Research Proposal and Literature Review

An Application of Copulas to Modelling Financial Assets with Group-wise dependence Structure

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1 Introduction

The concept of cryptocurrency first emerged in 2008 with the introduction of Bitcoin in a paper titled "Bitcoin: A Peer-to-Peer Electronic Cash System"[16], sent through a cryptography mailing list under the pseudonym, Satoshi Nakamoto. In this influential paper, Satoshi Nakamoto introduced the concept of Bitcoin as a decentralized payment system built on a network of distributed ledgers. According to Nakamoto, this cryptographic-based system eliminates the need for a trusted central authority.[16] This idea gained momentum as the aftermaths of the 2008 financial crisis manifested, leading to the emergence of many other cryptocurrencies, including Ethereum. The crypto market experienced another boom towards the end of 2020, as central banks adopted extensive quantitative easing measures in response to the COVID-19 pandemic. With the emergence of more crypto assets and derivative product, this market has come to represent a significant financial innovation of the 2000s.

The cryptocurrency market is subject to significantly less regulation than other financial markets, and is therefore characterized by high volatility and sporadic periods of extremely high and low returns. For instance, the total market capitalization was around USD$185 billion dollars and the trading volume was around USD$70 billion in January 2020. In November 2021, the total market capitalization grew to an astonishing USD$2.86 Trillion and trading volume of around 150 billion, representing an annualized return of more than 320% [1].

Advocates of Bitcoin argue that Bitcoin has certain safe heaven properties which is commonly observed in gold. This concept has inspired various research into the benefits of using Bitcoin and other cryptocurrencies in financial portfolio diversification or hedging. For example, Bhuiyan et al. [7] concluded by examining data from January 9th, 2014 to May 31st, 2022, that Bitcoin exhibited low correlation with traditional equity markets and provided consistent diversification benefits to several equity markets, especially during normal economic conditions. However, it is widely accepted that non-mainstream cryptocurrencies cannot be regarded as secure assets. In fact, mainstream cryptocurrencies, namely Bitcoin and Ethereum, have been found to act as a price reference for other cryptocurrencies. For instance, Briola and Aste [9] found that Bitcoin and Ethereum price movements have strong influence on the returns of other crypto assets.

The distinct characteristics of cryptocurrencies, namely, the pronounced volatility, the presence of aperiodic cyclical patterns of volatility, and a time-varying, alternating dependence structure with both the stock markets and traditional safe-heaven assets render them an interesting subject for analysis.

2 Literature Reiview

2.1 Existing Research on Price Dependencies Between Cryptocurrencies

As the concept of cryptocurrencies started to attract more attention from traditional financial markets, a trend evidenced by the increasing amount of investments in crypto assets made by traditional financial institutions including banks, hedge funds, and private equities, there has been a surge of academic research dedicated to examining various aspects of cryptocurrency market returns.

Kumar and Taufeeq examined the co-movement patterns of four mainstream cryptocurrencies using wavelet-based methods. They found that cryptocurrency prices are mainly driven by Bitcoin and shocks will have a ripple effect within the market. Moreover, the correlation between these four cryptocurrencies exhibited a cyclical pattern that was also aperiodic. [17].

Stosic et al. [18] looked the cross correlation of prices changes between 119 publicly traded cryptocurrencies from August 2016 to January 2018 using concepts from random matrix theory and minimum spanning trees. Their objective was to search for a hierarchical structure in the cryptocurrencies market. They found that there are varying degrees of contribution to the eigenvectors of the correlation matrix by different cryptocurrencies, leading to non-trivial hierarchical structures, which indicates the existence of grouping of cryptocurrency pairs. Furthermore, their study revealed that there exists an significantly stable community structure amongst the cryptocurrencies.

James and Menzie [12] analyzed returns behavior of 52 cryptocurrencies with the highest market capitalization from Jan 1st 2019 to 30th June 2020, a period containing significant volatilities, with the crypto market having experienced both a significant drawback and a boom in price during this period. In their analysis, James and Menzie first identified market regime changes using a turning point algorithm. An inverse relationship was found between the market size and collective behavior amongst cryptocurrencies. They found that there is a increased uniformity in the relationship between size and volatility than size and return. Finally, they found an increase level of volatility across crypto markets during crashes by examining the Wasserstein distances between probability density functions of rolling volatility[12].

Briola and Aste [9] studied the resulted structures of Minimum Spanning Trees and Triangulated Maximally Filtered Graph when applied to 25 cryptocurrencies at different time intervals. Most interestingly, they found that Ethereum can be regarded as a reference node in the hierarchy for many other assets and this reference property persists in time. Briola also observed that the dynamics of cryptocurrencies as described by Minimum Spanning Trees are strongly influenced by Bitcoin and Ethereum. [9].

