International Economics 158 (2019) 77–90

Contents lists available at ScienceDirect

International Economics

journal homepage: www.elsevier.com/locate/inteco

# Vine copula-based dependence and portfolio value-at-risk analysis of the cryptocurrency market

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ARTICLE INFO

*JEL classification:*
E31
E42
G12

*Keywords:*
Cryptocurrency
Vine copula
Dependence
Value-at-risk

ABSTRACT

In this paper, we use vine copula approaches to model the co-dependence and portfolio value-at-risk (VaR) of six cryptocurrencies using data of daily periodicity from September 2015 to June 2018. We establish evidence of strong dependencies among the virtual currencies with a dynamic dependency structure. We find that among the class of cryptocurrencies examined, Ethereum offers the best optimal and economically risk-reward trade-off subject to a no-shorting constraint for portfolio investors using the efficient frontier. Given the paucity of empirical research on the cryptocurrency markets, this paper provides new insights, which could be useful in developing dependence and risk strategies for investment and hedging purposes, especially during more volatile periods in the markets.

## 1. Introduction

If the potential of cryptocurrencies to find a significant place among financial assets remains controversial, it appears increasingly strong. Cryptocurrency markets have grown so rapidly that some experts suggest that they are already a new category of investment assets (Corbet et al., 2018). The price of Bitcoin, the world's most valuable cryptocurrency, though erratic, most recently jumped from US$4395 (27 August 2017) to US$6531 (15 September 2018). As of July 19, 2018, the total cryptocurrency market was valued at US$295 billion. In fact, key financial markets players, including CNBC billionaire investor Marc Larsry, speculate that the price of bitcoin could reach US$40,000.

It is not necessary to embrace such optimistic forecasts to reckon the rapidly evolving virtual currency market, coupled with its potential to become a less expensive alternative to traditional currencies. They have attracted the attention of not only investors and other key players in the financial markets sector, but also of researchers (Demir et al., 2018). Existing research on the cryptocurrency market includes studies focusing on the price formation of Bitcoin (e.g. Grinberg, 2011; Barber et al., 2012; Kroll et al., 2013; Moore and Christin, 2013; Bouoiyour et al., 2014; Kristoufek, 2015; Ciaian et al., 2018), the patterns of information transmission across cryptocurrencies markets and other asset classes (e.g. Corbet et al., 2018), technical aspects and stylized facts of cryptocurrency markets (e.g. Dwyer, 2015; Bariviera et al., 2017), hedging and safe haven properties of cryptocurrencies (e.g. Dyhrberg, 2016; Bouri et al., 2017), return-volume relationships (e.g. Balciar et al., 2017), speculation (e.g., Yermack, 2014; Glaser et al., 2014; Blau, 2017; Corbet et al., 2018), volatility of Bitcoin returns (Katsiampa, 2017), market efficiency (e.g., Urquhart, 2016; Bariviera, 2017; Nadarajah and Chu, 2017), and their transaction cost (Kim, 2017).

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https://doi.org/10.1016/j.inteco.2019.03.002
Received 1 November 2018; Received in revised form 4 March 2019; Accepted 4 March 2019
Available online 11 March 2019

While these studies provide rich and useful analyses, they have not so far accounted for the structural linkages and interdependencies among the various cryptocurrencies, a notable exception being the paper by Ciaian et al. (2018). As with any other financial assets, such as stocks, commodities, bonds, and others, understanding how cryptocurrencies inter-relate and depend on each other is useful for investors seeking to hold a diversified portfolio and crucial to understanding the microstructure of the cryptocurrency market.

Therefore, our paper attempts to bridge the literature and knowledge gaps by examining the price dependency relationships and portfolio value-at-risk (VaR) of Bitcoin and five key altcoins (Dash, Ethereum, Litecoin, Ripple, and Stellar) using regular vine (R-vine) copulas that allow for the modelling of composite risk of financial assets.

Specifically, the paper addresses the questions: (a) do cryptocurrencies depend on each other at all? (b) do price developments in one cryptocurrency cause any discernible impacts in the price movements of other cryptocurrencies? Providing answers to these questions is crucial as they hold significant implications, especially in relation with potential contagion and risk spillover effects among cryptocurrencies.

Our paper is technically quite distinct from that of Ciaian et al. (2018), which uses the autoregressive distributed lag (ARDL) model to analyse the relationships between 17 virtual currencies (VCs) and two altcoin price indices from 2013 to 2017, suggesting that Bitcoin and altcoin markets are interdependent. Our application of the R-vine copula method shows robustness, as it is able to provide greater flexibility and permit the modelling of complex dependency patterns using the rich variety of bivariate copulas that may be arranged and analysed in a tree structure to explore multiple dependencies. To further show the capabilities of this flexible modelling technique, we use R-vine copulas to quantify VAR for a weighted portfolio of six cryptocurrency indices. This helps to demonstrate the useful application of both C-vine and R-vine measures of co-dependency at the time of extreme financial stress and their effectiveness in teasing out changes in co-dependency (Allen et al., 2017). Applying the copula theory to model VAR enables us to construct a flexible multivariate distribution with different margins and different dependency structures that allow the joint distribution of a portfolio to be free from assumptions of normality and linear correlations. With this, our paper becomes the first to model copula-based extreme VAR for a combined portfolio of cryptocurrencies.

The rest of the paper is structured as follows: Section 2 outlines the empirical model; Section 3 presents the data, stylized facts, and descriptive statistics. Sections 4 and 5 detail the empirical analysis and conclusion, respectively.

## 2. Empirical models

### 2.1. Basic concept

As espoused by Sklar (1959), a multivariate distribution function has the following components: marginal distributions that capture the individual characteristics of each series and a copula that exhaustively shows the dependence between them. Moreover, a copula has the ability to link any given set of marginal distributions to construct a joint distribution, offering a great deal of flexibility in the specification of the marginal distribution and the dependence structure between them (Delatte and Lopez, 2013). Basically, copula functions possess one-dimension margins with an interval of [0,1], which have invariant monotonic increasing transformations of the marginal (Nelsen, 2006). The functionality of copulas assumes the existence of a vector of $X$ random variables with marginal distribution functions of $F_i(X_i), i = 1, 2, \ldots, d$. The set of transformation $U_i = F_i(X_i)$ defines a dependent and uniformly distributed vector of random variables $\boldsymbol{U} = (U_1, \ldots, U_d)$ on $[0,1]^{\mathrm{d}}$.

The view of Nelson is that given that the function $F_i(X_i)$ is continuous in nature, the joint distribution of $X$ can be expressed, in the form below:

$$F(x) = C(F_1(x_i), \ldots, F_d(x_d)) = C(U_1, \ldots, U_d) \tag{1}$$

where C(U) is the copula of the distribution, C:$[0,1]^{\mathrm{d}} \rightarrow [0,1]$ and U = $(U_1, \ldots, U_d)$. The copula C could also be likened to a joint distribution function of vector U. Equation (1) is Sklar's theorem. By expansion we could define copular C(U) as:

$$C(u) = F\left(F_1^{-1}(u_1), \ldots, F_d^{-1}(u_d)\right) \tag{2}$$

The accompanying copula density is given as:

$$c(u) = \partial^d C(U_1, \ldots, U_d) / \partial(U_1, \ldots, U_d) \tag{3}$$

Lebrun and Dutfoy (2009a) define the joint probability density function of X, $f_X(x_d) = f_X(x_1, \ldots, x_d)$ as:

$$f_X(x_1, \ldots, x_d) = c\{F_1(x_1), \ldots, F_d(x)\} \prod_{i=1}^{d} f(x_1) \tag{4}$$

where $f_i(x_i)$ is the marginal probability density function of $X_i$. Equation (4) combines the marginal distributions and copula density, which contains all relevant information about the dependence structure of the random variables. Genest et al. (2009) and Lebrun and Dutfoy (2009b) calculate the conditional marginal distributions of $\boldsymbol{X}$ as:

$$F_{i|1,\ldots,j-1}(x_i|x_i, \ldots, x_{i-1}) = C_{i|1,\ldots i-1}(u_i|u_i, \ldots, u_{i-1}) \tag{5}$$

where

$$C_{i|1,\dots i-1}(u_i|u_i,\dots,u_{i-1}) = \frac{\partial^{i-1}C(U_1,\ \dots,U_i,1,\dots,1)}{\partial(U_1,\dots,U_i)} \Big/ \frac{\partial^{i-1}C(U_1,\dots,U_i,\ \ U_{i-1},1,\dots,1)}{\partial(U_1,\dots,U_i)} \tag{6}$$

