ROMEN SAMUEL R. WABINA
Data Science for Healthcare and Clinical Informatics

1. If $A = \begin{bmatrix} 5 & -2 & -4 \\ -4 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ $B = \begin{bmatrix} 1 & -4 \\ -1 & 5 \\ 2 & -4 \end{bmatrix}$

Please find the below items if it is valid.

(1.1) $A - A = \begin{bmatrix} 5 & -2 & -4 \\ -4 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} - \begin{bmatrix} 5 & -2 & -4 \\ -4 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$

$$A - A = \begin{bmatrix} 5-5 & -2+2 & -4+4 \\ -4+4 & 3-3 & 4-4 \\ 1-1 & -2+2 & -3+3 \end{bmatrix}$$

$$A - A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(1.2) $A + A = \begin{bmatrix} 5 & -2 & -4 \\ -4 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} + \begin{bmatrix} 5 & -2 & -4 \\ -4 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$

$$A + A = \begin{bmatrix} 5+5 & -2-2 & -4-4 \\ -4-4 & 3+3 & 4+4 \\ 1+1 & -2-2 & -3-3 \end{bmatrix}$$

$$A + A = \begin{bmatrix} 10 & -4 & -8 \\ -8 & 6 & 8 \\ 2 & -4 & -6 \end{bmatrix}$$

(1.3) $A^T = \begin{bmatrix} 5 & -2 & -4 \\ -4 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}^T = \begin{bmatrix} 5 & -4 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{bmatrix}$

(1.4) $A^2 = \begin{bmatrix} 5 & -2 & -4 \\ -4 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} \begin{bmatrix} 5 & -2 & -4 \\ -4 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$

where $A \in \mathbb{R}^{(3,3)}$, hence $A^2 \in \mathbb{R}^{(3,3)}$.

$$A^2 = \begin{bmatrix} (5)(5)+(-2)(-4)+(-4)(1) & (5)(-2)+(-2)(3)+(-4)(-2) & (5)(-4)+(-2)(4)+(-4)(-3) \\ (-4)(5)+(3)(-4)+(4)(1) & (-4)(-2)+(3)(3)+(4)(-2) & (-4)(-4)+(3)(4)+(4)(-3) \\ (1)(5)+(-2)(-4)+(-3)(1) & (1)(-2)+(-2)(3)+(-3)(-2) & (1)(-4)+(-2)(4)+(-3)(-3) \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 25+8-4 & -10-6+8 & -20-8+12 \\ -20-12+4 & 8+9-8 & 16+12-12 \\ 5+8-3 & -2-6+6 & -4-8+9 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 29 & -8 & -16 \\ -28 & 9 & 16 \\ 10 & -2 & -3 \end{bmatrix}$$

(1.5)
$$A^{-1} = \begin{bmatrix} 5 & -2 & -4 \\ -4 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}^{-1} = \left[\begin{array}{ccc|ccc} 5 & -2 & -4 & 1 & 0 & 0 \\ -4 & 3 & 4 & 0 & 1 & 0 \\ 1 & -2 & -3 & 0 & 0 & 1 \end{array}\right]$$

$$\frac{1}{5}R_1 \rightarrow R_1 \left[\begin{array}{ccc|ccc} 1 & -2/5 & -4/5 & 1/5 & 0 & 0 \\ -4 & 3 & 4 & 0 & 1 & 0 \\ 1 & -2 & -3 & 0 & 0 & 1 \end{array}\right]$$

$$R_2-(-4)R_1 \rightarrow R_1 \left[\begin{array}{ccc|ccc} 1 & -2/5 & -4/5 & 1/5 & 0 & 0 \\ 0 & 7/5 & 4/5 & 4/5 & 1 & 0 \\ 1 & -2 & -3 & 0 & 0 & 1 \end{array}\right]$$

$$R_3-(1)R_1 \rightarrow R_3 \left[\begin{array}{ccc|ccc} 1 & -2/5 & -4/5 & 1/5 & 0 & 0 \\ 0 & 7/5 & 4/5 & 4/5 & 1 & 0 \\ 0 & -8/5 & -11/5 & -1/5 & 0 & 1 \end{array}\right]$$

$$\left(\frac{5}{7}\right)R_2 \rightarrow R_2 \left[\begin{array}{ccc|ccc} 1 & -2/5 & -4/5 & 1/5 & 0 & 0 \\ 0 & 1 & 4/7 & 4/7 & 5/7 & 0 \\ 0 & -8/5 & -11/5 & 0 & 0 & 1 \end{array}\right]$$

$$R_3-\left(-\frac{8}{5}\right)R_2 \rightarrow R_3 \left[\begin{array}{ccc|ccc} 1 & -2/5 & -4/5 & 1/5 & 0 & 0 \\ 0 & 1 & 4/7 & 4/7 & 5/7 & 0 \\ 0 & 0 & -9/7 & 5/7 & 8/7 & 1 \end{array}\right]$$

$$\left(-\frac{7}{9}\right)R_3 \rightarrow R_3 \left[\begin{array}{ccc|ccc} 1 & -2/5 & -4/5 & 1/5 & 0 & 0 \\ 0 & 1 & 4/7 & 4/7 & 5/7 & 0 \\ 0 & 0 & 1 & -5/9 & -8/9 & -7/9 \end{array}\right]$$

$$R_2-\left(\frac{4}{7}\right)R_3 \rightarrow R_2 \left[\begin{array}{ccc|ccc} 1 & -2/5 & -4/5 & 1/5 & 0 & 0 \\ 0 & 1 & 0 & 8/9 & 11/9 & 4/9 \\ 0 & 0 & 1 & -5/9 & -8/9 & -7/9 \end{array}\right]$$

$$R_1-\left(-\frac{4}{5}\right)R_3 \rightarrow R_3 \left[\begin{array}{ccc|ccc} 1 & -2/5 & 0 & -11/45 & -32/45 & -28/45 \\ 0 & 1 & 0 & 8/9 & 11/9 & 4/9 \\ 0 & 0 & 1 & -5/9 & -8/9 & -7/9 \end{array}\right]$$

$$R_1-\left(-\frac{2}{5}\right)R_2 \rightarrow R_1 \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/9 & -2/9 & -4/9 \\ 0 & 1 & 0 & 8/9 & 11/9 & 4/9 \\ 0 & 0 & 1 & -5/9 & -8/9 & -7/9 \end{array}\right]$$

