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S. I. Hussain, N. Ruza, N. Masseran, et al.

Dependence Structure Between Index Stock Market and Bitcoin Using Time-Varying Copula and Extreme Value Theory

S. I. Hussain[1, a)], N. Ruza[2], N. Masseran[3] and M. A. M. Safari[4]

[1,2,3,4]Department of Mathematical Sciences,
Faculty of Science and Technology,
Universiti Kebangsaan Malaysia
43600 Bangi, Selangor, Malaysia

[a)]*Corresponding author: sih@ukm.edu.my*

Abstract. Dependence structure between financial assets plays an important role in risk management. This research investigates the dependence pattern between the stock market and the potential of cryptocurrency. We employed time-varying copula and Extreme Value Theory (EVT) to model the extreme dependence between the United States (US) index stock market (S&P500) and Bitcoin. Empirical results show risk diversification for holdings of the S&P500 and Bitcoin during extreme events seem to be effective. This paper contributes to a better understanding of the dependence structure of the financial market during extreme events. This information is useful for investors who are seeking for the cross-market diversification.

INTRODUCTION

Dependence structure between financial markets has been a debated issue among the financial industry and academic for a long time. The information regarding the linkages between financial assets will help risk managers or portfolio management to identify the opportunities for better portfolio management and risk diversification in terms of pricing and asset allocation. However, most of the tools for dependence measurement is not accurate to work with financial data under extreme settings.

Many kinds of literature have shown financial data is not suitable with normality assumption. Financial returns tend to be fat-tail phenomena and less accurate to stipulate the multivariate distribution relating more than one return series with the normality framework. The copula could be one alternative to overcome the drawbacks of linear dependence. It is shown in many finance literature copula models have significant advantages over other modeling methods in understanding the dependence structures of financial return series.

Worldwide, the market for cryptocurrencies is growing fast. As of April, 2018 there are about USD325 billions for the total market capitalization [1]. The rapid increase in these number has attracted the attention of the public, media, investors, and the government. For this research, we will focus on Bitcoin which is the most popular and the largest cryptocurrency by market volume and capitalization.

Bitcoin is the first and the largest cryptocurrency, introduced in 2009 via a protocol proposed by [2]. The total trading volume and market capitalization is account almost 29% of the market capitalization. History, Bitcoin start to be traded on the Mt. Gox exchange in 2010 and could be traded anytime in many exchanges around the world.

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The growth of Bitcoin demand for the last few years has been significant. There is an extreme upward movement for Bitcoin as a $1,000 USD investment in July 2010 for 7 years could have accumulated to $81,000,000 [3]. Bitcoin is also known as digital gold as it could be seen as an alternative to mainstream currencies as some people might lose trust with the main currencies [4]. The unique characteristics of Bitcoin there is no central authority having control over it or guaranteeing it. All of this works based on sophisticated protocol and fully decentralized. The supply of Bitcoin is limited and cannot be recreated.

This research is motivated by the extreme behavior found in Bitcoin's price movements, which has to attract academic and financial player attention. Based on [5], Bitcoin seems to face scrutiny as being speculative. According to [6], the Bitcoin market is still in its infancy and inefficient. The previous study shows the empirical data tend to exhibit heavy tail, long memory, dynamic volatility and leverage. This paper seeks to remedy by expanding our study to cover the period during 2017 and 2018 which consider a very volatile period for cryptocurrency. This paper attempts to show on the potential of the hedge properties and risk diversification of Bitcoin for portfolio and asset diversification.

It has been suggested Bitcoin could be ideal for risk-averse investors and risk diversification during turbulence period. A study by [7] demonstrated that America dollar could be hedged by using Bitcoin for the short-term strategy. In the same vein, Bitcoin is found to be uncorrelated with other financial assets either during a financial crisis or normal period [8]. Details study has shown the financial characteristics for Bitcoin are totally different from financial assets. More study regarding Bitcoin behavior and predictability could be found in [9] and [10].

