Value-at-Risk and Expected Shortfall in Cryptocurrencies' Portfolio: A Vine Copula–based Approach*

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Abstract

Risk management is an important and helpful process for investors, hedge funds, traders and market makers. One of its key points is the appropriate estimation of risk measures which can improve the investment decisions and trading strategies. The high volatility of cryptocurrencies turns them a really risky investment and consequently, appropriate risk measures estimation is extremely necessary. In this paper, we deal with the estimation of two widely-used risk measures such as Value-at-Risk and Expected Shortfall in a cryptocurrency context. To face the presence of outliers and the correlation between cryptocurrencies, we propose a methodology based on vine copulas and robust volatility models. Our procedure is illustrated in a seven-dimensional equal-weight cryptocurrency portfolio and displays good performance.

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1 Introduction

Since the onset of the global financial crisis in 2008, individuals have lost trust in the mainstream monetary system. It is widely believed that the financial crisis facilitated the emergence of the digital currencies, known as cryptocurrencies (Dyhrberg, 2016). Bouri et al. (2017a,b) argued that the popularity of cryptocurrencies stems from investors' view that digital currency can be considered a shelter against economic and financial turmoil. Luther and Salter (2017) provided evidence that the demand for Bitcoin increased rapidly after the Cyprus bailout news in March 2013. Selmi et al. (2018) stated that Bitcoin has displayed substantial resilience against periods of economic and financial shock. In April 2018, the total market value of cryptocurrencies had exceeded 295 billion, and the total number of this class of currency exceeded 1,600 cryptocurrencies (Yi et al., 2018). Moreover, 32 countries have authorised the use of cryptocurrencies (Trabelsi, 2018). Thus, it is unsurprising that such a phenomenon has attracted the attention of researchers, economists and policy makers.

Early studies focused on the nature and characteristics of the digital currencies, and whether they may be classified as a traditional asset, such as equities, bonds or commodities, or whether they may be considered a currency—for example, see Dyhrberg (2016), Wu and Pandey (2014) and Yermack (2015). The empirical results have indicated ambiguous conclusions about the nature of the cryptocurrencies.

The second strand of the literature on cryptocurrencies focuses on their usage in investment strategies as hedge or safe haven, and whether investors can benefit for risk-mitigation from their inclusion in portfolios, alongside equities, bonds and commodities –see Briere et al. (2015), Bouoiyour and Selmi (2017), Bouri et al. (2017a,b), Baur et al. (2018), Feng et al. (2018), Selmi et al. (2018), Ehlers and Gauer (2019), Guesmi et al. (2019), Borri (2019), Platanakis and Urquhart (2019), among others. The results vary greatly in terms of whether cryptocurrencies can be used as a hedge or safe haven in periods of turmoil, or whether it is no more than a diversifier.

The most recent strand of literature focuses on cryptocurrencies' portfolio (Jiang and Liang, 2017; Liu, 2019; Brauneis and Mestel, 2019). In this context, the appropriate estimation of risk measures in cryptocurrencies' portfolio constitutes an essential tool to enable improved investment decisions and trading strategies. While risk measures estimation in a univariate cryptocurrency context has been studied by Katsiampa (2017), Stavroyiannis (2017), Troster et al. (2019) and Trucíos (2019), among others, its multivariate counterpart has been little exploited. Some works dealing with cryptocurrencies in a multivariate framework are Katsiampa (2019a), Katsiampa (2019b) and Chaim and Laurini (2019); however, these papers focus on describing the volatility dynamics of cryptocurrencies, rather than estimating risk measures which is the main contribution of our paper.

Our current work goes one step further by proposing a Value-at-Risk (VaR) and Expected Shortfall (ES) estimation strategy in cryptocurrencies' portfolios. To accomplish this task, we use vine copula and robust univariate volatility models. Robust univariate volatility models have displayed better performance in the presence of extreme observations than have non-robust models—see Carnero et al. (2012), Trucíos et al. (2017), Chen (2018), Troster et al. (2019), Charles and Darné (2019a), Charles and Darné (2019b), Tabasi et al. (2019) and Trucíos (2019). In addition, vine copulas have been shown to outperform other modelling approaches when employed to model the dependence structures of multiple classes of assets (Dissmann et al., 2013; Abbara and Zevallos, 2018; Yu et al., 2018). The vine copula approach can deal with the restrictive natures of the bivariate copulas (Brechmann and Czado, 2013), thereby allowing us to model sophisticated dependency structures because of non-linearities. In such a setting, vine copula–based modelling of VaR and ES becomes very meaningful to investors.

To the best of our knowledge, our work is the first to investigate VaR and ES in portfolios based only in cryptocurrencies. The proposed methodology deals with the correlation between cryptocurrencies and the presence of outliers jointly, an approach never used before in cryptocurrencies' portfolios. Our results reveal that vine copulas combined with robust univariate volatility models can be successfully used to estimate the VaR and ES in cryptocurrencies' portfolios.

The rest of this paper is organised as follows. In Section 2, we begin with a literature review on risk measures. Section 3 presents the methods used, while Section 4 contains the data description and empirical results. Finally, Section 5 presents the main conclusions and future work.

2 Literature review

The literature on estimation procedures of risk measures in a cryptocurrency context is quite restrictive, and most papers on this topic have focused on univariate volatility forecast, leaving out other important and widely-used risk measures, such as VaR and ES, as well as risk measures in a multivariate context.

In a VaR context, Likitratcharoen et al. (2018) estimated the VaR of Bitcoin and other cryptocurrencies using historical and Gaussian parametric VaR. A further step was made by Osterrieder and Lorenz (2017) and Gkillas and Katsiampa (2018) to consider extreme value theory to estimate VaR. Other approaches take into account the time-varying volatility of cryptocurrencies, such as the performed by Ardia et al. (2019), Stavroyiannis (2018), Troster et al. (2019), Pele and Mazurencu-Marinescu-Pele (2019) and Trucíos (2019).

Ardia et al. (2019) used generalised autoregressive conditionally heteroscedastic (GARCH) (Engle, 1982; Bollerslev, 1986) and GJR (Glosten et al., 1993) models to estimate the VaR considering one, two and three regimes. They concluded that the VaR 1% considering two and three regimes achieved better performance than when considering a single regime. These results were independent of the unconditional distribution used. For the VaR 5%, the results depended on the distribution assumption being the best performance achieved, using a two-regime GARCH model with Student's t-distribution.

Stavroyiannis (2018) used the GJR model and Pearson type-IV distribution (Stavroyiannis et al., 2012; Stavroyiannis and Zarangas, 2013) to compute VaR, and obtained good performance for VaR 1%, yet poor performance for VaR 2.5% and 5%. Troster et al. (2019) compared VaR estimation considering several GARCH-type and generalised autoregressive score (GAS) (Creal et al., 2013; Harvey, 2013) models. The main conclusion was that GAS models are better for estimating VaR than are GARCH-type models. Trucíos (2019) compared several procedures to forecast the daily volatility of Bitcoin, and compared different methodologies to estimate the VaR. He concluded that the robust-based residual bootstrap procedure of Trucíos et al. (2017) yielded a better estimation of the VaR than did GARCH-type and GAS models.

In contrast, Pele and Mazurencu-Marinescu-Pele (2019) used high-frequency data and proposed a new procedure based on entropy to forecast the daily VaR of Bitcoin. They also provided a comparison between their VaR forecasting procedure and other traditional methods, such as historical, normal and Student's t GARCH(1,1). They concluded that the proposed procedure performed better than the other ones.

Nevertheless, some papers have focused on the multivariate cryptocurrency context, such as Jiang and Liang (2017), Liu (2019), Katsiampa (2019a), Katsiampa (2019b), Koutmos (2018), Borri (2019), Brauneis and Mestel (2019) and Chaim and Laurini (2019). However, none of these works focused on VaR and ES, the risk measures commonly used by investors and regulators to evaluate the riskiness of asset portfolios. This lack in the literature is the focus of our paper.

