

3EJ4 Lab1

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Part1:

For the NPN-BJT 2N3904 characterized, if we want to bias this device to conduct a collector current $I_C \approx 1.0 \text{ mA}$ at the lowest V_{CE} value, answer the following questions.

Q1. (7 Points) Based on the simulated data in Steps 1.2-1.4, use the bias condition giving the closest I_C value to the desired collector current, find out

V+ (VCC)	V- (VE)	V(Q1C) (Volt)	V(Q1B) (Volt)	VCE (Volt)	VBEon (Volt)	RC (ohm)	RB (ohm)	IC (A)	IB (A)	$\beta = I_C/I_B$	VA (V)	$r_o = V_A/I_C$	$g_m = I_C/25\text{mV}$	$r_\pi = 25\text{mV}/I_B$
0.5	-1.5	3.98E-01	-8.79E-01	1.898	0.621	100	1.00E+05	0.00102	8.79E-06	117		9.76E+05	4.10E-02	2845

(1) What are the simulated V_{BEon} in volt and the base current I_B in μA ?

Ans: $V_{BEon} = 0.621\text{V}$, $I_B = 8.79\mu\text{A}$

(2) What is the $\beta = I_C/I_B$ value at this I_C ?

Ans: 117

(3) What is the early voltage $|V_A|$ in volt?

Ans: 1000V

(4) What is the output resistance r_o in $\text{k}\Omega$?

Ans: 976 $\text{k}\Omega$

(5) What is the transconductance g_m in mS ?

Ans: 41mS

(6) What is the input resistance r_π in $\text{k}\Omega$?

Ans: 2.845 $\text{k}\Omega$

Q2. (8 Points) Based on the measured data in Step 1.8, use the same bias condition used in Q1 (or the first reliable data if that bias condition is an outlier), find out

V+ (VCC)	V- (VE)	Channel 1 (VC)	Channel 2 (VB)	VCE (Volt)	VBEon (Volt)	RC (ohm)	RB (ohm)	IC (A)	IB (A)	$\beta = I_C/I_B$	VA (V)	$r_o = V_A/I_C$	$g_m = I_C/25\text{mV}$	$r_\pi = 25\text{mV}/I_B$
0.5	-1.5	3.20E-01	-8.17E-01	1.82	0.6826	100	1.00E+05	0.0018	8.17E-06	220		1.00E+05	7.20E-02	3058

(1) How much is the measured collector current I_C in mA ?

Ans: 1.8mA

(2) What are the measured V_{BEon} in volt and the base current I_B in μA ?

Ans: $V_{BEon} = 0.6826\text{V}$, $I_B = 8.17\mu\text{A}$

(3) What is the $\beta = I_C/I_B$ value at this I_C ?

Ans: 220

(4) What is the early voltage $|V_A|$ in volt?

Ans: 180V

(5) What is the output resistance r_o in $\text{k}\Omega$?

Ans: 100k Ω

(6) What is the transconductance g_m in mS ?

Ans: 72mS

(7) What is the input resistance r_π in $\text{k}\Omega$?

Ans: 3.058k Ω

Part2:

For the PNP-BJT 2N3906 characterized, if we want to bias this device to conduct a collector current $I_C \approx 1.0 \text{ mA}$ at the lowest V_{EC} value, answer the following questions.

Q3. (7 Points) Based on the simulated data in Steps 2.2-2.4, use the bias condition giving the closest I_C value to the desired collector current, find out

V+ (V)	V- (VCC)	V(Q1C) (V)	V(Q1B) (V)	VEC (V)	VEBon (V)	RC (ohm)	RB (ohm)	IC (A)	IB (A)	$\beta = I_C/I_B$	VA (V)	$r_o = V_A/I_C$	$g_m = I_C/25\text{mV}$	$r_\pi = 25\text{mV}/I_B$
1.5	-0.5	-3.97E-01	8.40E-01	1.90	0.660	100	1.00E+05	1.03E-03	8.40E-06	123		1.39E+05	4.12E-02	2976

(1) What are the simulated V_{EBon} in volt and the base current I_B in μA ?