Therefore,
$$A^{-1} = \begin{bmatrix} 5 & -2 & -4 \\ -4 & 3 & 4 \\ 1 & -2 & 3 \end{bmatrix}^{-1} = \begin{bmatrix} 1/9 & -2/9 & -4/9 \\ 8/9 & 11/9 & 4/9 \\ -5/9 & -8/9 & -7/9 \end{bmatrix}$$

(1.6)
$$A-B = \begin{bmatrix} 5 & -2 & -4 \\ -4 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} - \begin{bmatrix} 1 & -4 \\ -1 & 5 \\ 2 & -4 \end{bmatrix} =$$

Solution does not exists since the number of rows and columns of matrices don't match, where matrix $A \in \mathbb{R}^{(3\times3)}$ and $B \in \mathbb{R}^{(3\times2)}$.

(1.7)
$$BA = \begin{bmatrix} 1 & -4 \\ -1 & 5 \\ 2 & -4 \end{bmatrix} \begin{bmatrix} 5 & -2 & -4 \\ -4 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} =$$

The dimensions of A and B are $A \in \mathbb{R}^{(3\times3)}$ and $B \in \mathbb{R}^{(3\times2)}$, respectively. Since matrix B has $n=2$ and matrix A has $m=3$, the solution does not exist since n, m from matrices B and A does not match.

(1.8)
$$AB = \begin{bmatrix} 5 & -2 & -4 \\ -4 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} \begin{bmatrix} 1 & -4 \\ -1 & 5 \\ 2 & -4 \end{bmatrix} =$$

Unlike in (1.7), matrix multiplication is possible for AB since the number of columns in matrix A is equal to the number of rows in B. The expected dimension of AB is (3×2).

$$\begin{bmatrix} 5 & -2 & -4 \\ -4 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} \begin{bmatrix} 1 & -4 \\ -1 & 5 \\ 2 & -4 \end{bmatrix} = \begin{bmatrix} (5)(1)+(-2)(-1)+(-4)(2) & (5)(-4)+(-2)(5)+(-4)(-4) \\ (-4)(1)+(3)(-1)+(4)(2) & (-4)(-4)+(3)(5)+(4)(-4) \\ (1)(1)+(-2)(-1)+(-3)(2) & (1)(-4)+(-2)(5)+(-3)(-4) \end{bmatrix}$$

$$AB = \begin{bmatrix} 5+2-8 & -20-10+16 \\ -4-3+8 & 16+15-16 \\ 1+2-6 & -4-10+12 \end{bmatrix} = \begin{bmatrix} -1 & -14 \\ 1 & 15 \\ -3 & -2 \end{bmatrix}$$

Therefore,
$$AB = \begin{bmatrix} -1 & -14 \\ 1 & 15 \\ -3 & -2 \end{bmatrix}$$

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(2.1) From $A = \begin{bmatrix} 1 & 2 \\ 4 & 5 \\ 7 & 8 \end{bmatrix}$ find E if $EA = \begin{bmatrix} 1 & 2 \\ 1 & -1 \\ 7 & 8 \end{bmatrix}$

Consider the system $AX=B$ with coefficient matrix A and augmented matrix $[A|B]$. For this problem, suppose $X=A$ and $B=EA$. Hence, $EA=B$. By standard calculation, we compute $E=A^{-1}B$. However, A is non-square matrix so A^{-1} is difficult to calculate. We need to determine $\text{rank}(A)$ and $\text{rank}(A|B)$ to verify whether the system of equation is consistent or inconsistent.

$$\text{rank}(A) = \text{rank}\left(\begin{bmatrix} 1 & 2 \\ 4 & 5 \\ 7 & 8 \end{bmatrix}\right)$$

$$R_2 - 4R_1 \to R_2 \begin{bmatrix} 1 & 2 \\ 0 & -3 \\ 7 & 8 \end{bmatrix} \quad R_3 - 7R_1 \to R_3 \begin{bmatrix} 1 & 2 \\ 0 & -3 \\ 0 & -6 \end{bmatrix}$$

$$\left(-\frac{1}{3}\right)R_2 \to R_2 \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 0 & -6 \end{bmatrix} \quad \begin{matrix} R_3 + 6R_2 \to R_3 \\ \text{and} \\ \text{restore } R_2 \end{matrix} \begin{bmatrix} 1 & 2 \\ 0 & 2 \\ 0 & 0 \end{bmatrix}$$