3 Methodology

In this section, we briefly describe the methodology used to estimate the VaR and ES. Our approach is based on vine copulas and robust univariate volatility models. Section 3.1 describes the univariate volatility models used, while Section 3.2 introduces the vine copulas. Finally, Section 3.3 describes the proposed algorithm to estimate the VaR and ES.

3.1 Univariate volatility models

The presence of outliers in cryptocurrency returns has been observed by Catania et al. (2018); Troster et al. (2019); Chaim and Laurini (2019); Charles and Darné (2019a,b); Trucíos (2019), among others. Two approaches that have been successfully used to model and forecast the daily volatility of cryptocurrencies in a univariate context are the robust GARCH model of Boudt et al. (2013) and the GAS model of Creal et al. (2013) and Harvey (2013). Both approaches are robust to extreme observations and perform much better than non-robust approaches—for example, see Catania et al. (2018), Troster et al. (2019) and Trucíos (2019). These models are briefly introduced here.

3.1.1 Robust GARCH model

The GARCH model (Engle, 1982; Bollerslev, 1986) is quite popular in financial econometrics and constitutes a benchmark to model and forecast the daily volatility of financial returns. Its estimation and forecasting performance in the presence of outliers has been well documented in the literature—see Charles and Darné (2005), Muler and Yohai (2008), Carnero et al. (2012), Hotta and Tsay (2012), Trucíos et al. (2015) and Hotta and Trucíos (2018) for good examples.

Let $r_t = \sigma_t \epsilon_t$ be the observed return at time t, with σ_t being its corresponding volatility and ϵ_t being a white noise process. Under the GARCH(1,1) model, the evolution of the squared volatility is given by:

$$\sigma_t^2 = \omega + \alpha r_{t-1}^2 + \beta \sigma_{t-1}^2, \tag{1}$$

with the parameters ω, α and β satisfying stationary conditions.[1] If $r_t = y_t + a_t$, where y_t and a_t are the GARCH and jump components, respectively, Equation (1) is drastically affected by the jump process. To mitigate the effect of a_t, Boudt et al. (2013), based on the idea of Muler and Yohai (2008), proposed using a volatility equation robust to outliers. This equation is given by:

$$\sigma_t^2 = \omega + \alpha \gamma_c r_c \left(\frac{r_{t-1}^2}{\sigma_{t-1}^2} \right) \sigma_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{2}$$

with ω, α and β satisfying the same conditions in (1), γ_c a constant to guarantee Fisher consistency and $r_c(\cdot)$ a robust filter to mitigate the effect of outliers. To achieve better performance in a forecasting context, Trucíos et al. (2017) proposed using the filter $r_c(\cdot)$, as in Carnero et al. (2012). This filter is given by $r_c(x) = \begin{cases} 1, & \text{if } x > c, \\ x, & \text{if } x \leq c, \end{cases}$, with c being a threshold value. Details about the estimation procedure and its performance can be found in Boudt et al. (2013), Trucíos et al. (2015) and Trucíos et al. (2017).

The robust procedure of Boudt et al. (2013), with the modification suggested by Trucíos et al. (2017), has reported better performance to forecast volatility in

[1] Sufficient conditions for stationarity are given by $\omega > 0$, α, $\beta \geq 0$ and $\alpha + \beta < 1$.

the presence of additive outliers than have alternative robust approaches (Trucíos et al., 2015). This procedure was successfully applied by Trucíos (2019) to forecast the daily volatility of Bitcoin.

3.1.2 GAS model

The family of GAS models (Creal et al., 2013; Harvey, 2013) is a flexible alternative to the classical GARCH-type models, since they provide a very general framework for modelling time-varying parameters. GAS models have been successfully applied to model daily volatility by Derbali and Sy (2016), Gao and Zhou (2016), Catania et al. (2018), Troster et al. (2019) and Trucíos (2019), among many others.

The general expression of the GAS model is given by:

$$f_{t+1} = \omega + \beta f_t + \alpha S_t \left[\frac{\partial \log p(r_t|f_t)}{\partial f_t} \right], \tag{3}$$

with S_t being a scaling function for the score and f_t being a time-varying parameter. S_t is usually used as $\mathcal{I}_t^{-\lambda}$, where $\mathcal{I}_t$ is the Fisher information and the usual choices of λ are 0, 1/2 or 1. The main difference with the classical GARCH model is in the volatility equation ($f_t = \sigma_t^2$), which depends on the past values of the score of the conditional distribution, instead of the squared returns. Given that the score depends on the complete density, instead of only second-order moments, the GAS models allow us to better exploit the dynamic structure of the data.

Although well-known models, such as GARCH and EGARCH (Nelson, 1991), can be obtained as special cases of GAS model by using $p(\cdot)$ as the Gaussian density and appropriate choices of S_t and f_t. The main advantage of the GAS model derives from using $p(\cdot)$ as a non-Gaussian density. An interesting and well-studied choice for $p(\cdot)$ is the Student's t-density. This choice is particularly attractive for its flexibility to capture the volatility dynamics and for its robustness to heavy tails and outliers—for example, see Harvey and Chakravarty (2008), Harvey and Sucarrat (2014), Blazsek and Villatoro (2015) and Blasques et al. (2017).

3.2 Vine copula

Copulas have been widely used in financial applications, such as risk management, portfolio allocation and derivative pricing—for example, see Palaro and Hotta (2006), Huang et al. (2009), Brechmann and Czado (2013), Boubaker and Sghaier (2013), Weiß (2013), Kakouris and Rustem (2014), Han et al. (2017), Abbara and Zevallos (2018) and Yu et al. (2018), to name only a few. Its attractiveness derives from an early result from Sklar (1959), which stated that, for a random vector $X = (X_1, \cdots, X_d)$ with joint distribution F and marginals $F_1, \cdots, F_d$, there exists a copula[2] function C, such that:

$$F(x_1, \cdots, x_d) = C(F_1(x_1), \cdots, F_d(x_d)). \tag{4}$$

Conversely, also hold that:

$$C(u_1, \cdots, u_d) = F(F_1^{-1}(u_1), \cdots, F_d^{-1}(u_d)), \tag{5}$$

where $u_i = F_i(x_i)$. Sklar's theorem is a keystone in copulas theory, which has been successfully applied in many fields, such as the climate, medicine and finance fields.

For the bivariate case, there are numerous copulas proposed in the literature, and their choice depends on the dependence structure of the data—see Joe (2014) and Schepsmeier et al. (2018) for details.

When the dimension d is larger than two, copula modelling is challenging, and several methods to deal with this have been proposed in the literature (see Fischer et al. (2009) for an empirical comparison of those methods), in which vine copula[3] (Joe, 1996, 1997) is the most prominent one. This approach proposes decomposing the multivariate distribution into a cascade of pair-copulas.

In a d-dimensional case ($d \geq 3$), there are a number of possible pair-copula constructions. In this sense, Bedford and Cooke (2001, 2002) introduced regular vine copulas—a graphical model to help organise the possible pair-copulas. Regular vine copulas are very general and include special cases, such as canonical (C-vine) and D-vine copulas. For $d \geq 5$, there are regular vine copulas that are neither C-vine nor D-vine copulas (Aas et al., 2009). The difference between vine copulas relies on how the graphical method is constructed. For further discussion on pair-copulas, see Bedford and Cooke (2002), Aas et al. (2009) and Czado (2019).

[2]If F and the marginals $F_1, \cdots, F_d$ are absolutely continuous, then the copula C is unique.
[3]Also called pair-copula construction.

