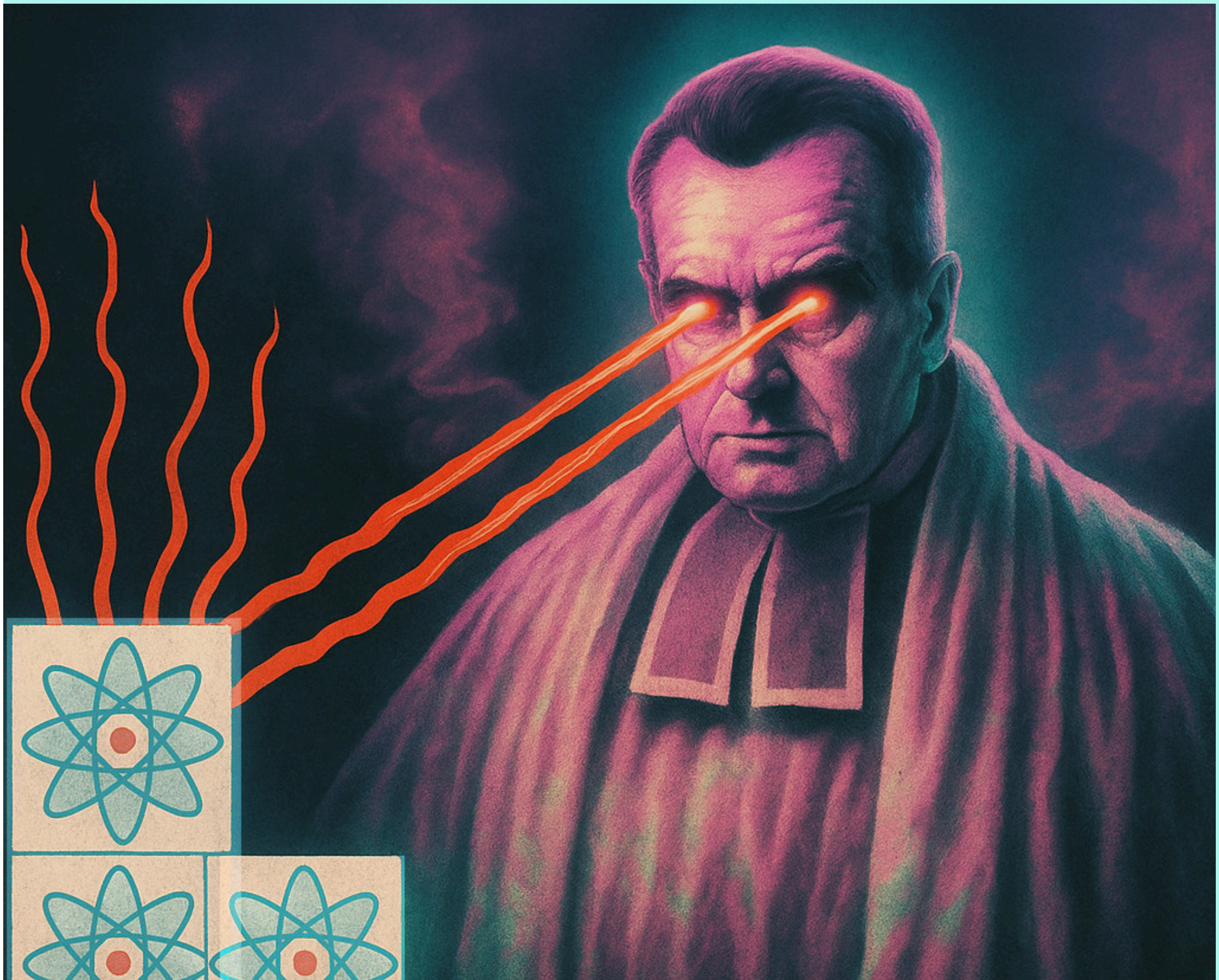
BAYESUVIUS QUANTICO

A VISUAL DICTIONARY OF QUANTUM BAYESIAN NETWORKS



ROBERT R. TUCCI

Bayesuvius Quantico,
a visual dictionary of Quantum Bayesian Networks

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This book is constantly being expanded and improved. To download the latest version, go to
https://github.com/rrtucci/bayes-quantico

