Documentation

Prreamble

Winter 2023

Contents

Installation and usage

For those who want to quickly get their hands on this, simply write

```
\usepackage{prreamble}
```

See the section *Options* for options of this package.

If you do not want to load the entire package, you can download the source, and `\input` the parts that you like.

1 Options

This package has many options. They can be used like options for any other package. For example

```
\usepackage[modern]{prreamble}
```

General typesetting

- **debug**. This option prints an information page to the first page of the document. It illustrates which options have been selected.
- **nobreaks**. Unless this option is passed, `prreamble` will overwrite the `\ldots` command to `\allowbreak \ldots\allowbreak`. Overwriting `\ldots` is particularly useful when using `\ldots` within math environments which have the habit of being very long and going into the margin.
- **printlinks**. This option will create a footnote for every link added with the `\phref` macro. This means inline clickable links can be used, but if the document is ever printed, the reader may still access the link.
- **print**. This option passes the parameter `draft` to the `hyperref` package, which disables all link colouring. Links will be the usual text colour. This can be desirable for (greyscale) printing. Implies `printlinks`.
- **centertitles**. This option passes the parameter `center` to the `titlesec` package, causing titles to be centred.
- **centerstacktitles**. This option rewrites the `section` definition using `titlesec`. The title is centred, and the section number is above the title.
- **nomath**. Do not define any math-type macros (like `set{}`). This option is useful for preventing clashes with existing macros.
- **noletters**. Do not define macros for special letters (blackboard, calligraphic, fractur or monospace) in math mode. This option is useful for preventing clashes with existing macros.

Typefaces

- **concroman**. This option enables the use of the Concrete Roman font.
- **concbold**. This option enables the use of Daniel Flipo's Concrete Roman Bold typeface. Implies `concroman`.
- **euler**. This option enables the AMS Euler math typeface.
- **concmath**. This option enables Ulrik Vieth's concmath font. Implies `concroman`.

For environments

- **scdefs**. Uses `scshape` for definitions.
- **scthms**. Uses `scshape` for theorems.
- **scdefobjs**. Uses `schape` for text rendered with the `\define` macro.

- **scemph**. Uses `scshape` for emphasis, that is, text rendered with the `\emph` macro.
- **uldefs**. Underlines definitions.
- **ulthms**. Underlines theorems..
- **uldefobjs**. Underlines text rendered with the `\define` macro.

Pre-defined groups of options

- **original**. Equivalent to `concroman`, `euler`.

  The resulting document will look similar to how Knuth originally typset *Concrete Mathematics*.

- **classic**. Equivalent to `concbold`, `concroman`, `concmath`, `scdefobjs`, `scemph`, `uldefs`, `ulthms` and setting the line stretch to 1.75.

  This evokes a style similar to that of Matsumura's *Commutative Algebra* [Mat80], whose body text was typeset on a typewriter with the math being added by hand after the fact. When using shorter titles in a document with larger margins, I think adding **centertitles** to these two options completes this traditional look.

- **modern**. Equivalent to `concroman`, `concmath`.

  This is the style I personally use the most, and is the style used to produce this document.

2 Concrete Mathematics: A brief history

1989 Donald Knuth (inventor of TeXand METAFONT) develops a square-serif font *Concrete Roman* [Knu89] to use together the then newly produced *AMS Euler* math typeface in the book *Concrete Mathematics* [KGP94]. Most notably, since *Concrete Roman* was produced as an experiment, it does not provide a bold typeface.

1995 Ulrik Vieth extends the Concrete Roman typeface to include a math typeface and distributes this combined font as *Concmath* in the METAFONT format.
ctan.org/pkg/concmath-fonts
Vieth also produces a package `concmath` to provide LaTeX support for this font.
ctan.org/pkg/concmath
Concmath still does not have a boldface, but no longer needs an accompanying math typeface: AMS Euler is no longer needed.