3.3 VaR and ES

To compute the VaR and ES, we follow a methodology similar to Brechmann and Czado (2013). The steps are detailed as follows[4]:

- **Step 1:** For each return series, estimate the volatility $\hat{\sigma}_t$ $(t = 1, \ldots, T+1)$ in a robust way, and obtain the standardised residuals, $\hat{\epsilon}_t = r_t/\hat{\sigma}_t$.

- **Step 2:** For the standardised residuals obtained in Step 1, fit a heavy-tail distribution and obtain the estimated error distribution function, $\hat{F}_t = P(x \leq \hat{\epsilon}_t)$.

- **Step 3:** Fit a vine copula to the matrix of estimated error distribution $(\hat{F}^1, ..., \hat{F}^7)$, where $\hat{F}^j$ is the estimated error distribution of the jth series obtained in Step 2.

- **Step 4:** Using the estimated copula parameters, simulate $m = 1, \cdots, M$ vectors of one-step-ahead standardised residuals, $\hat{\epsilon}^m_{T+1} = (\hat{\epsilon}^m_{T+1,1}, ..., \hat{\epsilon}^m_{T+1,7})$. For each vector, obtain the portfolio return, $R^m_{T+1} \approx \sum_{j=1}^{7} \omega_j \hat{r}^m_{T+1,j}$, where $\hat{r}^m_{T+1,j} = \hat{\sigma}_{T+1,j}\hat{\epsilon}^m_{T+1,j}$ is the one-step-ahead simulated jth return and ω_j its corresponding weight in the portfolio.[5]

- **Step 5:** Finally, estimate the one-step-ahead VaR and ES by:

$$\widehat{\text{VaR}}^{\alpha}_{T+1} = \{q : \sum_{m=1}^{M} \frac{I(R^m_{T+1} \leq q)}{M} = \alpha\},$$

$$\widehat{\text{ES}}^{\alpha}_{T+1} = J^{-1} \sum_{m=1}^{M} R^m_{T+1} I(R^m_{T+1} < \text{VaR}^{\alpha}_{\text{T+1}}),$$

  where $J = \sum_{m=1}^{M} I(R^m_{T+1} < \text{VaR}^{\alpha}_{\text{T+1}})$ and $I(\cdot)$ is the indicator function.

[4]For the sake of simplicity, we assume that the returns are zero-mean. In practise, the returns are filtered by an ARMA model or only centred to have zero-mean.

[5]Note that $\hat{\sigma}_{T+1,j}$ was obtained in Step 1 and $\hat{\epsilon}^m_{T+1,j}$ are the one-step-ahead standardised residuals obtained in Step 4.

4 Data and Results

We examined the daily closing prices (in US dollars) of cryptocurrencies traded from 1 January 2015 to 14 June 2019 (1,625 observations)[6]. The data is freely available at Coin Metrics.[7] We only considered cryptocurrencies with no missing values, which resulted in seven cryptocurrencies: Bitcoin (BTC), Dash (DASH), DigiByte (DGB), Dogecoin (DOGE), Litecoin (LTC), MaidSafeCoin (MAID) and Vertcoin (VTC).

For each cryptocurrency, daily returns were computed by $r_t = \log(P_t/P_{t-1}) \times 100$, with P_t being the closing price at day t. Table 1 reports the descriptive statistics of the seven cryptocurrencies, while Figure 1 plots the correlation among them.

Table 1: Descriptive statistics of daily returns

Crypto.	Mean	Std. Dev.	Min	Q_1	Med.	Q_3	Max	Skewness	Kurtosis
BTC	0.0020	0.0386	-0.2356	-0.0106	0.0023	0.0171	0.2235	-0.2617	8.4704
DASH	0.0027	0.0585	-0.2434	-0.0251	-0.0012	0.0275	0.3831	0.7969	8.1097
DGB	0.0035	0.0970	-0.4304	-0.0407	-0.0032	0.0375	1.1523	2.4058	26.4218
DOGE	0.0017	0.0634	-0.4851	-0.0217	0.0000	0.0194	0.5211	0.9966	15.7085
LTC	0.0024	0.0599	-0.5193	-0.0180	0.0000	0.0192	0.5185	0.6807	16.4857
MAID	0.0008	0.0667	-0.4021	-0.0326	0.0015	0.0356	0.3398	-0.1291	6.2217
VTC	0.0020	0.1012	-0.6141	-0.0429	-0.0034	0.0370	0.8652	1.4681	14.1660

In accordance with Ardia et al. (2019), Conrad et al. (2018), Chaim and Laurini (2018, 2019) and Trucíos (2019), among others, these cryptocurrencies were highly volatile and presented large kurtosis. VTC and DGB reported the largest standard deviation, while BTC was the least volatile cryptocurrency. The large values in the standard deviation and kurtosis can be explained by the presence of large returns, as observed in Figure 2, where extreme observations were observed in all cases. BTC and LTC were the most correlated cryptocurrencies, while DGB and VTC were the least correlated.

Figure 3 displays the sample autocorrelation function of returns (left panel) and squared returns (right panel) with their corresponding 95% confidence bands.

[6]As a Trade-off between the size of historical data and the number of cryptocurrencies we choose to start from 2015. Series with missing values were excluded, ending up with seven cryptocurrencies in our analysis.

[7]https://coinmetrics.io/data-downloads/

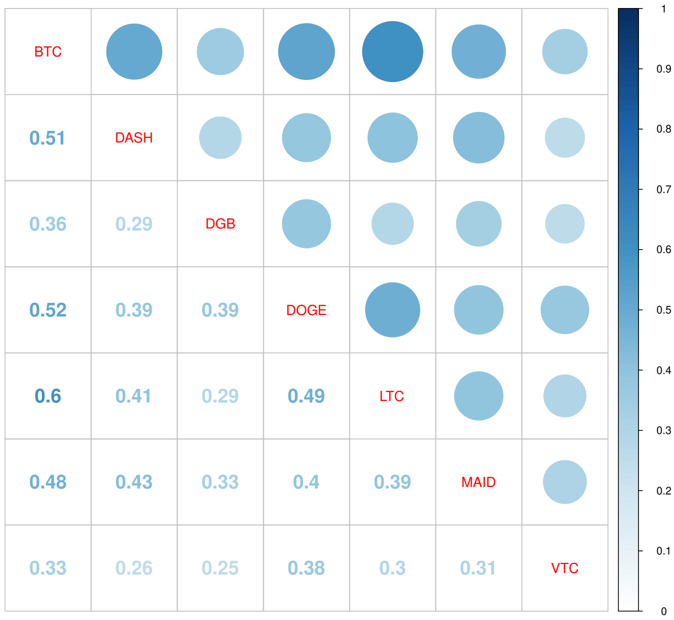

Figure 1: Unconditional correlation of returns among Bitcoin (BTC), Dash (DASH), DigiByte (DGB), Dogecoin (DOGE), Litecoin (LTC), MaidSafeCoin (MAID) and Vertcoin (VTC).

Following Trucíos (2019) and in accordance with Francq and Zakoïan (2009), the confidence bands for the autocorrelations of returns were computed using the generalised non-parametric Bartlett's (Francq and Zakoïan, 2009) formula, while the confidence band for the autocorrelations of squared returns was obtained using Bartlett's (Bartlett, 1946) formula. Given that the returns did not exhibit serial correlation, no ARMA filter was applied to the data, so that each series was only centred to have zero-mean. To capture the dynamic of the second-order moments, we used a vine copula approach, with marginals being modelled by both GAS and robust GARCH models. The pair-copula families were selected using the AIC criterion and were estimated sequentially by maximum likelihood. For implementation details, see Schepsmeier et al. (2018).