Ans: $V_{EBon} = 0.660\text{V}$, $I_B = 8.40\mu\text{A}$

(2) What is the $\beta = I_C/I_B$ value at this I_C ?

Ans: 123

(3) What is the early voltage $|V_A|$ in volt?

Ans: 133

(4) What is the output resistance r_o in $k\Omega$?

Ans: 139k Ω

(5) What is the transconductance g_m in mS?

Ans: 41.2mS

(6) What is the input resistance r_π in $k\Omega$?

Ans: 2.976k Ω

Q4. (8 Points) Based on the measured data in Step 2.8, use the same bias condition used in Q3 (or the first reliable data if that bias condition is an outlier), find out

V+ (V)	V- (VCC)	Channel 1 (VC)	Channel 2 (VB)	VEC (V)	VEBon (V)	RC (ohm)	RB (ohm)	IC (A)	IB (A)	$\beta = I_C/I_B$	VA (V)	$r_o = V_A/I_C$	$g_m = I_C/25\text{mV}$	$r_\pi = 25\text{mV}/I_B$
1	-0.5	-4.01E-01	3.54E-01	1.40	0.646	100	1.00E+05	9.90E-04	3.54E-06	280		3.36E+04	3.96E-02	7066

(1) How much is the measured collector current I_C in mA?

Ans: 0.990mA

(2) What are the measured V_{EBon} in volt and the base current I_B in μA ?

Ans: $V_{EBon} = 0.646\text{V}$, $I_B = 3.54\mu\text{A}$

(3) What is the $\beta = I_C/I_B$ value at this I_C ?

Ans: 280

(4) What is the early voltage $|V_A|$ in volt?

Ans: 33V

(5) What is the output resistance r_o in $k\Omega$?

Ans: 33.6k Ω

(6) What is the transconductance g_m in mS?

Ans: 39.6mS

(7) What is the input resistance r_π in $k\Omega$?

Ans: 7.066k Ω

Part3:

Q5. (10 Points) Express the base current I_B as a function of V_{BB} , R_{BB} , V_{BEon} , R_3 , V_{EE} , and β

$$\text{Ans: } I_B = \frac{V_{BB} - (V_{EE} + V_{BEon})}{R_{BB} + (\beta + 1)R_3} = \frac{R_2}{R_1 + R_2} \frac{-(V_{EE} + \Delta V_{EE})}{R_{BB} + (\beta + 1)R_3} - \frac{V_{BEon}}{R_{BB} + (\beta + 1)R_3}$$

Solution:

Q5: Apply loop analysis from x_3 to x_1 From Figure 7

$$V_{BB} - R_{BB}I_B - V_{BEon} - \underline{I_E R_3} - V_{EE} = 0$$

$$I_E = (\beta + 1) \cdot I_B$$

$$V_{BB} - V_{BEon} - V_{EE} = I_B (R_{BB} + R_3 \cdot (\beta + 1))$$

$$I_B = \frac{V_{BB} - V_{BEon} - V_{EE}}{R_{BB} + R_3 \cdot (\beta + 1)}$$

$$(1) V_{BB} = \frac{R_1}{R_1 + R_2} V_{EE}$$

$$(2) R_{BB} = R_1 || R_2 = \frac{R_1 R_2}{R_1 + R_2}$$

$$(3) I_B = \frac{V_{BB} - V_{EE} - V_{BEon}}{R_{BB}}$$

$$I_B = \frac{V_{BB} - V_{BEon} - V_{EE}}{R_{BB} + R_3 (\beta + 1)} = \frac{\frac{R_1}{R_1 + R_2} V_{EE} - V_{EE} - V_{BEon}}{R_{BB} + R_3 (\beta + 1)}$$
$$= \frac{V_{EE} \left(\frac{R_1}{R_1 + R_2} - 1 \right) - V_{BEon}}{R_{BB} + R_3 (\beta + 1)} \quad \rightarrow \frac{R_1 + R_2}{R_1 + R_2} = \frac{-R_2}{R_1 + R_2}$$