Bayes Quantico
by Robert R. Tucci

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Appendix A

Spectral Decomposition and Eigenvalue Projection Operators: COMING SOON

$M \in \mathbb{C}^{d\times d}$

$$M|v\rangle = \lambda|v\rangle \tag{A.1}$$

If M is Hermitian ($H^\dagger = H$), its eigenvalues are real. ($\lambda = \langle\lambda|M\lambda\rangle \in \mathbb{R}$)

$$cp(\lambda) \stackrel{\text{def}}{=} \det(M-\lambda) = 0 \tag{A.2}$$

If M is a Hermitain matrix, then there exists a unitary matric ($CC^\dagger = C^\dagger C = 1$) such that

$$CMC^\dagger = \begin{bmatrix} D_{\lambda_1} & 0 & 0 & 0 \\ 0 & D_{\lambda_2} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & D_{\lambda_r} \end{bmatrix} \tag{A.3}$$

where

$$D_{\lambda_i} = \mathrm{diag}\underbrace{(\lambda_i, \lambda_i, \dots, \lambda_i)}_{d_i \text{ times}} \tag{A.4}$$

$$d = \sum_{i=1}^{r} d_i \tag{A.5}$$

$$CMC^\dagger = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \tag{A.6}$$

$$CP_1C^\dagger = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \frac{CMC^\dagger - \lambda_2}{\lambda_1 - \lambda_2} \tag{A.7}$$

$$CP_2C^\dagger = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \frac{CMC^\dagger - \lambda_1}{\lambda_2 - \lambda_1} \tag{A.8}$$

If $I^{d_i \times d_i}$ is the d_i dimensional unit matrix,

$$\begin{aligned} P_i &= C^\dagger diag(0,\ldots,0,I^{d_i\times d_i},0,\ldots,0)C && \text{(A.9)} \\ &= \prod_{j\neq i} \frac{M-\lambda_j}{\lambda_i - \lambda_j} && \text{(A.10)} \end{aligned}$$

Note that P_i are Hermitian ($P_i^\dagger = P_i$) because M is Hermitian and its eigenvalues are real.)

Note that P_i and M commute

$$[P_i, M] = P_i M - M P_i = 0 \tag{A.11}$$

orthogonal

$$P_i P_j = \delta(i,j) P_j \tag{A.12}$$

complete

$$\sum_i P_i = 1 \tag{A.13}$$

$$M = \sum_{i=1}^r P_i M P_i \tag{A.14}$$

$$d_i = \mathrm{tr} P_i \tag{A.15}$$

$$\begin{aligned} CMP_1C^\dagger &= \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} && \text{(A.16)} \\ &= \lambda_1 \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} && \text{(A.17)} \end{aligned}$$

$$MP_i = \lambda_i P_i \text{ (no } i \text{ sum)} \tag{A.18}$$

$$f(M)P_i = f(\lambda_i)P_i \text{ (no } i \text{ sum)} \tag{A.19}$$

$M^{(1)}, M^{(2)}$

$$[M^{(1)}, M^{(2)}] = 0 \tag{A.20}$$

Use $M^{(1)}$ to decompose V into $\bigoplus_i V_i$. Use $M^{(2)}$ to decompose V_i into $\bigoplus_j V_{i,j}$. If $M^{(1)}$ and $M^{(2)}$ don't commute, let $P_i^{(1)}$ be the eigenvalue projection operators of $M^{(1)}$. The replace $M^{(2)}$ by $P_i^{(1)} M^{(2)} P_i^{(1)}$

$$[M^{(1)}, P_i^{(1)} M^{(2)} P_i^{(1)}] = 0 \tag{A.21}$$

Appendix B

Quantum Bayesian Networks and their Birdtracks: COMING SOON

$$
\begin{tikzcd}[row sep=small]
 & & a \arrow[dl] & a & \\
\lambda \arrow[r, dashed, dash] & C^\lambda \arrow[dr] & b \arrow[l] & b & (C^\dagger)^\lambda \arrow[l] \arrow[ul] \arrow[r, dashed, dash] & \lambda \\
 & & c & c \arrow[ur] &
\end{tikzcd}
$$

$$
\begin{tikzcd}
 & b \arrow[d] \\
A \arrow[r, dashed, dash] & T^A \arrow[d] \\
 & a
\end{tikzcd}
$$

$$
\begin{tikzcd}[row sep=small]
a \arrow[r, dash] & \mathcal{A} \arrow[r, dash] \arrow[dd, equal] & x \\
b \arrow[r, dash] & {} \arrow[r, dash] & y \\
c \arrow[r, dash] & {} \arrow[r, dash] & z
\end{tikzcd}
$$