Assaf et al. [3] adopted transfer entropy to measure the information transmission between Bitcoin, Ethereum and Ripple, capturing events separately according to the level of frequency. This enables them to capture the non-linear market dependencies, especially with respect to the tail distributions of the returns. They found that there exists a bidirectional mechanism for information transmission between Bitcoin and Ripple, suggesting that the influence is mutual, whereas information flows one directional from Ripple to Ethereum.

2.2 Theoretical Work on Copulas

In [4], Bedford and Cooke introduced a vine copulas structure as a generalized version of Markov trees that can be applied to model the dependence structure of multivariate data. They argue that vines differ from Markov trees and Bayesian approaches in that they relax the concept of conditional independence, allowing for various forms of conditional dependence among variables. Bedford and Cooke presented vines as a simple way of modelling a multivariate distribution, and the model can be constructed by specifying pair-wise marginal distributions. They also found that Gibbs sampling can be a promising method for sampling from a vine structure. In their subsequent work [5]. Bedford and Cooke states that vine copulas can have practical use in assessing the level of sensitivity of a model to uncertainties in its parameters.

While vine copulas offers a flexible framework for modelling high-dimensional dependencies, its computational cost increases exponentially with the number of dimensions. Berchmann et al[8] proposed a way to reduce the computational complexity of regular vine copulas by employing statistical techniques for pairwise and joint simplification of vine copulas, resulting in a truncated vine copula structure. Their paper also conducted empirical studies of the proposed model using a 19-dimensional data set and concluded the resulting model was significantly more computationally efficient.

In [13], Krupskii and Joe proposed a new way to model data with tail asymmetry or dependence using factor copulas. This approach is particularly useful when the multivariate normality assumption is violated and a few latent variables determines the dependence structure in the observed variables. Traditionally, the Gaussian factor model assumes normally distributed unobserved factors with linear relationships amongst themselves. The factor copula structure models the dependence structure in observed data with latent variables and allows for a factor structure in the correlation matrix with partial correlation. Another popular approach in recent years has been the vine copulas, but it requires $d(d-1)$ number of bivariate copulas for d variables and typically involves $O(d^2)$ number of parameters. The factor copulas model proposed in this paper has a simpler dependence structure and reduces the number of parameters involved from increasing exponentially to linearly with number of dimensions.

In a multivariate copulas model, the d dimensional variables of interest, $X_1, ..., X_d$ are transformed to uniform random variables, $U_1, ..., U_d$ with their respective univariate marginal distributions. A d-dimensional copulas C then applied on the transformed variables $\mathbf{U}$, leading to the joint distribution $C(u_1, .., u_d)$. In the copula model with p factors, the transformed variables $U_1, ..., U_d$ are regarded as independent conditional on p latent variables $V_1, ..., V_p$. Then we have:

$$C(u_1, ...u_d) = \underbrace{\int .. \int}_{\text{p}} \prod_{k=1}^{d} F_{k|V_1,...V_p}(u_k|v_1, ...v_p)dv_1...dv_p \tag{2.2.1}$$

Where $F_{k|V_1,...V_p}(u_k|v_1, ...v_p)$ is a sequence of bi-variate copulas, conditional distribution of $\mathbf{U}$ given latent variables $\mathbf{V}$.

Hua and Joe [11] studied the tail patterns and tail asymmetry of certain copulas families, introduced the concept of tail order functions and presented fundamental properties for analysing the tail structures. In particular, they provided discussions of the Archimedean copula with Laplace transformation of tail heavy positive random variables, proposed a new single parameter Archimedean copula model and studied the tail orders of some special copulas .

In [2], Aas and Berg conducted empirical analysis and compared two copula models, namely, the nested Archimedean construction(NAC) and pair-copula construction, for modelling multivariate data with complex dependence structures using two four-dimensional data sets. Aas and Berg argues that the PCC is less restrictive than NAC for the fact that every bivariate pair of copulas in the NAC needs to be Archimedean and that there are also significant limitations to the degree of dependency within the levels of NAC. Their study provides evidence that the second model is superior in fitting high dimensional data sets due to its computational efficiency and allowance for higher degree of dependence.