Extreme Value Theory (EVT) is one of the statistical methods that focus on extreme data rather than the whole of a distribution. EVT seems to appear to be a robust approach and natural for extreme risk management. It focuses only on extreme data which could yield a more accurate for risk management tools. Further, this method seems to be a more natural and robust approach to study the risk associated with tails.

The earliest studies on EVT could be traced in [11] and [12]. Several attempts have been made to understand the extreme characteristic in finance markets. [13] used EVT to examine extreme movements in the U. S. stock market. While [14] and [15] applied EVT to emerging stock markets, details application of EVT in finance regarding Chinese stock market also could be found in [16,17,18]. For this paper, the tail distribution for each series will be model via peaks over threshold (POT) method. POT method is based on clustered phenomena. Our study will focus on high volatility periods in the cryptocurrency market which is between 2017 and 2018.

To date various methods have been developed and introduced to understand the linkages between two series of financial data. The aim of this paper is to contribute to the field regarding the dependence structure between this financial asset via EVT and copula in a new and growing market. For this project, copula models are used to analyze the dependence pattern beyond linear correlation concept. It able to works for time-varying nonlinear dependence with a high degree of flexibility. In contrast to other methods, there is no requirement for normality assumptions in marginal distributions. Interestingly, copula able to estimate marginal distributions and the joint distribution separately without to loss of important information during the procedure. The linear correlation works on the level of dependence which only demonstrates the overall strength of the relationship and it not able to describe the behavior in which the two markets are related. This research is devoted to the behavior between the stock market and cryptocurrency. Extreme volatility in cryptocurrency makes copula features more attractive than the traditional dependence tools to fit with our objective.

The extreme volatility movements in the price for the Bitcoin also point to the importance of idiosyncratic effects on the stock market. This project provided an interesting opportunity to see how far the dependence structure containing these both financial assets can go in the process to reduce risk. First, this study investigates the different dependence structures between S&P500 and Bitcoin using constant copula models. To the best of our knowledge, this is the first attempt to examine the linkages between the stock market and Bitcoin using EVT and copula. Second, this study will focus on investigating the dynamic of overall dependence and tail linkages between series using time-varying copula models.

To end, in this research we consider the time-varying dependence between S&P500 with Bitcoin. The findings of this study show that Bitcoin is poorly correlated with S&P500 which bring to the potential of diversification between these both assets.

EVT AND COPULA BACKGROUND

This part explains the details of the EVT and copula background.

Extreme Value Theory

Extreme Value Theory (EVT) is one of the parametric estimations for the tail distribution rather than the whole sample. EVT could be referred to as the extreme observations asymptotic behavior or pattern of a random variable. Over a particular period of time, there are two different approaches to recognize extremes values in real data.

First, the block maxima method (BMM) approach divides the sample into periods or blocks. This approach examines the maximum of a variable in consecutive blocks or periods, for instance weekly, months quarterly or years. The maxima method only considers the maximum value in each block or periods. BMM is the classic approach to identify the extremes of each event and has been used by a considerable amount of literature in structural engineering, geological engineering, hydrology and many other fields. Yet, due to the existing volatility clustering as well as the tendency to remove from the analysis a great amount number of important and relevant data points, BMM is not suitable for financial data. This leads to the second method for EVT that efficiently employs data from financial time series.

Next is the Peaks over threshold (POT) approach. Without heavily remove the data, POT is able to revamp the modeling of tail distribution by identifies extreme observation that exceeds the threshold. The only concern in POT is to identify a suitable threshold to be used. A trade-off between bias and variance need to be considered to achieve an optimal threshold. Then, the Generalized Pareto Distribution (GPD) will be used to fit the observed excesses above a given threshold. All exceed values are considered as extreme values and fit the characteristic of data. The formulation for POT could be written as below:

The observations $(x_1, x_2, \dots)$ be a sequence of identically and independent distributed random variables with the distribution function F. Then for a large class of underlying distribution F and large u, the conditional excess distribution function F_u can be written to approximate with GPD, that is,

$$F_u(y) \approx G_{\xi,\alpha}(y), u \to \infty$$

where

$$G_{\xi,\alpha}(y) = \begin{cases} 1 - \left(1 + \frac{\xi y}{\alpha}\right)^{\frac{-1}{\xi}} & if\ \xi \neq 0 \\ 1 - e^{\frac{-y}{\alpha}} & if\ \xi = 0 \end{cases} \qquad (1)$$

For $y \geq 0$ when $\xi \geq 0$ and $0 \leq y \leq (\frac{-\alpha}{\xi})$ when $\xi < 0$. Notation of α is known as the scale parameter, while ξ is the shape parameter for the GPD.