There are two non-zero rows, therefore,

$\text{rank}(A) = 2$

$$\text{rank}(A|B) = \text{rank}\left(\left[\begin{array}{cc|cc} 1 & 2 & 1 & 2 \\ 4 & 5 & 1 & -1 \\ 7 & 8 & 7 & 8 \end{array}\right]\right)$$

$$R_2 - 4R_1 \to R_2 \left[\begin{array}{cc|cc} 1 & 2 & 1 & 2 \\ 0 & -3 & -3 & -9 \\ 7 & 8 & 7 & 8 \end{array}\right] \quad R_3 - 7R_1 \to R_3 \left[\begin{array}{cc|cc} 1 & 2 & 1 & 2 \\ 0 & -3 & -3 & -9 \\ 0 & -6 & 0 & -6 \end{array}\right]$$

$$\left(-\frac{1}{3}\right)R_2 \to R_2 \left[\begin{array}{cc|cc} 1 & 2 & 1 & 2 \\ 0 & 1 & 1 & 3 \\ 0 & -6 & 0 & -6 \end{array}\right] \quad \begin{matrix} R_3 + 6R_2 \to R_3 \\ \text{and} \\ \text{restore } R_2 \end{matrix} \left[\begin{array}{cc|cc} 1 & 2 & 1 & 2 \\ 0 & -3 & -3 & -9 \\ 0 & 0 & 6 & 12 \end{array}\right]$$

Therefore, $\text{rank}(A|B) = 3$

Given the linear system, since $\text{rank}(A) = 2 < \text{rank}(A|B) = 3$, then the system is <u>inconsistent</u>. Therefore, matrix E and this system of linear equations has <u>NO SOLUTION</u>.

(2.2) From $A = \begin{bmatrix} 1 & 2 \\ 4 & 5 \\ 7 & 8 \end{bmatrix}$ find E if $AE = \begin{bmatrix} -1 & 2 \\ -1 & 5 \\ -1 & 8 \end{bmatrix}$

Consider the system $AX=B$ with coefficient matrix A and augmented matrix $[A|B]$. For this problem, suppose $X=E$ and $B=AE$. Hence, $AE=B$. Similar to (2.1), we need to determine $\text{rank}(A)$ and $\text{rank}(A|B)$ to verify whether the system or inconsistent.

$$\text{rank}(A) = \text{rank}\left(\begin{bmatrix} 1 & 2 \\ 4 & 5 \\ 7 & 8 \end{bmatrix}\right)$$

$$R_2 - 4R_1 \to R_2 \begin{bmatrix} 1 & 2 \\ 0 & -3 \\ 7 & 8 \end{bmatrix} \quad R_3 - 7R_1 \to R_3 \begin{bmatrix} 1 & 2 \\ 0 & -3 \\ 0 & -6 \end{bmatrix}$$

$$\left(-\frac{1}{3}\right)R_2 \to R_2 \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 0 & -6 \end{bmatrix} \quad \begin{matrix} R_3 + 6R_2 \to R_3 \\ \text{and} \\ \text{restore } R_2 \end{matrix} \begin{bmatrix} 1 & 2 \\ 0 & 2 \\ 0 & 0 \end{bmatrix}$$

Therefore, $\text{rank}(A) = 2$

$$\text{rank}(A|B) = \text{rank}\left(\left[\begin{array}{cc|cc} 1 & 2 & -1 & 2 \\ 4 & 5 & -1 & 5 \\ 7 & 8 & -1 & 8 \end{array}\right]\right)$$

$$\begin{matrix} R_2 - 4R_1 \to R_2 \\ R_3 - 7R_1 \to R_3 \end{matrix} \left[\begin{array}{cc|cc} 1 & 2 & -1 & 2 \\ 0 & -3 & 3 & -3 \\ 0 & -6 & 6 & -6 \end{array}\right] \quad \left(-\frac{1}{3}\right)R_2 \to R_2 \left[\begin{array}{cc|cc} 1 & 2 & -1 & 2 \\ 0 & 1 & -1 & 1 \\ 0 & -6 & 6 & -6 \end{array}\right]$$

$$\begin{matrix} (-6)R_2 \to R_2 \\ R_3 - (-6)R_2 \to R_3 \end{matrix} \left[\begin{array}{cc|cc} 1 & 2 & -1 & 2 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\text{Restore } R_2 \left[\begin{array}{cc|cc} 1 & 2 & -1 & 2 \\ 0 & -3 & 3 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Therefore, $\text{rank}(A|B) = 2$

From the given linear system, since $\text{rank}(A) = 2 = \text{rank}(A|B) = 2$, then the system is <u>consistent</u>. However, a consistent system of equation does not mean a unique solution, that is, a consistent system may have a unique or infinite solution.

Since $\text{rank}(A) < n = 3$ where n is the number of unknowns, therefore the consistent system of equation have <u>INFINITE SOLUTIONS</u>.

(3) If $A = \begin{bmatrix} 1 & 3 & 5 & 0 \\ 3 & 1 & 9 & 4 \\ 0 & 4 & 3 & 5 \\ 0 & 2 & -1 & 1 \end{bmatrix}$ find $|A|$

$$|A| = (1)\begin{bmatrix} 1 & 9 & 4 \\ 4 & 3 & 5 \\ 2 & -1 & 1 \end{bmatrix} - (3)\begin{bmatrix} 3 & 9 & 4 \\ 0 & 3 & 5 \\ 0 & -1 & 1 \end{bmatrix} + (5)\begin{bmatrix} 3 & 1 & 4 \\ 0 & 4 & 5 \\ 0 & 2 & 1 \end{bmatrix} - (0)\begin{bmatrix} 3 & 1 & 9 \\ 0 & 4 & 3 \\ 0 & 2 & -1 \end{bmatrix}$$