2022 Daniel Flipo converts Vieth's METAFONT*Concmath* sources to an OTF format and adds a boldface. This font is available through the LaTeX package `concmath-otf`.
ctan.org/pkg/concmath-otf

This history, has created somewhat confusing nomenclature:

- *Concrete Mathematics* is the textbook written by Knuth, Ronald Graham and Oren Patashnik.
- *Concrete Roman* is the typeface created by Knuth for the book *Concrete Mathematics*.
- *Concmath* is Vieth's combined Concrete Roman typeface with a math typeface.
- Now `concmath-otf` is the Package by Daniel Flipo that also provides a boldface to concrete Roman.

Some passages that I found interesting.

> *During 1987 and 1988 I prepared a textbook entitled Concrete Mathematics [...] it would be the first major use of a new typeface [AMS Euler] by Hermann Zapf, commissioned by the American Mathematical Society [...] Our original intention ... was to use Computer Modern Roman for the text and AMS Euler for the mathematics. But Roy noticed that AMS Euler was somewhat darker [...] When I saw Roy's samples, I decided to pursue ... settings for the parameters of Computer Modern that would produce an "Egyptian" (square-serif) style. [...] I decided to call the resulting font Concrete Roman, because of its general solid appearance and because it was first used in the book Concrete Mathematics.*
>
> — Donald Knuth in *Typesetting Concrete Mathematics* [Knu89].

> *The Concrete Math fonts (also known as "concmath" fonts) were developed by [myself] in early 1995, originally for use in a poster presentation. When the use of magnified sizes of Computer Modern math fonts printed at low resolution turned out to be unsatisfactory for comfortable reading in poster sizes, the need for a somewhat darker alternative became apparent. Since the only existing alternative would have been to use the AMS Euler fonts in math mode, which were deemed a little too exotic for the intended application, [I] set out to investigate the possibilities of generating a complete set of Concrete Math fonts by applying systematic changes to the METAFONT parameter files.*
>
> — Ulrik Vieth on the origins of Concrete Math [Vie99]

3 Overview

Here is an overview of all the available commands. Please refer to the appropriate section later in the document for more detailed documentation.
When a command is followed by {n}, it means that it has n arguments. When followed by [n], it means that there are n optional arguments.