More details about theorem could be referred to as [19] and [20].

Copula

Copula method is first introduced in finance during the early 2000s [21]. Based on the previous literature, financial returns are often not normal. This leads to the using of copula models because of its unique features. This approach has been applied to understand the linkages or relationship between financial assets. Copula method works based nonnormality assumption which makes it attractive to the user.

Based on the theory, copula works by decomposing an n-dimensional joint distribution into its marginal distributions. The combination of the probability distribution functions able to help copula to extract information regarding dependence. A formula bivariate copula that is applied in this research can be defined as:

$$F(X_1, X_2) = C(F_1(X_1), F_2(X_2)) \quad (2)$$

where F is the combination probability distribution function from $F_1(X_1), F_2$, while C is the copula function and F_1 (F_2) is the marginal distribution with the random variables of X_1 (X_2). There are many copulas could be applied depending on their properties and characteristics. This study employed Normal and Symmetrised Joe–Clayton (SJC) copulas to serve our objectives. The overall pattern between both financial assets could be captured via the Normal model. . While SJC copula is employed to understand the pattern of the tails (upper and lower) .

METHODOLOGY

This section describes the econometric modeling procedure is used. Our methodology can be divided into two main sections: First, fitting the data with the marginal distribution. Next, we employ copula to model the linkages of the series.

Marginal Distribution Estimation

Generalized autoregressive conditional heteroscedastic (GARCH) models have been widely used to model volatility in financial data [22]. For this research, we applied the GJR-GARCH model with the mean return modeled as an AR(1) process [23,24]. The return is denoted as r_t and the conditional variance is denoted as h_t^2 for the following period of t. This model can be defined as follows:

$$r_t = \mu + \vartheta r_{t-1} + \varepsilon_t, \quad (3)$$

$$h_t^2 = c + \gamma h_{t-1}^2 + \eta_1 \varepsilon_{t-1}^2 + \eta_2 s_{t-1} \varepsilon_{t-1}^2, \quad (4)$$

where $s_{t-1} = 1$ when ε_{t-1} is 0 and negative otherwise. df is represented the degree of freedom and symbol of Ω_{t-1} is known as the previous information set. According to [25], Bitcoin has several features properties including to follow Student-t error distributions. Based on this fact, Student's t-distribution can be applied to model the standardized residual of the series z_t:

$$z_t|\Omega_{t-1} = \sqrt{\frac{df}{\sigma_t^2(df-2)}}\,\varepsilon_t z_t \sim iid\ t_{df} \quad (5)$$

GPD distribution is applied to model the residuals that exceed a predefined threshold. More details regarding the procedure to find suitable threshold value u could be found in [26,27,28]. For this study, we employ 10th and 90th percentile of the empirical data as a study by [29]. The integration between GJR-GARCH for the central part and EVT for the tails is also known as conditional EVT.

To be specific, the combination function of GARCH-EVT can be defined as follows:

$$F(z) = \begin{cases} \frac{k^L}{n}\left(1 + \xi \frac{u^L - z}{\alpha}\right)^{-1/\xi} & \text{for } z < u^L, \\ \emptyset(z) & \text{for } u^L < z < u^U \\ 1 - \frac{k^U}{n}\left(1 + \xi \frac{z - u^U}{\alpha}\right)^{-1/\xi} & \text{for } z > u^U, \end{cases} \quad (6)$$

where scale parameter is represented by α , ξ is the shape parameter, u^L (u^U) is for the lower (upper) threshold, while the number of observations is represented by n , the empirical distribution function is represented by $\emptyset$ and $k^L(k^U)$ is the number of observations below(exceeding) the threshold u^L (u^U).