$$|A| = (1)\begin{bmatrix} 1 & 9 & 4 & 1 & 9 \\ 4 & 3 & 5 & 4 & 3 \\ 2 & -1 & 1 & 2 & -1 \end{bmatrix} - (3)\begin{bmatrix} 3 & 9 & 4 & 3 & 9 \\ 0 & 3 & 5 & 0 & 3 \\ 0 & -1 & 1 & 0 & -1 \end{bmatrix} + (5)\begin{bmatrix} 3 & 1 & 4 & 3 & 1 \\ 0 & 4 & 5 & 0 & 4 \\ 0 & 2 & 1 & 0 & 2 \end{bmatrix} - (0)\cancel{\begin{bmatrix} 3 & 1 & 9 & 3 & 1 \\ 0 & 4 & 3 & 0 & 4 \\ 0 & 2 & -1 & 0 & 2 \end{bmatrix}}$$

$$|A| = (1)\left[(1)(3)(1)+(9)(5)(2)+(4)(4)(-1)-(2)(3)(4)-(-1)(5)(1)-(1)(4)(9)\right] -$$

$$(3)\left[(3)(3)(1)+(9)\cancel{(5)(0)}+(4)\cancel{(0)}(1)-(0)\cancel{(3)}(4)-(-1)(5)(3)-(1)\cancel{(0)}(9)\right] +$$

$$(5)\left[(3)(4)(1)+(1)\cancel{(5)}(0)+(4)\cancel{(0)}(2)-(0)\cancel{(4)}(4)-(2)(5)(3)-(1)\cancel{(0)}(1)\right] - 0$$

$$|A| = (1)(3+90-16-24+5-36)-(3)(9+15)+5(12-30)$$

$$|A| = 22-72-90 = -140$$

$$|A| = -140$$

(3.2) $[5]A = [5]\begin{bmatrix} 1 & 3 & 5 & 0 \\ 3 & 1 & 9 & 4 \\ 0 & 4 & 3 & 5 \\ 0 & 2 & -1 & 1 \end{bmatrix} = \begin{bmatrix} (5)(1) & (5)(3) & (5)(5) & (5)(0) \\ (5)(3) & (5)(1) & (5)(9) & (5)(4) \\ (5)(0) & (5)(4) & (5)(3) & (5)(5) \\ (5)(0) & (5)(2) & (5)(-1) & (5)(1) \end{bmatrix}$

$$[5]A = \begin{bmatrix} 5 & 15 & 25 & 0 \\ 15 & 5 & 45 & 20 \\ 0 & 20 & 15 & 25 \\ 0 & 10 & -5 & 5 \end{bmatrix}$$

(3.3) $AA^{-1} = \begin{bmatrix} 1 & 3 & 5 & 0 \\ 3 & 1 & 9 & 4 \\ 0 & 4 & 3 & 5 \\ 0 & 2 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 & 5 & 0 \\ 3 & 1 & 9 & 4 \\ 0 & 4 & 3 & 5 \\ 0 & 2 & -1 & 1 \end{bmatrix}^{-1}$

$$\begin{bmatrix} 1 & 3 & 5 & 0 \\ 3 & 1 & 9 & 4 \\ 0 & 4 & 3 & 5 \\ 0 & 2 & -1 & 1 \end{bmatrix}^{-1} = \left[\begin{array}{cccc|cccc} 1 & 3 & 5 & 0 & 1 & 0 & 0 & 0 \\ 3 & 1 & 9 & 4 & 0 & 1 & 0 & 0 \\ 0 & 4 & 3 & 5 & 0 & 0 & 1 & 0 \\ 0 & 2 & -1 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \quad R_2-3R_1 \rightarrow R_2 \left[\begin{array}{cccc|cccc} 1 & 3 & 5 & 0 & 1 & 0 & 0 & 0 \\ 0 & -8 & -6 & 4 & -3 & 1 & 0 & 0 \\ 0 & 4 & 3 & 5 & 0 & 0 & 1 & 0 \\ 0 & 2 & -1 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$-\frac{1}{8}R_2 \rightarrow R_2 \left[\begin{array}{cccc|cccc} 1 & 3 & 5 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 3/4 & -1/2 & 3/8 & -1/8 & 0 & 0 \\ 0 & 4 & 3 & 5 & 0 & 0 & 1 & 0 \\ 0 & 2 & -1 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \quad R_3-4R_2 \rightarrow R_3 \left[\begin{array}{cccc|cccc} 1 & 3 & 5 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 3/4 & -1/2 & 3/8 & -1/8 & 0 & 0 \\ 0 & 0 & 0 & 7 & -3/2 & 1/2 & 1 & 0 \\ 0 & 2 & -1 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$R_4-2R_2 \rightarrow R_4 \left[\begin{array}{cccc|cccc} 1 & 3 & 5 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 3/4 & -1/2 & 3/8 & -1/8 & 0 & 0 \\ 0 & 0 & 0 & 7 & -3/2 & 1/2 & 1 & 0 \\ 0 & 0 & -5/2 & 2 & -3/4 & 1/4 & 0 & 1 \end{array}\right] \quad R_4 \leftrightarrow R_3 \left[\begin{array}{cccc|cccc} 1 & 3 & 5 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 3/4 & -1/2 & 3/8 & -1/8 & 0 & 0 \\ 0 & 0 & -5/2 & 2 & -3/4 & 1/4 & 0 & 1 \\ 0 & 0 & 0 & 7 & -3/2 & 1/2 & 1 & 0 \end{array}\right]$$

$$-\frac{2}{5}R_3 \rightarrow R_3 \left[\begin{array}{cccc|cccc} 1 & 3 & 5 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 3/4 & -1/2 & 3/8 & -1/8 & 0 & 0 \\ 0 & 0 & 1 & -4/5 & 3/10 & -1/10 & 0 & -2/5 \\ 0 & 0 & 0 & 7 & -3/2 & 1/2 & 1 & 0 \end{array}\right] \quad \frac{1}{7}R_4 \rightarrow R_4 \left[\begin{array}{cccc|cccc} 1 & 3 & 5 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 3/4 & -1/2 & 3/8 & -1/8 & 0 & 0 \\ 0 & 0 & 1 & -4/5 & 3/10 & -1/10 & 0 & -2/5 \\ 0 & 0 & 0 & 1 & -3/14 & 1/14 & 1/7 & 0 \end{array}\right]$$

