

# BAYESUVIUS QUANTICO

A VISUAL DICTIONARY OF  
QUANTUM BAYESIAN NETWORKS



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# **Bayesuvious Quantico,** a visual dictionary of Quantum Bayesian Networks

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the latest version, go to

<https://github.com/rrtucci/bayes-quantico>

## **Bayes Quantico**

by Robert R. Tucci

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# Chapter 9

## Lie Algebras

$$i \in \mathbb{Z}_{[1,N]}, a, b \in \mathbb{Z}_{[1,n]}$$

$$(C_{Adj}^i)_b^a = \frac{1}{\sqrt{K}} (T^i)_b^a = i \rightsquigarrow C_{Adj}^i \begin{array}{c} a \\ \downarrow \\ b \end{array} \quad (9.1)$$

Note that we list the indices of  $T^i$  in the counter-clockwise (CC) direction, starting at the  $i$  leg. The matrices  $T^i$  are called the generators. It's customary to choose them so that they are Hermitian and  $K = \frac{1}{2}$ .<sup>1</sup>

$$\boxed{(T^i)_b^a (T^j)_a^b = \text{tr}(T^i T^j) = K \delta(i, j)} \quad i \rightsquigarrow T^i \begin{array}{c} \xrightarrow{\sum a} \\ \xleftarrow{\sum b} \end{array} T^j \rightsquigarrow j = K \leftarrow \bullet \quad (9.2)$$

$$(P_{Adj})_{b,d}^{a,c} = \sum_i \frac{1}{K} (T^i)_b^a (T^i)_d^c = \frac{1}{K} \begin{array}{c} a \\ \downarrow \\ b \end{array} \rightsquigarrow \begin{array}{c} d \\ \uparrow \\ c \end{array} \quad (9.3)$$

$$H \in V^n \otimes \bar{V}^n$$

$$(P_{Adj})_{bd}^{ac} H_c^d = \sum_i (T^i)_b^a \underbrace{\left[ \frac{1}{K} (T^i)_d^c H_c^d \right]}_{h_i \in \mathbb{R}} \quad (9.4)$$

---

<sup>1</sup>For  $SU(2)$ , it is customary to use  $T^i = \frac{1}{2} \sigma_i$ , where  $\sigma_i$  for  $i = 1, 2, 3$  are the Pauli matrices. For  $SU(3)$ , it is customary to choose  $T^i = \frac{1}{2} \lambda_i$  where  $\lambda_i$  for  $i = 1, 2, \dots, 8$  are the Gell-Mann matrices.

$$G = 1 + iD \in \mathcal{G}$$

$$\epsilon_i \in \mathbb{R}, |\epsilon_i| \ll 1$$

$$D = \sum_i \epsilon_i T^i = V^n \otimes \bar{V}^n$$

Recall Eq.(A.28). If  $x \in V^{n^p} \otimes \bar{V}^{n^q}$ ,  $\mathbb{G} \in \mathcal{G} \subset GL(n^{p+q}, \mathbb{C})$ ,

$$(x')_{a:p}{}^{b:q} = \mathbb{G}_{a:p}{}^{b:q}{}_{rev(c:q)}{}^{rev(d:p)} x_{d:p}{}^{c:q}, \quad (x'_\alpha = \mathbb{G}_\alpha{}^\beta x_\beta) \quad (9.5)$$

where we define

$$\mathbb{G}_\alpha{}^\beta \stackrel{\text{def}}{=} \prod_{i=1}^p G_{a_i}{}^{d_i} \prod_{i=1}^q G^{b_i}{}_{c_i} \quad (9.6)$$

$$\mathbb{G}_\alpha{}^\beta = 1 + i \sum_j \epsilon_j (\mathbb{T}^j)_\alpha{}^\beta \quad (9.7)$$

$$G_{a_i}{}^{d_i} = 1 + i \sum_j \epsilon_j (T^j)_{a_i}{}^{d_i} \quad (9.8)$$

$$G^{b_i}{}_{c_i} = (G^*)_{b_i}{}^{c_i} = 1 - i \sum_j \epsilon_j (T^{j*})_{b_i}{}^{c_i} = 1 - i \sum_j \epsilon_j (T^j)^{b_i}{}_{c_i} \quad (9.9)$$

When  $x'_\alpha = x_\alpha$ , to first order in  $\epsilon_i$ ,

$$0 = (\mathbb{T}^j)_\alpha{}^\beta x_\beta = \left[ (T^j)_{a_i}{}^{d_i} \frac{1}{\delta_{a_i}^{d_i}} - (T^j)^{b_i}{}_{c_i} \frac{1}{\delta_{c_i}^{b_i}} \right] \delta_{a:p}^{d:p} \delta_{c:q}^{b:q} x_{d:p}{}^{c:q} \quad (9.10)$$

$$(\mathbb{T}^j)_\alpha{}^\beta = \left[ (T^j)_{a_i}{}^{d_i} \frac{1}{\delta_{a_i}^{d_i}} - (T^j)^{b_i}{}_{c_i} \frac{1}{\delta_{c_i}^{b_i}} \right] \delta_{a:p}^{d:p} \delta_{c:q}^{b:q} \quad (9.11)$$

