

BAYESUVIUS QUANTICO

A VISUAL DICTIONARY OF
QUANTUM BAYESIAN NETWORKS



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Bayesuvious Quantico, a visual dictionary of Quantum Bayesian Networks

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This book is constantly being expanded and improved. To download
the latest version, go to

<https://github.com/rrtucci/bayes-quantico>

Bayes Quantico

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Appendices

Appendix A

Spectral Decomposition and Eigenvalue Projection Operators: COMING SOON

$$M \in \mathbb{C}^{d \times d}$$

$$M|v\rangle = \lambda|v\rangle \quad (\text{A.1})$$

If M is Hermitian ($H^\dagger = H$), its eigenvalues are real. ($\lambda = \langle \lambda | M | \lambda \rangle \in \mathbb{R}$)

$$cp(\lambda) \stackrel{\text{def}}{=} \det(M - \lambda) = 0 \quad (\text{A.2})$$

If M is a Hermitain matrix, then there exists a unitary matrix ($CC^\dagger = C^\dagger C = 1$) such that

$$CMC^\dagger = \begin{bmatrix} D_{\lambda_1} & 0 & 0 & 0 \\ 0 & D_{\lambda_2} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & D_{\lambda_r} \end{bmatrix} \quad (\text{A.3})$$

where

$$D_{\lambda_i} = \text{diag}(\underbrace{\lambda_i, \lambda_i, \dots, \lambda_i}_{d_i \text{ times}}) \quad (\text{A.4})$$

$$d = \sum_{i=1}^r d_i \quad (\text{A.5})$$

$$CMC^\dagger = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \quad (\text{A.6})$$

$$CP_1C^\dagger = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \frac{CMC^\dagger - \lambda_2}{\lambda_1 - \lambda_2} \quad (\text{A.7})$$

$$CP_2C^\dagger = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \frac{CMC^\dagger - \lambda_1}{\lambda_2 - \lambda_1} \quad (\text{A.8})$$

If $I^{d_i \times d_i}$ is the d_i dimensional unit matrix,

$$P_i = C^\dagger \text{diag}(0, \dots, 0, I^{d_i \times d_i}, 0, \dots, 0)C \quad (\text{A.9})$$

$$= \prod_{j \neq i} \frac{M - \lambda_j}{\lambda_i - \lambda_j} \quad (\text{A.10})$$

Note that P_i are Hermitian ($P_i^\dagger = P_i$) because M is Hermitian and its eigenvalues are real.)

Note that P_i and M commute

$$[P_i, M] = P_i M - M P_i = 0 \quad (\text{A.11})$$

orthogonal

$$P_i P_j = \delta(i, j) P_j \quad (\text{A.12})$$

complete

$$\sum_i P_i = 1 \quad (\text{A.13})$$

$$M = \sum_{i=1}^r P_i M P_i \quad (\text{A.14})$$

$$d_i = \text{tr} P_i \quad (\text{A.15})$$

$$C M P_1 C^\dagger = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad (\text{A.16})$$

$$= \lambda_1 \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad (\text{A.17})$$

$$M P_i = \lambda_i P_i \text{ (no } i \text{ sum)} \quad (\text{A.18})$$

$$f(M) P_i = f(\lambda_i) P_i \text{ (no } i \text{ sum)} \quad (\text{A.19})$$

$M^{(1)}, M^{(2)}$

$$[M^{(1)}, M^{(2)}] = 0 \quad (\text{A.20})$$

Use $M^{(1)}$ to decompose V into $\bigoplus_i V_i$. Use $M^{(2)}$ to decompose V_i into $\bigoplus_j V_{i,j}$. If $M^{(1)}$ and $M^{(2)}$ don't commute, let $P_i^{(1)}$ be the eigenvalue projection operators of $M^{(1)}$. The replace $M^{(2)}$ by $P_i^{(1)} M^{(2)} P_i^{(1)}$

$$[M^{(1)}, P_i^{(1)} M^{(2)} P_i^{(1)}] = 0 \quad (\text{A.21})$$

Appendix B

Birdtracks: COMING SOON

B.1 Classical Bayesian Networks and their Instantiations

TPM (Transition Probability Matrix) $P(y|x) \in [0, 1]$ where $x \in \text{val}(\underline{x})$ and $y \in \text{val}(\underline{y})$

$$\sum_{y \in \text{val}(\underline{y})} P(y|x) = 1 \quad (\text{B.1})$$

$$\mathcal{C} = \begin{array}{ccc} & b & \\ \swarrow & & \nwarrow \\ \underline{c} & \longleftarrow & \underline{a} \end{array} \quad (\text{B.2})$$

$$\mathcal{C}(a, b, c) = P(c|b, a)P(b|a)P(a) = \begin{array}{ccc} & b & \\ \swarrow & & \nwarrow \\ c & \longleftarrow & a \end{array} P(a) \quad (\text{B.3})$$

$$a^2 = (a_1, a_2)$$

$$\mathcal{C}' = \begin{array}{ccc} & b & \\ \swarrow & & \nwarrow \\ \underline{c} & \longleftarrow \underline{a_2} & \underline{a^2} \end{array} \quad (\text{B.4})$$

$$\mathcal{C}'(a^2, b, c) = P(c|b, a_2)P(a_2|a^2)P(b|a_1)P(a_1|a^2)P(a^2) = \begin{array}{ccc} & b & \\ \swarrow & & \nwarrow \\ c & \longleftarrow a_2 & a^2 \end{array} P(a^2) \quad (\text{B.5})$$