To evaluate the out-of-sample performance of the VaR and ES strategies described in Section 3, we used a rolling windows scheme with a window size of 750

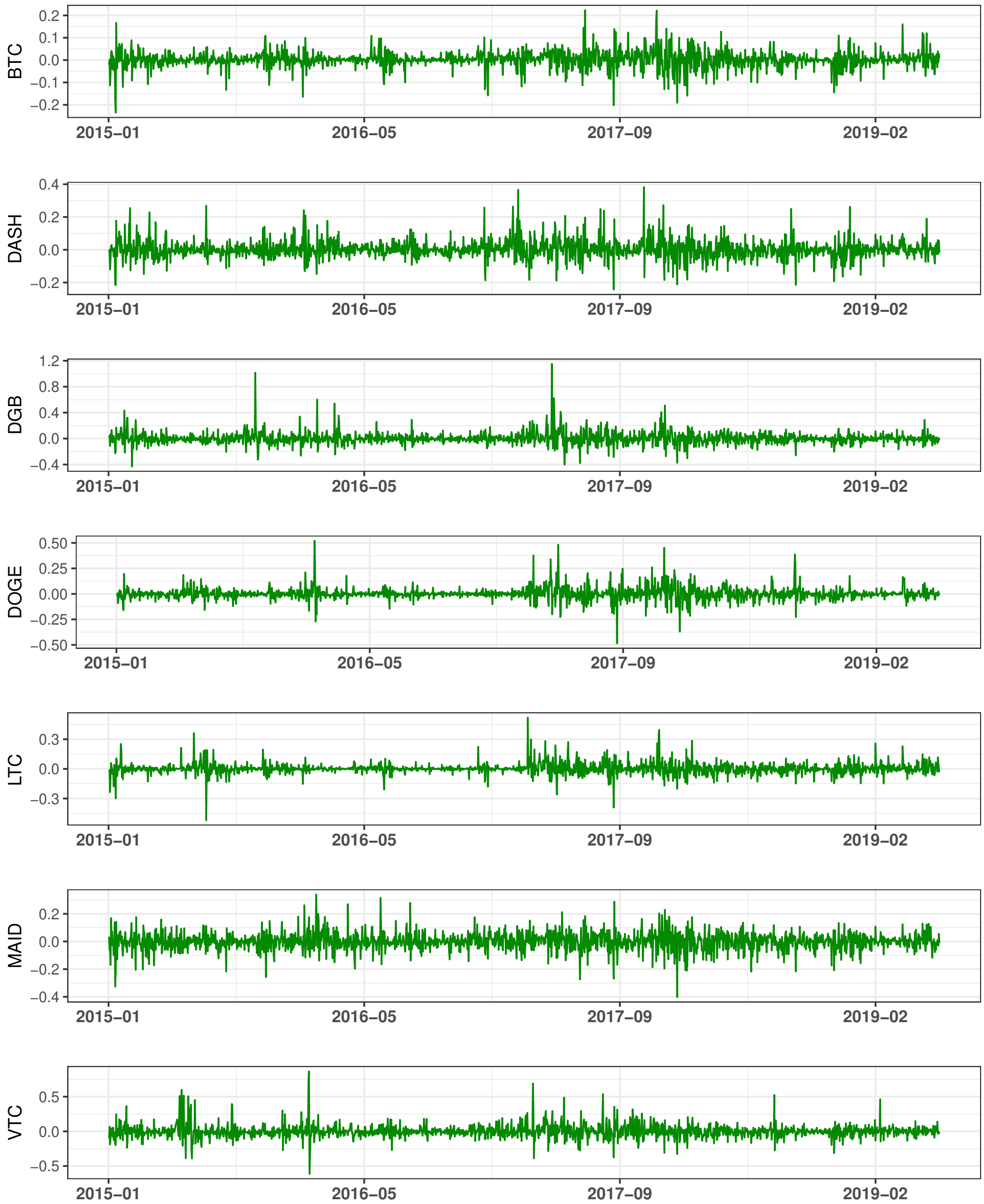

Figure 2: Daily returns of Bitcoin (BTC), Dash (DASH), DigiByte (DGB), Dogecoin (DOGE), Litecoin (LTC), MaidSafeCoin (MAID) and Vertcoin (VTC).

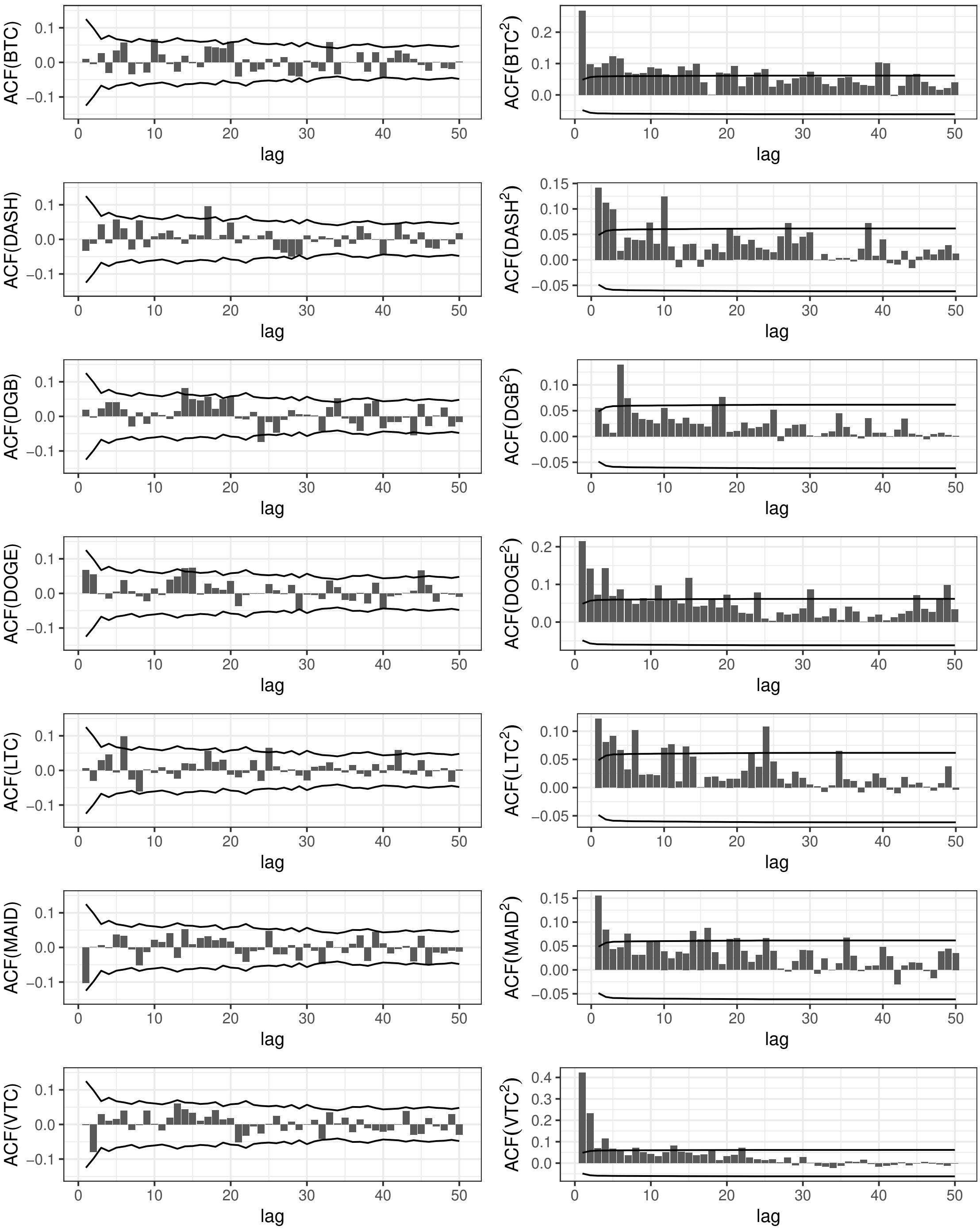

Figure 3: Sample autocorrelation function of returns with their corresponding generalised non-parametric Bartlett 95% confidence bands (left panel) and sample autocorrelation of squared returns with their corresponding Bartlett 95% confidence bands (right panel).

days and 875 days in the out-of-sample period[8]. We used the R packages *VineCopula* (Schepsmeier et al., 2018), *CDVine* (Brechmann and Schepsmeier, 2013), *GAS* (Catania et al., 2017) and *RobGARCHBoot* (Trucios, 2019) to fit the vine copula, GAS and robust GARCH models, respectively.