2.3 Applications of Copulas for Modelling Dependence Structure in Cryptocurrencies

In [10], Gong and Huser examined the dependence structure between 5 cryptocurrencies with the largest market capitalization. In this paper, Gong and Huser applied a copula model that is flexible in both tails and is able to capture and control for the dependence structures within each joint tail such that the transition between dependence classes is smooth. They also utilized a local likelihood method for quantifying the dependence among cryptocurrencies from a temporal perspective. Their studies found that the dependence in the lower tail of the joint returns distribution has become increasingly strong in recent years, representing a higher correlation in extreme low returns, whereas the dependence structure in the joint upper tails have remained relatively moderate.

BenSaïda [6] applied regular (R)-vine copulas to examine the linkage between Bitcoin and fiat currencies of developed and emerging currencies and compared the results to that of the t copula and the dynamic conditional correlation GARCH model. In his application, BenSaïda paid special attention to a series of shock events to both the crypto market and traditional financial market for signs of changes in the level of dependency. Bensaïda found that the dependence between Bitcoin and fiat currencies were better captured by the model when information on shock transmission is provided, and R-vine outperformed the other two models considered [6].

Naeem et al. [15] examined dependencies between the trading volumes and returns of Bitcoin, Ethereum and Litecoin under the GARCH-copula framework. They found that the tail returns of these three cryptocurrencies are closely related to extreme trading volumes. An asymmetric tail dependence structure was found in the joint return-volume distribution, namely, the dependence appears to be stronger in the lower tail than in the upper tail. Suggesting that during periods of market downturn, volume is likely to remain at low levels, which is in line with what is observed in traditional financial markets. However, volume and returns are exhibit less correlations in the upper tails, suggesting that the arrival of negative information will have limited effect on returns. Naeem proposed that this phenomenon can be explained by the presence of irrational, over-optimistic investors in the cryptocurrency market who are known for chasing after highs and not closing their positions in hope to eventually avoid the losses amidst market downturns.

In [19], Tiwari et al. modeled the dependencies and contagion risks between three cryptocurrencies, Bitcoin, Ripple and Litecoin from August 4th, 2013 to June 17th, 2018 by applying the mixture copulas framework and methods for a full-range tail dependence copulas. They found that using the mixture model, there is a stronger upper-tail dependence between bitcoin and Litecoin, while the lower-tail dependence is more prevalent in other cryptocurrency pairs. While under the method of full-range tail dependence copulas, there is significant upper and lower tail dependencies between all pairs of cryptocurrencies.

3 Research Plan

3.1 Motivation

3.1.1 Group Construction

In their study on the effect of fundamental and speculative drivers of cryptocurrency prices, Kukacka and Kristoufek [14] used the cusp catastrophe model to analyse the drivers of several cryptocurrencies, namely Bitcoin, Ethereum, Litecoin, XRP and dogecoin. They found that both fundamental and speculative elements have a strong impact on Bitcoin and that it is priced from a long-term perspective. In addition, indicators of investor interest, such as google search volumes and Wikipedia page views as well as transaction volume have a particularly significant impact on Bitcoin prices, indicating a pivotal role of Bitcoin in driving the overall cryptocurrencies market dynamics. Other research suggested similar conclusions of Bitcoin and Ethereum as market leaders. For instance, Briola and Aste [9] found that Bitcoin and Ethereum price movements can act as price reference for other cryptocurrencies.

Another interesting aspect found in Kukacka and Kristoufek's research is that since many other cryptoassets are based on the Ethereum network (hereafter referred to as the ETH-based coins), the success of these cryptoassets often causes cyclicality in the price of Ethereum. As the transaction volume of ETH-based coins increases and thus more transaction is conducted on the Ethereum blockchain, the price of Ethereum and other coins in the Ethereum ecosystem experiences a positive shock. [14]

Kukacka and Kristoufek's study also found that the price of XRP showed stronger correlation with speculative factors rather than fundamental components. In addition, their study suggest that prices of smaller crytoassets, such as Litecoin are more susceptible to changes in traditional financial market movements and are driven by their own fundamental and speculative components as well as the market leader and that prices of pure speculative coins, such as Dogecoin have shown to be driven primarily by the general crypto market sentiment. [14]

This research draws on the findings of previous academic works on some specific dependency structures within cryptocurrencies and divide them into three groups and aims to construct a copula structure that provides an in-depth insight into the dependency structure within cryptocurrency groups. The first group will contain Bitcoin, whose returns has been shown to be a fundamental driver of other crytocurrency prices. The second group will contain coins rested in the Ethereum ecosystem, with strong returns dependence found within this group. The third group will contain speculative coins, whose price movements are driven primarily by the general crypto market sentiment.