Equation (6) can also be considered for the bivariate case, with $u_1 = u$ and $u_2 = v$

$$F_{X2|X1}(x_2|x_!) = C(v|u) = \partial C(u,v)/_{\partial u} \tag{7}$$

If a random sample of size $n$ has a corresponding vector of variables $X$, i.e. $x_1 = (x_{i1},\dots,x_{id})(i=1,\dots,n)$ is given, then, Genest et al. (1995) and Genest and Favre (2007) define the estimated copular as:

$$C_n(u) = \frac{1}{n}\sum_{i=1}^{n} 1(S_{i1} \leq U_1,\dots,S_{id} \leq U_d) \tag{8}$$

where $S_{ij} = r_{ir}/(n+1)$ are the pseudo-observations and $r_{ir}$ are the ranks associated with the sample. The pseudo observations can be grouped into vector $S_i = (S_{i1},\dots,S_{id})$.

### 2.2. *Vine-copulas*

Aas et al. (2006) defined vine copulas as any pair-copula construction. The joint probability density function if X, $f_x(x) = f_x(X_1,\dots,X_d)$ can be expressed in terms of conditional density functions as:

$$f(x) = f_1(x_1)f_{2|1}(x_2|x_1)f_{3|1,2}(x_3|x_1,x_2),\dots,f_{d|1,\dots,d-1}(x_d|x_1,\dots,x_{d-1}) \tag{9}$$

It should be noted that Equation (9) is not a representation of a unique decomposition. It is one of the many ways in which the function $f_X(x_d)$ can be decomposed. However, the bi-variate decomposition is represented as:

$$f_{12}(x_1,x_2) = C_{12}(F_1(x_1),F_2(x_2))f_1(x_1)f_2(x_2) \tag{10}$$

where $C_{12}$ is the copula density between variable $X_1$ and $X_2$. Subjecting Equation (10) to conditional probability, we have the:

$$f_{2|1}(x_1|x_2) = C_{12}(F_1(x_1),F_2(x_2))f_2(x_2) \tag{11}$$

We can extend Equation (11) to a three random variable dimension and express it as:

$$f_{3|1,2}(x_3|x_1,x_2) = C_{23|1}\left(F_{2|1}(x_2|x_1),F_{3|1}(x_3|x_1)\right)f_{3|1}(x_3|x_1) \tag{12}$$

Here, it is assumed in Equation (12) that $C_{23|1}$ is independent of $X_1$, which allows for flexible modelling as a result of it being considered a good approximation (Brechmann, 2010). However, an alternative specification of Equation (12) could be:

$$f_{3|1,2}(x_3|\ x_1,x_2) = C_{13|2}\left(F_{1|2}(x_1|x_2),F_{3|2}(x_3|x_2)\right)f_{3|2}(x_3|x_2) \tag{13}$$

With both Equations (12) and (13), we can decompose the conditional densities into appropriate pair copulas and marginal (conditional) densities. The possibility of further decomposition of Equation (13) is explored, as illustrated in Equation (11), which thus leads to:

$$f_{3|1,2}(x_3|\ x_1,x_2) = C_{13|2}\left(F_{1|2}(x_1|x_2),F_{3|2}(x_3|x_2)\right)C_{23}(F_2(x_2),\ F_3(x_3))f_3(x_3) \tag{14}$$

In a situation in which two-pair-copulas are involved — for example Equation (9) — each term can be decomposed into appropriate products of conditional pair-copulas multiplied by the conditional densities using the formula below:

$$f(x|v) = c_{x,v_j|v_{-j}}\left(F\left(x|v_{-j}\right),F\left(v_j|v_{-j}\right)\right)f\left(x|v_{-j}\right) \tag{15}$$

where $v_j$ is an arbitrarily-selected component of a vector $v$, $v_j$ is v, excluding such component, and $c_{x,v_j|v_{-j}}$ is the conditional bivariate copula. The pair-copula that involves marginal conditional distribution can also be obtained using a recursive approach based on the formula below:

$$F(x|v) = \ \partial c_{x,v_j|v_{-j}}\left(F\left(x|v_{-j}\right),F\left(v_j|v_{-j}\right)\right),\ F\left(x|v_{-j}\right)/\partial F\left(x|v_{-j}\right) \tag{16}$$

There is a special case of univariate **V** which can be expressed as:

$$F(x|v) = \ \partial c_{x,v}(F(x),F(v))/\partial F(v) \tag{17}$$

Following Aas et al. (2009), the C-vines copula is expressed as:

$$F_{i|1,\ \dots,\ i-1}(x_i|x_1,\dots,x_{i-1}) = \partial C_{i,\ i-1|1,\dots,\ 1-2}(F(x_i|x_1,\ \dots,\ x_{i-2}),F(x_{i-1}|x_1,\ \dots,\ x_{i-2})),\ \ /\partial F((x_i|x_{1-1}|x_1,\ \dots,\ x_{i-2})) \tag{18}$$

### 2.3. Regular-vine copulas

Because of the lack of flexibility of the C-Vine pair copula in dealing with complicated models, Dissman (2010) proposes an alternative by constructing an R-vine using diagram algorithm. The R-vine has a few shortfalls, including its static nature. Also, the computational effort required to estimate the model grows exponentially with the dimension.

A section of the literature suggests addressing these problems by simplifying or truncating the model. Allen et al. (2017) define the truncation of an R-vine at level $K$ as the situation in which any pair-copulas equal to or larger than $K$ are replaced with independent copulas. The independent copulas are regarded as Gaussian copulas, which are easier to specify than other variants of copulas and easier to interpret in terms of the correlation parameter. Vuong (1989) proposes that the statistics that could be used are Alkaike information criterion (AIC), Bayesian information criterion (BIC), and likelihood-ratio based tests. The general specification of an R-vine is expressed below:

$$f\left(x_1,\dots,x_d = \left[\prod_{k=1}^{d} f_x(x_k)\right] * \left[\prod_{i=1}^{d-1}\prod_{e\in E_i} c_{j(e),k(e)|D(e)}\left(F\left(x_{j(e)}\middle|x_{D(e)}\right)\right), \left(F\left(x_{k(e)}\middle|x_{D(e)}\right)\right)\right]\right) \tag{19}$$

## 3. Data, stylized facts and descriptive statistics

### 3.1. Data

The paper uses daily logarithmic returns of six virtual currencies (cryptocurrencies): Bitcoin and five altcoin (Dash, Ethereum, Litecoin, Ripple, and Stellar) from September 2015 to June 2018 (1045 observations).[1] The data for all virtual currencies/cryptocurrencies are obtained from CryptoCompare.com. The selection of all cryptocurrencies is based on data availability and their relative verve in terms of market capitalization on the virtual currency markets.