As usual in finance, we estimated the one-step-ahead VaR and ES at both 2.5% and 1% levels. The results for the back-testing are reported in Tables 2 and 3, respectively. In cases where the GAS model was used, the Student's t and asymmetric Student's t with left-tail decay parameter distributions were assumed to be as suggested by Troster et al. (2019). In contrast, when the robust GARCH model was implemented, the Student's t, GED, skew Student's t and skew GED distributions were considered in Step 2 of the algorithm,[9] as described in Section 3.3.

Table 2 reports the percentage of hits (returns smaller than VaR), as well as the p-values of the unconditional coverage (UC) (Kupiec, 1995), independence (IND) (Christoffersen, 1998), conditional coverage (CC) (Christoffersen, 1998), dynamic quantile (DQ) (Engle and Manganelli, 2004) and VaR quantile (VQ) (Gaglianone et al., 2011) tests. Additionally, the average quantile loss function of González-Rivera et al. (2004) is also reported. When using an appropriate VaR estimation, we expect that the null hypothesis were not rejected in all cases, indicating, roughly speaking, that the estimated proportion of hits is equal to the nominal one and that the hits occur independently. Furthermore, we would also like a procedure that yields to the smaller average quantile loss function which means that, between the procedures using in the comparison, the best procedure reports the smaller distance between observed returns and VaR estimation.

For VaR 2.5%, only five models failed to reject the null hypothesis at 5% of significance in all cases, with all of them being vine copulas with robust GARCH marginals. For the D-vine copulas case, the heavy-tail distributions used in Step 2 of the proposed algorithm was the GED while for the C-vine copula case, were Student's t and GED. For the R-vine copulas, the distributions were skew Student's t, skew

[8]In order to avoid convergence problems we use 750 days (approximately three years data in traditional markets) and a rolling window scheme as commonly used in the literature.

[9]In the robust GARCH procedure, an M-estimator was used Boudt et al. (2013) and, consequently, no distributional assumptions were made. The distributions referred here were only used in Step 2 of the algorithm.

GED and GED. Note that the C/R-vine copula with robust GARCH marginal and skew GED, as well as the R-vine copula with robust GARCH marginal and Student's t-distribution, presented p-values larger than 1%. Additionally, using the model confidence set (MCS) approach Hansen et al. (2011) on the quantile loss function, we identified a set of models with superior performance (small quantile loss function). These models are presented in Table 2 in bold.

For the VaR 1%, almost all models reported p-values larger than 5% in all cases, with exceptions observed in the vine copulas with GAS marginal and Student's t-distribution, in which case the null hypothesis in the VQ test was rejected. Other exceptions were the D/R-vine copula with robust GARCH marginals and skew GED, in which cases the p-value of the UC and CC as well as VQ test were smaller than 5%, yet larger than 1%, respectively. The set of models with minimum loss quantile function selected using the MCS are presented in bold.

Note that regardless of the vine copula used, the robust GARCH marginal using the GED in Step 2 of the algorithm yielded to a good performance in all VaR cases.

Table 3 reports the p-value of the tests of McNeil and Frey (2000) (McF), Nolde and Ziegel (2017) (NZ) and Bayer and Dimitriadis (2018) (BD). We expect to not reject the null hypothesis in all cases when using an appropriate ES estimation. The statistical tests used in Table 3 verify whether the expected value of the exceedance residuals is zero (McNeil and Frey, 2000) and whether the estimated ES is correctly specified relative to a series of realised returns (Nolde and Ziegel, 2017; Bayer and Dimitriadis, 2018).

For the ES 2.5% and 1%, the use of robust GARCH marginals with GED and Skew GED distributions in Step 2 of our algorithm fails to reject the null hypothesis at 5% of significance in all cases, regardless the vine copula used. Additionally, when C/R vine copulas are combined with astd-GAS models we also fail to reject the null hypothesis in all cases (also at 5% of significance). At 1% of significance, the robust GARCH marginals with Skew Student's t-distribution in Step 2 also fails to reject the null hypothesis when C and R vine copulas are used. These results show that use of the algorithm described in Section 3.3 along with robust GARCH marginals and GED or Skew GED in the step 2 of the algorithm yield good ES estimates.

Additionally, note that McF and NF test simultaneously the VaR and ES, and

Table 2: Back-testing for the VaR 2.5% (top panel) and 1% (bottom panel) of the cryptocurrency equal-weight portfolio. Out-of-sample period from 21 January 2017 to 14 June 2019. p-values larger than 0.05 are in shadow cells, while the models with best performance selected by the MCS are in bold.

		Dist.	% Hits	UC	IND	CC	DQ	VQ	Av. Loss
					VaR 2.5 %				
D-Vine	GAS	std	1.2571	0.0093	0.5964	0.0295	0.1994	0.0011	0.5195
		astd	3.6571	0.0401	0.8661	0.1198	0.4243	0.0000	0.4157
	RGARCH	std	2.6286	0.8091	0.2648	0.5216	0.9033	0.0334	**0.3897**
		sstd	3.7714	0.0249	0.1075	0.0221	0.0793	0.0005	**0.4108**
		ged	2.4000	0.8487	0.3092	0.5855	0.9149	0.0753	**0.3937**
		sged	4.0000	0.0088	0.7105	0.0303	0.0822	0.0167	**0.4117**
C-Vine	GAS	std	1.2571	0.0093	0.5964	0.0295	0.2029	0.0014	0.5190
		astd	3.6571	0.0401	0.1188	0.0360	0.3310	0.0006	**0.4126**
	RGARCH	std	2.4000	0.8487	0.3092	0.5855	0.4929	0.0977	**0.3885**
		sstd	3.3143	0.1414	0.1582	0.1253	0.5807	0.0001	**0.3991**
		ged	2.0571	0.3869	0.3842	0.4710	0.9034	0.0593	**0.3898**
		sged	3.6571	0.0401	0.1188	0.0360	0.1037	0.0285	**0.4084**
R-Vine	GAS	std	1.2571	0.0093	0.5964	0.0295	0.2028	0.0016	0.5196
		astd	3.7714	0.0249	0.1075	0.0221	0.2561	0.0041	**0.4127**
	RGARCH	std	2.7429	0.6504	0.2443	0.4582	0.5894	0.0182	**0.3909**
		sstd	3.3143	0.1414	0.1582	0.1253	0.5809	0.0753	**0.3989**
		ged	2.2857	0.6804	0.3331	0.5751	0.9191	0.0560	**0.3895**
		sged	3.8857	0.0150	0.0971	0.0131	0.0600	0.0298	**0.4088**
					VaR 1 %				
D-Vine	GAS	std	0.8000	0.5379	0.7367	0.7817	0.7716	0.0000	0.3049
		astd	1.7143	0.0539	0.4692	0.1201	0.1228	0.0427	**0.2146**
	RGARCH	std	1.0286	0.9326	0.6652	0.9074	0.9527	0.8202	**0.2012**
		sstd	1.4857	0.1780	0.5309	0.3317	0.6925	0.0502	**0.2108**
		ged	1.1429	0.6780	0.6304	0.8171	0.9453	0.6138	**0.2048**
		sged	1.9429	0.0131	0.4115	0.0328	0.0570	0.0306	**0.2123**
C-Vine	GAS	std	0.6857	0.3219	0.7733	0.5874	0.8108	0.0000	0.3047
		astd	1.6000	0.1009	0.4996	0.2072	0.1199	0.2807	**0.2129**
	RGARCH	std	1.0286	0.9326	0.6652	0.9074	0.9929	0.6985	**0.2024**
		sstd	1.1429	0.6780	0.6304	0.8171	0.9345	0.1447	**0.2056**
		ged	1.1429	0.6780	0.6304	0.8171	0.9455	0.7045	**0.2051**
		sged	1.6000	0.1009	0.4996	0.2072	0.6034	0.3483	**0.2102**
R-Vine	GAS	std	0.9143	0.7960	0.7006	0.8982	0.6723	0.0000	0.3029
		astd	1.3714	0.2959	0.5632	0.4900	0.8747	0.1636	**0.2116**
	RGARCH	std	0.9143	0.7960	0.7006	0.8982	0.9983	0.6682	**0.2007**
		sstd	1.1429	0.6780	0.6304	0.8171	0.9086	0.6208	**0.2069**
		ged	1.1429	0.6780	0.6304	0.8171	0.9707	0.8634	**0.2010**
		sged	1.4857	0.1780	0.5309	0.3317	0.7259	0.0154	**0.2109**