3.1 General Typesetting

Blackboard		Calligraphic		Fractur upper		Fractur lower		Monospace	
\bA	$\mathbb{A}$	\cA	$\mathcal{A}$	\fA	$\mathfrak{A}$	\fa	$\mathfrak{a}$	\mA	$\mathtt{A}$
\bA	$\mathbb{A}$	\cA	$\mathcal{A}$	\fA	$\mathfrak{A}$	\fa	$\mathfrak{a}$	\mA	$\mathtt{A}$
\bB	$\mathbb{B}$	\cB	$\mathcal{B}$	\fB	$\mathfrak{B}$	\fb	$\mathfrak{b}$	\mB	$\mathtt{B}$
\bC	$\mathbb{C}$	\cC	$\mathcal{C}$	\fC	$\mathfrak{C}$	\fc	$\mathfrak{c}$	\mC	$\mathtt{C}$
\bD	$\mathbb{D}$	\cD	$\mathcal{D}$	\fD	$\mathfrak{D}$	\fd	$\mathfrak{d}$	\mD	$\mathtt{D}$
\bE	$\mathbb{E}$	\cE	$\mathcal{E}$	\fE	$\mathfrak{E}$	\fe	$\mathfrak{e}$	\mE	$\mathtt{E}$
\bF	$\mathbb{F}$	\cF	$\mathcal{F}$	\fF	$\mathfrak{F}$	\ff	$\mathfrak{f}$	\mF	$\mathtt{F}$
\bG	$\mathbb{G}$	\cG	$\mathcal{G}$	\fG	$\mathfrak{G}$	\fg	$\mathfrak{g}$	\mG	$\mathtt{G}$
\bH	$\mathbb{H}$	\cH	$\mathcal{H}$	\fH	$\mathfrak{H}$	\fh	$\mathfrak{h}$	\mH	$\mathtt{H}$
\bI	$\mathbb{I}$	\cI	$\mathcal{I}$	\fI	$\mathfrak{I}$			\mI	$\mathtt{I}$
\bJ	$\mathbb{J}$	\cJ	$\mathcal{J}$	\fJ	$\mathfrak{J}$	\fj	$\mathfrak{j}$	\mJ	$\mathtt{J}$
\bK	$\mathbb{K}$	\cK	$\mathcal{K}$	\fK	$\mathfrak{K}$	\fk	$\mathfrak{k}$	\mK	$\mathtt{K}$
\bL	$\mathbb{L}$	\cL	$\mathcal{L}$	\fL	$\mathfrak{L}$	\fl	$\mathfrak{l}$	\mL	$\mathtt{L}$
\bM	$\mathbb{M}$	\cM	$\mathcal{M}$	\fM	$\mathfrak{M}$	\fm	$\mathfrak{m}$	\mM	$\mathtt{M}$
\bN	$\mathbb{N}$	\cN	$\mathcal{N}$	\fN	$\mathfrak{N}$	\fn	$\mathfrak{n}$	\mN	$\mathtt{N}$
\bO	$\mathbb{O}$	\cO	$\mathcal{O}$	\fO	$\mathfrak{O}$	\fo	$\mathfrak{o}$	\mO	$\mathtt{O}$
\bP	$\mathbb{P}$	\cP	$\mathcal{P}$	\fP	$\mathfrak{P}$	\fp	$\mathfrak{p}$	\mP	$\mathtt{P}$
\bQ	$\mathbb{Q}$	\cQ	$\mathcal{Q}$	\fQ	$\mathfrak{Q}$	\fq	$\mathfrak{q}$	\mQ	$\mathtt{Q}$
\bR	$\mathbb{R}$	\cR	$\mathcal{R}$	\fR	$\mathfrak{R}$	\fr	$\mathfrak{r}$	\mR	$\mathtt{R}$
\bS	$\mathbb{S}$	\cS	$\mathcal{S}$	\fS	$\mathfrak{S}$	\fs	$\mathfrak{s}$	\mS	$\mathtt{S}$
\bT	$\mathbb{T}$	\cT	$\mathcal{T}$	\fT	$\mathfrak{T}$	\ft	$\mathfrak{t}$	\mT	$\mathtt{T}$
\bU	$\mathbb{U}$	\cU	$\mathcal{U}$	\fU	$\mathfrak{U}$	\fu	$\mathfrak{u}$	\mU	$\mathtt{U}$
\bV	$\mathbb{V}$	\cV	$\mathcal{V}$	\fV	$\mathfrak{V}$	\fv	$\mathfrak{v}$	\mV	$\mathtt{V}$
\bW	$\mathbb{W}$	\cW	$\mathcal{W}$	\fW	$\mathfrak{W}$	\fw	$\mathfrak{w}$	\mW	$\mathtt{W}$
\bX	$\mathbb{X}$	\cX	$\mathcal{X}$	\fX	$\mathfrak{X}$	\fx	$\mathfrak{x}$	\mX	$\mathtt{X}$
\bY	$\mathbb{Y}$	\cY	$\mathcal{Y}$	\fY	$\mathfrak{Y}$	\fy	$\mathfrak{y}$	\mY	$\mathtt{Y}$
\bZ	$\mathbb{Z}$	\cZ	$\mathcal{Z}$	\fZ	$\mathfrak{Z}$	\fz	$\mathfrak{z}$	\mZ	$\mathtt{Z}$

3.2 Sets

Command	Symbol	Example	Notes
`\set {1}`	$\{ \}$	$\{a, b, c\}$	Set
`\cset {2}`	$\{ \mid \}$	$\{x \in \mathbb{C} \mid x^2 = 1\}$	Conditional set
`\bigset {1}`	$\big\{ \big\}$	$\big\{\frac{1}{2}, b, c\big\}$	Big set
`\bigcset {2}`	$\big\{ \mid \big\}$	$\big\{x \in \mathbb{C} \mid x^2 = 1\big\}$	Big conditional set
`\tup {1}`	$( )$	$(1, 0, 0)$	Tuple
`\Tr`	Tr	$\mathrm{Tr}(A)$	Trace
`\one`	$\mathbb{1}$	$\mathbb{1}_E(x)$	Set-characteristic function
`\subset`	$\subseteq$	$A \subseteq B$	Subset (Overrides `\subseteq`)
`\supset`	$\supseteq$	$A \supseteq B$	Superset (Overrides `\supseteq`)
`\complement {1}`	^{c}	$A^c = X \setminus A$	Complement
`\closure {1}`	$\overline{\ }$	$\overline{A}$	Closure
`\interior {1}`	$^\circ$	$\mathring{A}$	Interior
`\card {1}`	$\lvert \ \rvert$	$\lvert A\rvert$	Cardinality
`\until`	$, \dots ,$	$1, \dots , n$	Listing