$$= \frac{-\frac{R_2}{R_1 + R_2} V_{EE} - V_{BEon}}{R_{BB} + R_3 (\beta + 1)}$$

$$= \frac{R_2}{R_1 + R_2} \cdot \frac{(-V_{EE})}{R_{BB} + R_3 (\beta + 1)} - \frac{V_{BEon}}{R_{BB} + R_3 (\beta + 1)}$$

$$\therefore I_B = \frac{V_{BB} - V_{BEon} - V_{EE}}{R_{BB} + R_3 (\beta + 1)} = \frac{R_2}{R_1 + R_2} \frac{(-V_{EE})}{R_{BB} + R_3 (\beta + 1)} - \frac{V_{BEon}}{R_{BB} + R_3 (\beta + 1)}$$

Q6. (10 Points) Comparing the IB expression obtained in Q5 with (3), what is the difference between these two equations? For a change ΔV_{EE} in the power supply VEE, derive equations for the resulted change in the base current ΔI_B using (3) and the IB expression obtained in Q5. Show that the emitter resistor R3 reduces the change in the base current ΔI_B as a result of the change ΔV_{EE} in the power supply VEE

Ans:

The difference between these two equations is there is a following Resistance equation $(\beta + 1)R_3$ in Q5 if we don't assume R3 is zero from the Fig.7.

Equations for the resulted change in the base current ΔI_B .

Using expression (3):
$$\Delta I_B = \frac{R_2}{R_1 + R_2} \frac{-\Delta V_{EE}}{R_{BB}}$$

Using expression (Q5):
$$\Delta I_B = \frac{R_2}{R_1 + R_2} \frac{-\Delta V_{EE}}{R_{BB} + (\beta + 1)R_3}$$

As seen from the two expressions, if there is a change ΔV_{EE} in the power supply, all other resistors stay constant, the emitter resistor R3 (Resistor must be positive) will increase the denominator values in order to reduce the overall change of current ΔI_B .

Solution:

Q6:
$$(5) I_B = \frac{V_{BB} - V_{BEON} - V_{EE}}{R_{BB} + R_3(\beta + 1)} = \frac{R_2}{R_1 + R_2} \frac{(-V_{EE})}{R_{BB} + R_3(\beta + 1)} \cdot \frac{V_{BEON}}{R_{BB} + R_3(\beta + 1)}$$

(3)
$$I_B = \frac{V_{BB} - (V_{EE} + V_{BEON})}{R_{BB}} = \frac{R_2}{R_1 + R_2} \frac{(-V_{EE})}{R_{BB}} \cdot \frac{V_{BEON}}{R_{BB}}$$

Using expression (3), if a change ΔV_{EE} , $V_{EE} + \Delta V_{EE}$

$$\therefore I_{B\text{final}} = \frac{R_2}{R_1 + R_2} \frac{-(V_{EE} + \Delta V_{EE})}{R_{BB}} \cdot \frac{V_{BEON}}{R_{BB}}$$

$$\Delta I_B = I_{B\text{final}} - I_B = \frac{R_2}{R_1 + R_2} \frac{-(V_{EE} + \Delta V_{EE})}{R_{BB}} \cdot \frac{V_{BEON}}{R_{BB}} - \frac{R_2}{R_1 + R_2} \frac{(-V_{EE})}{R_{BB}} \cdot \frac{V_{BEON}}{R_{BB}}$$

$$= \frac{R_2}{R_1 + R_2} \left(\frac{-V_{EE} - \Delta V_{EE} + V_{BEON} + V_{EE} - V_{BEON}}{R_{BB}} \right)$$

