

# BAYESUVIUS QUANTICO

A VISUAL DICTIONARY OF  
QUANTUM BAYESIAN NETWORKS



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# **Bayesuvious Quantico,** a visual dictionary of Quantum Bayesian Networks

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This book is constantly being expanded and improved. To download  
the latest version, go to

<https://github.com/rrtucci/bayes-quantico>

## **Bayes Quantico**

by Robert R. Tucci

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# Appendices

# Appendix A

## Spectral Decomposition and Eigenvalue Projection Operators: COMING SOON

$$M \in \mathbb{C}^{d \times d}$$

$$M|v\rangle = \lambda|v\rangle \quad (\text{A.1})$$

If  $M$  is Hermitian ( $H^\dagger = H$ ), its eigenvalues are real. (  $\lambda = \langle \lambda | M | \lambda \rangle \in \mathbb{R}$  )

$$cp(\lambda) \stackrel{\text{def}}{=} \det(M - \lambda) = 0 \quad (\text{A.2})$$

If  $M$  is a Hermitain matrix, then there exists a unitary matric ( $CC^\dagger = C^\dagger C = 1$ ) such that

$$CMC^\dagger = \begin{bmatrix} D_{\lambda_1} & 0 & 0 & 0 \\ 0 & D_{\lambda_2} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & D_{\lambda_r} \end{bmatrix} \quad (\text{A.3})$$

where

$$D_{\lambda_i} = \text{diag}(\underbrace{\lambda_i, \lambda_i, \dots, \lambda_i}_{d_i \text{ times}}) \quad (\text{A.4})$$

$$d = \sum_{i=1}^r d_i \quad (\text{A.5})$$

$$CMC^\dagger = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \quad (\text{A.6})$$

$$CP_1C^\dagger = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \frac{CMC^\dagger - \lambda_2}{\lambda_1 - \lambda_2} \quad (\text{A.7})$$

$$CP_2C^\dagger = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \frac{CMC^\dagger - \lambda_1}{\lambda_2 - \lambda_1} \quad (\text{A.8})$$

If  $I^{d_i \times d_i}$  is the  $d_i$  dimensional unit matrix,

$$P_i = C^\dagger \text{diag}(0, \dots, 0, I^{d_i \times d_i}, 0, \dots, 0)C \quad (\text{A.9})$$

$$= \prod_{j \neq i} \frac{M - \lambda_j}{\lambda_i - \lambda_j} \quad (\text{A.10})$$

Note that  $P_i$  are Hermitian ( $P_i^\dagger = P_i$ ) because  $M$  is Hermitian and its eigenvalues are real.)

Note that  $P_i$  and  $M$  commute

$$[P_i, M] = P_i M - M P_i = 0 \quad (\text{A.11})$$

orthogonal

$$P_i P_j = \delta(i, j) P_j \quad (\text{A.12})$$

complete

$$\sum_i P_i = 1 \quad (\text{A.13})$$

$$M = \sum_{i=1}^r P_i M P_i \quad (\text{A.14})$$

$$d_i = \text{tr} P_i \quad (\text{A.15})$$

$$C M P_1 C^\dagger = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad (\text{A.16})$$

$$= \lambda_1 \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad (\text{A.17})$$