### 3.2. Stylized facts and descriptive statistics

The cryptocurrency market has witnessed substantial growth rebounds since its inception in 2009. From a market capitalization of around $1.5 billion at the beginning of 2013, it has grown to around $295 billion by end of June 2018. What is true about the cryptocurrency market is the fact that Bitcoin, the world's most valuable virtual currency, is the main driver of the price formations of all other cryptocurrencies, accounting for about 80% of the total market share. Despite the absolute dominance of Bitcoin, the performance of altcoin has also improved significantly over time. The share of all altcoin in the total market share of virtual currencies grew from 5.5% in 2013 to around 20% in 2016 (Ciaian et al., 2018). The most dominant of all altcoins is Ethereum, as shown in Table 1.

Fig. 1 displays the evolution of return dynamics for all cryptocurrencies examined. The graphical inspection shows signs of commonalities in the returns of all the six virtual currencies. All six VCs display visible patterns of volatility clustering dynamics over time.

In terms of currency composition used for the trading of altcoins, the data in Table 1 shows that Bitcoin dominates, posting, on average about 68.1% of all altcoin purchases. This is followed by the US dollar (14.2%), the Chinese yuan (14.4%), and other altcoins (0.9%). The relative dominance of Bitcoin as the medium of exchange in altcoin sales transactions give credence to possible interdependencies between Bitcoin and altcoin prices.[2]

In Table 2, we follow Ciaian et al. (2018) to show features of some virtual currencies. Like Bitcoin, most virtual currencies (e.g. Ripple, LiteCoin, NEM, Dash) have a fixed maximum supply of the coins that can be minted (put in circulation). The marginal supply growth of minted coins decreases over time and converges to zero when the maximum level is reached, the speed of which varies from currency to currency. In terms of the transaction execution/validation mechanism, different virtual currencies have different blockchain-generating mechanisms. Virtual currencies such as Bitcoin, Ethereum, Litecoin, Monero, Dash, etc. use the proof-of-work (P.o.W) mechanism. Miners that successfully complete the PoW receive a reward (new coins and transaction fees). The main principle of the PoW is that it is costly in terms of the computing power to produce but easy to verify by network participants. Miners must complete the PoW before their proposed block of transactions can be accepted by the network. That is, the PoW needs to be performed by a miner in order a new set of transactions (block) can be added to the distributed transaction database (block chain). The PoW mechanism has the disadvantage of leading to a large investment in the computer power and the energy consumption with the only benefit to verify transactions (Farell, 2015).

Others such as PeerCoin, NovaCoin, and MintCoin use the hybrid system of Proof of Stake (PoS) and PoW — see Ciaian et al. (2018) for a detailed analysis of the functions of these mechanisms. As noted by Ciaian et al. (2018) few currencies use a hybrid system of the PoS and PoW mechanisms, for example, PeerCoin, NovaCoin, MintCoin (see Table 2). The hybrid system based on the PoS and PoW can address the problem of the initial coin distribution as the hybrid PoW/PoS system uses the PoW mechanism for the initial coin distribution to miners. Afterwards, the PoS mechanism gradually takes over the PoW mechanism (e.g. PeerCoin, MintCoin). Thus, the combination of the PoS and PoW avoids the initial distribution problem inherent to the PoS, while reducing the overall cost of the system characteristics for the PoW mechanism (Krawisz, 2013; Farell, 2015). For example, MintCoin used the PoW during the first five weeks

[1] Choice of variables and sample size are informed by data availability and relative verve of variables in the oversell cryptocurrency market.
[2] Ciaian et al. (2018).

**Table 1**
**Currency composition of the global altcoin trading volume (%):** Adopted from Ciaian et al. (2018).

| | BTC | USD | CNY | EUR | Altcoin | Other currencies |
|---|---|---|---|---|---|---|
| Ethereum (ETH) | 47.01 | 26.13 | 2.31 | 8.28 | | 16.27 |
| LiteCoin (LTC) | 40.74 | 22.59 | 33.69 | 2.57 | | 0.41 |
| DogeCoin (DOGE) | 57.75 | | 41.68 | | | 0.57 |
| Monero (XMR) | 84.40 | 11.44 | | 4.16 | | 0.00 |
| Ripple (XRP) | 84.27 | 7.11 | 5.20 | 3.42 | | 0.00 |
| DigitalCash (DASH) | 78.95 | 18.10 | 1.26 | | | 1.69 |
| NEM (XEM) | 76.72 | | 23.28 | | | 0.00 |
| PeerCoin (PPC) | 58.37 | 28.33 | 13.22 | | | 0.08 |
| BitShares (BTS) | 71.87 | | 28.13 | | | 0.00 |
| NxT (NXT) | 49.78 | 3.74 | 46.42 | | | 0.06 |
| NameCoin (MINT) | 27.74 | 71.05 | 1.21 | | | 0.00 |
| NovaCoin (NMC) | 18.93 | 81.07 | | | | 0.00 |
| CounterParty (XCP) | 99.77 | | | | | 0.23 |
| Qora (QORA) | 99.79 | | | | | 0.21 |
| MintCoin (MINT) | 84.23 | | | | 15.77 | 0.00 |
| BitShares (BTS) | 72.42 | | 27.58 | | | 0.00 |
| FeatherCoin (FTC) | 93.84 | | 5.70 | | | 0.46 |
| PrimeCoin (XLB) | 51.65 | | 48.35 | | | 0.00 |
| Lisk (LSK) | 97.06 | | | | 2.90 | 0.04 |
| ALT 19 | 68.17 | 14.19 | 14.63 | 0.97 | 0.98 | 1.05 |

This table shows the percentage of the individual currencies used in the trading (purchasing) of altcoins. The table reports the percentage of altcoin purchases done in BitCoin (BTC), US dollar (USD), the Chinese mainland currency (CNY), the European Union currency (EUR), other altcoin, and other standard currencies, respectively. For each altcoin, the sum of the shares of BTC, USD, CNY, EUR, other altcoin, and other currencies adds up to 100%.

after its introduction, while afterwards it has almost completely switched to the PoS system.

Table 3 displays the descriptive statistics of all virtual currencies (VCs) examined in the paper. All currencies record positive means and positive standard deviations. With the exception of Bitcoin and Ethereum, the VCs are positively skewed. Additionally, all series have fat-tails and non-normally distributed, consistent with most financial assets. The results further show that all series are stationary. Again, the null hypothesis of no ARCH effects is rejected, as shown by the ARCH-LM and Box-Q tests.

## 4. Empirical results

We analyse the empirical results in two main strands. First, we model the dependence structure of the considered VCs in the sample using the C- and R-vine copulas. The second analysis focuses on the modelling of portfolio VaR of a weighted portfolio of all examined VCs using R-vine Copulas.

### 4.1. *Modelling dependence using vine copulas*

Prior to modelling the dependence structure for the six VCs we obtained suitably standardized marginal distributions using the AR(1)-GARCH(1,1) as the base marginal model. Residuals from the GARCH model were standardized, and the margins were obtained using the Ranks method. The margins were then used as inputs for the appropriate copula selection. We select the best fit copula using the Alkaike information criterion (AIC) criterion.

#### 4.1.1. *C-vine copulas*

In Fig. 2, we show the structure of C-vines. Here the chosen root node is the one that maximizes the sum of pair-wise dependencies to this node. We begin by connecting all the virtual currencies to Litecoin (Tree 1) which is at the centre of the Tree.

For selection purposes, we use a range of six, utilizing AIC as the criterion to select from the following copulas: 1 = Gaussian copula, 2 = Student-*t* copula (*t*-copula), 3 = Clayton copula, 4 = Gumbel copula, 5 = Frank copula, 6 = Joe copula. Next, we work out transformed observations from the estimated pair copulas, which are used as input parameters for Tree 2. We obtain these by constructing a graph similar to the C-vine construction principles of proximity, and finding a maximum dependence tree.

Panels A and B of Table 4 show the C-vine copula structure and specification matrix, respectively. For brevity, we discuss results in Panel B, which indicates the type of copula fitted to capture the dependencies between the various pairs of VCs. We observe from the bottom of column 1 that the Clayton copula (number 3) is fit to capture the dependency between Ripple and Bitcoin, conditioned by the relationship with Ethereum which also uses Clayton, and so forth. Only three out of the six copula families are used with the dominant copula family being three (Clayton), appearing five timer, followed by Frank (number 5), which appears 4 times.