$$= \frac{R_2}{R_1 + R_2} \frac{(-\Delta V_{EE})}{R_{BB}}$$

Using expression (5), if a change ΔV_{EE} , $V_{EE} + \Delta V_{EE}$

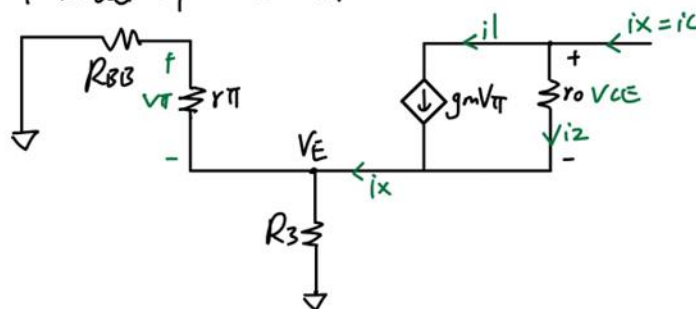
$$\therefore I_{B\text{final}} = \frac{R_2}{R_1 + R_2} \frac{-(V_{EE} + \Delta V_{EE})}{R_{BB} + R_3(\beta + 1)} \cdot \frac{V_{BEON}}{R_{BB} + R_3(\beta + 1)}$$

$$\begin{aligned}
 \Delta I_B &= I_{B \text{ final}} - I_B = \frac{R_2}{R_1 + R_2} \frac{-(V_{EE} + \Delta V_{EE})}{R_{B3} + R_3(\beta + 1)} \frac{V_{BEON}}{R_{B3} + R_3(\beta + 1)} \\
 &\quad - \frac{R_2}{R_1 + R_2} \frac{(-V_{EE})}{R_{B3} + R_3(\beta + 1)} \frac{V_{BEON}}{R_{B3} + R_3(\beta + 1)} \\
 &= \frac{R_2}{R_1 + R_2} \left(\frac{-V_{EE} - \Delta V_{EE} + V_{BEON} + V_{EE} - V_{BEON}}{R_{B3} + R_3(\beta + 1)} \right) = \frac{R_2}{R_1 + R_2} \frac{(-\Delta V_{EE})}{R_{B3} + R_3(\beta + 1)} \\
 &\quad R_3 \uparrow, \Delta I_B \downarrow, R_3 \rightarrow \text{resistor must be positive}
 \end{aligned}$$

Q7. (15 Points) Inserting the feedback R_3 at the emitter of the BJT not only stabilizes the I_B but also improves (or increases) the output resistance R_o of the current sink shown in Fig. 6/ Fig. 7 (i.e., I_o is more stable when there is a change in V_{CE}). Using a π -model for the BJT, prove that the output resistance of the current sink is $R_o = r_o + [R_3 || (R_{BB} + r_\pi)] \left[1 + g_m r_o \left(\frac{r_\pi}{R_{BB} + r_\pi} \right) \right]$.

Ans:

Q7. π -model of the BJT



$$R_o = \frac{V_x}{i_x}$$

$$i_l = g_m v_\pi, \quad i_2 = i_x - i_l = i_x - g_m v_\pi, \quad V_{CE} = r_o \cdot i_2 = r_o \cdot (i_x - g_m v_\pi)$$

$$V_E = i_x \cdot (R_{BB} + r_\pi) || R_3$$

$$v_\pi = -V_E \cdot \frac{r_\pi}{R_{BB} + r_\pi}$$

$$V_x = V_{CE} + V_E,$$

$$= r_o \cdot (i_x - g_m \cdot (-V_E \cdot \frac{r_\pi}{R_{BB} + r_\pi})) + i_x \cdot (R_{BB} + r_\pi) || R_3$$

$$= r_o \cdot \cancel{i_x} + r_o \cdot g_m \cdot \cancel{i_x} \cdot (R_{BB} + r_\pi) || R_3 \cdot \frac{r_\pi}{R_{BB} + r_\pi} + \cancel{i_x} \cdot (R_{BB} + r_\pi) || R_3$$

$$= i_x \left(r_o + r_o g_m \frac{(R_{BB} + r_\pi) || R_3 \cdot r_\pi}{R_{BB} + r_\pi} + (R_{BB} + r_\pi) || R_3 \right)$$

$$= \cancel{i_x} \left(r_o + (R_{BB} + r_\pi) || R_3 \right) \left(g_m r_o \cdot \frac{r_\pi}{R_{BB} + r_\pi} + 1 \right)$$

$$\therefore R_o = \frac{V_x}{i_x} = r_o + [R_3 || (R_{B3} + r_{\pi})] \left[1 + g_{mro} \left(\frac{r_{\pi}}{R_{B3} + r_{\pi}} \right) \right]$$