3.3 Topology

Command	Symbol	Example	Notes
`\homot`	$\simeq$	$A \simeq B$	Homotopy
`\weak`	*	Weak*convergence	Weak*convergence

3.4 Measure and Probability

Command	Symbol	Example	Notes
`\Var`	Var	$\mathrm{Var}(X)$	Variance
`\Cov`	Cov	$\mathrm{Cov}(X)$	Covariance
`\Leb`	Leb	$\mathrm{Leb}(X)$	Lebesgue Measure

3.5 Trigonometric functions

Command	Symbol	Example	Notes
`\arsinh`	arsinh	$\mathrm{arsinh}(\alpha)$	Hyperbolic arcsine
`\arcosh`	arcosh	$\mathrm{arcosh}(\alpha)$	Hyperbolic arccosine
`\artninh`	artanh	$\mathrm{artanh}(\alpha)$	Hyperbolic arctangent

3.6 Analysis

Command	Symbol	Example Usage	Notes
`\ceiling {1}`	$\lceil\ \rceil$	$x \leq \lceil x \rceil < x + 1$	Ceiling function
`\de`	d	$\mathrm{d}\, f$	Total Derivative
`\De`	D	$\mathrm{D}\, f$	Total Derivative
`\divergence`	div	$\operatorname{div}(f) = \int \cdots$	Divergence
`\dvol`	dvol	$\int_A f(x)\, \mathrm{dvol}$	Volumetric integration
`\euler`	e	$\mathrm{e}^{2\pi\iota} + 1 = 0$	Euler's constant
`\floor {1}`	$\lfloor\ \rfloor$	$x - 1 < \lfloor x \rfloor \leq x$	Floor function
`\graph`	graph	$\operatorname{graph}(g) = \{(x, f(x)) \mid x \in X\}$	Graph
`\ii`	ι	$z = x + \iota y$	Imaginary unit
`\laplace`	$\mathit{\Delta}$	$\mathit{\Delta}(g)$	Laplace operator
`\metric`	d	$\mathrm{d}(x, y)$	Metric
`\re`	Re	$\operatorname{Re}(z)$	Real part
`\restr {2}`		$f\|_A$	Restriction
`\rot`	rot	$\operatorname{rot}(f) = \int \cdots$	Rotation/ curl
`\support [1]`	supp	$\operatorname{supp}_A(f) = \overline{\{x \in A \mid f(x) \neq 0\}}$	Support of a function
`\vc {1}`		$\mathbf{v} = (v_1, \ldots, v_n)$	Vector (bold upright)
`\vol`	vol	$\operatorname{vol}(P) = \int_P \mathrm{dvol}$	Volume