Panels C and D depict the copula co-dependence estimates for the VCs. The bottom row entries indicate the existence of strong positive bivariate dependencies between subsets of the series. This is similar to the results established by Ciaian et al. (2018) and Corbet et al. (2018). In both panels, the bottom entry of the first column shows a strong co-dependence between Ripple and Bitcoin. Across all columns and rows, the dependencies are all positive with the highest dependency established between Bitcoin and Ethereum. These two

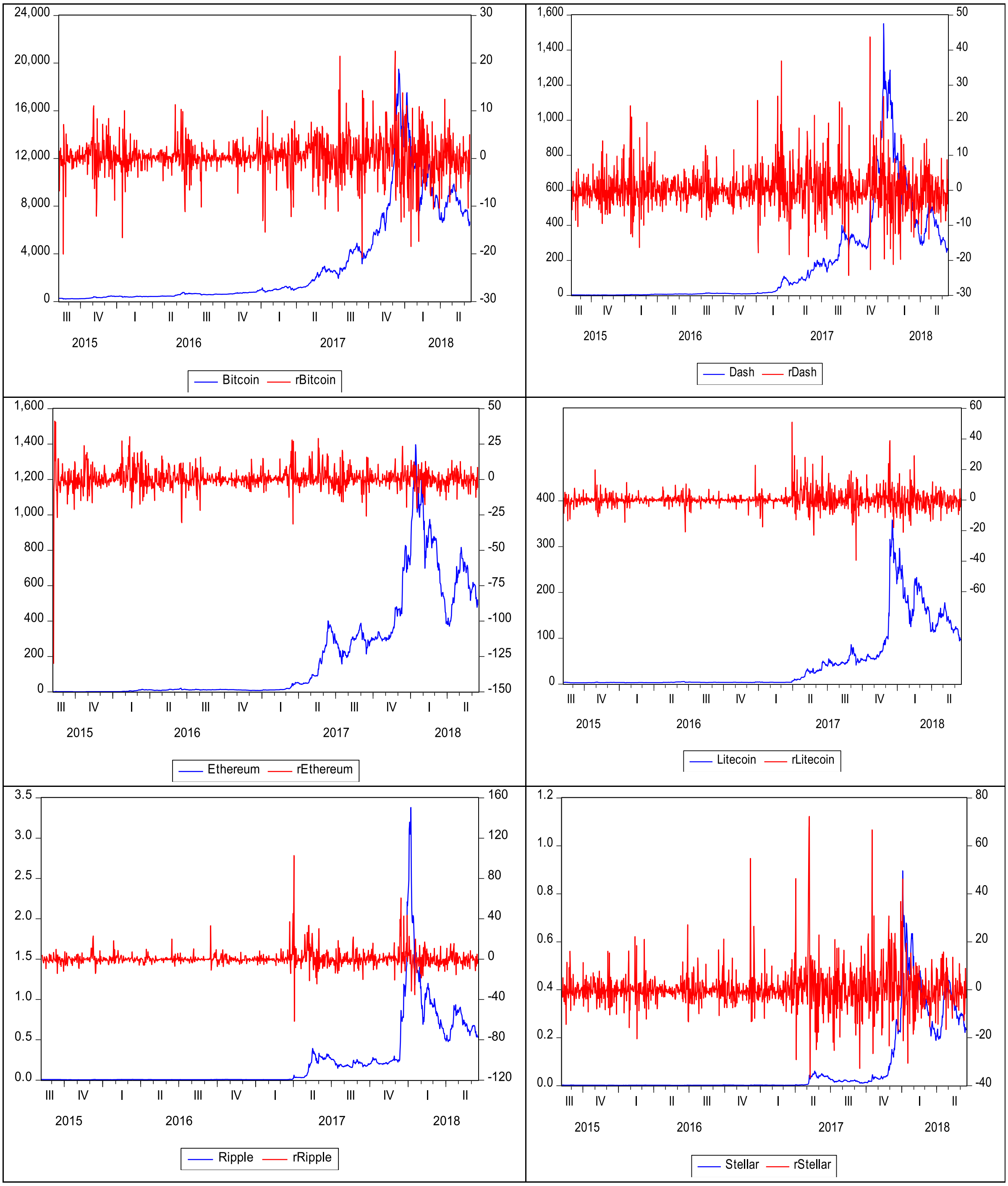


**Fig. 1.** Evolution of Cryptocurrency level and returns in the sample: From Sept. 2015 to June 2018.

VCs are the most liquid and highly capitalized in the cryptocurrency markets. Establishing such evidence of strong correlation among them is thus not surprising.

### *4.1.2. R-vine copulas*

Given that R-vine copulas provide more flexibility than C-vine copulas, we now discuss R-vine copulas for a more comprehensive comparison of the two approaches. Trees 1–3 for the R-vine copulas are shown in Fig. 3. It can be observed that R-vine copula structures are more flexible than C-vine structures. Tree 1 shows two categories of cryptocurrencies. All three trees indicate that the R-Vine

**Table 2**
Characteristics of selected virtual currencies: Adopted from Ciaian et al. (2018).

| Name | Symbol | Date of release | Duration of a block creation | Blocks generation mechanism | Growth of supply | Maximum supply |
|---|---|---|---|---|---|---|
| **Major VC** | | | | | | |
| BitCoin | BTC | 2009 | 10 min | PoW | Decreasing rate (halved every 210,000 blocks) | 21 Million |
| Ethereum | ETH | | | PoW** | Smoothly decreasing (in relative terms) (when reaching 72 million units the supply will stay at max 18 million of new coins per year) | Unlimited |
| Ripple | XRP | 2012 | 3–5 s | BC | Undefined: half of all units will be released for circulation, while OpenCoin will retain the rest. | 100 billion |
| LiteCoin | LTC | 2011 | 2.5 min | PoW | Decreasing rate (halved every 840,000 blocks) | 84 million |
| Monero | XMR | 2014 | 2 min | PoW | Smoothly decreasing (in relative terms) (when reaching 18.4 million units, the supply will stay constant at 0.6 new coins per 2-min block) | Unlimited |
| Dash | DASH | 2015 | 2.5 min | PoW | Decreasing rate (newly minted coins decrease 7.1% annually) | 22 million |
| NEM | | 2015 | 1 min | PoI | Zero (100% of coins were distributed when launched) | 9 billion |
| **Minor VC** | | | | | | |
| DogeCoin | DOGE | 2013 | 1 min | PoW | Smoothly decreasing rate (in relative terms) (when reaching 100 billion units, the supply will stay constant at 5.256 billion of new coins per year) | Unlimited |
| PeerCoin | PPC | 2012 | 10min | Hybrid (PoS & PoW) | Semi-constant (the supply increases at a rate of up to 1% per year but partially offset by destruction of 0.01 PPC per transaction) | Unlimited |
| NameCoin | NMC | 2011 | 10 min | PoW | Decreasing (halved every 4 years) | 21 million |
| NovaCoin | NVC | 2013 | 10 min | Hybrid (PoS &PoW) | Dynamic inflation | Unlimited |
| NxT | NXT | 2013 | 1 min | PoS | Zero (100% of coins were distributed when launched) | 1 billion |
| CounterParty | XCP | 2014 | 10 min | PoB | Negative growth at decreasing rate (coins are destroyed) | 2.6 million |
| MintCoin | MINT | 2014 | 30 s | Hybrid (PoS & PoW) | Decreasing rate (PoS: first year: 20%; second year: 15%; third year: 10%; fourth year and after: 5%. PoW reward: halved every week in first 5 weeks, 1 coin per block afterwards) | 70 billion |
| Qora | QORA | 2014 | 1–5 min | PoS | Undefined | 10 billion |
| SuperNET | UNITY | 2014 | – | Basket of virtual currencies | Zero | 816061 |
| BitShares | BTS | 2014 | 5–10 s | DPoS | Variable | 3.7 billion |

The table presents the characteristics of selected virtual currencies. The blocks-generation mechanism can be any of the following: Proof-of-Work (PoW); Proof-of-Stake (PoS); Delegated Proof-of-Stake (DPoS); Proof-of-Importance (PoI); Byzantine Consensus (BC); Proof-of-burn (PoB); *OpenCoin is the company that developed Ripple; **Ethereum plans to move to PoS protocol.