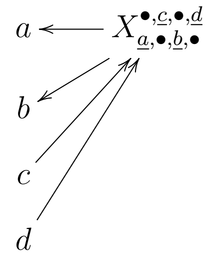
$$
\begin{tikzcd}[row sep=small]
a \arrow[r, dash] & \mathcal{S} \arrow[r, dash] \arrow[dd, equal] & x \\
b \arrow[r, dash] & {} \arrow[r, dash] & y \\
c \arrow[r, dash] & {} \arrow[r, dash] & z
\end{tikzcd}
$$

B.1 Quantum Bayesian Networks

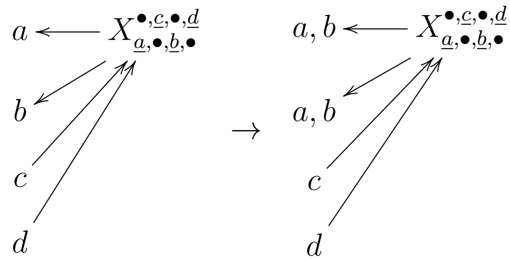
B.2 Birdtracks

$$\delta(b,a) = \mathbb{1}(a=b) = \delta^b_a = \; a \leftarrow\!\!\bullet\!\!-\!\!- b \tag{B.1}$$

$$\langle a, b | X^{\bullet,\underline{c},\bullet,\underline{d}}_{\underline{a},\bullet,\underline{b},\bullet} | c, d\rangle = X^{\bullet,c,\bullet,d}_{a,\bullet,b,\bullet} = \tag{B.2}$$

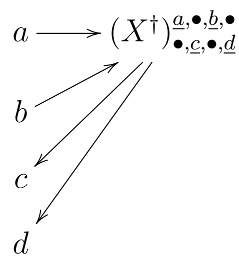


$$\tag{B.3}$$

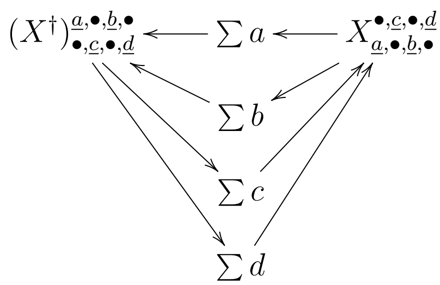


$X^{\bullet,\underline{c},\bullet,\underline{d}}_{\underline{a},\bullet,\underline{b},\bullet} \in V^2 \otimes V_2$. Sometimes, we will omit denote this node simply b y X. This if okay as long as we are not using, X to also denote a different version of $X^{\underline{a},\bullet,\underline{b},\bullet}_{\bullet,\underline{c},\bullet,\underline{d}}$ with some of the indices raised or lowered or their order has been changed.

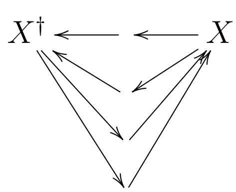
$$(X^\dagger)^{a,\bullet,b,\bullet}_{\bullet,c,\bullet,d} = \tag{B.4}$$



$$(X^\dagger)^{a,\bullet,b,\bullet}_{\bullet,c,\bullet,d} X^{\bullet,c,\bullet,d}_{a,\bullet,b,\bullet} = \tag{B.5}$$



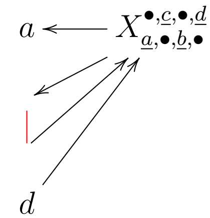
$$= \tag{B.6}$$



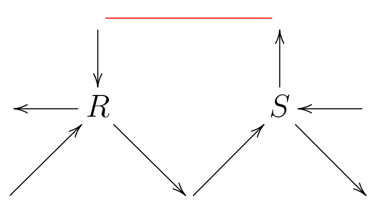
$a^m \in \mathbb{Z}^m_+$

$$R^{a_3^{m_3}, b_2^{n_2}}_{b_3^{n_3}, a_2^{m_2}} S^{a_2^{m_2}, b_1^{n_1}}_{b_2^{n_2}, a_1^{m_1}} = \begin{tikzcd} b_3^{n_3} & R \arrow[l] \arrow[dr] & \sum b_2^{n_2} \arrow[l] & S \arrow[l] \arrow[dr] & b_1^{n_1} \arrow[l] \\ a_3^{m_3} \arrow[ur] & & \sum a_2^{m_2} \arrow[ur] & & a_1^{m_1} \end{tikzcd} \tag{B.7}$$