Marginalizer nodes $\underline{a_1}$ and $\underline{a_2}$ have the TPMs

$$P(a'_i|\underline{a^2} = (a_1, a_2)) = \delta(a'_i, a_i) \quad (\text{B.6})$$

for $i = 1, 2$

B.2 Quantum Bayesian Networks and their Instantiations

TPM (Transition Probability Matrix) $A(y|x) \in \mathbb{C}$ where $x \in \text{val}(\underline{x})$ and $y \in \text{val}(\underline{y})$

$$\sum_{y \in \text{val}(\underline{y})} |A(y|x)|^2 = 1 \quad (\text{B.7})$$

$$\mathcal{Q} = \begin{array}{c} \underline{b} \\ \swarrow \quad \searrow \\ \underline{c} \longleftarrow \underline{a} \end{array} \quad (\text{B.8})$$

$$\mathcal{Q}(a, b, c) = A(c|b, a)A(b|a)A(a) = \begin{array}{c} b \\ \swarrow \quad \searrow \\ c \longleftarrow a \end{array} A(a) \quad (\text{B.9})$$

$$a^2 = (a_1, a_2)$$

$$\mathcal{Q}' = \begin{array}{c} \underline{b} \\ \swarrow \quad \searrow \\ \underline{c} \longleftarrow \underline{a_2} \quad \underline{a^2} \end{array} \quad (\text{B.10})$$

$$\mathcal{Q}'(a^2, b, c) = A(c|b, a_2)A(a_2|a^2)A(b|a_1)A(a_1|a^2)A(a^2) = \begin{array}{c} b \\ \swarrow \quad \searrow \\ c \longleftarrow a_2 \quad a^2 \end{array} A(a^2) \quad (\text{B.11})$$

Marginalizer nodes $\underline{a_1}$ and $\underline{a_2}$ have the TAMs

$$A(a'_i|\underline{a^2} = (a_1, a_2)) = \delta(a'_i, a_i) \quad (\text{B.12})$$

for $i = 1, 2$

B.3 Birdtracks

$$\delta(b, a) = \mathbb{1}(a = b) = \delta_a^b = a \leftarrow \bullet \longrightarrow b \quad (\text{B.13})$$

$$\langle a, b | X_{\underline{a}, \bullet, \underline{b}, \bullet}^{\bullet, \underline{c}, \bullet, \underline{d}} | c, d \rangle = X_{\underline{a}, \bullet, \underline{b}, \bullet}^{\bullet, \underline{c}, \bullet, \underline{d}} = \begin{array}{c} a \longleftarrow X_{\underline{a}, \bullet, \underline{b}, \bullet}^{\bullet, \underline{c}, \bullet, \underline{d}} \\ \swarrow \quad \nearrow \\ b \quad c \\ \swarrow \quad \nearrow \\ d \end{array} \quad (\text{B.14})$$

$$\begin{array}{ccc}
a \longleftarrow X_{\underline{a}, \bullet, \underline{b}, \bullet}^{\bullet, \underline{c}, \bullet, \underline{d}} & & a, b \longleftarrow X_{\underline{a}, \bullet, \underline{b}, \bullet}^{\bullet, \underline{c}, \bullet, \underline{d}} \\
b \nearrow & \rightarrow & a, b \nearrow \\
c \nearrow & & c \nearrow \\
d \nearrow & & d \nearrow
\end{array} \quad (\text{B.15})$$

$X_{\underline{a}, \bullet, \underline{b}, \bullet}^{\bullet, \underline{c}, \bullet, \underline{d}} \in V^2 \otimes V_2$. Sometimes, we will omit denote this node simply by X . This is okay as long as we are not using, X to also denote a different version of $X_{\bullet, \underline{c}, \bullet, \underline{d}}^{\underline{a}, \bullet, \underline{b}, \bullet}$ with some of the indices raised or lowered or their order has been changed.

$$\begin{array}{ccc}
a \longrightarrow (X^\dagger)_{\bullet, \underline{c}, \bullet, \underline{d}}^{\underline{a}, \bullet, \underline{b}, \bullet} & & \\
b \nearrow & & \\
c \nearrow & & \\
d \nearrow & &
\end{array} \quad (\text{B.16})$$

$$\begin{array}{ccc}
(X^\dagger)_{\bullet, \underline{c}, \bullet, \underline{d}}^{\underline{a}, \bullet, \underline{b}, \bullet} \longleftarrow \sum a \longleftarrow X_{\underline{a}, \bullet, \underline{b}, \bullet}^{\bullet, \underline{c}, \bullet, \underline{d}} & & \\
\searrow \quad \swarrow & \swarrow & \\
\sum b & & \\
\searrow \quad \swarrow & \swarrow & \\
\sum c & & \\
\searrow \quad \swarrow & \swarrow & \\
\sum d & &
\end{array} \quad (\text{B.17})$$