**Table 3**
Descriptive statistics for the return's series of all cryptocurrencies [Full Sample].

| | Bitcoin | Dash | Ethereum | Litecoin | Ripple | Stellar |
|---|---|---|---|---|---|---|
| Mean | 0.301064 | 0.421139 | 0.497288 | 0.299323 | 0.399162 | 0.436218 |
| Median | 0.297250 | −0.010696 | −0.047304 | 0.000000 | −0.356607 | −0.379301 |
| Maximum | 22.51190 | 43.77458 | 41.23373 | 51.03482 | 102.7356 | 72.30553 |
| Minimum | −20.75298 | −24.32250 | −130.2106 | −39.51508 | −61.62727 | −36.63577 |
| Std. Dev. | 4.104105 | 6.082920 | 8.190081 | 5.912513 | 7.865335 | 8.821986 |
| Skewness | −0.255451 | 0.915357 | −3.517362 | 1.367465 | 3.065364 | 2.038405 |
| Kurtosis | 7.785554 | 8.990679 | 66.45691 | 16.07008 | 41.82002 | 17.47755 |
| Jarque-Bera | 1008.536*** | 1708.564*** | 177487.4*** | 7763.781*** | 67253.59*** | 9849.986*** |
| ARCH LM-test (lags 12) | 90.819*** | 62.558*** | 110.38*** | 64.427*** | 110.12*** | 188.09*** |
| Box Q test (lags 20) | 198.5487*** | 88.1989*** | 21.8271*** | 105.861*** | 154.139*** | 277.405*** |
| ADF test (lags 20) | −5.9414*** | −5.4620*** | −5.0492*** | −6.5592*** | −5.9445*** | −5.8115*** |
| Observations | 1045 | 1045 | 1045 | 1045 | 1045 | 1045 |

*** denotes the ejection of the null hypothesis at 1% level of significance.

structure is more flexible. Tree-1 clearly underscores the difference in Bitcoin relative to all other five VCs, which are classified as altcoins. In Tree 1, we also observe that Litecoin, Ripple, and Dash are the most connected to Bitcoin, with Litecoin having the only direct dependence with Bitcoin. This phenomenon can partly be explained by the fact that among the category of altcoins in our sample, only Litecoin, Ripple, and Dash are similar to Bitcoin in terms of supply characteristics and type of valuation mechanism (Ciaian et al., 2018).

Similar to the C-vine copula analysis, Panels A–D of Table 5 show the R-vine copula structure, specification matrix, parameter estimates, and tau matrix, respectively. The advantage of using the R-vines to capture the complex patterns of dependency is seen in Panels C and D. Unlike the C-vine copula where only two copula families are used, in the R-Vine copula (Panel B of Table 5), all six copula

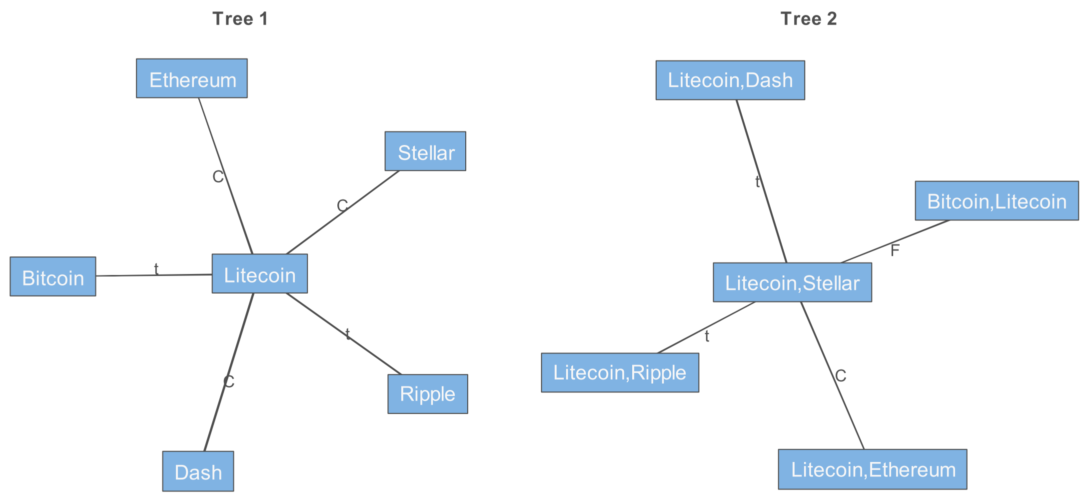


**Fig. 2.** C-Vine Trees 1 and 2.

**Table 4**
Results for C-Vine copula estimates.

| | Ripple | Dash | Stellar | Litecoin | Ethereum | Bitcoin |
|---|---|---|---|---|---|---|
| *Panel a: C-Vine Copula Structure.* | | | | | | |
| Ripple | 5 | 0 | 0 | 0 | 0 | 0 |
| Dash | 6 | 1 | 0 | 0 | 0 | 0 |
| Stellar | 1 | 6 | 2 | 0 | 0 | 0 |
| Litecoin | 2 | 2 | 6 | 3 | 0 | 0 |
| Ethereum | 3 | 3 | 3 | 6 | 4 | 0 |
| Bitcoin | 4 | 4 | 4 | 4 | 6 | 6 |
| *Panel b: C-Vine Copula Specification Matrix.* | | | | | | |
| Ripple | 0 | 0 | 0 | 0 | 0 | 0 |
| Dash | 5 | 0 | 0 | 0 | 0 | 0 |
| Stellar | 5 | 3 | 0 | 0 | 0 | 0 |
| Litecoin | 2 | 5 | 2 | 0 | 0 | 0 |
| Ethereum | 3 | 2 | 2 | 5 | 0 | 0 |
| Bitcoin | 3 | 2 | 3 | 3 | 2 | 0 |
| *Panel c: C-Vine Copula Parameter Estimates.* | | | | | | |
| Ripple | 0 | 0 | 0 | 0 | 0 | 0 |
| Dash | 0.321106 | 0 | 0 | 0 | 0 | 0 |
| Stellar | 0.692103 | 0.094649 | 0 | 0 | 0 | 0 |
| Litecoin | 0.28113 | 0.600774 | 0.11651 | 0 | 0 | 0 |
| Ethereum | 0.336054 | 0.397769 | 0.241859 | 0.487291 | 0 | 0 |
| Bitcoin | 0.579894 | 0.422772 | 0.629283 | 0.656521 | 0.718199 | 0 |
| *Panel d: C-Vine Copula Tau matrix.* | | | | | | |
| Ripple | 0 | 0 | 0 | 0 | 0 | 0 |
| Dash | 0.035642 | 0 | 0 | 0 | 0 | 0 |
| Stellar | 0.076535 | 0.045186 | 0 | 0 | 0 | 0 |
| Litecoin | 0.181419 | 0.066513 | 0.074342 | 0 | 0 | 0 |
| Ethereum | 0.143855 | 0.260431 | 0.155514 | 0.054015 | 0 | 0 |
| Bitcoin | 0.224774 | 0.277886 | 0.239336 | 0.247136 | 0.510066 | 0 |

families are used. It is also apparent that at different dependencies, conditioned across the same node, various copulas are used. For instance, in column 1, the first copula used is the Student-t copula (no. 2), followed by the Clayton copula (no. 3), then the Frank copula (no. 5), the Clayton (no. 3), again the Student-t copula (no. 2). Overall, the results indicate the usefulness in capturing tail risk and fat-tailed distributions in financial and economic downturns, as the Student-t copula, which captures asymmetric behaviour dominates, appearing six times followed by Frank (no.5), which appears five times.