3.7 Algebra

Command	Symbol	Example	Notes
\Ann[1]	Ann_R		Annihilator
\Ass[1]	Ass	$\mathrm{Ass}(M)$	Associated primes
\Aut	Aut	$\mathrm{Aut}(F)$	Automorphisms
\Bil[1]	Bil	$\mathrm{Bil}(X \times Y, Z)$	Bilinear maps
\Char	char	$\mathrm{char}(k)$	(Field) Characteristic
\cl{1}	$\overline{a}$	$\overline{2} \equiv \overline{5} \pmod 3$	Class of representatives
\divides	$\mid$	$2 \mid 4$	Divides
\End	End	$\mathrm{End}(R)$	Endomorphisms
\ev	ev		Evaluation map (Overriden from physics)
\eval	eval		Evaluation map (Overriden from physics)
\Frac	Frac	$\mathrm{Frac}(\mathbb{Z}) = \mathbb{Q}$	Fraction field
\frac{2}	$\cdot/\cdot$	$1/2 = 2/4$	Fraction, use \defaultfrac for original
\Gal	Gal	$\mathrm{Gal}(L/k)$	Galois group
\height[1]	ht_B	$\mathrm{ht}(p)$	Prime ideal height
\Hom	Hom	$\mathrm{Hom}(A, B)$	Homomorphisms
\hull{1}	$\langle\ \rangle$	$\langle S \rangle = W$	Span/Linear hull
\id	id		Identity
\ifrac{2}	$\cdot/\cdot$	$1/2 = 2/4$	Inline fraction
\im	im		Image
\into[1]	$\xhookrightarrow{\iota}$		Injection/ into map
\jac	Jac	$\mathrm{Jac}(R)$	Jacobson Radical
\ker	ker		Kernel
\krulldim	$\dim_{\mathrm{Krull}}$	$\dim_{\mathrm{Krull}}(R)$	Krull dimension
\lcm	lcm	$\mathrm{lcm}(a, b)$	Lowest common denominator
\Mat	Mat	$\mathrm{Mat}_{n \times n}(k)$	Matrices
\nil	nil	$\mathrm{nil}(R)$	Nilradical
\Obj	Obj	$\mathrm{Obj}(\mathcal{C})$	Object
\onto[1]	$\overset{\pi}{\twoheadrightarrow}$		Surjection/ onto map
\Orb	Orb	$\mathrm{Orb}_G(x)$	Orbit
\ord	ord	$\mathrm{ord}(g) \mid \lvert G \rvert$	Order
\quot{2}	$\cdot/\cdot$	R/I	Quotient, alias \q
\rad	rad	$\mathrm{rad}(R)$	Jacobson Radical
\scalarprod{2}	$\langle \cdot, \cdot \rangle$	$\langle x, y \rangle$	Scalar Product
\sgn	sgn		Sign

`\Span`	Span	$\operatorname{Span}(S) = W$	Span
`\Stab`	Stab	$\operatorname{Stab}_G(x)$	Stabiliser
`\Sym`	Sym	$\operatorname{Sym}(T)$	Symmetric
`\tens[1]`	$\otimes_R$	$a \otimes b$	Tensor product
`\Tens[1]`	$\bigotimes_R$	$A \bigotimes_R B$	Big Tensor Product
`\to[1]`	$\xrightarrow{f}$		Overloaded regular `\to` for named arrow
`\tor`	tor	$\operatorname{tor}(G)$	Torsion
`\trdeg[1]`	trdeg_k	$\operatorname{trdeg}(L/k)$	Transcendence degree

3.8 Computing

Command	Symbol	Problem
`\SAT`	SAT	(Clause) Satisfiability problem
`\CLIQUE`	CLIQUE	Clique finding problem

4 General Typsetting

4.1 Blackboard letters

Syntax:

```
\begin{align*}
  \ba ,\bb ,\bc ,\bd ,\be ,\bf ,\bg ,\bh ,\bi ,\bj ,\bk ,\bl ,\bm ,
  \bn ,\bo ,\bp ,\bq ,\br ,\bs \bt ,\bu ,\bv ,\bw ,\bx ,\by ,\bz
\end{align*}
```

Compiled:

$$\mathbb{A}, \mathbb{B}, \mathbb{C}, \mathbb{D}, \mathbb{E}, \mathbb{F}, \mathbb{G}, \mathbb{H}, \mathbb{I}, \mathbb{J}, \mathbb{K}, \mathbb{L}, \mathbb{M}, \mathbb{N}, \mathbb{O}, \mathbb{P}, \mathbb{Q}, \mathbb{R}, \mathbb{S}\mathbb{T}, \mathbb{U}, \mathbb{V}, \mathbb{W}, \mathbb{X}, \mathbb{Y}, \mathbb{Z}$$

