \tag{18.42}$$

For $p = 2 \le n$,

$$d_{\mathcal{A}_2} = \binom{n}{2} \tag{18.43}$$

$$\mathcal{A}_p = 0 \text{ if } n < p \tag{18.44}$$

For example, for $n = 2$ and $p = 3$



(18.45)

$$\mathcal{A}_3|a,a,b\rangle = \frac{1}{6}\left(\begin{array}{l} |a,a,b\rangle + |a,b,a\rangle + |b,a,a\rangle \\ -|a,b,a\rangle - |a,a,b\rangle - |b,a,a\rangle \end{array} \right) \tag{18.46}$$

$$= 0 \tag{18.47}$$

Bibliography