$$M P_i = \lambda_i P_i \text{ (no } i \text{ sum)} \quad (\text{A.18})$$

$$f(M) P_i = f(\lambda_i) P_i \text{ (no } i \text{ sum)} \quad (\text{A.19})$$

$M^{(1)}, M^{(2)}$

$$[M^{(1)}, M^{(2)}] = 0 \quad (\text{A.20})$$

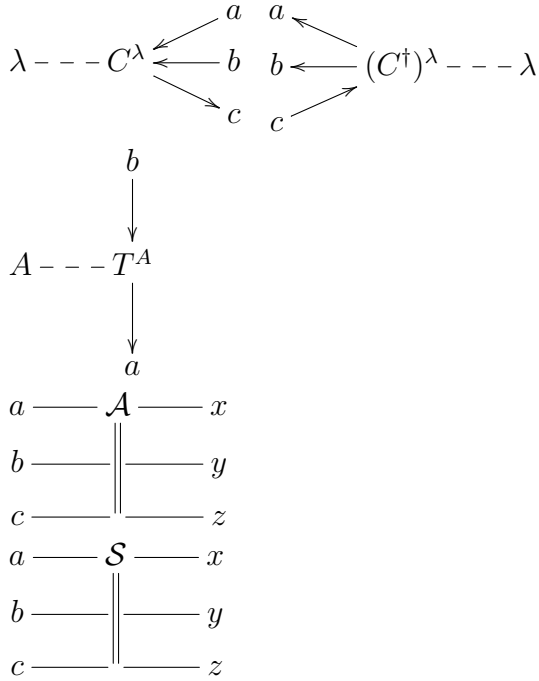
Use  $M^{(1)}$  to decompose  $V$  into  $\bigoplus_i V_i$ . Use  $M^{(2)}$  to decompose  $V_i$  into  $\bigoplus_j V_{i,j}$ . If  $M^{(1)}$  and  $M^{(2)}$  don't commute, let  $P_i^{(1)}$  be the eigenvalue projection operators of  $M^{(1)}$ . The replace  $M^{(2)}$  by  $P_i^{(1)} M^{(2)} P_i^{(1)}$

$$[M^{(1)}, P_i^{(1)} M^{(2)} P_i^{(1)}] = 0 \quad (\text{A.21})$$



# Appendix B

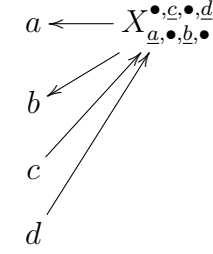
## Quantum Bayesian Networks and their Birdtracks: COMING SOON

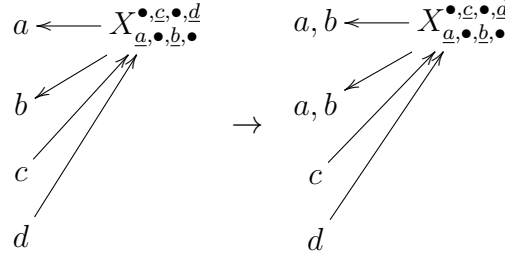


### B.1 Quantum Bayesian Networks

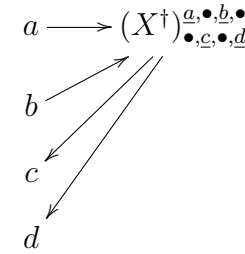
### B.2 Birdtracks

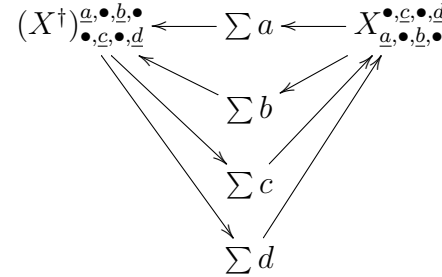
$$\delta(b, a) = \mathbb{1}(a = b) = \delta_a^b = a \leftarrow \bullet \rightarrow b \quad (\text{B.1})$$

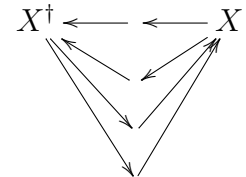
$$\langle a, b | X_{\underline{a}, \bullet, \underline{b}, \bullet}^{\bullet, \underline{c}, \bullet, \underline{d}} | c, d \rangle = X_{\underline{a}, \bullet, \underline{b}, \bullet}^{\bullet, \underline{c}, \bullet, \underline{d}} =$$

(B.2)


(B.3)

$X_{\underline{a}, \bullet, \underline{b}, \bullet}^{\bullet, \underline{c}, \bullet, \underline{d}} \in V^2 \otimes V_2$ . Sometimes, we will omit denote this node simply by  $X$ . This is okay as long as we are not using,  $X$  to also denote a different version of  $X_{\underline{a}, \bullet, \underline{b}, \bullet}^{\bullet, \underline{c}, \bullet, \underline{d}}$  with some of the indices raised or lowered or their order has been changed.

$$(X^\dagger)^{a, \bullet, b, \bullet}_{\bullet, \underline{c}, \bullet, \underline{d}} =$$

(B.4)

$$(X^\dagger)^{a, \bullet, b, \bullet}_{\bullet, \underline{c}, \bullet, \underline{d}} X_{\underline{a}, \bullet, \underline{b}, \bullet}^{\bullet, \underline{c}, \bullet, \underline{d}} =$$