4.2 Calligraphic letters

Syntax:

```
\begin{align*}
  \cA ,\cB ,\cC ,\cD ,\cE ,\cF ,\cG ,\cH ,\cI ,\cJ ,\cK ,\cL ,\cM ,
  \cN ,\cO ,\cP ,\cQ ,\cR ,\cS \cT ,\cU ,\cV ,\cW ,\cX ,\cY ,\cZ
\end{align*}
```

Compiled:

$$\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E}, \mathcal{F}, \mathcal{G}, \mathcal{H}, \mathcal{I}, \mathcal{J}, \mathcal{K}, \mathcal{L}, \mathcal{M}, \mathcal{N}, \mathcal{O}, \mathcal{P}, \mathcal{Q}, \mathcal{R}, \mathcal{S}\mathcal{T}, \mathcal{U}, \mathcal{V}, \mathcal{W}, \mathcal{X}, \mathcal{Y}, \mathcal{Z}$$

4.3 MathFrak letters

Syntax:

```
\begin{align*}
  \fA ,\fB ,\fC ,\fD ,\fE ,\fF ,\fG ,\fH ,\fI ,\fJ ,\fK ,\fL ,\fM ,
  \fN ,\fO ,\fP ,\fQ ,\fR ,\fS \fT ,\fU ,\fV ,\fW ,\fX ,\fY ,\fZ
\end{align*}
```

Compiled:

$$\mathfrak{A}, \mathfrak{B}, \mathfrak{C}, \mathfrak{D}, \mathfrak{E}, \mathfrak{F}, \mathfrak{G}, \mathfrak{H}, \mathfrak{I}, \mathfrak{J}, \mathfrak{K}, \mathfrak{L}, \mathfrak{M}, \mathfrak{N}, \mathfrak{O}, \mathfrak{P}, \mathfrak{Q}, \mathfrak{R}, \mathfrak{S}\mathfrak{T}, \mathfrak{U}, \mathfrak{V}, \mathfrak{W}, \mathfrak{X}, \mathfrak{Y}, \mathfrak{Z}$$

5 Sets

5.1 Regular set

Definition 5.1 (Symmetric group)**.** The set of all bijections on the set $\{1, \ldots, n\}$ is denoted S_n and called the *symmetric group*.

```
\begin{definition}[Symmetric group]
  The set of all bijections on the set $\set{1, \ldots, n}$ is
  denoted $S_n$ and called the \define{symmetric group}.
\end{definition}
```

Note: The size of the surrounding braces is automatically adjusted.

5.2 Conditional set

Definition 5.2 (Group Center)**.** The *center* of a group G denoted $Z(G)$ is the subgroup of all elements of G that commute with all other elements. That is,

$$Z(G) = \{g \in G \mid \forall h \in G, gh = hg\}$$

```
\begin{definition}[Group Center]
  The \define{center} of a group $G$ denoted $Z(G)$ is the subgroup
  of all elements of $G$ that commute with all other elements. That
  is,
  %
  \begin{align*}
    Z(G) = \cset{g \in G}{\forall h \in G, gh = hg}
  \end{align*}
\end{definition}
```

Note: The size of the surrounding braces is automatically adjusted.

5.3 Tuples

Example. *Even though the sequences*

$$(1, 1, 1, \ldots)$$

and

$$\left(1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\right)$$

converge, the composite sequence

$$\left(1, \frac{1}{2}, 1, \frac{1}{3}, 1, \frac{1}{4}, \ldots\right)$$

does not.

```
  \begin{example*}
    Even though the sequences
    \begin{align*}
      \tup{1, 1, 1, \ldots}
```

```
    \end{align*}
    and
    \begin{align*}
      \tup{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots}
    \end{align*}
    converge, the composite sequence
    \begin{align*}
      \tup{1, \frac{1}{2}, 1, \frac{1}{3}, 1, \frac{1}{4}, \ldots}
    \end{align*}
    does not.
  \end{example*}
```

Note: The size of the surrounding parenthesis is automatically adjusted.

