3EJ4 Lab5

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Part1:

Q1. (20 Points) (1) Find the transfer function of the first-order LPF, its low-frequency gain, and its -3dB frequency f_c. (2) Compare the calculated low-frequency gain and the -3dB frequency f_c with the simulated data from Step 1.3 and the measured data from Step 1.8, respectively. Justify/discuss the observation and comparison.

(1):

(1)

$$T(s) = \frac{V_o(s)}{V_{in}(s)}$$

$\because V_+ = V_-$

$$V_- = V_o(s) \cdot \frac{R_3}{R_3 + R_2}$$

and $$\frac{V_{in} - V_+}{R_1} = \frac{V_+}{\frac{1}{sC_1}}$$

$$\therefore \frac{V_{in} - V_o(s) \cdot \frac{R_3}{R_3+R_2}}{R_1} = V_o(s) \cdot \frac{R_3}{R_3+R_2} \cdot sC_1$$

Rearrange the equation and get $T(s) = \frac{V_o(s)}{V_{in}(s)} = \frac{R_3+R_2}{R_3(R_1 s C_1 + 1)}$

(2) At low-frequency, $f < f_c$, $\therefore \frac{V_{out}}{V_{in}} \cong A_F = 1 + R_2/R_3 = 1.5$

At the -3dB (cutoff) frequency, $f_c = \frac{1}{2\pi R_1 C_1} = 1591\ Hz$

(2):

Both the simulated and the experimental transfer function graphs are very close to each other though they have different Vin input ac signal amplitudes, the simulation one has 1V and the experimental one has 100mV. The input ratio is 10:1 which is exactly the same ratio of the output Vout amplitude. Though the input value will change the output values, but based on the graph and the formula derived from (1), the gain of the active low pass filter will always be Vout/Vin which in this case, stays constant if there no components being switched.

Q2. (20 Points) Derive the transfer function and calculate the low-frequency gain. Verify the calculated gain using the simulated data obtained in Step 2.2 and the measured data obtained in Step 2.6, respectively.

Apply KCL at Vx:

$$\frac{V_x - V_{in}}{R_1} + \frac{V_x - 0}{R_2 + Z_2} + \frac{V_x - V_{out}}{Z_1} = 0$$

and also

$$V_- = V_+ = V_{out} \cdot \frac{R_3}{R_3 + R_4} = \frac{V_{out}}{2}$$

Rearrange those equations we can get:

$$T(s) = \frac{V_o}{V_{in}} = \frac{2}{R_1(C_1 + C_2 + sC_1)(1 + sR_2C_2) - 2sR_1C_1 - R_1C_2}$$

$$= \frac{2}{R_1R_2C_1C_2s^2 + [R_1(C_1 + C_2)R_2C_2 + R_1C_1 - 2R_1C_1]s + [R(C_1 + C_2) - R_1C_2]}$$

$$= \frac{2}{R_1R_2C_1C_2s^2 + (R_1C_2 + R_2C_2 - R_1C_1)s + 1}$$

$$= \frac{\frac{2}{R_1R_2C_1C_2}}{s^2 + \frac{R_1C_2 + R_2C_2 - R_1C_1}{R_1R_2C_1C_2}s + \frac{1}{R_1R_2C_1C_2}}$$

$$= \frac{91 \times 10^6}{s^2 + 15.5 \times 10^3 s + 45.5 \times 10^6} = \frac{a_o}{s^2 + \frac{w_o}{Q}s + w_o^2}$$

$\therefore\ w_o^2 = 45.5 \times 10^6 \Rightarrow w_o = 6745.37\ \text{rad/s} = 6745.37/2\pi = 1.07\ \text{kHz}$

$\frac{w_o}{Q} = 15.5 \times 10^3$, $\therefore\ Q = w_o / 15.5 \times 10^3 = 0.435 \simeq 0.44$

$$\text{DC gain} = \frac{a_o}{w_o^2} = \frac{91 \times 10^6}{45.5 \times 10^6} = 2,\ 20 \cdot \log(2) = 6.02\ \text{dB}$$

Frequency	db(v(vout))	ph(v(vout))
Hz	dBV	deg
1.00E+00	6.0205895	-1.23E-01

The calculated gain matches the simulated gain which is 6.02dB as shown and very close to the measured gain

Frequency	Manitude	Phase
Hz	dB	degree
1	6.009183	-0.12825

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Q3. (20 Points) Calculate (1) the pole frequency f_o, (2) the cut-off frequency (or -3dB frequency) f_c, (3) the pole quality factor Q, (4) the peak value of the magnitude of the transfer function, and (5) the frequency f_{max} where the peak value of the magnitude of the transfer function happens. Verify the calculated results using the simulated data obtained in Step 2.2 and the measured data obtained in Step 2.6, respectively.