3.1.2 Copula Construction

Cryptocurrency returns have been shown to have hierarchical structures and complex interdependency structures, and is therefore suitable to be modelled under a copulas framework. Specifically, when the data demonstrates attributes of flexible and asymmetric tail dependence, vine copulas have been shown to be have exceptional performance in various studies [4][5].This research aims to construct a comprehensive model designed to capture the intricate dependence structure among three groups of cryptocurrencies using a combination of C-vine and D-vine.

The first group in our framework contains Bitcoin (BTC) and Ethereum (ETH), which are regarded as the cornerstone asset in the crypto market. Bitcoin and Ethereum has consistently been demonstrated to be the pivotal price benchmark, exerting significant influence over the valuation of other crypto currencies.

The second group is comprised of the top five Ethereum network coins with the highest market capitalization, including Chainlink (LINK), Unus Sed Leo (LEO), Uniswap (UNI), Maker (MKR) and Aave (AAVE). Prices of these coins have been observed to be highly correlated with the price of Ethereum and Bitcoin in the first group. This phenomenon can be partly explained by the fact that Ethereum was the first cryptocurrency utilizing smart contracts project and many of the subsequent smart-contract based coins operate within the Ethereum ecosystem and many rely on the Ethereum blockchain as their foundational infrastructure. In addition, as the first widely recognized cryptocurrency, Bitcoin has been found to drive the overall price dynamics of the crypto market. Prices of coins in the Ethereum network shows strong dependence with each other as excessive transaction volume generates transaction fees for miners in the system but also causes congestion.

The third category is comprised of two cryptocurrencies, Dogecoin (DOGE) and Shiba Inu (SHIB), referred to as ”penny coins” by the author. The term ”penny coins” has been adopted based on the concept of penny stocks in traditional financial markets. Penny stocks are refers to low-priced stocks often characterized by their speculative nature, hence the term is used primarily to describe a specific set of cryptocurrencies that do not have an inherent underlying value or utility and are primarily traded based on speculative hype.

3.2 Model Construction

3.2.1 Model 1

This research project will start out by building separate models for each group to model the dependency structure within each group separately. Since various variables has been shown to have an impact on the price movements of cryptocurrencies in previous research works, yet the exact effect of these variables remain unclear, we will employ a factor copulas model to capture the dependence structure within these groups conditioned on some latent factors. We construct separate copulas for each group because the latent factors impacting the returns of cryptocurrencies in different groups are not all the same. Furthermore, the multivariate normality assumption necessary for many other models, such as in the case of Gaussian factor model, is violated. A factor copula is appropriate in this case and we assume a few latent factors determine the dependence structure in the returns of cryptocurrencies in each group. In addition, the factor copulas model has a simpler dependence structure and the number of parameters increases linearly with the number of dimensions.

Questions: 1. How do I decide how many latent factors to include? 2. Are there existing code for bi-variate copulas for x number of factors? 3. Are there existing code for five-variate copulas for x number of factors?

3.2.2 Model 2

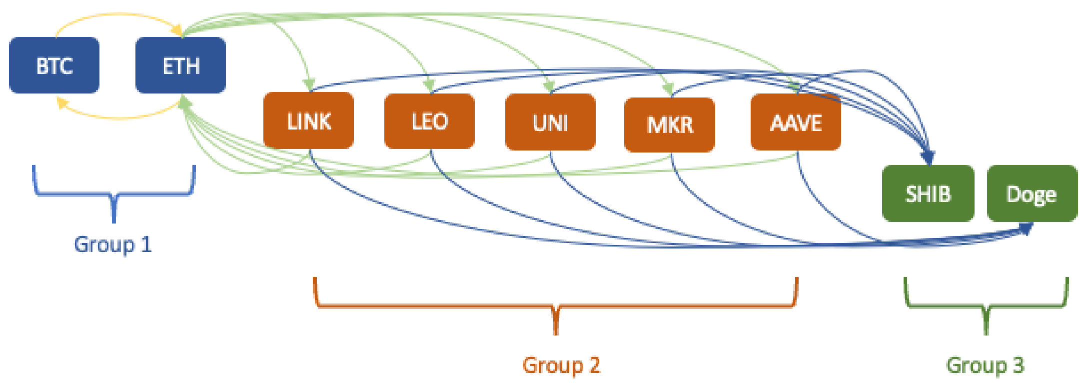

Figure 1: Inter-group Dependency Structure

3.3 Data Collection

The data used in this research project contains the hourly closing price of 8 cryptocurrencies from Jan 1st, 2019 to September 1st, 2023. These data are retrieved through the open-source Binance API. The coins are chosen such that they are representative of the three groups within our analysis framework and are selected by their market capitalization as of September 15th, 2023.

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