5.4 Set-theoretic characteristic function

Definition (Characteristic function)**.** Suppose E is a subset of an arbitrary set X. Then the *characteristic function* of E, $\mathbb{1}_E$, is defined as

$$\begin{aligned} \mathbb{1}_E \colon X &\to \{0,1\} \\ x &\mapsto \begin{cases} 1 & \mathrm{~if~} x \in E \\ 0 & \mathrm{~else~} \end{cases} \end{aligned}$$

```
\begin{definition*}[Characteristic function]
  Suppose $E$ is a subset of an arbitrary set $X$. Then the
  \define{characteristic function} on $E$, $\one_E$ is defined as
  %
  \begin{align*}
    \one_E \colon X & \to \set{0,1} \\
    x               & \mapsto
      \begin{cases}
        1 & \mathrm{~if~} x \in E \\
        0 & \mathrm{~else~}
      \end{cases}
  \end{align*}
\end{definition*}
```

6 Groups, Rings and Fields

Blackboard fonts

Syntax:

```
\begin{align*}
  \A, \C, \E, \F, \K, \N, \P, \Q, \Rk, \Rm, \R, \Z
\end{align*}
```

Compiled:

$$\mathbb{A}, \mathbb{C}, \mathbb{E}, \mathbb{F}, \mathbb{K}, \mathbb{N}, \mathbb{P}, \mathbb{Q}, \mathbb{R}^k, \mathbb{R}^m, \mathbb{R}, \mathbb{Z}$$

Important Groups

Syntax:

```
\begin{align*}
  \GL, \SL, \SO
\end{align*}
```

Compiled:

$$\mathrm{GL}, \mathrm{SL}, \mathrm{SO}$$

7 Algebra

7.1 Identity

Syntax:

```
\begin{theorem*}
  For $f \colon X \to Y$ a morphism of varieties, and $\id \colon Y \to
  Y$ the identity morphism, we have
  %
  \begin{align*}
    {(f, \id)}^{-1}(\laplace_{Y})
    = \cset{(x, y) \subset X \times Y}{f(x) = y}
    = \graph (f).
  \end{align*}
  %
  This shows that the graph of a morphism between varieties is closed.
\end{theorem*}
```

Compiled:

Theorem. *For* $f\colon X \to Y$ *a morphism of varieties, and* $\mathrm{id}\colon Y \to Y$ *the identity morphism, we have*

$$(f, \mathrm{id})^{-1}(\varDelta_Y) = \{(x, y) \subseteq X \times Y \mid f(x) = y\} = \mathrm{graph}(f).$$

$$d\varDelta a$$

This shows that the graph of a morphism between varieties is closed.

7.2 Matrices

Syntax:

```
\begin{theorem*}
  The $n \times n$ matrices over the field $K$, $\Mat_{n \times n} (K)$,
  form a $n^2$-dimensional $K$ vector space.
\end{theorem*}
```

Compiled:

Theorem. *The* $n \times n$ *matrices over the field* K, $\mathrm{Mat}_{n\times n}(K)$, *form a* n^2*-dimensional* K *vector space.*

7.3 Trace

Syntax:

```
\begin{definition*}
  For $A$ a square $n \times n$ matrix over the field $K$, then its
  \define{trace}, denoted $\Tr (A)$ is defined to be
  %
  \begin{align*}
    \Tr A := \sum A_{i,i}
  \end{align*}
\end{definition*}
```

Compiled:

Definition. For A a square $n \times n$ matrix over the field K, then it's *trace*, denoted $\operatorname{Tr}(A)$ is defined to be

$$\operatorname{Tr} A := \sum A_{i,i}$$

7.4 Span

Syntax:

```
\begin{definition*}[Span]
  We define the \define{span} of a subset $S$ of a vector space $V$ to be
  the smallest subspace of $V$ that contains $S$, denoted $\Span{S}$.
\end{definition*}
```

Compiled:

Definition (Span)**.** We define the *span* of a subset S of a vector space V to be the smallest subspace of V that contains S, denoted $\operatorname{Span} S$.