Copula Model

For copula estimation, this research employs both constant and dynamic version of copulas Normal and SJC) to describe the nature behavior of the dependence structure between S&P500 and Bitcoin.

Normal Copula

The Normal copula is associated with bivariate normality. It is a common type of copula and has been applied in many fields because of the simplicity. Only in condition if both u and v are normal, the random vector of random variables for u and v could be considered bivariate normal under bivariate copula. The Normal copula formulation relationship is:

$$C(u,v) = \int_{-\infty}^{\phi^{-1}(u)} \int_{-\infty}^{\phi^{-1}(v)} \frac{1}{2\pi\sqrt{1-\delta^2}} exp\left\{-\frac{x^2 - 2\delta xy + y^2}{2(1-\delta^2)}\right\} dxdy \quad (7)$$

$$C = \Phi_\delta(\Phi^{-1}(u), \Phi^{-1}(v)), -1 \le \delta \le 1 \quad (8)$$

where the standard normal CDF's are represented by Φ and Φ_δ and the value of the coefficient is shown as δ for linear correlation regarding u and v.

To capture the dynamic version of the copula, the Normal dependence parameter will be assumed to be dynamic and follow [30].

$$\delta_t = \lambda\left(\omega + \beta\delta_{t-1} + \alpha\frac{1}{10}\sum_{j=1}^{10}\left[\Phi^{-1}(u_{t-j})\Phi^{-1}(v_{t-j})\right]\right) \quad (9)$$

where the logistic transformation that has been modified is known as $\lambda = (1-e^{-x})/(1+e^{-x})$. This procedure is to ensure δ_t in the interval (-1,1). ARMA (1,10) process need to be assumed. The value of $\beta\delta_{t-1}$ will be used to understand the persistence effect and the variation effect of dependence is identified via the mean of the product of the last 10 observations of the transformed variables $\Phi^{-1}(u_{t-j})$ and $\Phi^{-1}(v_{t-j})$.

Symmetrized Joe–Clayton Copula (SJC Copula)

The SJC copula is a new version modification from the classic Joe–Clayton copula. The formula of Joe-Clayton copula could be written as

$$C_{JC}(u,v|\tau^u,\tau^L) = 1 - ([1-(1-u)^k]^{-\gamma} + [1-(v)^k]^{-\gamma}]1^{\frac{-1}{\gamma}})1/\mathrm{k} \quad (10)$$

where

$$k = \frac{1}{log_2(2-\tau^U)}, \quad (11)$$

$$\gamma = -\frac{1}{log_2(\tau^L)}, \quad (12)$$

and

$$\tau^U \epsilon (0,1), \tau^L \epsilon (0,1). \quad (13)$$

SJC copula has many advantages compared to others. For this case, SJC copula is applied to understand the behavior of asymmetric dependence. This advanced feature makes it more useful in comparison with Joe-Clayton copula, respectively. The formulation of SJC copula can be defined as:

$$C_{SJC}(u,v|\tau^U,\tau^L) = 0.5[C_{JC}(u,v|\tau^U,\tau^L) + C_{JC}(1-u,1-v|\tau^U,\tau^L) + u + v - 1] \quad (14)$$

where CJC is the basic of Joe–Clayton copula τ^L and τ^U represent the lower and upper tail dependence. The dynamic path for the SJC copula can be written as the following form:

$$\tau^{U/L} = \tilde{\lambda}\left(\omega^{U/L} + \beta^{U/L}\tau_{t-1} + \alpha^{U/L}\frac{1}{10}\sum_{i=1}^{10}\left|u_{1,t-i} - u_{2,t-i}\right|\right) \quad (15)$$

where the logistic transformation is $\tilde{\lambda}$ and $\tilde{\lambda}(x) = (1 + e^{-x})^{-1}$ is modified formulation to ensure the parameter for the $\tau^{U/L}$ is between the range zero and one.

Model Selection

Next, the best model of the copula is selected by using the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC).