$(\mathbb{T}^j)_{a_1 a_2}{}^{b_1}{}_{c_1}{}^{d_2 d_1} = (T^j)_{a_1}{}^{d_1} \delta_{a_2}^{d_2} \delta_{c_1}^{b_1} + \delta_{a_1}^{d_1} (T^j)_{a_2}{}^{d_2} \delta_{c_1}^{b_1} - \delta_{a_1}^{d_1} \delta_{a_2}^{d_2} (T^j)^{b_1}{}_{c_1}$

=

+

-

$b_1 \longrightarrow c_1 \qquad b_1 \longrightarrow c_1 \qquad b_1 \longrightarrow (T^j)^T \longrightarrow c_1$

(9.12)

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$$\boxed{0 = (\mathbb{T}^j x)_{a_1 a_2}{}^{b_1} = \left[ (T^j)_{a_1}{}^{d_1} \delta_{a_2}^{d_2} \delta_{c_1}^{b_1} + \delta_{a_1}^{d_1} (T^j)_{a_2}{}^{d_2} \delta_{c_1}^{b_1} - \delta_{a_1}^{d_1} \delta_{a_2}^{d_2} (T^j)^{b_1}{}_{c_1} \right] x_{d_1 d_2}{}^{c_1}}$$

$$0 = \begin{array}{c} j \\ \text{wavy line} \\ \swarrow \quad \searrow \\ a_1 \quad x \\ \nwarrow \quad \nearrow \\ a_2 \quad \mathbb{T}^j \\ \swarrow \quad \searrow \\ b_1 \end{array} = \begin{array}{c} j \\ \text{wavy line} \\ \swarrow \quad \searrow \\ a_1 \quad x \\ \nwarrow \quad \nearrow \\ a_2 \quad T^j \\ \swarrow \quad \searrow \\ b_1 \end{array} + \begin{array}{c} j \\ \text{wavy line} \\ \swarrow \quad \searrow \\ a_1 \quad x \\ \nwarrow \quad \nearrow \\ a_2 \quad T^j \\ \swarrow \quad \searrow \\ b_1 \end{array} - \begin{array}{c} j \\ \text{wavy line} \\ \swarrow \quad \searrow \\ a_1 \quad x \\ \nwarrow \quad \nearrow \\ a_2 \quad T^j \\ \swarrow \quad \searrow \\ b_1 \rightarrow (T^j)^T \end{array} \quad (9.13)$$

Since  $\mathbb{G} = 1 + i \sum_j \epsilon_j \mathbb{T}^j$ , generators decompose in the same way as the group elements

$$\boxed{\mathbb{T}^j = \sum_{\lambda} C_{\lambda}^{\dagger} T_{\lambda}^j C_{\lambda}}$$

$$\begin{array}{c} j \\ \text{wavy line} \\ \swarrow \quad \searrow \\ \nwarrow \quad \nearrow \\ \mathbb{T}^j \end{array} = \sum_{\lambda} \begin{array}{c} j \\ \text{wavy line} \\ \swarrow \quad \searrow \\ \nwarrow \quad \nearrow \\ C_{\lambda}^{\dagger} \leftarrow T_{\lambda}^j \leftarrow C_{\lambda} \end{array} \quad (9.14)$$

Clebsch-Gordan matrices are invariant matrices.

$$[G, C_{\lambda}] = G C_{\lambda} - C_{\lambda} G = 0 \quad (9.15)$$

$$C_{\lambda} = G^{\dagger} C_{\lambda} G = G_{\lambda}^{\dagger} C_{\lambda} G \quad (9.16)$$

$$0 = -T_{\lambda}^j C_{\lambda} + C_{\lambda} T^j \quad (9.17)$$

$$0 = \left\{ \begin{array}{l} - \begin{array}{c} j \\ \} \\ a \leftarrow T_\lambda^j \leftarrow C_\lambda \leftarrow c_2 \\ \swarrow \searrow \\ c_1 \quad b_1 \end{array} + \begin{array}{c} j \\ \} \\ a \leftarrow C_\lambda \leftarrow c_2 \\ \swarrow \searrow \\ T^j \leftarrow c_1 \\ b_1 \end{array} \\ + \begin{array}{c} j \\ \} \\ a \leftarrow C_\lambda \leftarrow T^j \leftarrow c_2 \\ \swarrow \searrow \\ c_1 \quad b_1 \end{array} - \begin{array}{c} j \\ \} \\ a \leftarrow C_\lambda \leftarrow c_2 \\ \swarrow \searrow \\ c_1 \quad (T^j)^T \rightarrow b_1 \end{array} \end{array} \right\} \quad (9.18)$$