7.5 Scalar product

Syntax:

```
\begin{definition*}[Scalar product]
  A \define{scalar product} on a vector space $V$, is a positive definite
  bilinear form, usually denoted $\scalarprod{\cdot}{\cdot} \colon V
  \times V \to K$.
\end{definition*}
```

Compiled:

Definition (Scalar product)**.** A *scalar product* on a vector space V, is a positive definite bilinear form, usually denoted $\langle \cdot, \cdot \rangle \colon V \times V \to K$.

7.6 Homomorphisms

Syntax:

```
\begin{theorem*}
  Finite dimensional vector spaces are isomorphic to their dual. That is,
  if $V$ is a finite dimensional vector space over the field $K$,
  %
  \begin{align*}
    V^* = \Hom_K (V, K) \cong V
  \end{align*}
\end{theorem*}
```

Compiled:

Theorem. *Finite dimensional vector spaces are isomorphic to their dual. That is, if V is a finite dimensional vector space over the field K,*

$$V^* = \operatorname{Hom}_K(V, K) \cong V$$

For named homomorphisms.
Syntax:

```
\begin{theorem*}
  Suppose $A, B$ are commutative algebras over the ring commutative ring
  $R$ and $I$ an ideal of $A$. Then we have a natural isomorphism
  %
  \begin{align*}
    \nHom{R}{alg}{\quot{A}{I}}{B}
      & \to \cset{f \in \nHom{R}{alg}{A}{B}}{I \subset \ker f} \\
    g
      & \mapsto g \circ \pi
  \end{align*}
\end{theorem*}
```

Compiled:

Theorem. *Suppose A, B are commutative algebras over the ring commutative ring R and I an ideal of A. Then we have a natural isomorphism*

$$\begin{aligned} \mathrm{Hom}_{(R-\mathrm{alg})}\left({}^{A}\!/_{I}, B\right) &\to \left\{ f \in \mathrm{Hom}_{(R-\mathrm{alg})}(A, B) \mid I \subseteq \ker f \right\} \\ g &\mapsto g \circ \pi \end{aligned}$$

For Automorphisms
Syntax:

```
\begin{theorem*}
  The group field automorphisms $\Aut(\R)$ of $\R$ is trivial.
\end{theorem*}
```

Compiled:

Theorem. *The group field automorphisms* $\mathrm{Aut}(\mathbb{R})$ *of* $\mathbb{R}$ *is trivial.*

7.7 Evaluation map

Syntax:

```
\begin{definition*}[Evaluation map]
  For $K$ a field and $a = (a_1, \ldots, a_n)$ a point in $K^n$ we define
  the \define{evaluation} map at $a$, $\ev_a \colon K[X_1, \ldots X_n]
  \to K$, to be the unique morphism of $K$-algebras that sends $X_i$ to
  $a_i$.
\end{definition*}
```

Compiled:

Definition (Evaluation map)**.** For K a field and $a = (a_1, \ldots, a_n)$ a point in K^n we define the *evaluation* map at a, $\mathrm{ev}_a\colon K[X_1, \ldots X_n] \to K$, to be the unique morphism of K-algebras that sends X_i to a_i.

8 Computing

8.1 Problems

Example (NP-Complete problems)**.** *Both,* SAT*, and* CLIQUE *are NP-complete problems.*

8.2 Time Complexity

Syntax:

```
\begin{definition*}[Time-complexity]
  The \define{time-complexity of the calculation of a multitape Turing
  machine $M$ on an input} $x$, $\Time _M(x)$, is given by the length of
  the calculation sequence bar the initial configuration. The
  \textbf{time-complexity of the multitape Turing machine} $M$, is the
  function $\Time _M \colon \N \to \N; n \mapsto \max \set{\Time _M (x)
  \colon x \in \Sigma ^n}$.
\end{definition*}
```

Compiled:

Definition (Time-complexity)**.** The *time-complexity of the calculation of a multitape Turing machine M on an input* x, $\mathrm{Time}_M(x)$, is given by the length of the calculation sequence bar the initial configuration. The *time-complexity of the multitape Turing machine M*, is the function $\mathrm{Time}_M \colon \mathbb{N} \to \mathbb{N}; n \mapsto \max\{\mathrm{Time}_M(x) \colon x \in \Sigma^n\}$.

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