Continuation (3.3)

$$R_3-\left(-\frac{4}{5}\right)R_4 \rightarrow R_3 \left[\begin{array}{cccc|cccc} 1 & 3 & 5 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 3/4 & -1/2 & 3/8 & -1/8 & 0 & 0 \\ 0 & 0 & 1 & 0 & 9/70 & -3/70 & 4/35 & -2/5 \\ 0 & 0 & 0 & 1 & -3/14 & 1/14 & 1/7 & 0 \end{array}\right] \quad R_2-\left(-\frac{1}{2}\right)R_4 \rightarrow R_2 \left[\begin{array}{cccc|cccc} 1 & 3 & 5 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 3/4 & 0 & 15/56 & -5/56 & 1/14 & 0 \\ 0 & 0 & 1 & 0 & 9/70 & -3/70 & 4/35 & -2/5 \\ 0 & 0 & 0 & 1 & -3/14 & 1/14 & 1/7 & 0 \end{array}\right]$$

$$R_2-\left(\frac{3}{4}\right)R_3 \rightarrow R_2 \left[\begin{array}{cccc|cccc} 1 & 3 & 5 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 6/35 & -2/35 & -1/70 & 3/10 \\ 0 & 0 & 1 & 0 & 9/70 & -3/70 & 4/35 & -2/5 \\ 0 & 0 & 0 & 1 & -3/14 & 1/14 & 1/7 & 0 \end{array}\right] \quad R_1-5R_3 \rightarrow R_1 \left[\begin{array}{cccc|cccc} 1 & 3 & 0 & 0 & 5/14 & 3/14 & -4/7 & 2 \\ 0 & 1 & 0 & 0 & 6/35 & -2/35 & -1/70 & 3/10 \\ 0 & 0 & 1 & 0 & 9/70 & -3/70 & 4/35 & -2/5 \\ 0 & 0 & 0 & 1 & -3/14 & 1/14 & 1/7 & 0 \end{array}\right]$$

$$R_1-(3)R_2 \rightarrow R_1 \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -11/70 & 27/70 & -37/70 & 11/10 \\ 0 & 1 & 0 & 0 & 6/35 & -2/35 & -1/70 & 3/10 \\ 0 & 0 & 1 & 0 & 9/70 & -3/70 & 4/35 & -2/5 \\ 0 & 0 & 0 & 1 & -3/14 & 1/14 & 1/7 & 0 \end{array}\right]$$

Therefore,
$$A^{-1} = \begin{bmatrix} -11/70 & 27/70 & -37/70 & 11/10 \\ 6/35 & -2/35 & -1/70 & 3/10 \\ 9/70 & -3/70 & 4/35 & -2/5 \\ -3/14 & 1/14 & 1/7 & 0 \end{bmatrix}$$

$$AA^{-1} = \begin{bmatrix} 1 & 3 & 5 & 0 \\ 3 & 1 & 9 & 4 \\ 0 & 4 & 3 & 5 \\ 0 & 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} -11/70 & 27/70 & -37/70 & 11/10 \\ 6/35 & -2/35 & -1/70 & 3/10 \\ 9/70 & -3/70 & 4/35 & -2/5 \\ -3/14 & 1/14 & 1/7 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} (1)\left(-\frac{11}{70}\right)+(3)\left(\frac{6}{35}\right)+(5)\left(\frac{9}{70}\right)+(0)\left(-\frac{3}{14}\right) & (1)\left(\frac{27}{70}\right)+(3)\left(\frac{-2}{35}\right)+(5)\left(\frac{-3}{70}\right)+(0)\left(\frac{1}{14}\right) & \left(\frac{-37}{70}\right)+\left(\frac{-3}{70}\right)+\left(\frac{20}{35}\right)+0 & \left(\frac{11}{10}\right)+\left(\frac{9}{10}\right)+\left(\frac{-10}{5}\right)+0 \\ (3)\left(\frac{-11}{70}\right)+(1)\left(\frac{6}{35}\right)+(9)\left(\frac{9}{70}\right)+(4)\left(-\frac{3}{14}\right) & (3)\left(\frac{27}{70}\right)+(1)\left(\frac{-2}{35}\right)+(9)\left(\frac{-3}{70}\right)+(4)\left(\frac{1}{14}\right) & (3)\left(\frac{-37}{70}\right)+(1)\left(\frac{-1}{70}\right)+(9)\left(\frac{4}{35}\right)+\left(\frac{4}{7}\right) & (3)\left(\frac{11}{10}\right)+(1)\left(\frac{3}{10}\right)+(9)\left(\frac{-2}{5}\right)+(4)(0) \\ (0)\left(\frac{-11}{70}\right)+(4)\left(\frac{6}{35}\right)+(3)\left(\frac{9}{70}\right)+(5)\left(-\frac{3}{14}\right) & (0)\left(\frac{27}{70}\right)+(4)\left(\frac{-2}{35}\right)+(3)\left(\frac{-3}{70}\right)+(5)\left(\frac{1}{14}\right) & (0)\left(\frac{-37}{70}\right)+(4)\left(\frac{-1}{70}\right)+(3)\left(\frac{4}{35}\right)+\left(\frac{5}{7}\right) & (0)\left(\frac{11}{10}\right)+(4)\left(\frac{3}{10}\right)+(3)\left(\frac{-2}{5}\right)+(5)(0) \\ (0)\left(\frac{-11}{70}\right)+(2)\left(\frac{6}{35}\right)+(-1)\left(\frac{9}{70}\right)+(1)\left(-\frac{3}{14}\right) & (0)\left(\frac{27}{70}\right)+(2)\left(\frac{-2}{35}\right)+(-1)\left(\frac{-3}{70}\right)+(1)\left(\frac{1}{14}\right) & (0)\left(\frac{-37}{70}\right)+(2)\left(-\frac{1}{70}\right)+(-1)\left(\frac{4}{35}\right)+\left(\frac{11}{10}\right) & (0)\left(\frac{11}{10}\right)+\left(\frac{6}{10}\right)+(-1)\left(\frac{-2}{5}\right)+(1)(0) \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{11}{70}+\frac{18}{35}+\frac{45}{70}+0 & \frac{27}{70}-\frac{6}{35}-\frac{15}{70}+0 & -\frac{37}{70}-\frac{3}{70}+\frac{20}{35}+0 & \frac{11}{10}+\frac{9}{10}-\frac{10}{5}+0 \\ -\frac{33}{70}+\frac{6}{35}+\frac{81}{70}-\frac{12}{14} & \frac{81}{70}-\frac{2}{35}-\frac{27}{70}+\frac{4}{14} & -\frac{111}{70}-\frac{1}{70}+\frac{36}{35}+\frac{4}{7} & \frac{33}{10}+\frac{3}{10}-\frac{18}{5}+0 \\ 0+\frac{24}{35}+\frac{27}{70}-\frac{15}{14} & 0-\frac{8}{35}-\frac{9}{70}+\frac{5}{14} & 0-\frac{4}{70}+\frac{12}{35}+\frac{5}{7} & 0+\frac{12}{10}-\frac{6}{5}+0 \\ 0+\frac{12}{35}-\frac{9}{70}-\frac{3}{14} & 0-\frac{4}{35}+\frac{3}{70}+\frac{1}{14} & 0-\frac{2}{70}-\frac{4}{35}+\frac{1}{7} & 0+\frac{6}{10}+\frac{2}{5}+0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Therefore, $AA^{-1} = \begin{bmatrix} 1 & 0 & 6 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = I_4$ (identity matrix)