Q8. (10 Points) Inserting the feedback R_3 at the emitter of the BJT improves the stabilization of the Q-point at the cost of increased $V_{o,min}$. What is the $V_{o,min}$ of the constant current sink when $R_3 \neq 0$?

Ans:

Given $V_{o,min}$ formula is $V_{o,min} = V_{EE} + 0.3V$, this is the minimum voltage if we assume when R_3 is zero. After inserting the feedback R_3 , the current will flow through the R_3 and generated a new voltage drop across R_3 , this voltage can be calculated as $V_{R3} = I_{R3} \cdot R_3$ by applying the Ohm's law. Since I_{R3} is the current flow through the emitter, there $I_{R3} = I_E$ and $V_{R3} = I_E \cdot R_3$.

As a result, after inserting the feedback R_3 , the $V_{o,min} = V_{EE} + 0.3V + V_{R3} = V_{EE} + 0.3V + I_E \cdot R_3$.

Q9. (15 Points) For $V_{EE} = -5V$, if we want to design a current sink with $I_o = 1.0 \text{ mA}$ and $V_{o,min} = -1V$ using the NPN-BJT 2N3094 characterized in Q1, what is the resistance value for R_3 ? To reduce the DC power consumption of R_1 and R_2 , we usually choose large resistance values (in tens or hundreds of $k\Omega$) for R_1 and R_2 . Suppose we choose $R_2 = 100 \text{ k}\Omega$, calculate R_1 in $k\Omega$. Verify the I_o vs. V_{CC} characteristics of the design by sweeping V_{CC} from $-5V$ to $5V$ with a $0.05V$ step and post the screenshot of the simulated I_o vs. V_{CC} characteristics.

Ans:

$R_3 = 3.67 \text{ k}\Omega$, $R_1 = 13.09 \text{ k}\Omega$

I_o vs. V_{CC} characteristics:



Solution:

Q9. Given $V_{EE} = -5V$, $I_o = 1.0 \text{ mA}$, $V_{o,min} = -1V$, $\beta = 117$

(Q8) $V_{o,min} = V_{EE} + 0.3V + I_E \cdot R_3$

$I_o = I_C = 1.0 \text{ mA}$, $I_C = \beta \cdot I_B$

$$1.0 = 117 \cdot I_B$$

$$I_B = \frac{1}{117} \text{ mA}$$

$$I_E = (\beta + 1) I_B$$

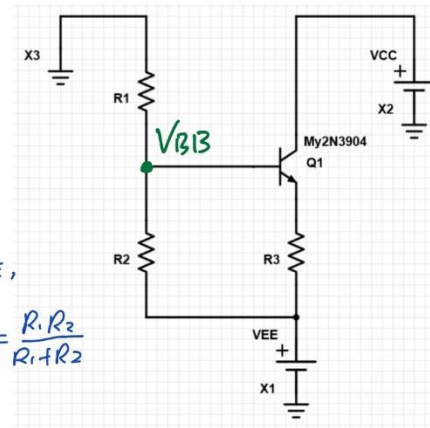
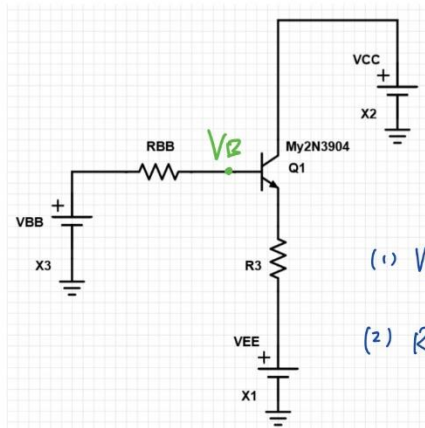
$$= 118 \cdot \frac{1}{117} = \frac{118}{117} \text{ mA}$$