The formula for AIC could be written as:

$$\text{AIC} = -2\ log(\ likelihood\ ) + 2k \quad (16)$$

The number of parameters employed in the model is represented by k.

AIC needs to consider the information lost and the complexities of the model. The best model of the copula is considered on the lowest value of AIC.

Formula for BIC can be written as

$$\text{BIC} = -2\ log(\ likelihood\ ) + k \cdot log(n) \quad (17)$$

where k is the number of parameters employed in the given model and the number of the sample is represented by n. Formulation between AIC and BIC is quite similar. However, BIC tends to penalize model complexity more heavily. A better fit of the model is based on the lowest value of BIC.

DESCRIPTIVE STATISTICS

For this research, we used daily S&P500 and Bitcoin prices from investing.com. Bitcoin is traded on several exchanges and operate every minute, including holidays, Saturdays and Sundays. A few of observation data Bitcoin needs to be removed to sync the data with S&P500 daily data. Daily data is used instead of weekly data for the period between Jan 2017 to December 2018 to generate reliable result by providing a large number of observations.

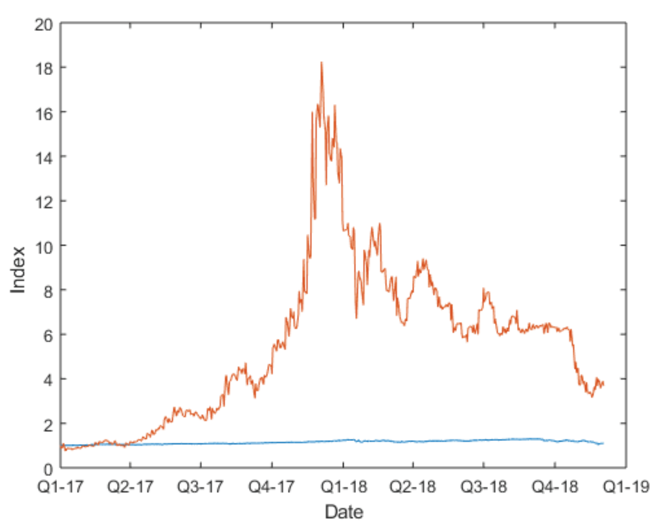

FIGURE 1..Price for S&P500 and Bitcoin

Figure 1 depicts the relative price level for Bitcoin and S&P500 series for the whole sample period. Bitcoin price is relatively far higher than S&P500. We could see the price for the Bitcoin increases and decrease significantly. Bitcoin has increased significantly during 2017. However, this trend change and the price have fallen by more than 50% after that. Compare to S&P500, the Bitcoin price has relatively increased and decreased higher than S&P500. As evidenced in Figure 1, Bitcoin is in the bubble phase and could be a product of speculation. In contrast, S&P500 seems to be relatively more stable over the sample period.

TABLE 1. Descriptive statistics

	Mean	Median	Std.	Kurtosis	Skew	Min	Max	Corr	JB Stat
S&P500	0.0209	0	1.141	7.4053	-1.1441	-6.4188	4.1843	0.1574	1531.40*
Bitcoin	0.2607	-0.3144	8.7322	4.8767	0.6807	-36.6427	51.713	-	981.30*

Notes: Std is a standard deviation,Corr is a correlation, JB Stat is Jarque Bera Statistics and '*' is significant at 0.01.

Table 1. demonstrates the descriptive statistics of daily returns for the S&P 500 and Bitcoin. We can see the value of the mean for S&P500 is 0.2009%. Bitcoin has a higher mean with 0.2607%. However, the median for the Bitcoin is negative. It is interesting to see the standard deviation for the Bitcoin is relatively almost eight times higher than S&P500. This indicates Bitcoin is riskier than S&P500 as the high value of standard deviation could be associated with high volatility.