(4) Please find x_1, x_2, x_3, x_4 by using Identity matrix

(4.1)

$$\begin{aligned} x_1 + x_2 + 2x_3 + x_4 &= 5 \\ 2x_1 + 3x_2 - x_3 - 2x_4 &= 2 \\ 4x_1 + 5x_2 + 3x_3 + x_4 &= 12 \end{aligned}$$

The given system of linear equations is the same as the matrix equation

$$\begin{bmatrix} 1 & 1 & 2 & 1 \\ 2 & 3 & -1 & -2 \\ 4 & 5 & 3 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \\ 12 \end{bmatrix}$$

The number of equations is less than the number of unknowns. Hence, let $u = x_1$ and consider u as a known quantity. Therefore, the matrix above is equivalent to the following matrix equation:

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & -1 & -2 \\ 5 & 3 & \cancel{2}1 \end{bmatrix}\begin{bmatrix} x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 5-u \\ 2-2u \\ 12-4u \end{bmatrix} \quad \text{where } A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & -1 & -2 \\ 5 & 3 & \cancel{2}1 \end{bmatrix};\ x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix};\ b = \begin{bmatrix} 5-u \\ 2-2u \\ 12-4u \end{bmatrix}$$

$$Ax = b$$

$$x = A^{-1}b = \begin{bmatrix} 1 & 1 & 1 \\ 3 & -1 & -1 \\ 5 & 3 & \cancel{2}1 \end{bmatrix}^{-1}\begin{bmatrix} 5-u \\ 2-2u \\ 12-4u \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & 2 & 1 \\ 3 & -1 & -2 \\ 5 & 3 & \cancel{2}1 \end{bmatrix}^{-1} = \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 3 & -1 & -2 & 0 & 1 & 0 \\ 5 & 3 & \cancel{2}1 & 0 & 0 & 1 \end{array}\right] \quad R_2 - (3)R_1 \rightarrow R_2 \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & -7 & -5 & -3 & 1 & 0 \\ 5 & 3 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$R_3 - 5R_1 \rightarrow R_3 \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & -7 & -5 & -3 & 1 & 0 \\ 0 & -7 & -4 & -5 & 0 & 1 \end{array}\right] \quad \left(-\frac{1}{7}\right)R_2 \rightarrow R_2 \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 5/7 & 3/7 & -1/7 & 0 \\ 0 & -7 & -4 & -5 & 0 & 1 \end{array}\right]$$

$$R_3 - (-7)R_2 \rightarrow R_3 \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 5/7 & 3/7 & -1/7 & 0 \\ 0 & 0 & 1 & -2 & -1 & 1 \end{array}\right] \quad R_2 - \left(\frac{5}{7}\right)R_3 \rightarrow R_2 \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & \frac{13}{7} & \frac{4}{7} & -\frac{5}{7} \\ 0 & 0 & 1 & -2 & -1 & 1 \end{array}\right]$$

$$R_1 - (1)R_3 \rightarrow R_1 \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 3 & 1 & -1 \\ 0 & 1 & 0 & \frac{13}{7} & \frac{4}{7} & -\frac{5}{7} \\ 0 & 0 & 1 & -2 & -1 & 1 \end{array}\right] \quad R_1 - (2)R_2 \rightarrow R_1 \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -5/7 & -1/7 & 3/7 \\ 0 & 1 & 0 & 13/7 & 4/7 & -5/7 \\ 0 & 0 & 1 & -2 & -1 & 1 \end{array}\right]$$