Looking at the dependency measures in Panels C and D, we observe a similar pattern of co-dependencies as found in the C-Vine copula, as the dependencies are all positive. Also, the strongest dependencies for Bitcoin are established with Ripple and Ethereum.

### *4.2. Portfolio value-at-risk analysis based on R-vine copula applications*

Until now, we have applied C-vine and R-vine copulas to capture the dependence structure among the virtual currencies. We have

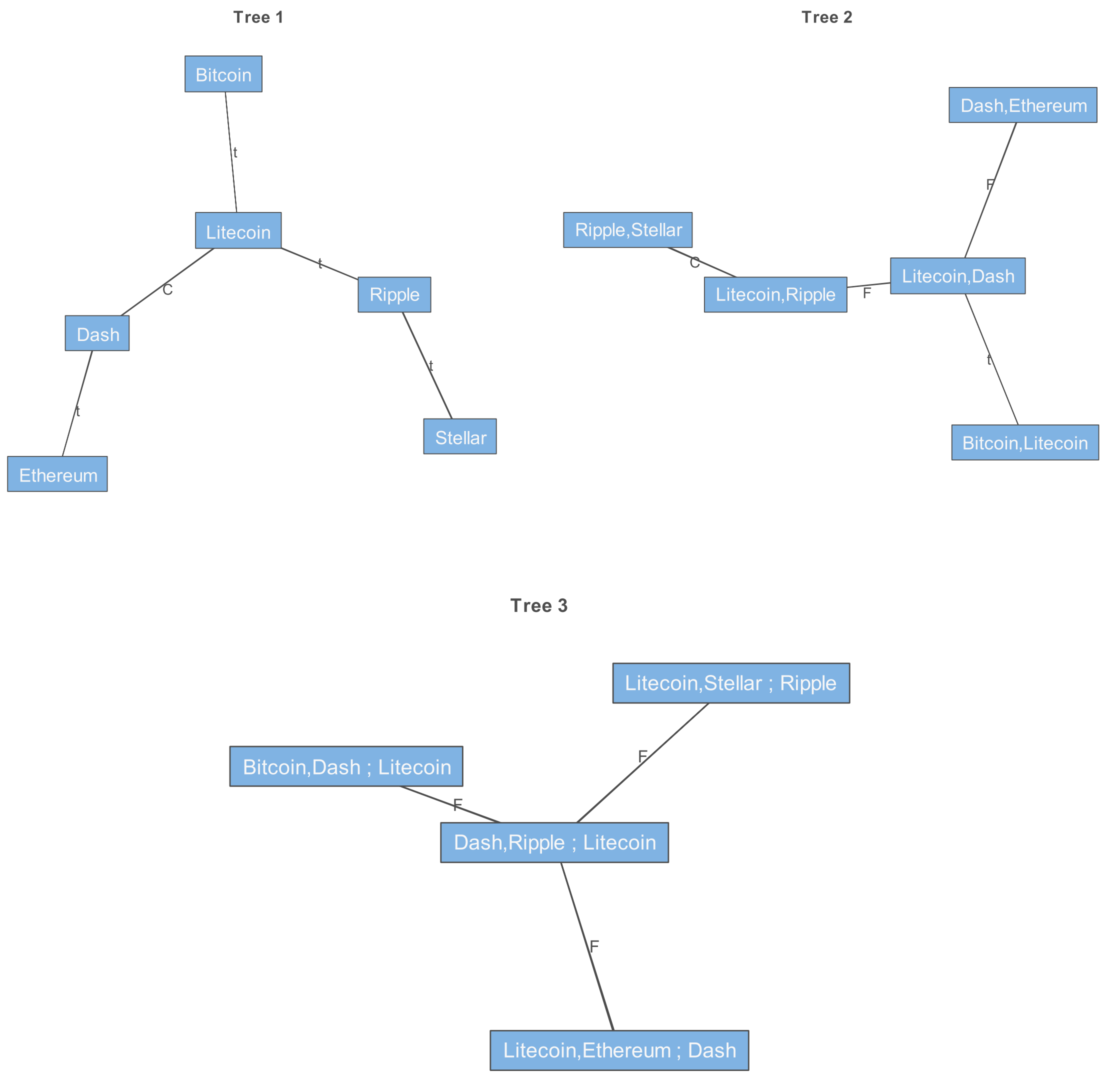


**Fig. 3.** R-Vine Trees-1, 2 and 3.

noticed that the R-vine copula approach provides more flexibility and better results than the standard bivariate copula framework because the copulas selected via the R-vine copula framework are more sensitive to the asset's return tail distributions and asymmetries of the series. The results could therefore be crucial for portfolio evaluation and risk modelling. The co-dependencies computed using R-vine copulas are useful for portfolio VaR analysis (Allen et al., 2017).

Akin to Allen et al. (2017), we create an equally-weighted portfolio of the six cryptocurrencies to explore the importance of Vine copulas in modelling VaR. We employ a 250 days moving window framework to forecast the VaR for this equally-weighted portfolio.[3]

We follow the following steps for the analysis on the VaR:

1. Convert the data for cryptocurrencies into log returns and select a 250 days moving window of returns.
2. Apply GARCH (1,1) with Student-*t* innovations to convert the log returns into an IID series. The same model is fitted in all the iterations to maintain uniformity in the approach which also makes the analysis a little less intensive.
3. Take the residuals from Step-2 and standardize them with the deviations obtained in Step 2.
4. Convert these residuals to student-*t* marginals for the estimation of copula. These steps are repeated for all the 6 cryptocurrencies to obtain a multivariate matrix of uniform marginals.

[3] To analyse the sensitivity of the results with respect to the rolling window size we also chose a 500-day forecast and found that the results are robust to the window size selected. Though, these results are not presented, they are available upon request.

**Table 5**
Results for R-Vine copula estimates.

| | Ripple | Dash | Stellar | Litecoin | Ethereum | Bitcoin |
|---|---|---|---|---|---|---|
| *Panel a: R-Vine structure* | | | | | | |
| Ripple | 3 | 0 | 0 | 0 | 0 | 0 |
| Dash | 6 | 5 | 0 | 0 | 0 | 0 |
| Stellar | 5 | 6 | 1 | 0 | 0 | 0 |
| Litecoin | 2 | 1 | 6 | 2 | 0 | 0 |
| Ethereum | 4 | 4 | 2 | 6 | 4 | 0 |
| Bitcoin | 1 | 2 | 4 | 4 | 6 | 6 |
| *Panel b. R-Vine Copula Specification Matrix.* | | | | | | |
| Ripple | 0 | 0 | 0 | 0 | 0 | 0 |
| Dash | 2 | 0 | 0 | 0 | 0 | 0 |
| Stellar | 3 | 5 | 0 | 0 | 0 | 0 |
| Litecoin | 5 | 5 | 5 | 0 | 0 | 0 |
| Ethereum | 3 | 5 | 5 | 2 | 0 | 0 |
| Bitcoin | 2 | 2 | 2 | 3 | 2 | 0 |
| *Panel c. R-Vine Copula Parameter Estimates.* | | | | | | |
| Ripple | 0 | 0 | 0 | 0 | 0 | 0 |
| Dash | 0.02743 | 0 | 0 | 0 | 0 | 0 |
| Stellar | 0.188782 | 0.366903 | 0 | 0 | 0 | 0 |
| Litecoin | 1.226027 | 1.160589 | 0.512777 | 0 | 0 | 0 |
| Ethereum | 0.376738 | 1.501614 | 1.197216 | 0.13281 | 0 | 0 |
| Bitcoin | 0.520782 | 0.428622 | 0.422772 | 0.629283 | 0.718199 | 0 |
| *Panel d: R-Vine Copula Tau matrix.* | | | | | | |
| Ripple | 0 | 0 | 0 | 0 | 0 | 0 |
| Dash | 0.017465 | 0 | 0 | 0 | 0 | 0 |
| Stellar | 0.08625 | 0.040712 | 0 | 0 | 0 | 0 |
| Litecoin | 0.134228 | 0.127256 | 0.056826 | 0 | 0 | 0 |
| Ethereum | 0.158511 | 0.163222 | 0.131163 | 0.0848 | 0 | 0 |
| Bitcoin | 0.348719 | 0.282001 | 0.277886 | 0.239336 | 0.510066 | 0 |