Table 3: Back-testing for the ES 2.5 % (left panel) and 1% (right panel) of the equal-weight cryptocurrency portfolio. Out-of-sample period from 21 January 2017 to 14 June 2019. Shadowed cells: p-values larger than 0.05.

			ES 2.5 %			ES 1 %		
		Dist.	McF	NF	BD	McF	NF	BD
D-Vine	GAS	std	0.2268	0.0000	0.0000	0.5731	0.0000	0.0000
		astd	0.7878	0.0392	0.1039	0.9101	0.0051	0.3451
	RGARCH	std	0.9610	0.0000	0.0275	0.9749	0.0000	0.0008
		sstd	0.9202	0.0164	0.3718	0.9058	0.0029	0.2196
		ged	0.4218	0.9068	0.6923	0.3573	0.8950	0.6078
		sged	0.2277	0.0577	0.0819	0.3080	0.1186	0.0531
C-Vine	GAS	std	0.3471	0.0000	0.0000	0.4823	0.0000	0.0000
		astd	0.7222	0.1654	0.2291	0.6879	0.2155	0.4408
	RGARCH	std	0.9536	0.0533	0.1069	0.9908	0.0003	0.0647
		sstd	0.9769	0.0432	0.3694	0.9489	0.0169	0.1880
		ged	0.3312	0.4293	0.6987	0.3088	0.8398	0.6148
		sged	0.2189	0.1313	0.1438	0.1840	0.2637	0.0580
R-Vine	GAS	std	0.3436	0.0000	0.0000	0.4960	0.0000	0.0000
		astd	0.7208	0.1185	0.3867	0.6919	0.4763	0.5722
	RGARCH	std	0.9511	0.0282	0.1706	0.9911	0.0010	0.0552
		sstd	0.9783	0.0419	0.3434	0.9501	0.0202	0.3012
		ged	0.3333	0.7483	0.8415	0.3152	0.9036	0.7768
		sged	0.2240	0.0881	0.1274	0.1824	0.3004	0.0662

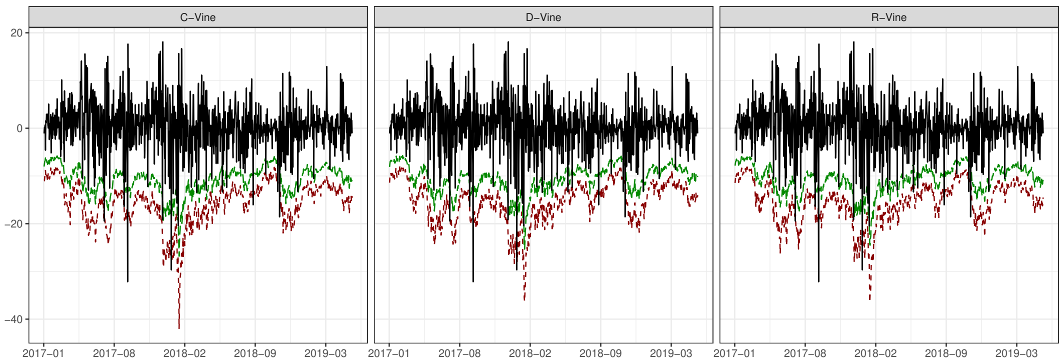

Figure 4: Portfolio returns in the out-of-sample period (black lines), VaR 1% (red dashed lines) and VaR 2.5% (green long dashed lines) using C-vine (left panel), D-vine (middle panel) and R-vine (right panel) copula.

can also be used to verify the estimation of the VaR, being complementary to the results in Table 2.

Taking into account our results in the VaR and ES estimation, only the D/C/R-vine copula with marginal robust GARCH and GED used in Step 2 of the proposed algorithm reported good performance in all cases. Note that these models also belongs to the set of models with best performance using the MCS on the quantile loss function.

Finally, Figure 4 displays the VaR 2.5% (green long dashed line) and VaR 1% (red dashed line) using the D/C/R-vine copula with marginal robust GARCH and GED in Step 2 of the algorithm. A visual inspection in the VaR 1% estimations shows that close to February 2018, a large value of VaR is observed in all cases, being the larger one when the C-vine copula is used. These results are in concordance with the average quantile loss function reported in Table 2.

5 Conclusion

This paper has proposed an algorithm based on vine copulas and robust volatility models to estimate the VaR and ES in a cryptocurrency portfolio. The proposed algorithm displays good performance in estimating both VaR and ES. The best performance was obtained when we combined vine copulas with marginals modelled by the robust GARCH procedure of Boudt et al. (2013) and GED, used in Step 2 of the algorithm. The algorithm was easy to implement and constituted a methodological tool for better estimation of risk measures in a cryptocurrency context.

Our methodology provides risk measures estimates with good performance which can be useful for investors, hedge funds, traders and market makers interested in cryptocurrency markets. Additionally, the risk measures estimation procedure proposed in this paper can improve investment decisions.

Finally, given the increasing number of cryptocurrencies nowadays, further research should include proper estimation of risk measures in both high-dimensional and high-frequency data, which is also on our research agenda.

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References

Aas, K., Czado, C., Frigessi, A., and Bakken, H. (2009). Pair-copula constructions of multiple dependence. *Insurance: Mathematics and economics*, 44(2):182–198.

Abbara, O. and Zevallos, M. (2018). Modeling and forecasting intraday VaR of an exchange rate portfolio. *Journal of Forecasting*, 37(7):729–738.

Ardia, D., Bluteau, K., and Rüede, M. (2019). Regime changes in Bitcoin GARCH volatility dynamics. *Finance Research Letters*, 29:266–271.

Bartlett, M. S. (1946). On the theoretical specification and sampling properties of autocorrelated time-series. *Supplement to the Journal of the Royal Statistical Society*, 8(1):27–41.

Baur, D. G., Dimpfl, T., and Kuck, K. (2018). Bitcoin, gold and the US dollar–a replication and extension. *Finance Research Letters*, 25:103–110.

Bayer, S. and Dimitriadis, T. (2018). Regression based expected shortfall backtesting. *arXiv preprint arXiv:1801.04112*.

Bedford, T. and Cooke, R. M. (2001). Probability density decomposition for conditionally dependent random variables modeled by vines. *Annals of Mathematics and Artificial intelligence*, 32(1-4):245–268.

Bedford, T. and Cooke, R. M. (2002). Vines–a new graphical model for dependent random variables. *The Annals of Statistics*, 30(4):1031–1068.