Therefore, $A^{-1} = \begin{bmatrix} -5/7 & -1/7 & 3/7 \\ 13/7 & 4/7 & -5/7 \\ -2 & -1 & 1 \end{bmatrix}$

$$x = A^{-1}b = \begin{bmatrix} -5/7 & -1/7 & 3/7 \\ 13/7 & 4/7 & -5/7 \\ -2 & -1 & 1 \end{bmatrix}\begin{bmatrix} 5-u \\ 2-2u \\ 12-4u \end{bmatrix} = \begin{bmatrix} \left(-\frac{5}{7}\right)(5-u) + \left(-\frac{1}{7}\right)(2-2u) + \left(\frac{3}{7}\right)(12-4u) \\ \left(\frac{13}{7}\right)(5-u) + \left(\frac{4}{7}\right)(2-2u) + \left(-\frac{5}{7}\right)(2-2u) \\ (-2)(5-u) + (-1)(2-2u) + 12-4u \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{25}{7} + \frac{5}{7}u - \frac{2}{7} + \frac{2}{7}u + \frac{36}{7} - \frac{12}{7}u \\ \frac{65}{7} - \frac{13}{7}u + \frac{8}{7} - \frac{8}{7}u - \frac{10}{7} - \frac{10}{7}u \\ -10 + 2u - 2 + 2u + 12 - 4u \end{bmatrix} = \begin{bmatrix} \frac{-5u+9}{7} \\ \frac{-u+13}{7} \\ 0 \end{bmatrix}$$

Therefore, $x_1 = u$; $x_2 = \frac{-5u+9}{7}$; $x_3 = \frac{-u+13}{7}$; $x_4 = 0$

(4.2)
$$\begin{aligned} x_1 - x_2 + x_3 - x_4 &= -4 \\ 4x_1 - x_2 + 3x_3 + x_4 &= -8 \\ 2x_1 + x_2 + x_3 - x_4 &= 0 \\ 3x_1 + 2x_2 + x_3 - 3x_4 &= 1 \end{aligned}$$

The given system of linear equations is the same as the matrix equation:

$$\begin{bmatrix} 1 & -1 & 1 & -1 \\ 4 & -1 & 3 & 1 \\ 2 & 1 & 1 & -1 \\ 3 & 2 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -4 \\ -8 \\ 0 \\ 1 \end{bmatrix}$$

Suppose $A = \begin{bmatrix} 1 & -1 & 1 & -1 \\ 4 & -1 & 3 & 1 \\ 2 & 1 & 1 & -1 \\ 3 & 2 & 1 & -3 \end{bmatrix}$; $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$; and $b = \begin{bmatrix} -4 \\ -8 \\ 0 \\ 1 \end{bmatrix}$ where $Ax = b$. Hence, $x = A^{-1}b$

$$A^{-1} = \left[\begin{array}{cccc|cccc} 1 & -1 & 1 & -1 & 1 & 0 & 0 & 0 \\ 4 & -1 & 3 & 1 & 0 & 1 & 0 & 0 \\ 2 & 1 & 1 & -1 & 0 & 0 & 1 & 0 \\ 3 & 2 & 1 & -3 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$R_2 - 4R_1 \rightarrow R_2 \left[\begin{array}{cccc|cccc} 1 & -1 & 1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 3 & -1 & 5 & -4 & 1 & 0 & 0 \\ 2 & 1 & 1 & -1 & 0 & 0 & 1 & 0 \\ 3 & 2 & 1 & -3 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$R_3 - 2R_1 \rightarrow R_3 \left[\begin{array}{cccc|cccc} 1 & -1 & 1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 3 & -1 & 5 & -4 & 1 & 0 & 0 \\ 0 & 3 & -1 & 1 & -2 & 0 & 1 & 0 \\ 3 & 2 & 1 & -3 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$R_4 - 3R_1 \rightarrow R_4 \left[\begin{array}{cccc|cccc} 1 & -1 & 1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 3 & -1 & 5 & -4 & 1 & 0 & 0 \\ 0 & 3 & -1 & 1 & -2 & 0 & 1 & 0 \\ 0 & 5 & -2 & 0 & -3 & 0 & 0 & 1 \end{array}\right]$$

$$\left(\frac{1}{3}\right) R_2 \rightarrow R_2 \left[\begin{array}{cccc|cccc} 1 & -1 & 1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1/3 & 5/3 & -4/3 & 1/3 & 0 & 0 \\ 0 & 3 & -1 & 1 & -2 & 0 & 1 & 0 \\ 0 & 5 & -2 & 0 & -3 & 0 & 0 & 1 \end{array}\right]$$

$$R_3 - 3R_2 \rightarrow R_3 \left[\begin{array}{cccc|cccc} 1 & -1 & 1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1/3 & 5/3 & -4/3 & 1/3 & 0 & 0 \\ 0 & 0 & 0 & -4 & 2 & -1 & 1 & 0 \\ 0 & 5 & -2 & 0 & -3 & 0 & 0 & 1 \end{array}\right]$$

$$R_4 - 5R_2 \rightarrow R_4 \left[\begin{array}{cccc|cccc} 1 & -1 & 1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1/3 & 5/3 & -4/3 & 1/3 & 0 & 0 \\ 0 & 0 & 0 & -4 & 2 & -1 & 1 & 0 \\ 0 & 0 & -1/3 & -25/3 & \frac{11}{3} & \frac{-5}{3} & 0 & 1 \end{array}\right]$$