Observation data for S&P500 is skewed to the left while Bitcoin data is skew to the right. The min value for the Bitcoin is also relatively low compared to S&P500. The same situation also takes place corresponding to the max value. Bitcoin relatively has a high value of max compared to S&P500. Bitcoin also seems to have higher volatility compare to S&P500 data. As we look into Jarque Bera statistic's value, both results show there is existing of heavy tail phenomena. There is strong evidence from both series of a non-linear characteristic and behavior of the extreme sample. Thus, any model-based assumption of normality will be less accurate. The framework which works based on linear and normal assumption will not be suitable to model both of these series.

The correlation is 0.1574 for S&P500 and Bitcoin pair. This value indicates a strong dependence pattern exists between both series. For the portfolio manager, the low correlation could be a good indicator for assets diversification. In contrast, a strong correlation indicates both financial assets are not suited during the volatility or crisis period. From this value, it is clear that during extreme volatility movement either one of the markets, Bitcoin might not be suitable to be paired with S&P500 as an effective diversification strategy. However, the value based on the linear method could be misleading. More insightful information about this relationship can be achieved by using copulas.

RESULTS AND DISCUSSION

The empirical findings are reported in this part. We present the estimation results for the copula models

Parameters Estimation

The main objective of this research is to explore the pattern of linkages between S&P500 and Bitcoin during 2017 and 2018. Normal and SJC copula with their time-varying versions are used to assess the behavior of the linkages. SJC copula is employed to provide information regarding the relationship of lower and upper tails. While to understand the overall dependence pattern, the Normal copula is applied.

The parameter results for the Normal and SJC copulas could be seen in Table 2. δ (Normal), τ^U and τ^L (SJC) are the main parameters for the constant copula while the critical parameters are ω (Normal), ω^U and ω^L (SJC), α (Normal), α^U and α^L (SJC) and β (Normal), β^U and β^L (SJC).
More details, ω (Normal), ω^U and ω^L (SJC) are applied to study the dynamic relationship of the structure. α (Normal), α^U and α^L (SJC) are used to represent the adjustment in the structure. While parameter β (Normal), β^U and β^L (SJC) are used to capture the power of persistence of relationship.

AIC and BIC are used to identify the best model. A lower value of AIC or BIC shows the model is a better fit with the empirical data.

TABLE 2. Constant and time-varying parameters

Constant		**Time-varying Normal**	
Normal Copula			
δ	-0.0431	ω	-0.1759
		α	-0.2608
		β	-1.9508
AIC	-0.3115	AIC	-0.3726
BIC	-0.3027	BIC	-0.3460
SJC Copula		**Time-varying SJC**	
τ^U	0.0358	ω^U	0.0080
τ^L	0.0000	α^U	-3.9738
		β^U	0.6801
		ω^L	-8.1621
		α^L	-2.6618
		β^L	9.8644
AIC	3.0909	AIC	12.2287
BIC	11.3921	BIC	37.1323

Based on Table 2, the constant values of overall dependence parameter δ is -0.0431. This value indicates a slightly negative relationship between S&P500 and Bitcoin. The value for the τ^U which could be associated with upper dependence is 0.0358. While, the value of τ^L which is associated with lower dependence is zero. This show S&P500 has almost no dependence with the Bitcoin during the negative extreme period. As we compared the static and time-varying model, it seems that the time-varying model provides a better estimation for Normal copula. In contrast, the static model for SJC copula is able to provide a better estimation than time-varying copula.

Figures II compares the path for the constant and time-varying parameters. For a dynamic path, the adjustment α is used to capture the adjustment in the structure link process and β is represent the degree of persistence and the value of ω is used to describe the dependence levels. Comparison between ω is important to understand the magnitude of level and possibility of diversification between them.