for cut off-frequency, $|T(j \cdot w_c)| = |T(0)| / \text{sqrt}(2)$

$$\frac{a_0}{-w_c^2 + \frac{w_0}{Q} w_c j + w_0^2} = \frac{1}{\sqrt{2}}$$

$$T(j \cdot w_c) = \frac{9.1 \times 10^7}{\sqrt{(4.55 \times 10^7 - w_c^2)^2 + (w_c^2 \times 1.55 \times 10^4)^2}} = \frac{1}{\sqrt{2}}$$

By solving this equation, we can find

$w_c^2 = 12,777,177.46$, $w^2 = -162,027,177.5$ ~~ge~~

$\therefore w_c = \sqrt{12,777,177.46} = 3574.5177$ rad/s

$= 568.9$ Hz

(1) The pole frequency can be found from the previous question which is 1.07kHz.
(2) The pole Quality factor Q = 0.44.
(3) The cutoff frequency is 568.9Hz as calculated shown above.
(4) Because the Q-value is less than 0.707(1/sqrt (2)) which the damping factor will be 1.42, it can conclude that the second order low pass filter is overdamped with the frequency response showing a long flat graph therefore there is no "bump" in the graph. Therefore, the peak value of the magnitude of the transfer function will happen when s=0, which when the frequency is 0 Hz. Which the peak value is the gain of this filter equal to 6.02dB.

(5)

To verify the calculated result: The (-3dB) cutoff frequency can be found by subtracting the gain (happens at the low frequency) by -3dB and find the correspond frequency values. Which it will happen when the gain is 3.02dB.

279	540.3476504	3.24309016	-5.72E+01
280	552.8079212	3.14136335	-5.82E+01
281	565.5555225	3.03672697	-5.92E+01
282	578.5970801	2.92914308	-6.02E+01

The simulation result as shown is 565.55Hz.

279	575.4399	3.10127	-58.16914
280	588.8437	2.994037	-59.19757
281	602.5596	2.886371	-60.23727

The measured result as show is 588.84Hz.

Based on simulation results and measured results, we can verify it is close to the calculated cut off frequency.

Q4. (20 Points) Derive the transfer function and calculate the center frequency gain. Verify the calculated gain using the simulated data obtained in Step 3.2 and the measured data obtained in Step 3.6, respectively.

1. $$\frac{V_1 - V_{in}}{R_1} + V_1 sC_2 + \frac{V_1 - V_o}{1/sC_1} = 0 \quad (1)$$

$$\frac{0 - V_1}{1/sC_2} + \frac{0 - V_o}{R_2} = 0$$

$$V_1 sC_2 = -\frac{V_o}{R_2}$$

$$V_1 = -\frac{V_o}{sC_2R_2} \quad (2)$$

$$\frac{-V_o \cdot s\cancel{C_1}}{s\cancel{C_1}R_2} - sC_1V_o$$

$$-\frac{V_oC_1}{C_2R_2}$$

$$\frac{\frac{-V_o}{sC_2R_2} - V_{in}}{R_1} \qquad \frac{V_o}{\cancel{sC_2}R_2}\cancel{sC_2} \qquad \frac{\frac{-V_o}{sC_2R_2} - V_o}{1/sC_1}$$

$$\frac{-V_o}{sC_2R_2R_1} - \frac{V_{in}}{R_1} - \frac{V_o}{R_2} - \frac{V_oC_1}{C_2R_2} - sC_1V_o = 0$$

$$-\frac{V_{in}}{R_1} = \frac{V_o}{sC_2R_2R_1} + \frac{V_o}{R_2} + \frac{V_oC_1}{C_2R_2} + sC_1V_o$$

$$-\frac{\cancel{V_{in}}\,1}{R_1} = \underbrace{V_o\left(\frac{1}{sC_2R_2R_1} + \frac{1}{R_2} + \frac{C_1}{C_2R_2} + sC_1\right)}_{V_{in}}$$

$$= -\frac{1}{R_1}$$

$$\frac{V_o}{V_{in}} = -\frac{1}{R_1}$$

$$-\frac{1}{R_1 \cdot (\quad)}$$

$$\frac{V_o}{V_{in}} = -\frac{1}{R_1} \cdot \left[\frac{1}{\frac{1}{sC_2R_2R_1} + \frac{1}{R_2} + \frac{C_1}{C_2R_2} + sC_1}\right]$$

$$= -\frac{1}{R_1} \cdot \left[\frac{1}{\frac{1}{sC_2R_2R_1} + \frac{sC_2R_1}{sC_2R_2R_1} + \frac{sC_1R_1}{sC_2R_2R_1} + \frac{sC_1C_2R_1R_2s}{sC_2R_2R_1}}\right]$$