5. Fit an R-Vine to the multivariate data so obtained and generate simulations using the fitted R-Vine model. We generate 1000 simulations per series for forecasting a day ahead VaR.
6. Convert the simulated uniform marginals to standardized residuals and simulate returns using GARCH simulations.
7. Generate a series of simulated daily portfolio returns to forecast 1% and 5% VaR.
8. Repeat steps 1 to 7 for a moving window.

Following the above methods provides the VaR forecasts that is time-invariant and have the advantage of being co-dependent on the cryptocurrencies in the portfolio. We use this framework as a manifestation of a practical application of the co-dependencies captured by the flexible Vine Copula approach applied to construct VaR forecasts. Fig. 4 depicts the 1% and 5% VaR forecasts along with original portfolio return series obtained from the method for the full sample based on a 250-day rolling window. The plot shows that the VaR forecasts closely follow the daily returns from the first quarter of 2016 to the early days of 2018 with few violations (see Fig. 5).

For a better comparison of the performance of our vine copula VaR forecasts, we also use our series of virtual currency returns

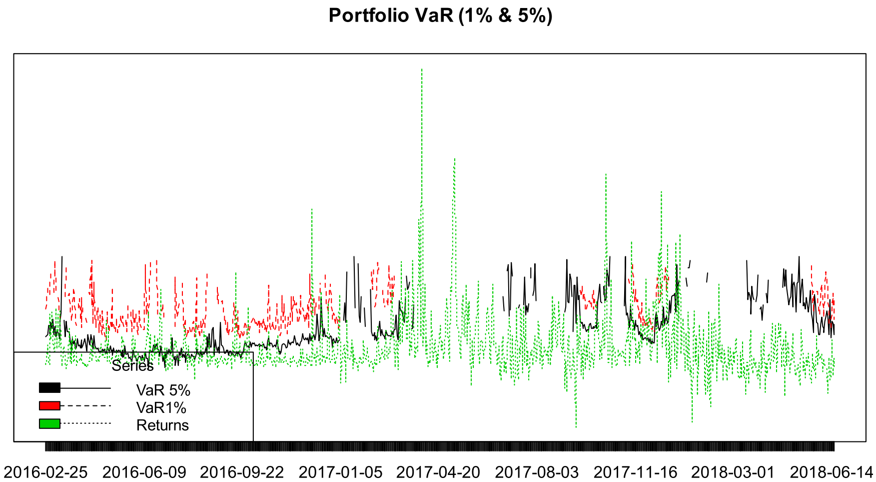


**Fig. 4.** Portfolio VaR analysis based on the application of vine copulas.

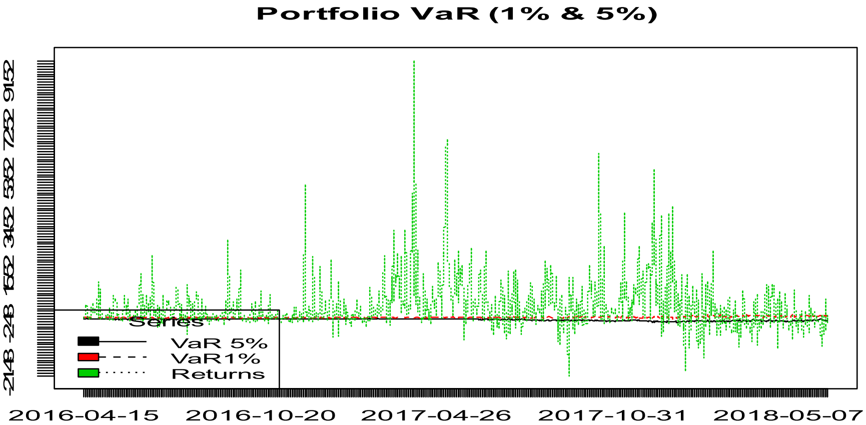


**Fig. 5.** Portfolio VaR analysis based on the application of GARCH (1,1) model.

combined into an equally-weighted portfolio to construct a simulation of a VaR analysis based on the use of a GARCH (1, 1) model with 200 daily rolling window. The relative number of violations of the VaR set at 1% and 5% indicate that our vine copula approach better captures the complex structure of dependencies and is better suited to VaR analysis.

Here also we follow the following steps:

1. Convert the data sample to log returns and select a moving window of 250 returns.[4]
2. Fit the GARCH (1,1) with Normal innovations to convert the log returns into an IID series.
3. Extract the fit from step 2 and simulate 1000 returns per asset.
4. Repeat step 2 and 3 for all cryptocurrencies followed by the computation of the portfolio return from the simulated series.
5. Generate a series of simulated daily portfolio returns to forecast 1% and 5% VaR.
6. Repeat step 1 to 5 for a moving window.

A close study of the plots (Fig. 6) reveal that the use of GARCH (1,1) model leads to multiple violations of the VaR 5% (black line) and VaR 1% (red line) compared to the VaR forecasts based on the application of vine copulas. Therefore, our vine copula models are best suited to compute the portfolio VaR during the considered time period.

### 4.3. *Analysis of efficient portfolios*

In this sub-section, we examine the mean-variance portfolio optimization framework using the efficient frontier of the portfolio under the short-selling constraints. In Fig. 8, we observe that only the two most liquid and highly capitalized VCs (Ethereum and Bitcoin) are located on the efficient frontier, representing the highest and the lowest expected returns among all the cryptocurrencies considered, respectively. The maximum Sharpe ratio and maximum utility can be achieved on the efficient frontier in Fig. 8, through rational asset allocation across the virtual currencies.[5]

In Fig. 9, we remove the short-selling constraints and plot the efficient frontier again. The figure shows that none of the virtual currencies now lie on the frontier.[6]

## 5. Conclusion

In this paper, we apply vine copula approaches to model the co-dependence and portfolio value-at-risk (VaR) of six cryptocurrencies using daily data from September 2015 to June 2018. We establish evidence of strong dependencies among the virtual currencies with a dependency structure that changes in a complicated manner. We find that among the class of cryptocurrencies examined, Ethereum offers the best optimal and economically risk-reward trade-off subject to a no-shorting constraint for portfolio investors using the efficient frontier. The findings indicate strong dependencies between Bitcoin and Ethereum, which represent the highly liquid and most capitalized cryptocurrencies. Further, we observe that Litecoin, Ripple and Dash are the most connected to Bitcoin with Litecoin having the only direct dependence with Bitcoin. This phenomenon can partly be explained by the fact that among the category of altcoins in our

[4] To analyse the sensitivity of the results with respect to the rolling window size we also chose 500 days rolling window and found that results robust to the window size selected. Though, these results are not presented, but are available upon request.

[5] Portfolio weights for the estimation of Fig. 6 are shown in Fig. 7.