(B.5)

$$=$$

(B.6)

$$a^m \in \mathbb{Z}_+^m$$

$$R_{b_3^{n_3}, a_2^{m_2}}^{a_3^{m_3}, b_2^{n_2}} S_{b_2^{n_2}, a_1^{m_1}}^{a_2^{m_2}, b_1^{n_1}} =$$
(B.7)

$$\text{tr}_{\underline{b}} X_{a, \bullet, \underline{b}, \bullet}^{\bullet, \underline{b}, \bullet, d} = \sum_b X_{a, \bullet, b, \bullet}^{\bullet, \underline{b}, \bullet, d} =$$
(B.8)

(B.9)

# Appendix C

## Clebsch-Gordan Coefficients: COMING SOON

$$\begin{bmatrix} 0 \\ C_\lambda^{d_\lambda \times d} \\ 0 \end{bmatrix}^{d \times d} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & I^{d_\lambda \times d_\lambda} & 0 \\ 0 & 0 & 0 \end{bmatrix}^{d \times d} C^{d \times d} \quad (\text{C.1})$$

Let  $b^{nb} = (b_1, b_2, \dots, b_{nb})$  where  $b_i \in \mathbb{Z}_{[0, nb_i]}$  and  $a^{na} = (a_1, a_2, \dots, a_{na})$  where  $a_i \in \mathbb{Z}_{[0, na_i]}$ . Hence,

$$d = \prod_{i=1}^{na} na_i, \quad d_\lambda = \prod_{i=1}^{nb} nb_i \quad (\text{C.2})$$

$$(C_\lambda)_{a^{na}}^{b^{nb}} = \lambda, a^{na} \longleftarrow C_\lambda \longleftarrow b_2 \quad (\text{C.3})$$

$$\begin{bmatrix} 0 & (C^\dagger)_\lambda^{d \times d_\lambda} & 0 \end{bmatrix}^{d \times d} = (C^\dagger)^{d \times d} \begin{bmatrix} 0 & 0 & 0 \\ 0 & I^{d_\lambda \times d_\lambda} & 0 \\ 0 & 0 & 0 \end{bmatrix}^{d \times d} \quad (\text{C.4})$$

$$(C_\lambda^\dagger)_{b^{nb}}^{a^{na}} = b_2 \longleftarrow (C_\lambda^\dagger) \longleftarrow \lambda, a^{na} \quad (\text{C.5})$$

Each  $b_i$  represents a different rep (or irrep)

$$(C_\lambda^\dagger)_{a^{na}}^{b^{nb}} (C_\lambda)_{(b')^{nb}}^{a^{na}} = (P_\lambda)_{(b')^{nb}}^{b^{nb}} \quad (\text{C.6})$$

$$\begin{array}{ccccccc}
& & & & (b')_1 & & \\
& & & & \swarrow & & \\
b_1 & \longleftarrow & & & & & \\
& & & & & & \\
b_2 & \longleftarrow & (C_\lambda^\dagger) & \longleftarrow & \lambda, \sum a^{na} & \longleftarrow & C_\lambda \\
& & \swarrow & & & & \swarrow \\
b_{nb} & & & & & & (b')_{nb}
\end{array}
= b^{nb} \longleftarrow P_\lambda \longleftarrow (b')^{nb}
\quad (\text{C.7})$$

$$(C_\lambda)_{b^{nb}}^{(a')^{na}} (C_\mu^\dagger)_{a^{na}}^{b^{nb}} = \delta(\lambda, \mu) \delta_{a^{na}}^{(a')^{na}} \quad (\text{C.8})$$

$$\begin{array}{ccccccc}
& & & & \sum b_1 & & \\
& & & & \swarrow & & \\
\lambda, a^{na} & \longleftarrow & C_\lambda & \longleftarrow & \sum b_2 & \longleftarrow & (C_\mu^\dagger) \\
& & \swarrow & & & & \swarrow \\
& & & & \sum b_{nb} & & 
\end{array}
\longleftarrow \mu, (a')^{na} = \lambda, a^{na} \longleftarrow \bullet \mu, (a')^{na}
\quad (\text{C.9})$$

# Bibliography