$$= -\frac{1}{R_1} \cdot \frac{sC_2R_2R_1}{1 + sC_2R_1 + sC_1R_1 + sC_1C_2R_1R_2s}$$

$$= \frac{-sC_2R_2}{1 + sC_2R_1 + sC_1R_1 + sC_1C_2R_1R_2s}$$

$$= \frac{-sC_2R_2}{1 + s(C_2R_1 + C_1R_1) + s^2(C_1C_2R_1R_2)}$$

$$= \frac{\frac{-sC_2R_2}{C_1C_2R_1R_2}}{s^2 + \frac{sC_2R_1}{C_1C_2R_1R_2} + \frac{sC_1R_1}{C_1C_2R_1R_2} + \frac{1}{C_1C_2R_1R_2}}$$

$$= \frac{-\frac{s}{R_1C_1}}{s^2 + s\left(\frac{1}{C_1R_2} + \frac{1}{C_2R_2}\right) + \frac{1}{C_1C_2R_1R_2}}$$

$$w_0^2 = \frac{1}{C_1C_2R_1R_2} \quad \therefore w_0 = \sqrt{\frac{1}{C_1C_2R_1R_2}}, \quad w_0 = \sqrt{\frac{1}{1n \cdot 1n \cdot 240k \cdot 240k}}$$

$$= 4166.7 \text{ or } 663\,Hz$$

w_0 is the center frequency

$$\frac{w_0}{Q} = \frac{1}{C_1R_1} + \frac{1}{C_2R_2}, \text{ given } w_0, \; R_1 = R_2 \quad \text{so} \quad \frac{w_0}{Q} = \frac{2}{CR}$$

$$Q = \frac{w_0CR}{2} = \frac{\sqrt{\frac{1}{C^2R^2}} \cdot CR}{2} = \frac{1}{2} = 0.5$$

$$-3\,dB: \quad w_1, w_2 = w_0\sqrt{1 + \frac{1}{4Q^2}} \pm \frac{w_0}{2Q}$$

$$w_1 = 1005.9.2, \quad w_2 = 1725.8898$$

$$BW = 833.33\,Hz$$

Based on the derivation shown above, the center frequency gain can be calculated as

$$C_g = \frac{-\frac{1}{RC} \cdot Q}{w_o}$$

$$C_g = -0.5$$

which is equal to -0.5 or -6.02dB. And the center frequency is 663Hz as calculated above.

Simulation data:

287	648.457008	-6.022752196	-1.79E+00
288	663.4102514	-6.020589658	1.80E+02
289	678.7083126	-6.022941891	1.79E+02

Measured data:

285	660.69345	-5.963705	-179.0542
286	676.08298	-5.961264	179.58029
287	691.83097	-5.961403	178.28871

By observing the center frequency, we can observe that both the simulation data have really close center frequency values as the calculated center frequency 663Hz and also the gain -6.02dB which verify the calculation.

Q5. (20 Points) Calculate (1) the center frequency ω_o, (2) the pole quality factor Q, (3) the two -3dB frequencies ω_1 and ω_2, and (4) the 3-dB bandwidth $BW = \omega_2 - \omega_1$. Verify the calculated results using the simulated data obtained in Step 3.2 and the measured data obtained in Step 3.6, respectively.

(1) The center frequency wo is equal to 663Hz as calculated above.
(2) The pole quality factor Q is 0.5 as calculated above.
(3) The two -3dB, w1 = 1725.9 and w2 = 10059.2 in radians. And w1=274.7Hz, w2=1601Hz
(4) The 3-dB BW= 8333.3 in radians. BW=1326.3Hz.

The center frequency is verified from the last question.
Based on the 3-dB frequency found above, the gain can be calculated as

$$f_3 = \frac{-\frac{1}{RC} \cdot Q}{\sqrt{2}\, w_o}$$

$$f_3 = -0.353553390593$$

$$20 \cdot \log(\mathrm{abs}(f_3))$$

$$= -9.03089986992$$

which is equal to -9.03dB.

And we can verify the gain from the graph as shown above. And their BW is close to the calculated value.
Simulation result:

272.6738966	-9.0759769	-1.35E+00

1614.064152	-9.081884149	1.35E+02

BW = 1342
Measured result:

275.4229	-9.02013	-134.676

1621.81	-8.9751	135.0685

BW = 1349

Since the Bandwidth = wo/Q, Q = wo/Bandwidth = 663/1342 = 0.49 -> Simulation
Q = 676/1349=0.50 ->measured which proves the Quality factor