$$\mathrm{tr}_{\underline{b}} X^{\bullet,\underline{b},\bullet,d}_{a,\bullet,\underline{b},\bullet} = \sum_b X^{\bullet,b,\bullet,d}_{a,\bullet,b,\bullet} = \tag{B.8}$$



(B.9)



Appendix C

Clebsch-Gordan Coefficients: COMING SOON

$$\begin{bmatrix} 0 \\ C_\lambda^{d_\lambda \times d} \\ 0 \end{bmatrix}^{d\times d} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & I^{d_\lambda \times d_\lambda} & 0 \\ 0 & 0 & 0 \end{bmatrix}^{d\times d} C^{d\times d} \tag{C.1}$$

Let $b^{nb} = (b_1, b_2, \ldots, b_{nb})$ where $b_i \in Z_{[0,nb_i]}$ and $a^{na} = (a_1, a_2, \ldots, a_{na})$ where $a_i \in \mathbb{Z}_{[0,na_i]}$. Hence,

$$d = \prod_{i=1}^{na} na_i, \quad d_\lambda = \prod_{i=1}^{nb} nb_i \tag{C.2}$$

$$(C_\lambda)^{b^{nb}}_{a^{na}} = \begin{tikzcd} & & b_1 \arrow[dl] \\ \lambda, a^{na} & C_\lambda \arrow[l] & b_2 \arrow[l] \\ & & b_{nb} \arrow[ul] \end{tikzcd} \tag{C.3}$$

$$\begin{bmatrix} 0 & (C^\dagger)_\lambda^{d\times d_\lambda} & 0 \end{bmatrix}^{d\times d} = (C^\dagger)^{d\times d} \begin{bmatrix} 0 & 0 & 0 \\ 0 & I^{d_\lambda \times d_\lambda} & 0 \\ 0 & 0 & 0 \end{bmatrix}^{d\times d} \tag{C.4}$$

$$(C^\dagger_\lambda)^{a^{na}}_{b^{nb}} = \begin{tikzcd} b_1 & & \\ b_2 & (C^\dagger_\lambda) \arrow[l] \arrow[ul] \arrow[dl] & \lambda, a^{na} \arrow[l] \\ b_{nb} & & \end{tikzcd} \tag{C.5}$$

Each b_i represents a different rep (or irrep)

$$(C^\dagger_\lambda)^{b^{nb}}_{a^{na}}(C_\lambda)^{a^{na}}_{(b')^{nb}} = (P_\lambda)^{b^{nb}}_{(b')^{nb}} \tag{C.6}$$

$$\begin{tikzcd}
b_1 & & & & (b')_1 \arrow[dl] & & & & \\
b_2 & (C^\dagger_\lambda) \arrow[l] \arrow[ul] \arrow[dl] & \lambda, \sum a^{na} \arrow[l] & C_\lambda \arrow[l] & (b')_2 \arrow[l] & = & b^{nb} & P_\lambda \arrow[l] & (b')^{nb} \arrow[l] \\
b_{nb} & & & & (b')_{nb} \arrow[ul] & & & &
\end{tikzcd} \tag{C.7}$$

$$(C_\lambda)^{(a')^{na}}_{b^{nb}}(C^\dagger_\mu)^{b^{nb}}_{a^{na}} = \delta(\lambda,\mu)\delta^{(a')^{na}}_{a^{na}} \tag{C.8}$$

$$\begin{tikzcd}
 & & \sum b_1 \arrow[dl] & & & & & \\
\lambda, a^{na} & C_\lambda \arrow[l] & \sum b_2 \arrow[l] & (C^\dagger_\mu) \arrow[l] \arrow[ul] \arrow[dl] & \mu, (a')^{na} \arrow[l] & = & \lambda, a^{na} & \mu, (a')^{na} \arrow[l, "\bullet" marking] \\
 & & \sum b_{nb} \arrow[ul] & & & & &
\end{tikzcd} \tag{C.9}$$

Bibliography