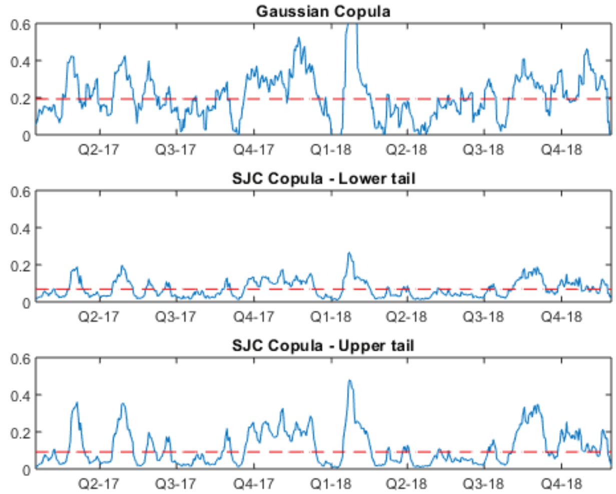

FIGURE 2. Constant and dynamic copula

Figure 2. presents the path of parameters regarding the linkages between S&P500 and Bitcoin. The parameter values for the constant could be seen in the dashed line while the time-varying copula in the solid line.

The time-varying dependence structure for the pair is considered in details. Figure 2. illustrates the time-varying paths of parameters for both Normal and SJC copulas for the overall, the upper and lower tail pattern of dependence structure. The coefficients, α (α^L or α^U) and β (β^L or β^U) are used to capture the time variations of dependence as shown in Equation (15).

A few interesting observations could be obtained from Figure 2. The pair has a strong correlation in terms of overall correlation. The correlation is positive and fluctuates between 0 and 0.4. As we looked into details, most of the time the values for the δ_t in the time-varying Normal copula model is moving between 0.1 and 0.3. There are several times the correlation is almost zero. A strong correlation also exists in early 2018. Thus, portfolio diversification strategy might not be suitable during this period. The dependence for the upper tails is between 0 and 0.4. While for the lower tail, the dependence is between 0. and 0.2. This result demonstrates the pair has stronger upper tail dependence than lower tail dependence. This also indicates that positive tail dependence can be observed at times for the upper and lower tail. The path for overall and both tails is quite volatile during the period measure. This is an interesting finding and a sharp contrast to most findings in the previous literature.

The values of ω in the SJC copula is the critical value to be observed in the process to understand the potential diversification between S&P500 and Bitcoin. Better risk management practices could be obtained to provide a suitable portfolio diversification [18]. In most of the financial and copula literature, the values of ω^L are normally to be relatively higher than ω^U. This indicates both series tend to have stronger dependence during the crisis compared to the normal period. In contrast, the values of ω for the upper is greater than and the value of ω for the lower. This could translate the pattern of dependence structure during a crisis period is not as strong as during a boom period. Thus lead to a better risk diversification for holding S&P500 and Bitcoin. This insight information is useful for the asset portfolio and risk manager to make a better decision.

Taken together, it is apparent that a strong relationship exists for the overall dependence between S&P500 and Bitcoin. This study also demonstrates the existence of upper tail dependence for this pair. This paper suggests that S&P500 and Bitcoin correlated in terms of positive extreme event and did not have a strong relationship or links during the crisis. Whilst this research does not provide the reasons behind these findings, it sheds light on the information regarding the dependence pattern between S&P 500 and Bitcoin during extreme volatility. This paper has confirmed the potential of diversification opportunities for investors by holding S&P500 and Bitcoin. To the best of our knowledge, this is the only study to offer some important insights regarding the potential of S&P500 and Bitcoin for risk diversification via copula and EVT.

6. CONCLUDING REMARKS

Bitcoin is the popular cryptocurrency and the largest by market capitalization. This study is deeply motivated by the extreme volatility and unique features found in Bitcoin data Until recently, the current finance and economy literature are still lacking empirical evidence on Bitcoin potential diversification with other financial assets despite the growing study. This study offers more understanding of the dependence behavior pattern especially during the extreme volatile period.

This study set out to determine the dependence structure between S&P500 and Bitcoin by using a combination of EVT and copulas. The integration between EVT and copula has shown to provide more insightful information to access the dependence structure beyond than the traditional approach. Based on the information regarding the tail behavior, it is found that the dependence pattern between S&P500 and Bitcoin pair during a crisis is not as strong as during a booming period. Thus, there is a potential of portfolio diversification between these both assets during an extreme negative period. At the same time, more information could be gained by using time-varying copula rather than linear correlation. The current findings can help the portfolio manager to seek better opportunities for risk diversification in both asset classes.

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