$$\begin{array}{ccc}
X^\dagger \longleftarrow \longleftarrow X & & \\
\searrow \quad \swarrow & \swarrow & \\
& &
\end{array} \quad (\text{B.18})$$

$$a^m \in \mathbb{Z}_+^m$$

$$\begin{array}{ccc}
b_3^{n_3} \longleftarrow R \longleftarrow \sum b_2^{n_2} \longleftarrow S \longleftarrow b_1^{n_1} & & \\
a_3^{m_3} \nearrow & \nearrow & \\
& \nearrow & \\
& \sum a_2^{m_2} & \\
& \nearrow & \\
& S & \\
& \searrow & \\
& a_1^{m_1} &
\end{array} \quad (\text{B.19})$$

$$\text{tr}_{\underline{b}} X_{a, \bullet, \underline{b}, \bullet}^{\bullet, b, \bullet, d} = \sum_b X_{a, \bullet, b, \bullet}^{\bullet, b, \bullet, d} = \begin{array}{c} a \longleftarrow X_{\underline{a}, \bullet, \underline{b}, \bullet}^{\bullet, \underline{c}, \bullet, \underline{d}} \\ \swarrow \nearrow \\ \text{---} \nearrow \\ d \end{array} \quad (\text{B.20})$$

$$\begin{array}{ccc} & \text{---} & \\ \downarrow & & \uparrow \\ \leftarrow R & & S \leftarrow \\ & \searrow \nearrow & \\ & & \searrow \nearrow \end{array} \quad (\text{B.21})$$

Appendix C

Clebsch-Gordan Coefficients: COMING SOON

$$\begin{bmatrix} 0 \\ C_\lambda^{d_\lambda \times d} \\ 0 \end{bmatrix}^{d \times d} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & I^{d_\lambda \times d_\lambda} & 0 \\ 0 & 0 & 0 \end{bmatrix}^{d \times d} C^{d \times d} \quad (\text{C.1})$$

Let $b^{nb} = (b_1, b_2, \dots, b_{nb})$ where $b_i \in \mathbb{Z}_{[0, nb_i]}$ and $a^{na} = (a_1, a_2, \dots, a_{na})$ where $a_i \in \mathbb{Z}_{[0, na_i]}$. Hence,

$$d = \prod_{i=1}^{na} na_i, \quad d_\lambda = \prod_{i=1}^{nb} nb_i \quad (\text{C.2})$$

$$(C_\lambda)^{b^{nb}}_{a^{na}} = \lambda, a^{na} \longleftarrow C_\lambda \longleftarrow b_2 \quad (\text{C.3})$$

$$\begin{bmatrix} 0 & (C^\dagger)_\lambda^{d \times d_\lambda} & 0 \end{bmatrix}^{d \times d} = (C^\dagger)^{d \times d} \begin{bmatrix} 0 & 0 & 0 \\ 0 & I^{d_\lambda \times d_\lambda} & 0 \\ 0 & 0 & 0 \end{bmatrix}^{d \times d} \quad (\text{C.4})$$

$$(C^\dagger_\lambda)^{a^{na}}_{b^{nb}} = b_2 \longleftarrow (C^\dagger_\lambda) \longleftarrow \lambda, a^{na} \quad (\text{C.5})$$

Each b_i represents a different rep (or irrep)

$$(C_\lambda^\dagger)_{a^{na}}^{b^{nb}} (C_\lambda)_{(b')^{nb}}^{a^{na}} = (P_\lambda)_{(b')^{nb}}^{b^{nb}} \quad (\text{C.6})$$

$$\begin{array}{ccccccc}
& & & & (b')_1 & & \\
& & & & \swarrow & & \\
b_1 & \longleftarrow & & & & & \\
& & & & & & \\
b_2 & \longleftarrow & (C_\lambda^\dagger) & \longleftarrow & \lambda, \sum a^{na} & \longleftarrow & C_\lambda \\
& & \swarrow & & & & \swarrow \\
b_{nb} & & & & & & (b')_{nb}
\end{array}
= b^{nb} \longleftarrow P_\lambda \longleftarrow (b')^{nb}
\quad (\text{C.7})$$

$$(C_\lambda)_{b^{nb}}^{(a')^{na}} (C_\mu^\dagger)_{a^{na}}^{b^{nb}} = \delta(\lambda, \mu) \delta_{a^{na}}^{(a')^{na}} \quad (\text{C.8})$$

$$\begin{array}{ccccccc}
& & \sum b_1 & & & & \\
& & \swarrow & & \swarrow & & \\
\lambda, a^{na} & \longleftarrow & C_\lambda & \longleftarrow & \sum b_2 & \longleftarrow & (C_\mu^\dagger) \\
& & \swarrow & & \swarrow & & \\
& & \sum b_{nb} & & & &
\end{array}
= \lambda, a^{na} \longleftarrow \bullet \mu, (a')^{na}
\quad (\text{C.9})$$

Bibliography