Blasques, F., Koopman, S. J., and Lucas, A. (2017). Maximum likelihood estimation for score-driven models. *Timbergen Institute Discussion Paper 14-029/III.*

Blazsek, S. and Villatoro, M. (2015). Is beta-t-EGARCH (1, 1) superior to GARCH (1, 1)? *Applied Economics*, 47(17):1764–1774.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*, 31(3):307–327.

Borri, N. (2019). Conditional tail-risk in cryptocurrency markets. *Journal of Empirical Finance*, 50:1–19.

Boubaker, H. and Sghaier, N. (2013). Portfolio optimization in the presence of dependent financial returns with long memory: A copula based approach. *Journal of Banking & Finance*, 37(2):361–377.

Boudt, K., Danielsson, J., and Laurent, S. (2013). Robust forecasting of dynamic conditional correlation GARCH models. *International Journal of Forecasting*, 29(2):244–257.

Bouoiyour, J. and Selmi, R. (2017). The Bitcoin price formation: Beyond the fundamental sources. *arXiv preprint arXiv:1707.01284*.

Bouri, E., Gupta, R., Tiwari, A. K., and Roubaud, D. (2017a). Does Bitcoin hedge global uncertainty? evidence from wavelet-based quantile-in-quantile regressions. *Finance Research Letters*, 23:87–95.

Bouri, E., Jalkh, N., Molnár, P., and Roubaud, D. (2017b). Bitcoin for energy commodities before and after the december 2013 crash: diversifier, hedge or safe haven? *Applied Economics*, 49(50):5063–5073.

Brauneis, A. and Mestel, R. (2019). Cryptocurrency-portfolios in a mean-variance framework. *Finance Research Letters*, 28:259–264.

Brechmann, E. C. and Czado, C. (2013). Risk management with high-dimensional vine copulas: An analysis of the Euro Stoxx 50. *Statistics & Risk Modeling*, 30(4):307–342.

Brechmann, E. C. and Schepsmeier, U. (2013). Modeling dependence with C- and D-vine copulas: The R package CDVine. *Journal of Statistical Software*, 52(3):1–27.

Briere, M., Oosterlinck, K., and Szafarz, A. (2015). Virtual currency, tangible return: Portfolio diversification with bitcoin. *Journal of Asset Management*, 16(6):365–373.

Carnero, M. A., Peña, D., and Ruiz, E. (2012). Estimating GARCH volatility in the presence of outliers. *Economics Letters*, 114(1):86–90.

Catania, L., Boudt, K., and Ardia, D. (2017). *GAS: Generalized Autoregressive Score Models*. R package version 0.2.5.

Catania, L., Grassi, S., and Ravazzolo, F. (2018). Predicting the volatility of cryptocurrency time-series. In *Mathematical and Statistical Methods for Actuarial Sciences and Finance*, pages 203–207. Springer.

Chaim, P. and Laurini, M. P. (2018). Volatility and return jumps in bitcoin. *Economics Letters*, 173:158–163.

Chaim, P. and Laurini, M. P. (2019). Nonlinear dependence in cryptocurrency markets. *The North American Journal of Economics and Finance*, 48:32–47.

Charles, A. and Darné, O. (2005). Outliers and GARCH models in financial data. *Economics Letters*, 86(3):347–352.

Charles, A. and Darné, O. (2019a). Volatility estimation for Bitcoin: Replication and robustness. *International Economics*, 157:23–32.

Charles, A. and Darné, O. (2019b). Volatility estimation for cryptocurrencies: Further evidence with jumps and structural breaks. *Economics Bulletin*, 39(2):954–968.

Chen, J. (2018). On exactitude in financial regulation: Value-at-Risk, expected shortfall, and expectiles. *Risks*, 6(2):61.

Christoffersen, P. F. (1998). Evaluating interval forecasts. *International Economic Review*, 29(4):841–862.

Conrad, C., Custovic, A., and Ghysels, E. (2018). Long-and Short-Term cryptocurrency volatility components: A GARCH-MIDAS analysis. *Journal of Risk and Financial Management*, 11(2):23.

Creal, D., Koopman, S. J., and Lucas, A. (2013). Generalized autoregressive score models with applications. *Journal of Applied Econometrics*, 28(5):777–795.

Czado, C. (2019). *Analyzing Dependent Data with Vine Copulas: A Practical Guide With R*, volume 222 of *Lecture Notes in Statistics*. Springer, Cham.

Derbali, A. and Sy, A. (2016). The volatility of exchange rate between the US dollar and African emerging currencies: analysing by GAS-GARCH-Student-t model. *International Journal of Critical Accounting*, 8(2):132–143.

Dissmann, J., Brechmann, E. C., Czado, C., and Kurowicka, D. (2013). Selecting and estimating regular vine copulae and application to financial returns. *Computational Statistics & Data Analysis*, 59:52–69.

Dyhrberg, A. H. (2016). Bitcoin, gold and the dollar–A GARCH volatility analysis. *Finance Research Letters*, 16:85–92.

Ehlers, S. and Gauer, K. (2019). Practical applications of beyond bitcoin: A statistical comparison of leading cryptocurrencies and fiat currencies and their impact on portfolio diversification. *The Journal of Alternative Investments*, 22(Supplement):1–5.

Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. *Econometrica*, 50(4):987–1007.

Engle, R. F. and Manganelli, S. (2004). CAViaR: Conditional autoregressive value at risk by regression quantiles. *Journal of Business & Economic Statistics*, 22(4):367–381.

Feng, W., Wang, Y., and Zhang, Z. (2018). Can cryptocurrencies be a safe haven: a tail risk perspective analysis. *Applied Economics*, 50(44):4745–4762.

Fischer, M., Köck, C., Schlüter, S., and Weigert, F. (2009). An empirical analysis of multivariate copula models. *Quantitative Finance*, 9(7):839–854.

Francq, C. and Zakoïan, J.-M. (2009). Bartlett's formula for a general class of nonlinear processes. *Journal of Time Series Analysis*, 30(4):449–465.

Gaglianone, W. P., Lima, L. R., Linton, O., and Smith, D. R. (2011). Evaluating Value-at-Risk models via quantile regression. *Journal of Business & Economic Statistics*, 29(1):150–160.

Gao, C.-T. and Zhou, X.-H. (2016). Forecasting VaR and ES using dynamic conditional score models and skew Student distribution. *Economic Modelling*, 53:216–223.

Gkillas, K. and Katsiampa, P. (2018). An application of extreme value theory to cryptocurrencies. *Economics Letters*, 164:109–111.

Glosten, L. R., Jagannathan, R., and Runkle, D. E. (1993). On the relation between the expected value and the volatility of the nominal excess return on stocks. *The Journal of Finance*, 48(5):1779–1801.

González-Rivera, G., Lee, T.-H., and Mishra, S. (2004). Forecasting volatility: A reality check based on option pricing, utility function, Value-at-Risk, and predictive likelihood. *International Journal of Forecasting*, 20(4):629–645.

Guesmi, K., Saadi, S., Abid, I., and Ftiti, Z. (2019). Portfolio diversification with virtual currency: Evidence from bitcoin. *International Review of Financial Analysis*, 63:431–437.

Han, Y., Li, P., and Xia, Y. (2017). Dynamic robust portfolio selection with copulas. *Finance Research Letters*, 21:190–200.

Hansen, P. R., Lunde, A., and Nason, J. M. (2011). The model confidence set. *Econometrica*, 79(2):453–497.

Harvey, A. and Chakravarty, T. (2008). Beta-t-(E)GARCH. *Cambridge Working Papers in Economics CWPE0840, University of Cambridge*.

Harvey, A. and Sucarrat, G. (2014). EGARCH models with fat tails, skewness and leverage. *Computational Statistics & Data Analysis*, 76:320–338.

Harvey, A. C. (2013). *Dynamic Models for Volatility and Heavy Tails: With Applications to Financial and Economic Time Series*, volume 52 of *Econometric Society Monographs*. Cambridge University Press, New York.