Multiplying on the left  $C_\lambda^\dagger$ , we get

$$\begin{array}{c} j \\ \} \\ a \leftarrow T_\lambda^j \leftarrow a' \end{array} = \begin{array}{c} j \\ \} \\ T^j \\ \swarrow \nwarrow \\ a \leftarrow C_\lambda \leftarrow C_\lambda^\dagger \leftarrow a' \end{array} + \begin{array}{c} j \\ \} \\ T^j \\ \swarrow \nwarrow \\ a \leftarrow C_\lambda \leftarrow T^j \leftarrow C_\lambda^\dagger \leftarrow a' \end{array} - \begin{array}{c} j \\ \} \\ T^j \\ \swarrow \nwarrow \\ a \leftarrow C_\lambda \leftarrow C_\lambda^\dagger \leftarrow a' \\ \searrow \nearrow \\ (T^j)^T \end{array} \quad (9.19)$$

$$\boxed{(\mathbb{T}^i)_{jk} = (T^i)_a{}^c (T^k)_c{}^b (T^j)_b{}^a - (T^i)_a{}^c (T^j)_c{}^b (T^k)_b{}^a}$$

$$\begin{array}{c} i \\ \} \\ \mathbb{T}^i \\ \swarrow \searrow \\ j \quad k \end{array} = \begin{array}{c} i \\ \} \\ T^i \\ \swarrow \nwarrow \\ \sum_a \quad \sum_c \\ T^j \rightarrow \sum_b \rightarrow T^k \\ \swarrow \searrow \\ j \quad k \end{array} - \begin{array}{c} i \\ \} \\ T^i \\ \swarrow \nwarrow \\ \sum_a \quad \sum_c \\ T^k \rightarrow \sum_b \rightarrow T^j \\ \swarrow \searrow \\ j \quad k \end{array} \quad (9.20)$$

Note that

$$\boxed{(\mathbb{T}^i)_{jk} = -(\mathbb{T}^i)_{kj}}$$

$$\begin{array}{c} i \\ \text{---} \\ \mathbb{T}^i \\ \text{---} \\ j \quad k \end{array} = - \begin{array}{c} i \\ \text{---} \\ \mathbb{T}^i \\ \text{---} \\ j \quad k \end{array} \quad (\text{Convention CC}) \quad (9.21)$$

$$\begin{array}{c} i \\ \text{---} \\ \mathbb{T}_{jk}^i \\ \text{---} \\ j \quad k \end{array} = - \begin{array}{c} i \\ \text{---} \\ \mathbb{T}_{kj}^i \\ \text{---} \\ j \quad k \end{array} \quad (\text{Convention L})$$

$$(\mathbb{T}^i)_{jk} \stackrel{\text{def}}{=} -iC_{ijk} = -\frac{1}{K}\text{tr}([T^i, T^j]T^k) \quad (9.22)$$

$C_{ijk}$ , called the bf structure constants, is totally antisymmetric

$$\boxed{\underbrace{T^i T^j - T^j T^i}_{[T^i, T^j]} = iC_{ijk} T^k} \quad (\text{Lie Algebra commutation relations})$$

$$\begin{array}{c} i \quad j \\ \text{---} \quad \text{---} \\ a \leftarrow T^i \leftarrow T^j \leftarrow c \end{array} - \begin{array}{c} i \quad j \\ \text{---} \quad \text{---} \\ a \leftarrow T^j \leftarrow T^i \leftarrow c \end{array} = (-i) \begin{array}{c} i \quad j \\ \text{---} \quad \text{---} \\ C_{ijk} \\ \text{---} \\ a \leftarrow T^k \leftarrow c \end{array} \quad (9.23)$$

$$\boxed{0 = (\mathbb{T}^i)_{jm} C_{mkl} - C_{ljm} (\mathbb{T}^i)_{mk} - C_{jkm} (\mathbb{T}^i)_{ml}}$$

$$0 = \begin{array}{c} i \\ \text{---} \\ \mathbb{T}^i \\ \text{---} \\ j \end{array} \sim \sum_m \begin{array}{c} l \\ \text{---} \\ C_{mkl} \\ \text{---} \\ k \end{array} - \begin{array}{c} i \quad l \\ \text{---} \quad \text{---} \\ C_{ljm} \sim \sum_m \mathbb{T}^i \\ \text{---} \quad \text{---} \\ j \quad k \end{array} - \begin{array}{c} i \quad l \\ \text{---} \quad \text{---} \\ \mathbb{T}^i \\ \text{---} \\ \sum_m C_{jkm} \\ \text{---} \\ j \quad k \end{array} \quad (9.24)$$

$$\boxed{C_{ijm}C_{mkl} - C_{ljm}C_{mki} = C_{iml}C_{jkm}}$$

$$\begin{array}{c} i \\ \text{wavy} \\ C_{ijm} \sim \sum m \sim \\ \text{wavy} \\ j \end{array}
\begin{array}{c} l \\ \text{wavy} \\ C_{mkl} \\ \text{wavy} \\ k \end{array}
-
\begin{array}{c} i \\ \text{wavy} \\ C_{ijm} \sim \sum m \sim \\ \text{wavy} \\ j \end{array}
\begin{array}{c} l \\ \text{wavy} \\ C_{mkl} \\ \text{wavy} \\ k \end{array}
=
\begin{array}{c} i \quad l \\ \text{wavy} \quad \text{wavy} \\ C_{iml} \\ \sum m \\ C_{jkm} \\ \text{wavy} \quad \text{wavy} \\ j \quad k \end{array}
\quad (9.25)$$

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