[6] Note that analysis of efficient portfolios is based on in-sample results only, as we estimate the mean and covariance matrix based on the entire sample, which is not feasible for real investment. Further, we have also attempted to do the same analysis on simulated data from the GARCH and copula models but we faced serious convergence issues.

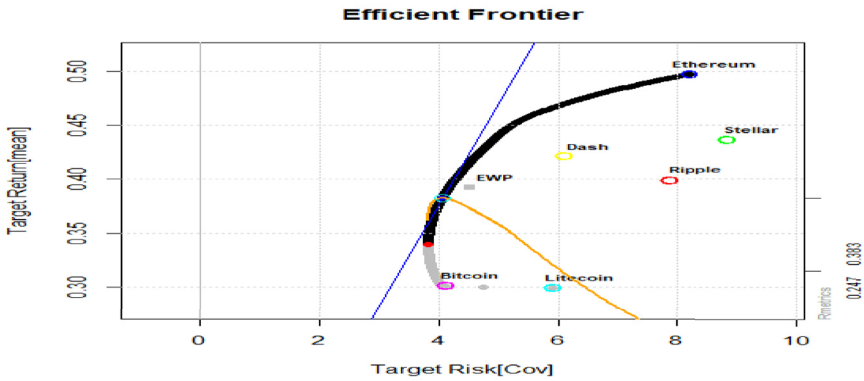


**Fig. 6.** Efficient frontier of cryptocurrencies portfolio with short-selling constraints.

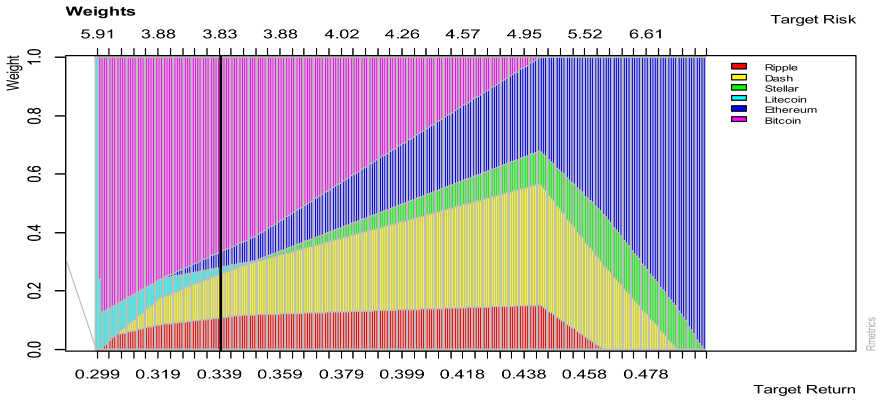


**Fig. 7.** Portfolio weights under different scenarios [5% minimum, 50% maximum allocation for each asset].

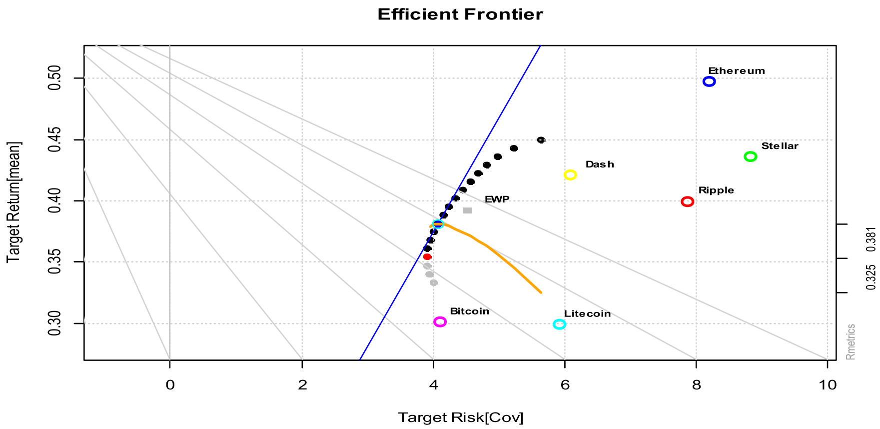


**Fig. 8.** Efficient frontier of cryptocurrencies' portfolio when short sales are allowed.

sample only Litecoin, Ripple, and Dash are similar to Bitcoin in terms of supply characteristics and type of valuation mechanism (Ciaian et al., 2018).

Relying on the flexibility of the R-vine copula and the preponderance of the Student-t copula family in modelling dependence, we model portfolio VaR based on the dependencies obtained. The results show that the VaR forecasts closely follow the daily returns with few violations. By comparing these with results obtained from the same data using GARCH (1,1) distribution, we conclude that our vine copula models are best suited to compute the portfolio VaR during the considered time period. Further, our analysis of the crypto-currency that will offer the best optimal and economically risk-reward trade-off subject to a shorting constraint for portfolio investors

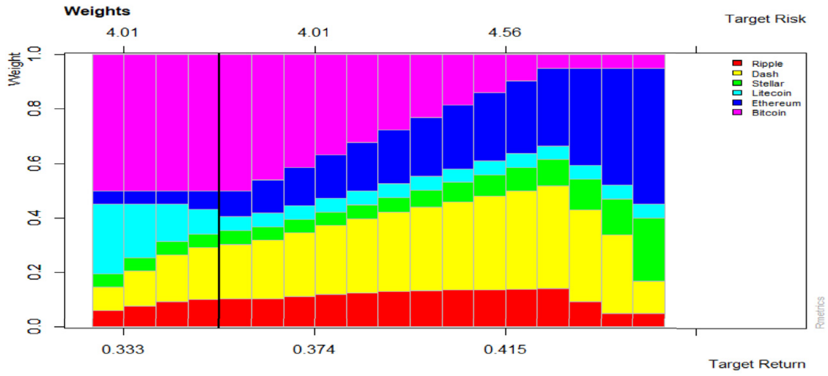


**Fig. 9.** Portfolio weights under different scenarios [5% minimum, 50% maximum allocation for each asset].

using the efficient frontier show that Ethereum is superior. Given the paucity of empirical research on the cryptocurrency markets, the paper provides new useful insights for investors willing to speculate or hedge positions using cryptocurrencies since our approach identifies parameters that could be useful in developing dependence risk and investment risk strategies for investment and hedging purposes, especially during more volatile periods in the markets. Dependence and relations are also crucial for regulators such as financial market authorities or central banks and for policymakers willing to reduce systemic risks.

The paper did not consider price breaks of 2017/2018. We would recommend for future studies to explore any such possible structural breaks.

## Appendix A

**Table 1A**
Descriptive statistics for all cryptocurrencies in level form.

| | BITCOIN | DASH | ETHEREUM | LITECOIN | RIPPLE | STELLAR |
|---|---|---|---|---|---|---|
| Mean | 3145.946 | 175.8781 | 200.4108 | 45.42374 | 0.228786 | 0.071925 |
| Median | 909.8950 | 13.60000 | 13.18000 | 4.095000 | 0.008105 | 0.002377 |
| Maximum | 19497.40 | 1550.850 | 1396.420 | 358.3400 | 3.380000 | 0.896227 |
| Minimum | 210.5000 | 2.060000 | 0.434829 | 2.630000 | 0.004090 | 0.001444 |
| Std. Dev. | 4065.650 | 268.6131 | 293.3687 | 69.52551 | 0.430982 | 0.144556 |
| Skewness | 1.634568 | 2.029330 | 1.581692 | 1.882112 | 3.250623 | 2.299154 |
| Kurtosis | 4.986783 | 7.213403 | 4.774310 | 5.960259 | 17.04698 | 7.826982 |
| Jarque-Bera | 637.8230*** | 1491.660*** | 573.3466*** | 999.4751*** | 10441.86*** | 1937.026*** |
| Observations | 1046 | 1046 | 1046 | 1046 | 1046 | 1046 |

*** denotes the ejection of the null hypothesis at 1% level of significance.

## Appendix B. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.inteco.2019.03.002.

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