Hotta, L. K. and Trucíos, C. (2018). Inference in (M)GARCH Models in the Presence of Additive Outliers: Specification, Estimation and Prediction. In Lavor, C. and Neto, F. A. M. G., editors, *Advances in Mathematics and Applications*, pages 179–202. Springer.

Hotta, L. K. and Tsay, R. S. (2012). Outliers in GARCH processes. In Bell, W. R., Holan, S. H., and McElroy, T. S., editors, *Economic Time Series: Modeling and Seasonality*, pages 337–358. Chapman and Hall.

Huang, J.-J., Lee, K.-J., Liang, H., and Lin, W.-F. (2009). Estimating value at risk of portfolio by conditional copula-GARCH method. *Insurance: Mathematics and economics*, 45(3):315–324.

Jiang, Z. and Liang, J. (2017). Cryptocurrency portfolio management with deep reinforcement learning. In *2017 Intelligent Systems Conference (IntelliSys)*, pages 905–913. IEEE.

Joe, H. (1996). Families of m-variate distributions with given margins and m (m-1)/2 bivariate dependence parameters. *Lecture Notes-Monograph Series*, pages 120–141.

Joe, H. (1997). *Multivariate models and multivariate dependence concepts*. Chapman and Hall/CRC, London.

Joe, H. (2014). *Dependence modeling with copulas*, volume 134. Chapman and Hall/CRC, Florida.

Kakouris, I. and Rustem, B. (2014). Robust portfolio optimization with copulas. *European Journal of Operational Research*, 235(1):28–37.

Katsiampa, P. (2017). Volatility estimation for Bitcoin: A comparison of GARCH models. *Economics Letters*, 158:3–6.

Katsiampa, P. (2019a). An empirical investigation of volatility dynamics in the cryptocurrency market. *Research in International Business and Finance*, 50:322–335.

Katsiampa, P. (2019b). Volatility co-movement between Bitcoin and Ether. *Finance Research Letters*, 30:221–227.

Koutmos, D. (2018). Return and volatility spillovers among cryptocurrencies. *Economics Letters*, 173:122–127.

Kupiec, P. H. (1995). Techniques for verifying the accuracy of risk measurement models. *The Journal of Derivatives*, 3(2):73–84.

Likitratcharoen, D., Ranong, T. N., Chuengsuksomboon, R., Sritanee, N., and Pansriwong, A. (2018). Value at risk performance in cryptocurrencies. *The Journal of Risk Management and Insurance*, 22(1):11–28.

Liu, W. (2019). Portfolio diversification across cryptocurrencies. *Finance Research Letters*, 29:200–205.

Luther, W. J. and Salter, A. W. (2017). Bitcoin and the bailout. *The Quarterly Review of Economics and Finance*, 66:50–56.

McNeil, A. J. and Frey, R. (2000). Estimation of tail-related risk measures for heteroscedastic financial time series: an extreme value approach. *Journal of Empirical Finance*, 7(3-4):271–300.

Muler, N. and Yohai, V. J. (2008). Robust estimates for GARCH models. *Journal of Statistical Planning and Inference*, 138(10):2918–2940.

Nelson, D. B. (1991). Conditional heteroskedasticity in asset returns: A new approach. *Econometrica: Journal of the Econometric Society*, pages 347–370.

Nolde, N. and Ziegel, J. F. (2017). Elicitability and backtesting: Perspectives for banking regulation. *The Annals of Applied Statistics*, 11(4):1833–1874.

Osterrieder, J. and Lorenz, J. (2017). A statistical risk assessment of Bitcoin and its extreme tail behavior. *Annals of Financial Economics*, 12(01):1750003.

Palaro, H. P. and Hotta, L. K. (2006). Using conditional copula to estimate value at risk. *Journal of Data Science*, 4:93–115.

Pele, D. T. and Mazurencu-Marinescu-Pele, M. (2019). Using high-frequency entropy to forecast Bitcoin's daily value at risk. *Entropy*, 21(2):102.

Platanakis, E. and Urquhart, A. (2019). Should investors include bitcoin in their portfolios? a portfolio theory approach. *The British Accounting Review*, page 100837.

Schepsmeier, U., Stoeber, J., Brechmann, E. C., Graeler, B., Nagler, T., and Erhardt, T. (2018). *VineCopula: Statistical Inference of Vine Copulas*. R package version 2.1.8.

Selmi, R., Mensi, W., Hammoudeh, S., and Bouoiyour, J. (2018). Is Bitcoin a hedge, a safe haven or a diversifier for oil price movements? a comparison with gold. *Energy Economics*, 74:787–801.

Sklar, M. (1959). Fonctions de repartition an dimensions et leurs marges. *Publ. inst. statist. univ. Paris*, 8:229–231.

Stavroyiannis, S. (2017). Value-at-Risk and Expected Shortfall for the major digital currencies. *arXiv preprint arXiv:1708.09343*.

Stavroyiannis, S. (2018). Value-at-risk and related measures for the Bitcoin. *The Journal of Risk Finance*, 19(2):127–136.

Stavroyiannis, S., Makris, I., Nikolaidis, V., and Zarangas, L. (2012). Econometric modeling and value-at-risk using the Pearson type-IV distribution. *International Review of Financial Analysis*, 22:10–17.

Stavroyiannis, S. and Zarangas, L. (2013). Out of sample value-at-risk and backtesting with the standardized Pearson type-IV skewed distribution. *Panoeconomicus*, 60(2):231–247.

Tabasi, H., Yousefi, V., Tamošaitienė, J., and Ghasemi, F. (2019). Estimating conditional value at risk in the Tehran stock exchange based on the extreme value theory using GARCH models. *Administrative Sciences*, 9(2):40.

Trabelsi, N. (2018). Are there any volatility spill-over effects among cryptocurrencies and widely traded asset classes? *Journal of Risk and Financial Management*, 11(4):66.

Troster, V., Tiwari, A. K., Shahbaz, M., and Macedo, D. N. (2019). Bitcoin returns and risk: A general GARCH and GAS analysis. *Finance Research Letters*, 30(6):187–193.

Trucíos, C. (2019). Forecasting Bitcoin risk measures: A robust approach. *International Journal of Forecasting*, 35(3):836–847.

Trucios, C. (2019). *RobGARCHBoot: Robust Bootstrap Forecast Densities for GARCH Models*. R package version 1.0.0.

Trucíos, C., Hotta, L. K., and Ruiz, E. (2015). Robust bootstrap forecast densities for GARCH models: returns, volatilities and value-at-risk. *UC3M Working Papers Statistics and Econometrics*, 15(22).

Trucíos, C., Hotta, L. K., and Ruiz, E. (2017). Robust bootstrap forecast densities for GARCH returns and volatilities. *Journal of Statistical Computation and Simulation*, 87(16):3152–3174.

Weiß, G. N. (2013). Copula-GARCH versus dynamic conditional correlation: an empirical study on VaR and ES forecasting accuracy. *Review of Quantitative Finance and Accounting*, 41(2):179–202.

Wu, C. Y. and Pandey, V. K. (2014). The value of Bitcoin in enhancing the efficiency of an investor's portfolio. *Journal of Financial Planning*, 27(9):44–52.

Yermack, D. (2015). Is Bitcoin a real currency? an economic appraisal. In *Handbook of digital currency*, pages 31–43. Elsevier.

Yi, S., Xu, Z., and Wang, G.-J. (2018). Volatility connectedness in the cryptocurrency market: Is Bitcoin a dominant cryptocurrency? *International Review of Financial Analysis*, 60:98–114.

Yu, W., Yang, K., Wei, Y., and Lei, L. (2018). Measuring Value-at-Risk and Expected Shortfall of crude oil portfolio using extreme value theory and vine copula. *Physica A: Statistical Mechanics and its Applications*, 490:1423–1433.