$$R_4 \leftrightarrow R_3 \left[\begin{array}{cccc|cccc} 1 & -1 & 1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1/3 & 5/3 & -4/3 & 1/3 & 0 & 0 \\ 0 & 0 & -1/3 & -25/3 & 11/3 & -5/3 & 0 & 1 \\ 0 & 0 & 0 & -4 & +2 & -1 & 1 & 0 \end{array}\right]$$

$$-3R_3 \rightarrow R_3 \left[\begin{array}{cccc|cccc} 1 & -1 & 1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1/3 & 5/3 & -4/3 & 1/3 & 0 & 0 \\ 0 & 0 & 1 & 25 & -11 & 5 & 0 & -3 \\ 0 & 0 & 0 & -4 & 2 & -1 & 1 & 0 \end{array}\right]$$

$$\left(-\frac{1}{4}\right) R_4 \rightarrow R_4 \left[\begin{array}{cccc|cccc} 1 & -1 & 1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1/3 & 5/3 & -4/3 & 1/3 & 0 & 0 \\ 0 & 0 & 1 & 25 & -11 & 5 & 0 & -3 \\ 0 & 0 & 0 & 1 & -1/2 & 1/4 & -1/4 & 0 \end{array}\right]$$

$$R_3 - 25R_4 \rightarrow R_3 \left[\begin{array}{cccc|cccc} 1 & -1 & 1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1/3 & 5/3 & -4/3 & 1/3 & 0 & 0 \\ 0 & 0 & 1 & 0 & 3/2 & -5/4 & 25/4 & -3 \\ 0 & 0 & 0 & 1 & -1/2 & 1/4 & -1/4 & 0 \end{array}\right]$$

$$R_2 - \left(\frac{5}{3}\right) R_4 \rightarrow R_2 \left[\begin{array}{cccc|cccc} 1 & -1 & 1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1/3 & 0 & -1/2 & -1/12 & 5/12 & 0 \\ 0 & 0 & 1 & 0 & 3/2 & -5/4 & 25/4 & -3 \\ 0 & 0 & 0 & 1 & -1/2 & 1/4 & -1/4 & 0 \end{array}\right]$$

$$R_1 - (-1)R_4 \rightarrow R_1 \left[\begin{array}{cccc|cccc} 1 & -1 & 1 & 0 & 1/2 & 1/4 & -1/4 & 0 \\ 0 & 1 & -1/3 & 0 & -1/2 & -1/12 & 5/12 & 0 \\ 0 & 0 & 1 & 0 & 3/2 & -5/4 & 25/4 & -3 \\ 0 & 0 & 0 & 1 & -1/2 & 1/4 & -1/4 & 0 \end{array}\right]$$

$$R_2 - \left(-\frac{1}{3}\right) R_3 \rightarrow R_2 \left[\begin{array}{cccc|cccc} 1 & -1 & 1 & -1 & 1/2 & 1/4 & -1/4 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1/2 & 5/2 & -1 \\ 0 & 0 & 1 & 0 & 3/2 & -5/4 & 25/4 & -3 \\ 0 & 0 & 0 & 1 & -1/2 & 1/4 & -1/4 & 0 \end{array}\right]$$

$$R_1 - (1)R_3 \rightarrow R_1 \left[\begin{array}{cccc|cccc} 1 & -1 & 0 & 0 & -1 & 3/2 & -13/2 & 3 \\ 0 & 1 & -1/3 & 0 & 0 & -1/2 & 5/2 & -1 \\ 0 & 0 & 1 & 0 & 3/2 & -5/4 & 25/4 & -3 \\ 0 & 0 & 0 & 1 & -1/2 & 1/4 & -1/4 & 0 \end{array}\right]$$

$$R_1 - (-1)R_2 \rightarrow R_1 \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -1 & 1 & -4 & 2 \\ 0 & 1 & 0 & 0 & 0 & -1/2 & 5/2 & -1 \\ 0 & 0 & 1 & 0 & 3/2 & -5/4 & 25/4 & -3 \\ 0 & 0 & 0 & 1 & -1/2 & 1/4 & -1/4 & 0 \end{array}\right]$$

Therefore, $A^{-1} = \begin{bmatrix} -1 & 1 & -4 & 2 \\ 0 & -1/2 & 5/2 & -1 \\ 3/2 & -5/4 & 25/4 & -3 \\ -1/2 & 1/4 & -1/4 & 0 \end{bmatrix}$

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Continuation (4.2)

$$X = \begin{bmatrix} -1 & 1 & -4 & 2 \\ 0 & -1/2 & 5/2 & -1 \\ 3/2 & -5/4 & 25/4 & -3 \\ -1/2 & 1/4 & -1/4 & 0 \end{bmatrix} \begin{bmatrix} -4 \\ -8 \\ 0 \\ 1 \end{bmatrix}$$

$$X = \begin{bmatrix} (-1)(-4) + (1)(-8) + (-4)(0) + (2)(1) \\ (0)(-4) + \left(-\frac{1}{2}\right)(-8) + \left(\frac{5}{2}\right)(0) + (-1)(1) \\ \left(\frac{3}{2}\right)(-4) + \left(-\frac{5}{4}\right)(-8) + \left(\frac{25}{4}\right)(0) + (-3)(1) \\ \left(-\frac{1}{2}\right)(-4) + \left(\frac{1}{4}\right)(-8) + \left(-\frac{1}{4}\right)(0) + (0)(1) \end{bmatrix} = \begin{bmatrix} 4 - 8 + 0 + 2 \\ 0 + 4 + 0 - 1 \\ -6 + 10 + 0 - 3 \\ 2 - 2 + 0 + 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ 1 \\ 0 \end{bmatrix}$$

Therefore, $x_1 = -2$, $x_2 = 3$, $x_3 = 1$, and $x_4 = 0$.