$$-1V = -5V + 0.3V + \frac{118}{117} \text{ mA} \cdot R_3$$

$$3.7V = \frac{118}{117} \text{ mA} \cdot R_3$$

$$R_3 = 3.7 \times \frac{117}{118} = 3.67 \text{ k}\Omega$$

$$R_2 = 100\text{ k}\Omega$$



$$(1) V_{BB} = \frac{R_1}{R_1 + R_2} V_{EE},$$

$$(2) R_{BB} = R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2}$$

$$V_{BB} = V_B + I_B \cdot R_{BB}$$

$$V_{BB} = \frac{R_1}{R_1 + R_2} V_{EE}$$

$$\therefore I_B \cdot R_{BB} + V_B = \frac{R_1}{R_1 + R_2} V_{EE}$$

$$I_B \cdot \frac{R_1 R_2}{R_1 + R_2} + V_B = \frac{R_1}{R_1 + R_2} V_{EE}$$

$$\frac{1}{117} \cdot \frac{100 R_1}{100 + R_1} - 0.67763 = \frac{R_1}{100 + R_1} \times -5$$

$$R_1 = 13.089 \approx 13.09\text{ k}\Omega$$

$$I_B = \frac{1}{117}\text{ mA}, I_E = \frac{118}{117}\text{ mA}$$

$$R_2 = 100\text{ k}\Omega$$

$$V_{EE} = -5$$

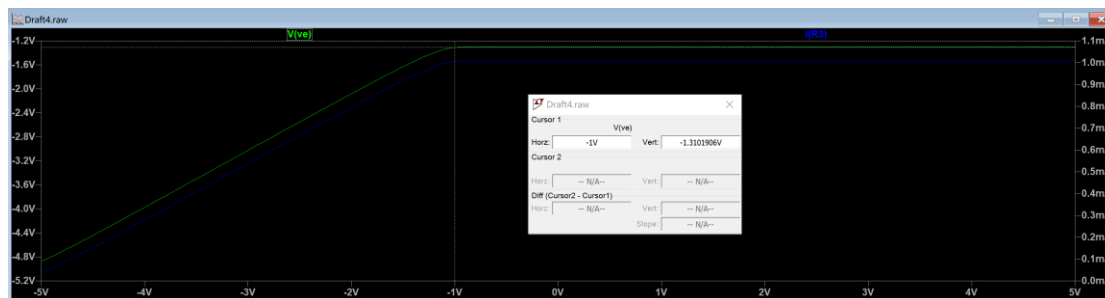
$$V_{EE} + I_E R_3 + V_{BEON} = V_B$$

$$-5 + \frac{118}{117} \times 3.67 + 0.62 = V_B$$

$$V_B = -0.67763$$

Q10. (10 Points) When designing the constant current sink shown in Fig. 6, we assume that $|V_{CE}| \geq 0.3\text{V}$ and Q_1 works in the active region. Based on the resistance values obtained in Q9, sweep V_{CC} in Fig. 6 from -5V to $+5\text{V}$ with a 0.05V step and measure V_E and I_C to determine the $|V_{CE}|$ required for Q_1 to work in the active region.

Ans:



As observed from the simulation graph, the horizontal axis is the V_{CC} DC sweep voltage, and the vertical axis is the V_E voltage. The active region is described as the horizontal line on the graph. The breakpoint observed of V_E and V_{CC} is -1.3101906V and -1V separately. Therefore, the V_{CE} can be calculated as $V_{CE} = V_{CC} - V_E = -1\text{V} + 1.3101906\text{V} = 0.3101906 \approx 0.31\text{V}$ which meets the assumption.

Circuit schematic for the Q9 and Q10:

