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Faculty of Medicine Ramathibodi Hospital

Department of Clinical Epidemiology and Biostatistics

Support Vector Machine (SVM)

RADI608: Data Mining and Machine Learning

RADI602: Data Mining and Knowledge Discovery

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Section of Data Science for Healthcare

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Introduction of Support Vector Machine

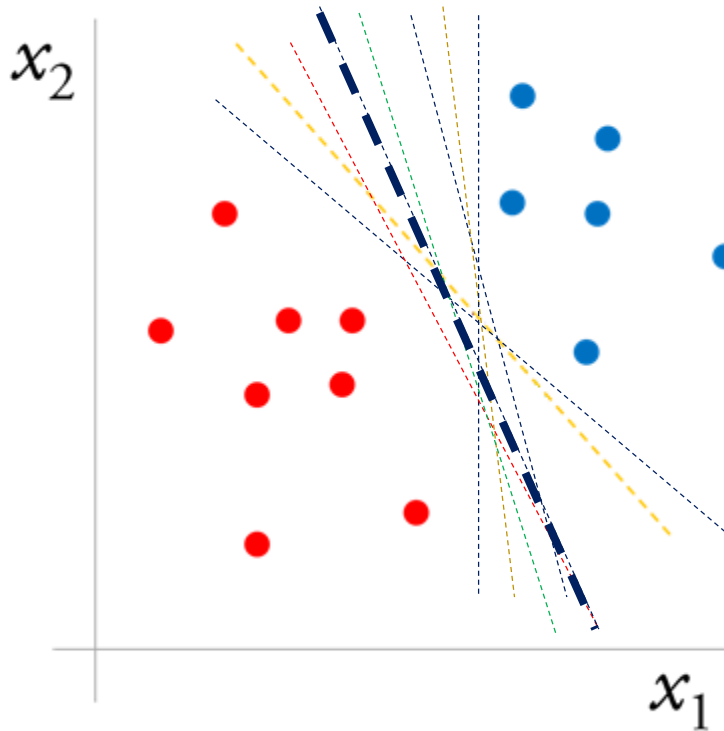
Support Vector Machine (SVM) was invented by Boser, Guyon, and Vapnik in 1992. SVM uses a machine learning methodology to enlarge the accuracy of classification and regression predictors with avoiding a model overfitting.

The SVM trained the input data based on optimization theory which constructed from statistical learning concepts and created a linear function in a high dimensional feature space.

Moreover, the objective of SVM is to search the best separating line or hyperplane that leaves the maximum margin from both classes (binary classification).



How does a support vector machine work?



find the best separating line that leaves the
maximum margin from both classes or keeps ● and
● as far away from each other as possible

http://efavdb.com/wp-content/uploads/2015/05/binaryclass_2d.png

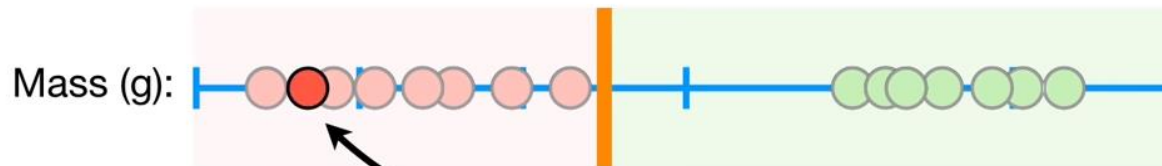


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How does a support vector machine work?



...we can classify it as **not obese**.

X = mass

Y = Obese, Not Obese

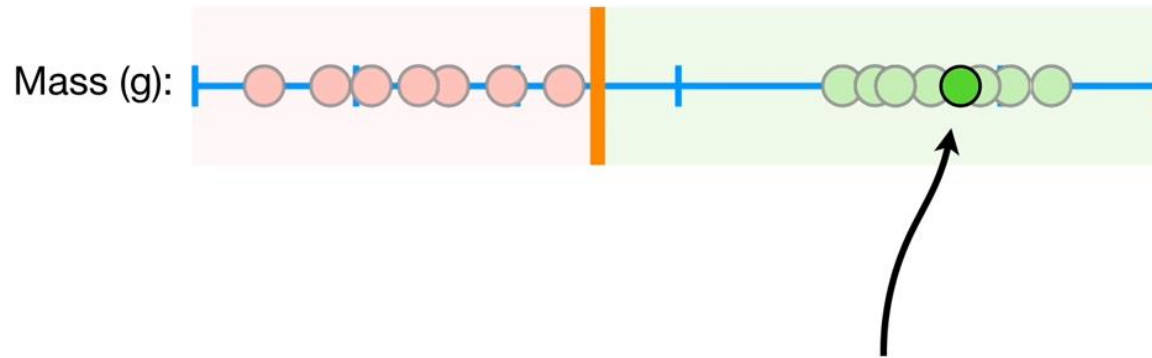


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How does a support vector machine work?



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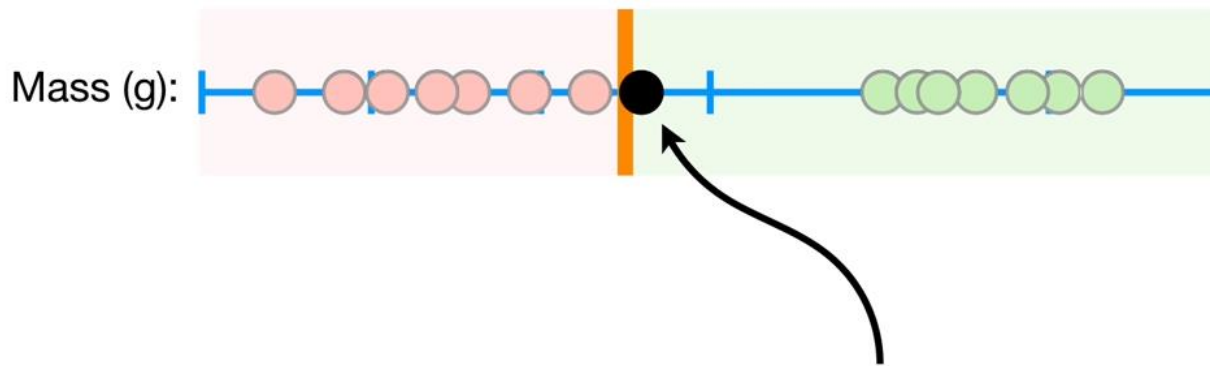


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How does a support vector machine work?



Because this observation has more mass than the threshold, we classify it as **obese**.

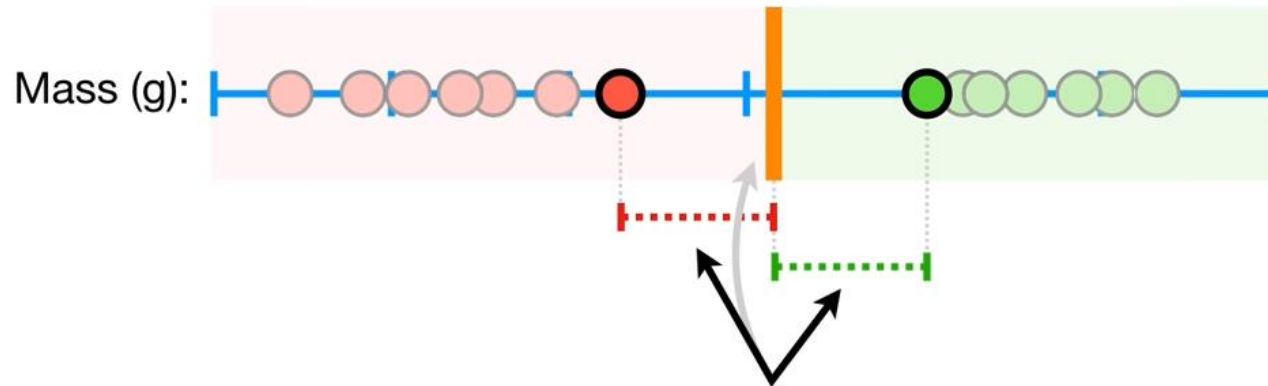


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How does a support vector machine work?



When the threshold is halfway between the two observations, the **margin** is as large as it can be.

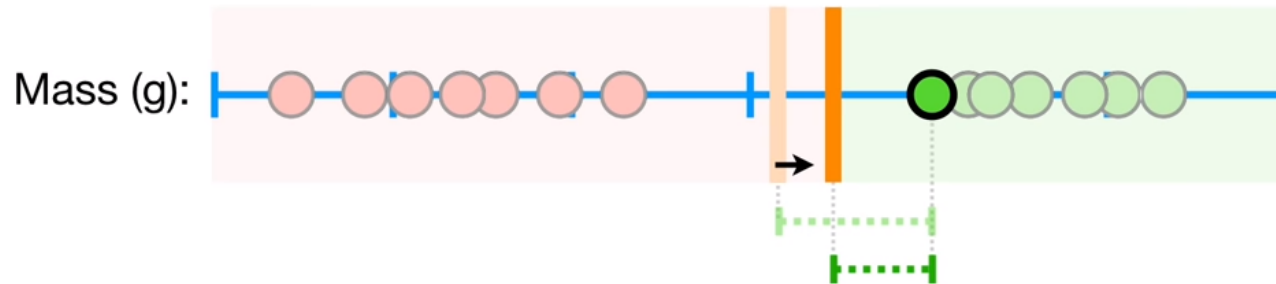


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How does a support vector machine work?



If we shift the threshold to the right

...then the distance between the
obese observation and the
threshold would get smaller...

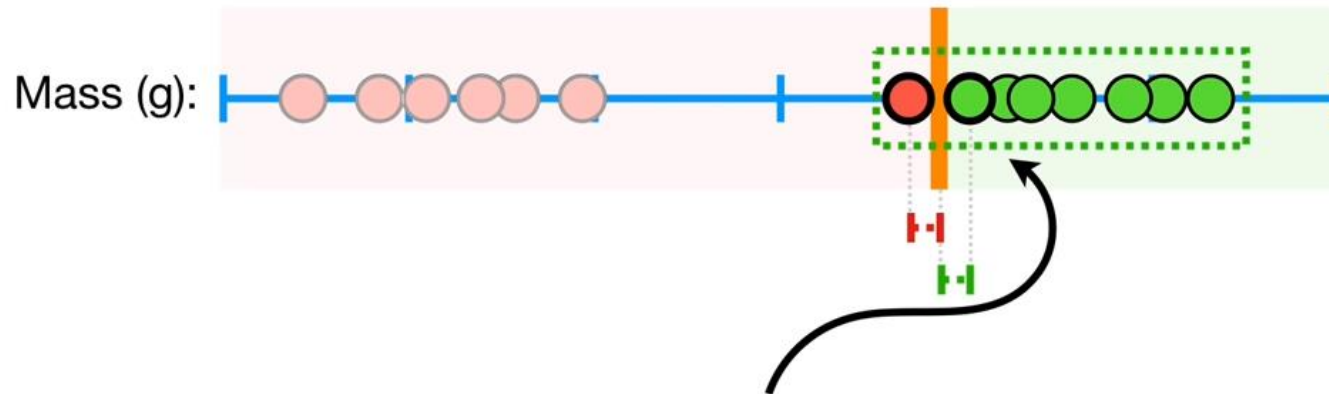


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How does a support vector machine work?



In this case, the **Maximum Margin Classifier** would be super close to the **obese** observations...

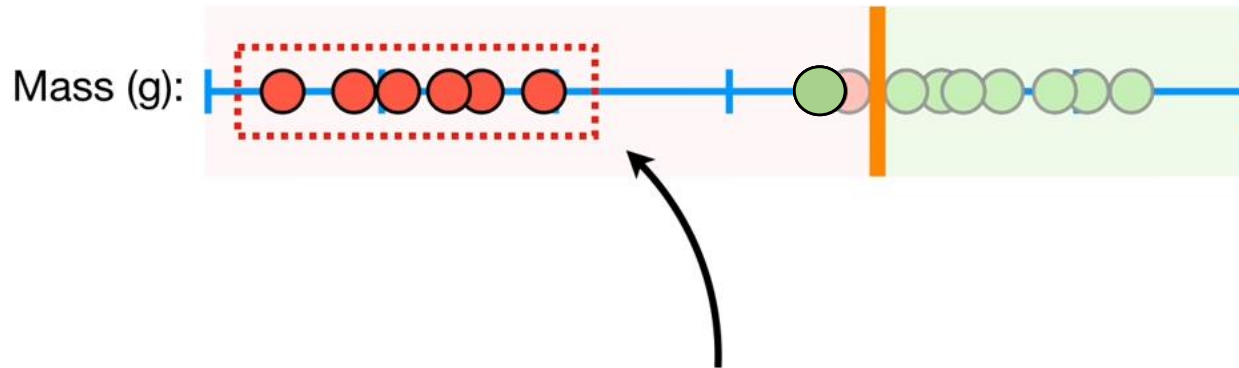


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How does a support vector machine work?



...we would classify it as **not obese**, even though most of the **not obese** observations are much further away than the **obese** observations.

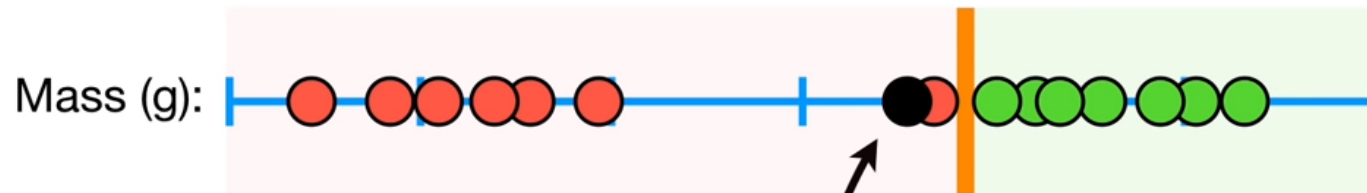


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How does a support vector machine work?



...and it performed poorly when
we got new data (high variance).

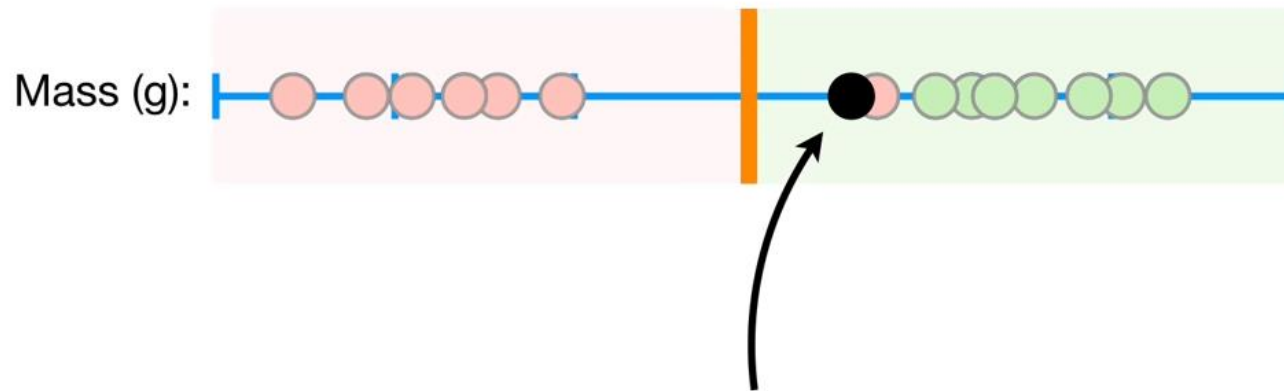


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How does a support vector machine work?



...it performed better when we got
new data (low variance).

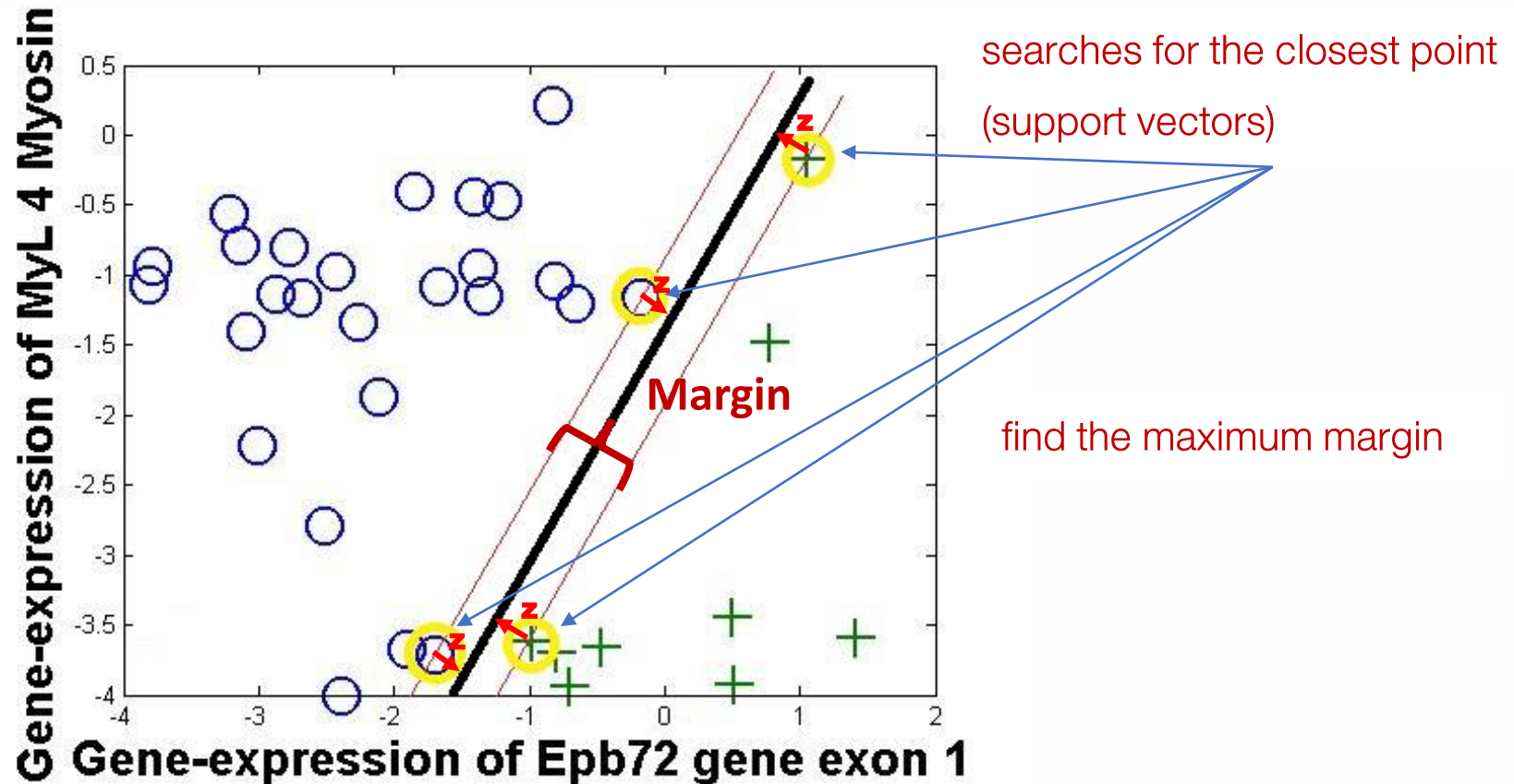


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How to create a discriminant line





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Maximum margin: Formalization

Classifiers: $f(x_i) = \text{sign}(w^\top x_i + b)$

Functional margin of x_i : $y_i(w^\top x_i + b)$

where w is a decision hyperplane normal vector, x_i is a data point i
and y_i is a class of data point i (+1 or -1)



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How to find the width of margin (1)

Since $w^T x + b = 0$ and $c(w^T x + b) = 0$ define the same plane, we could choose the normalization of w

Choose normalization such that $w^T x + b = +1$ and $w^T x + b = -1$ for the positive and negative support vectors respectively



How to find the width of margin (2)

Margin = Unit Vector . Difference Vector

$$= \frac{\mathbf{w}}{\|\mathbf{w}\|} \cdot (\mathbf{x}_+ - \mathbf{x}_-)$$

$$= \frac{\mathbf{w}^\top \mathbf{x}_+ - \mathbf{w}^\top \mathbf{x}_-}{\|\mathbf{w}\|}$$

$$\mathbf{w}^\top \mathbf{x}_+ + b = +1 \quad \mathbf{w}^\top \mathbf{x}_- + b = -1$$



$$\mathbf{w}^\top \mathbf{x}_+ = 1 - b$$



$$\mathbf{w}^\top \mathbf{x}_- = -1 - b$$

$$= \frac{1 - b + 1 + b}{\|\mathbf{w}\|} = \frac{2}{\|\mathbf{w}\|}$$



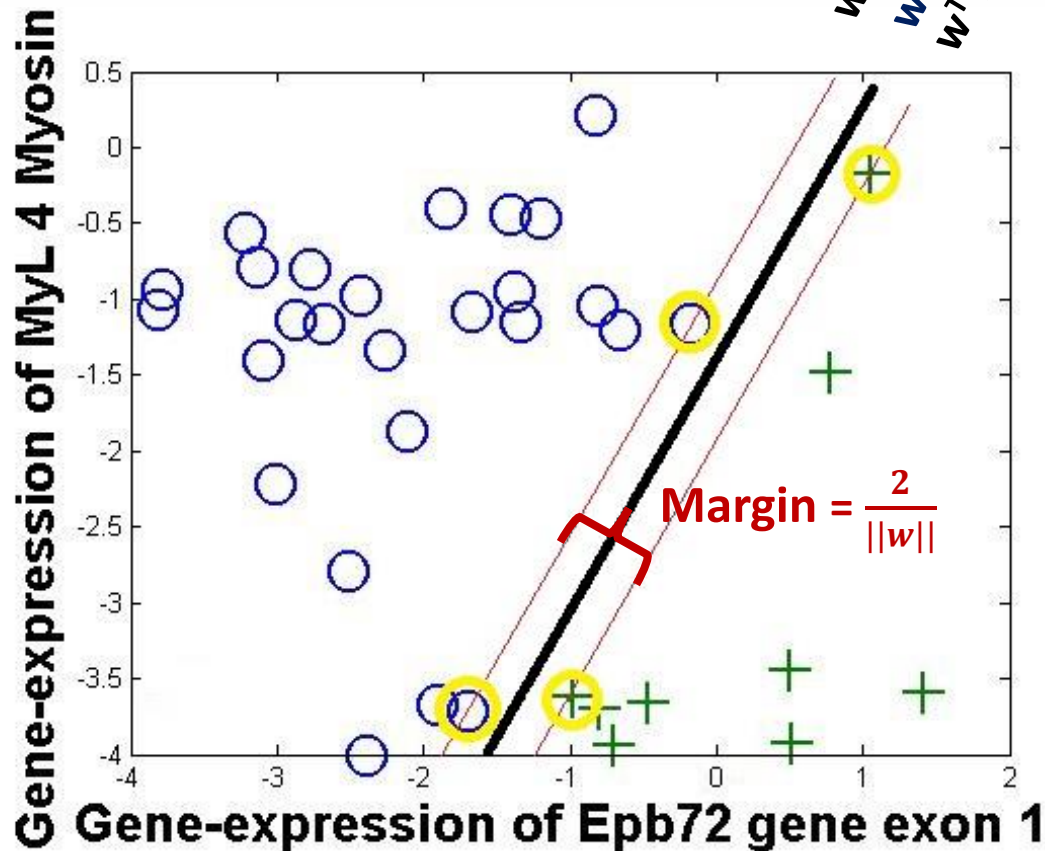
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How to find the width of margin (3)

$$\begin{aligned}w^T x + b &= -1 \\w^T x + b &= 0 \\w^T x + b &= +1\end{aligned}$$





Find the maximum margin **by minimizing w**

Learning the SVM can be formulated as an optimization:

$$\max_w \frac{2}{||w||} \text{ subject to } (w^\top x_i + b) \geq 1 \text{ if } y_i = +1 \text{ and } (w^\top x_i + b) \leq -1 \text{ if } y_i = -1 \text{ for } i = 1, \dots, N$$

Or equivalently

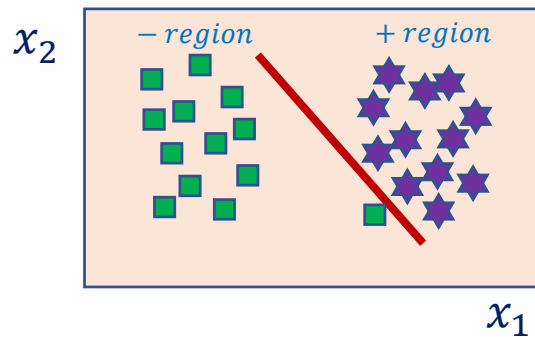
$$\min_w ||w||^2 \text{ subject to } y_i(w^\top x_i + b) \geq 1 \text{ for } i = 1, \dots, N$$

This is a quadratic optimization problem subject to linear constraints and there is a unique minimum

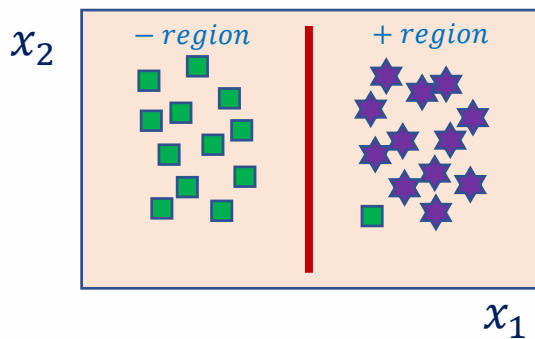




The trade off between the maximum margin and the number of mistakes



all points can be separated by a discriminant line with **narrow margin** or **hard margin**



one points can't be separated by a discriminant line but possibly the **large margin** or **soft margin**

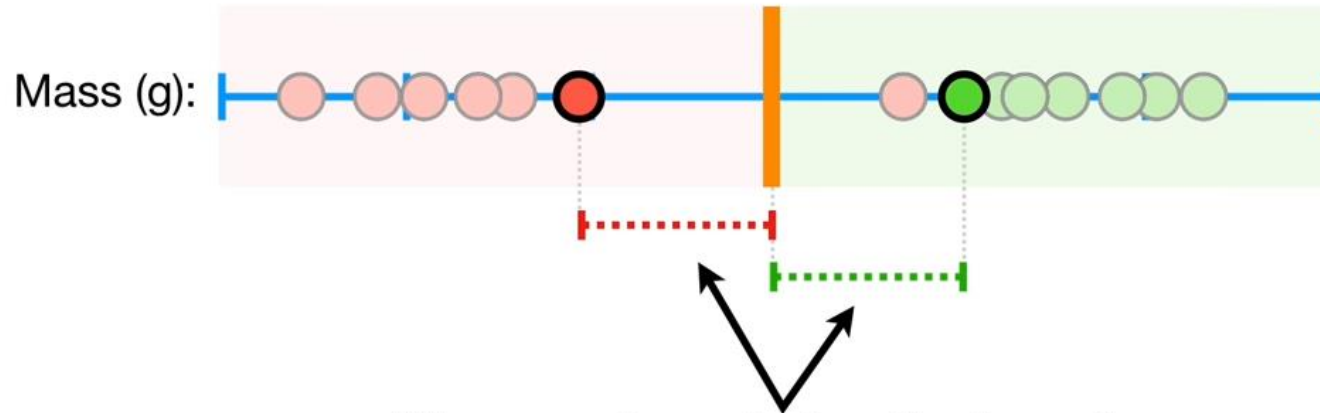


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The trade off between the maximum margin and the number of mistakes



When we allow misclassifications, the distance between the observations and the threshold is called a **Soft Margin**.

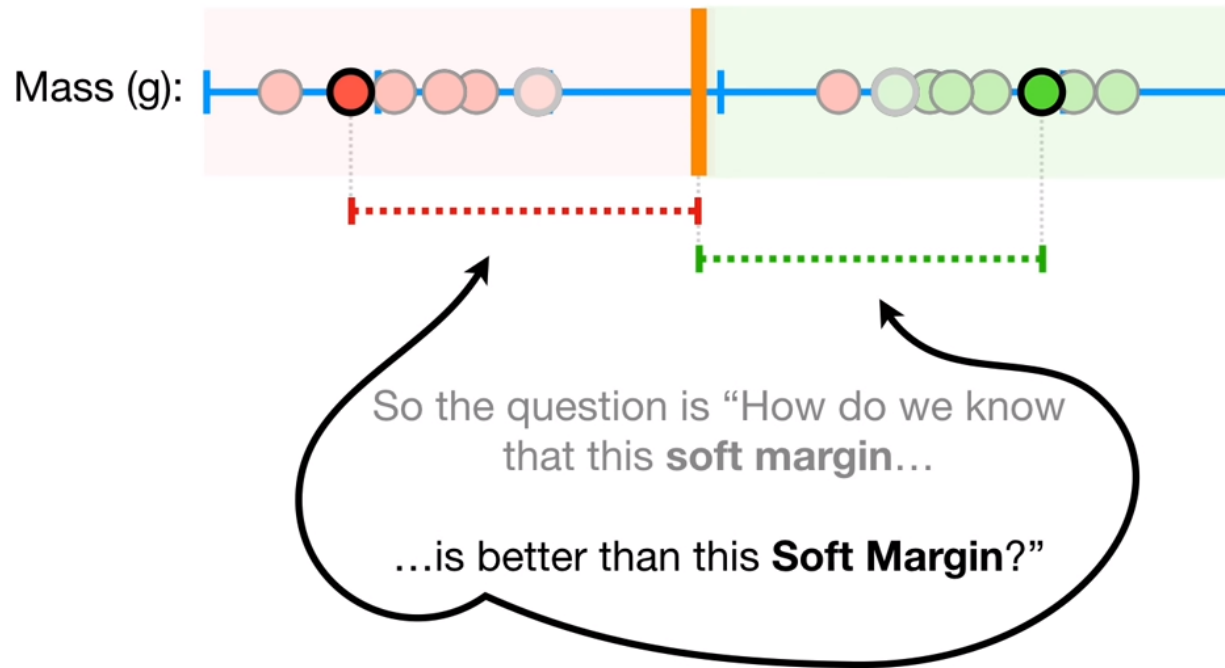


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The trade off between the maximum margin and the number of mistakes



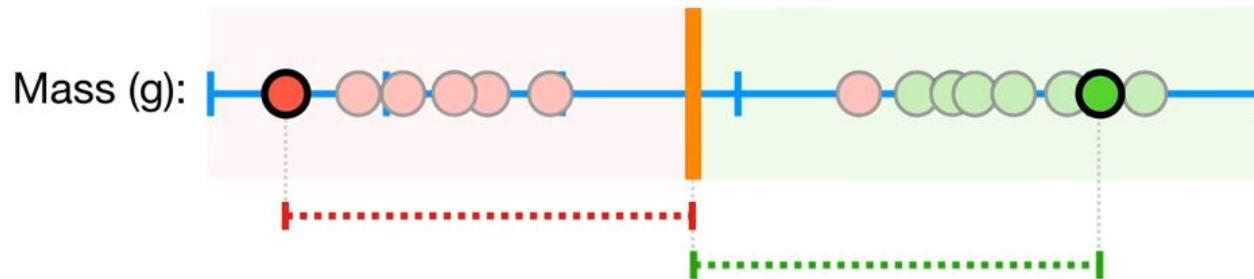


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The trade off between the maximum margin and the number of mistakes



The answer is simple: We use **Cross Validation** to determine how many misclassifications and observations to allow inside of the **Soft Margin** to get the best classification.



What is a large and a narrow margin (1)

Every constraint can be satisfied if is sufficiently large

C is a regularization parameter:

- Small **C** allows constraints to be easily ignored a large margin
- Large **C** makes constraints hard to ignore a narrow margin
- **C** = ∞ enforces all constraints to a hard margin



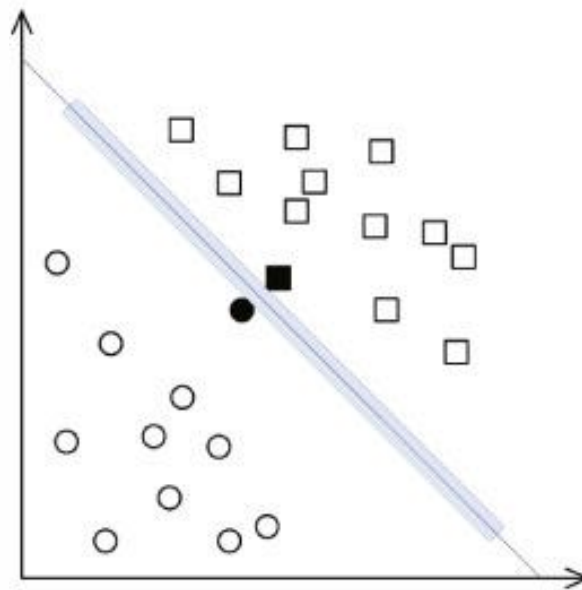
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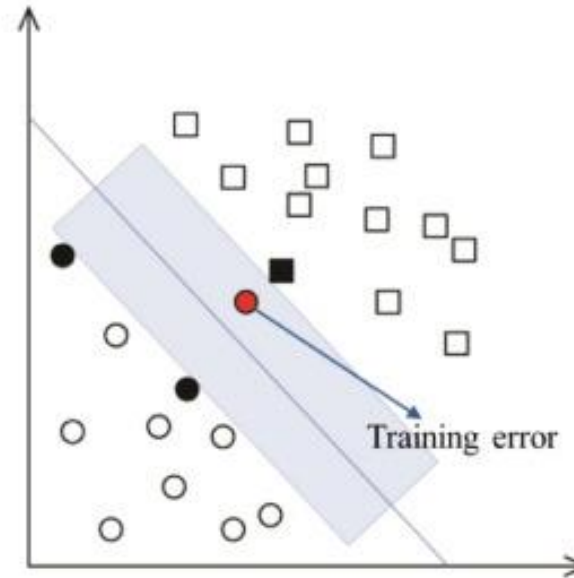
What is a large and a narrow margin (2)

$C = \text{infinity}$, Hard Solution



(a) LSVM with hard margin

$C = 10$, Soft Solution



(b) LSVM with soft margin

<https://ars.els-cdn.com/content/image/1-s2.0-S0926580516301297-gr4.jpg>



The SVM Optimization problem

$$\min_{w,b} \frac{\|w\|}{2} + C \sum_{i=1}^N \xi_i$$

Subject to: $y_i(w^\top x_i + b) \geq 1 - \xi_i$, $\xi_i \geq 0$, $i = 1, 2, \dots, N$

Could add even more flexibility by introducing a function ϕ that maps the original feature space to a higher dimensional feature space

Subject to: $y_i(w^\top \phi(x_i) + b) \geq 1 - \xi_i$, $\xi_i \geq 0$, $i = 1, 2, \dots, N$

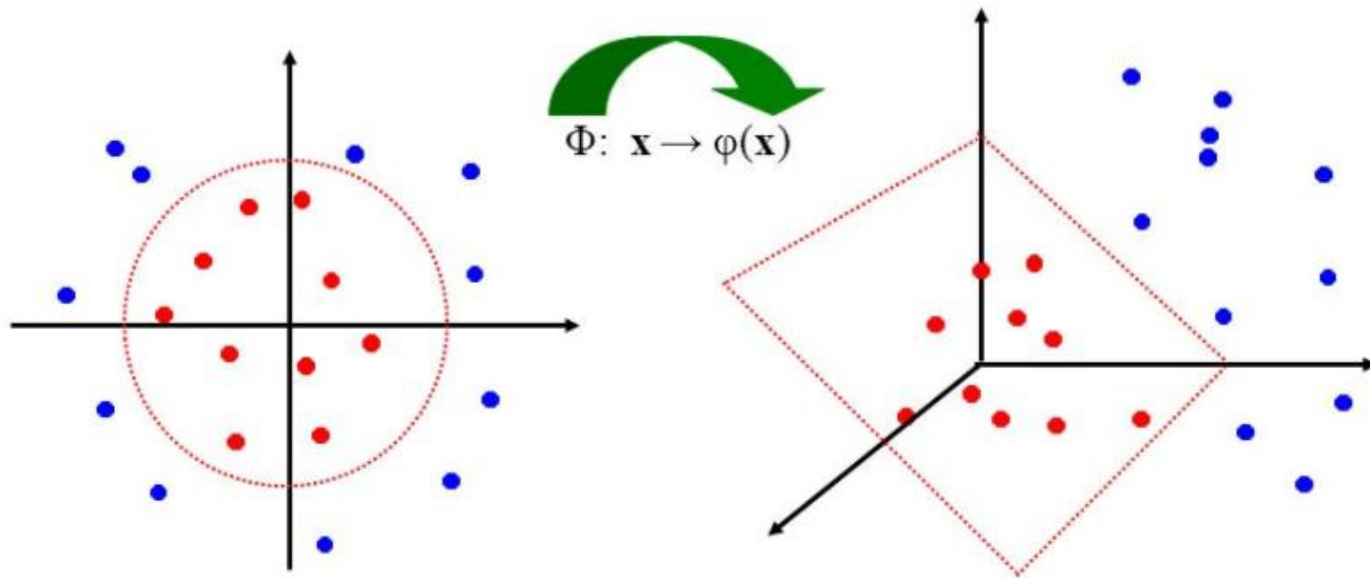


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Maps the original feature space to a higher dimensional feature space



The original space becomes a linear problem in high-dimensional space

Picture credit: Andrew W. Moore, http://www.cs.cmu.edu/~aarti/Class/10701_Spring14/slides/kernel_methods.pdf



Maps the original feature space to a higher dimensional feature space

Since **Maximal Margin Classifiers** and **Support Vector Classifiers** can't handle this data, it's high time we talked about...

Dosage (mg):



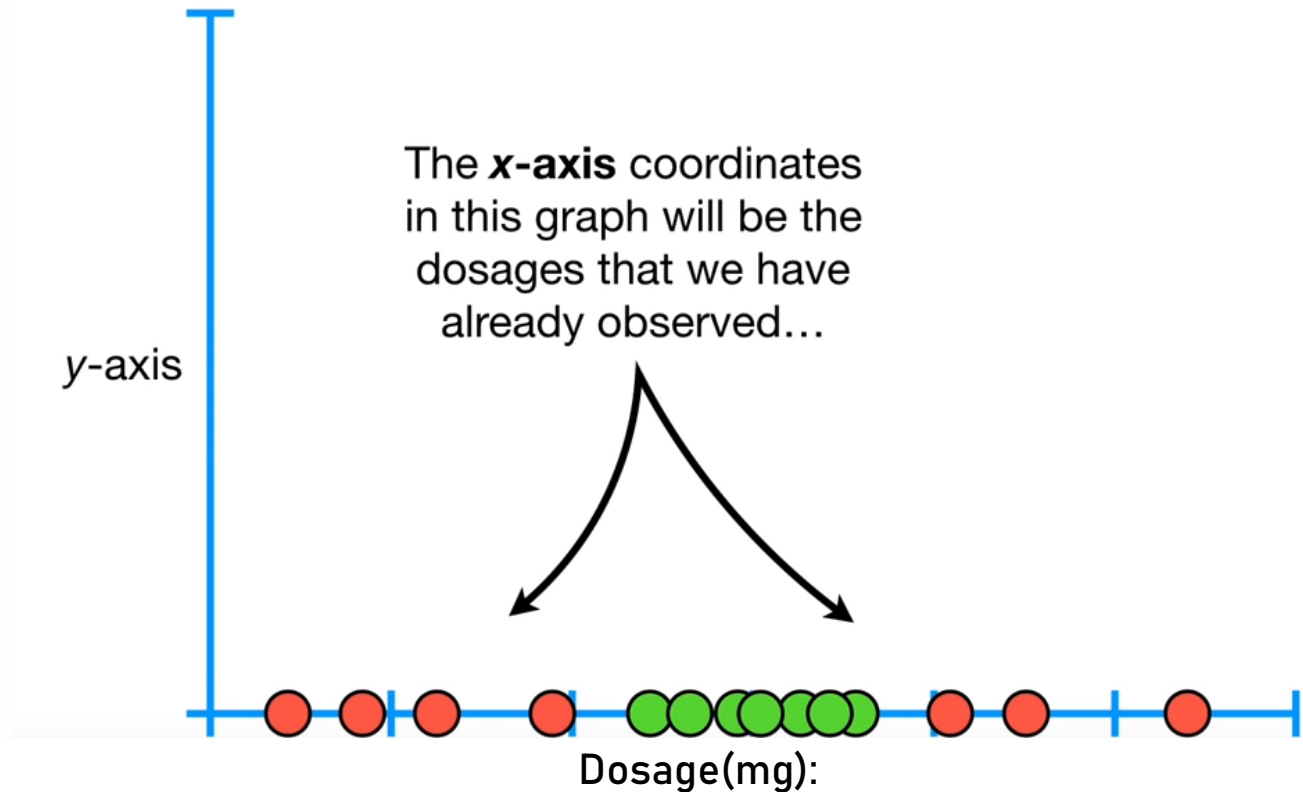


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Maps the original feature space to a higher dimensional feature space



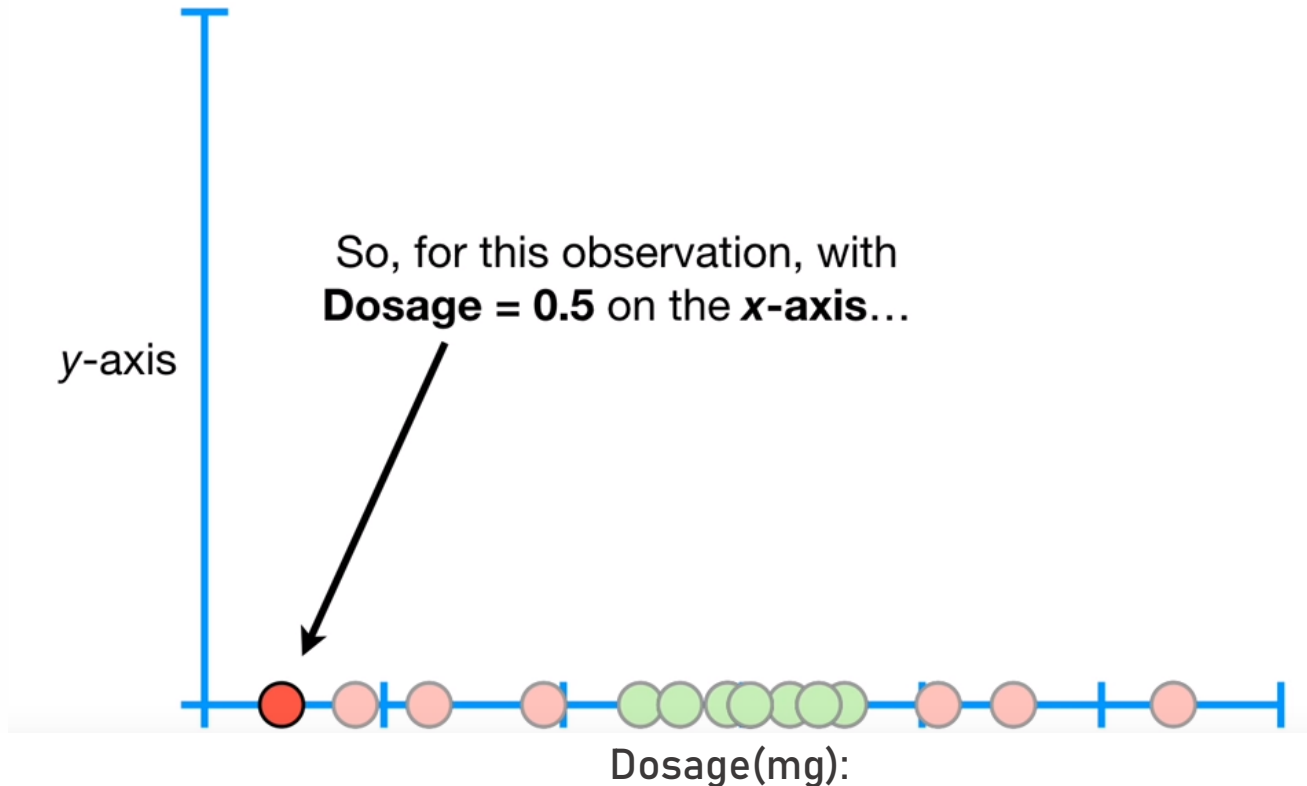


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Maps the original feature space to a higher dimensional feature space



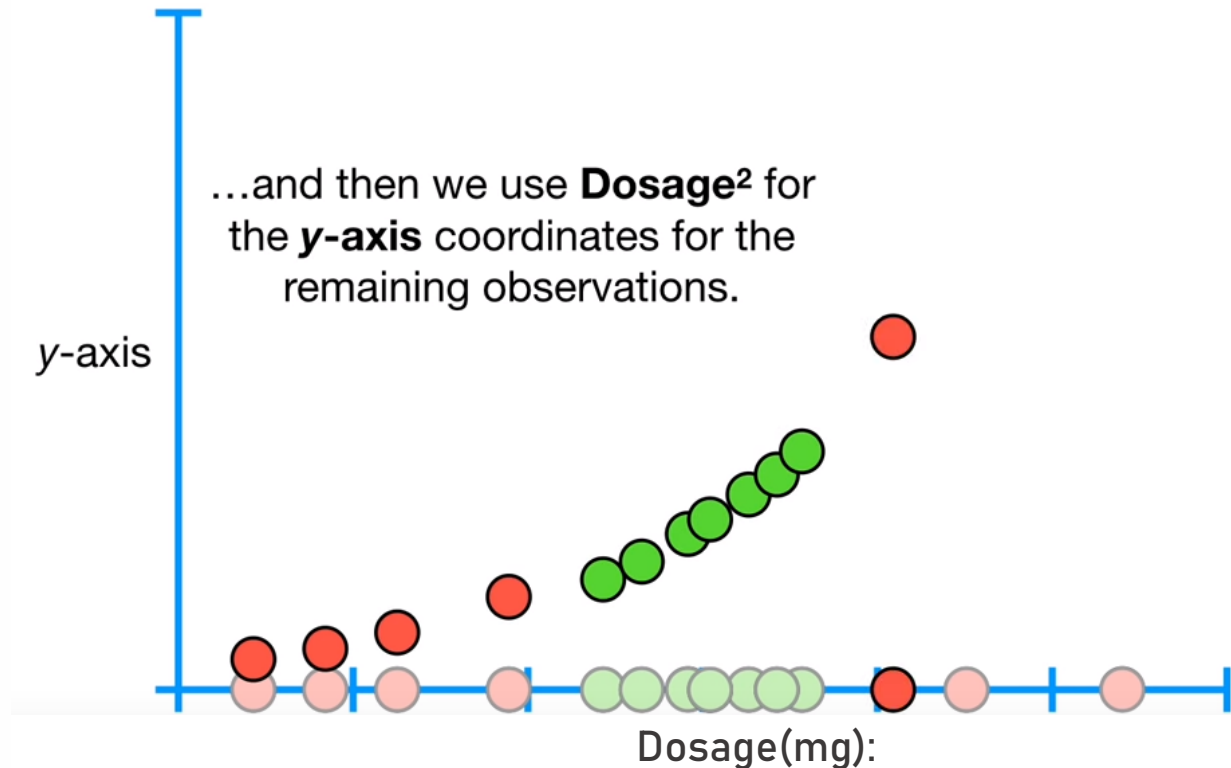


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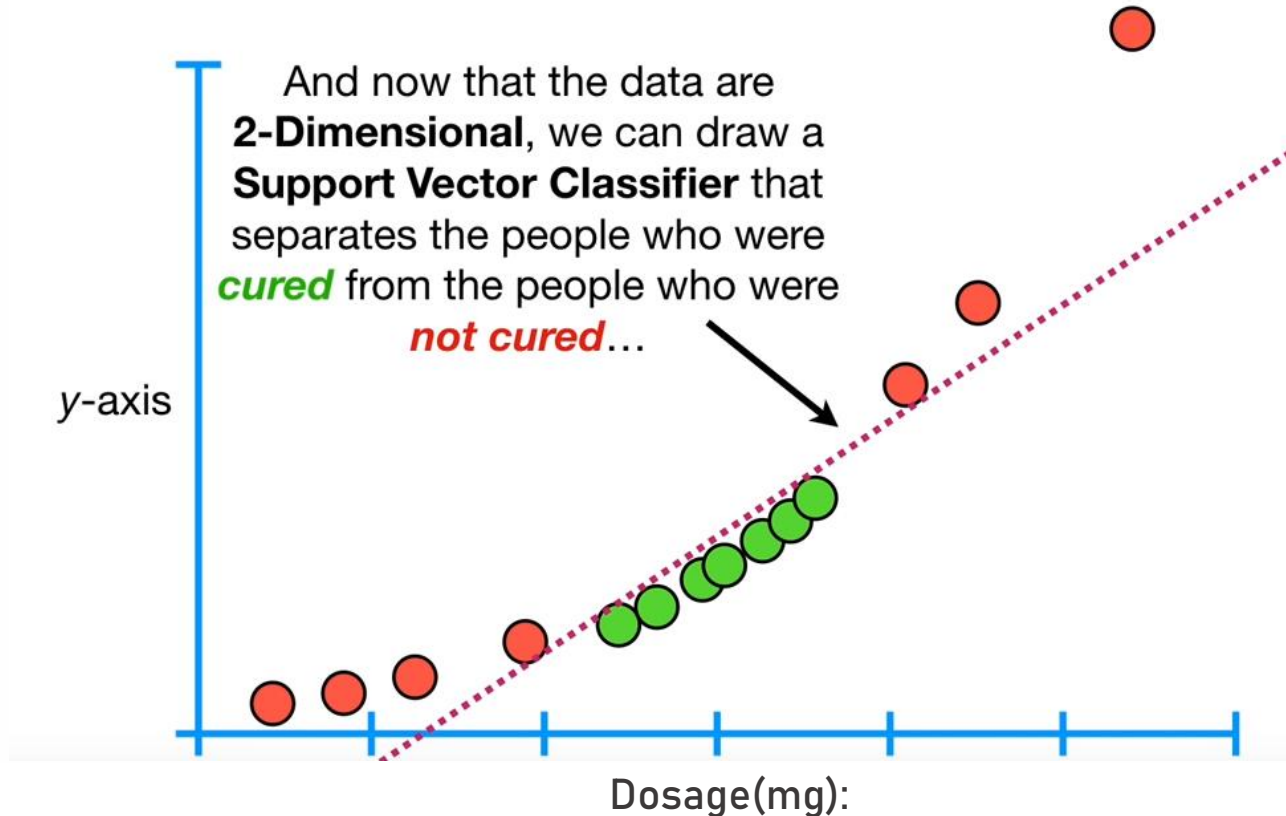
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Maps the original feature space to a higher dimensional feature space





Maps the original feature space to a higher dimensional feature space



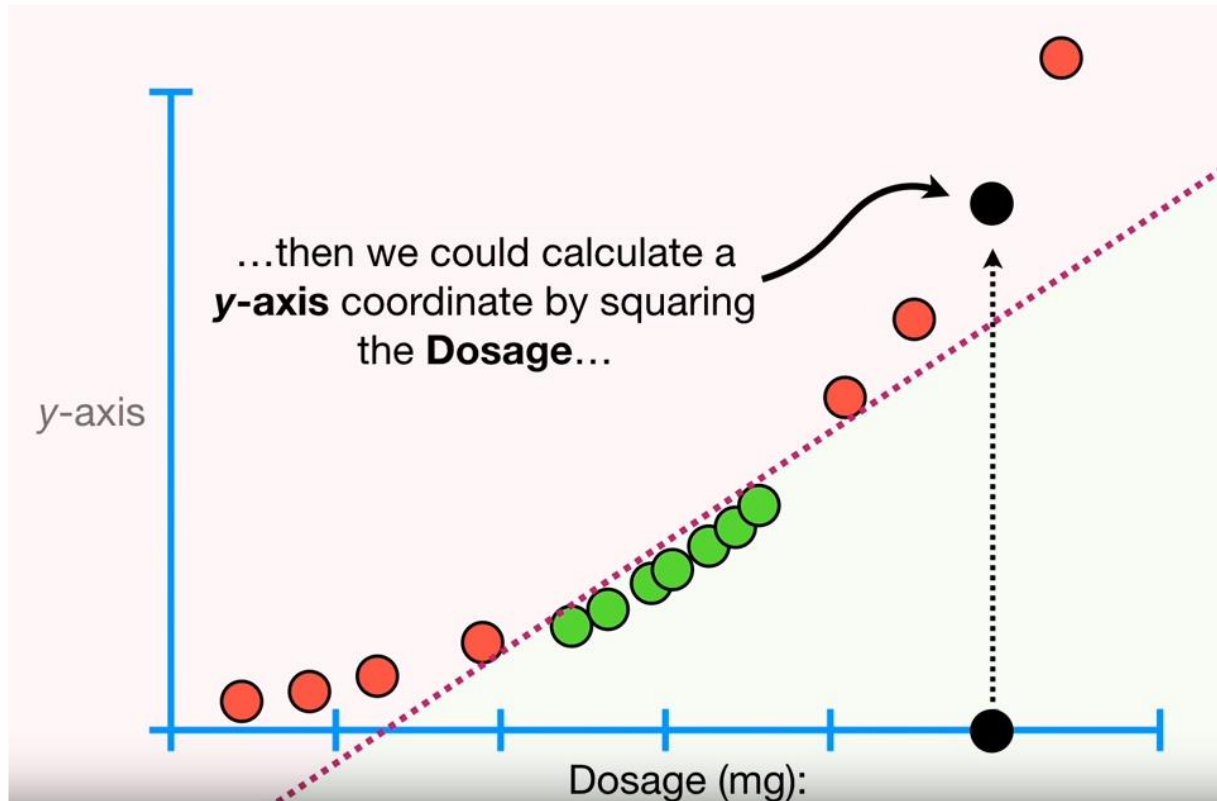


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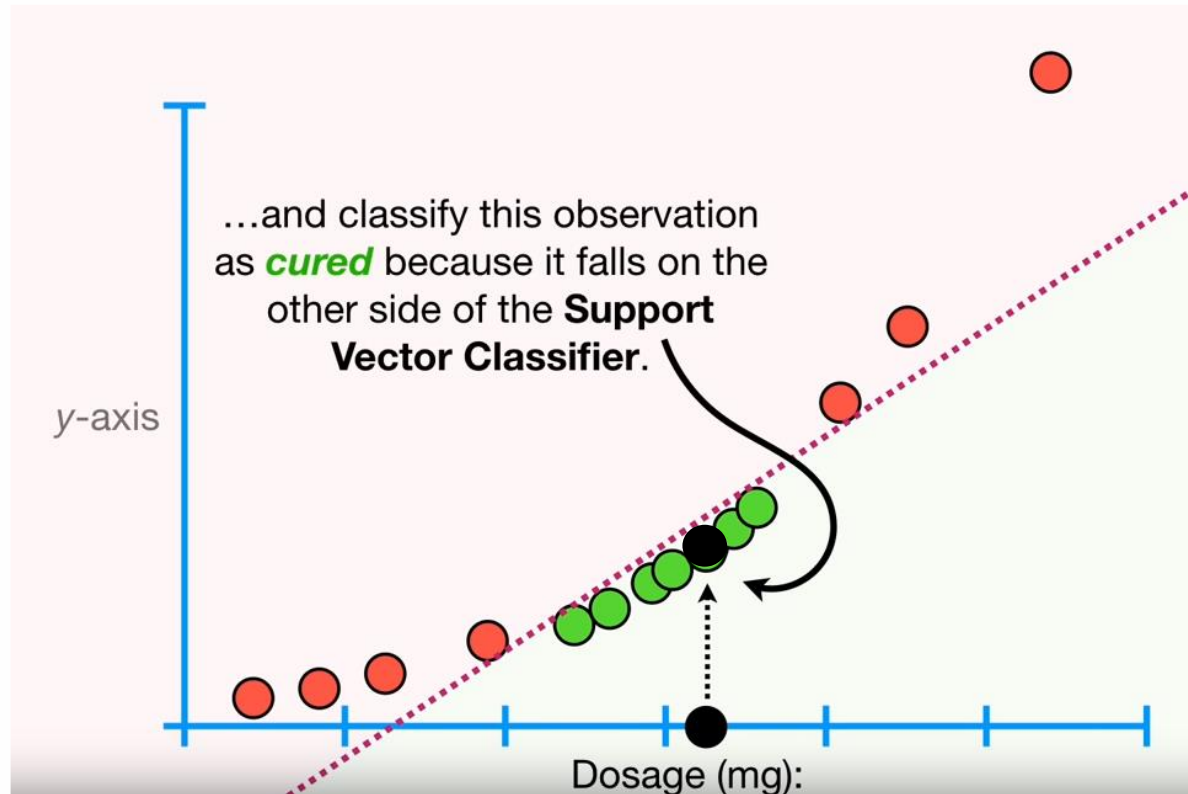


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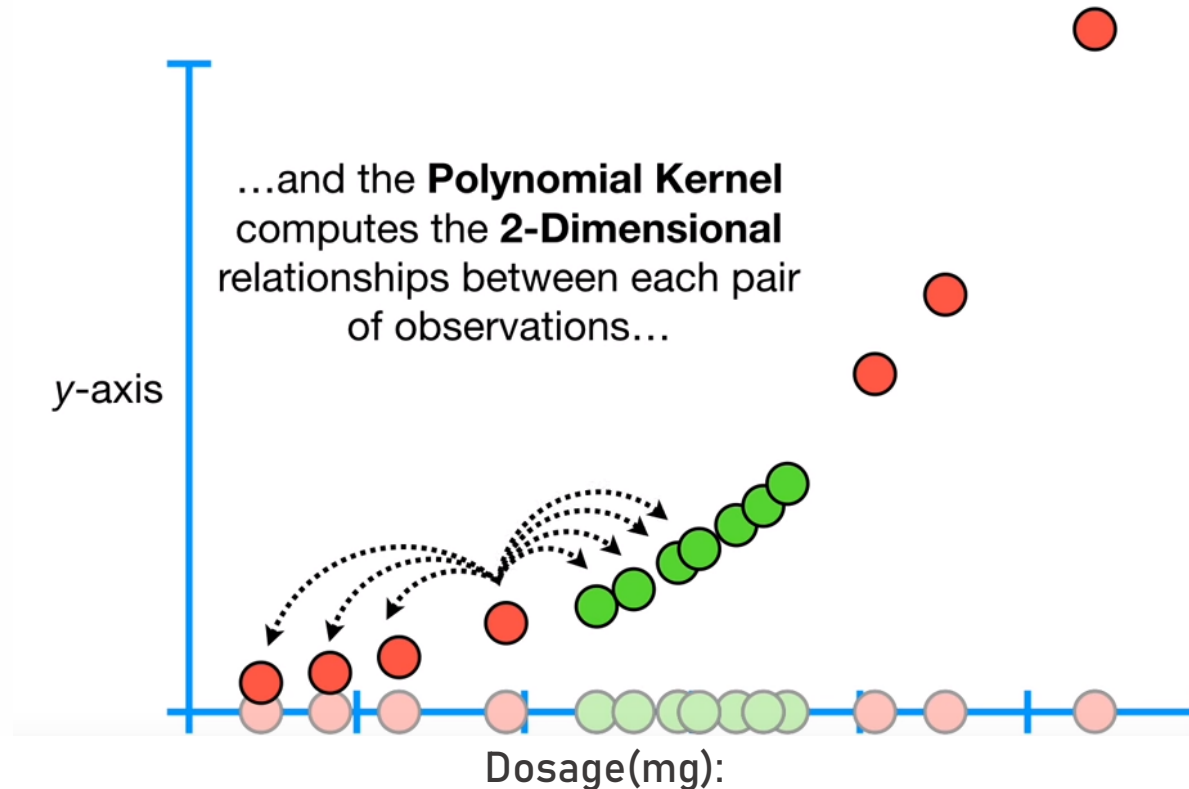


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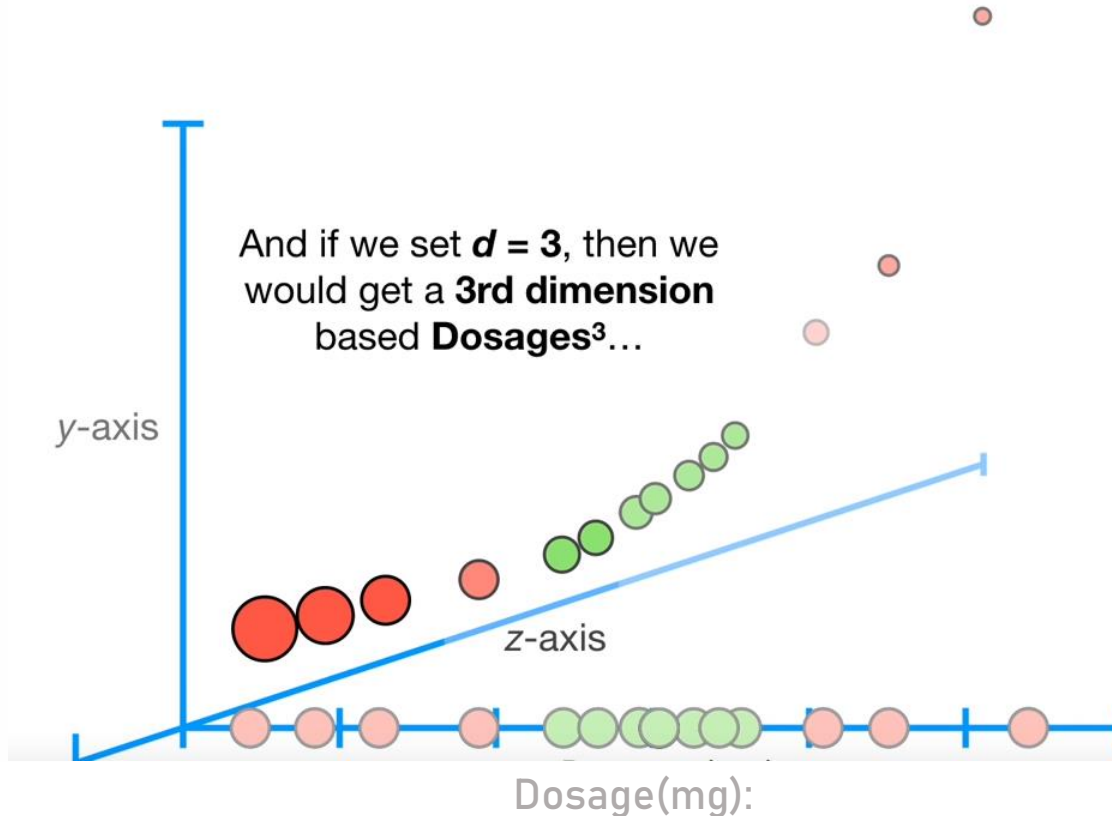


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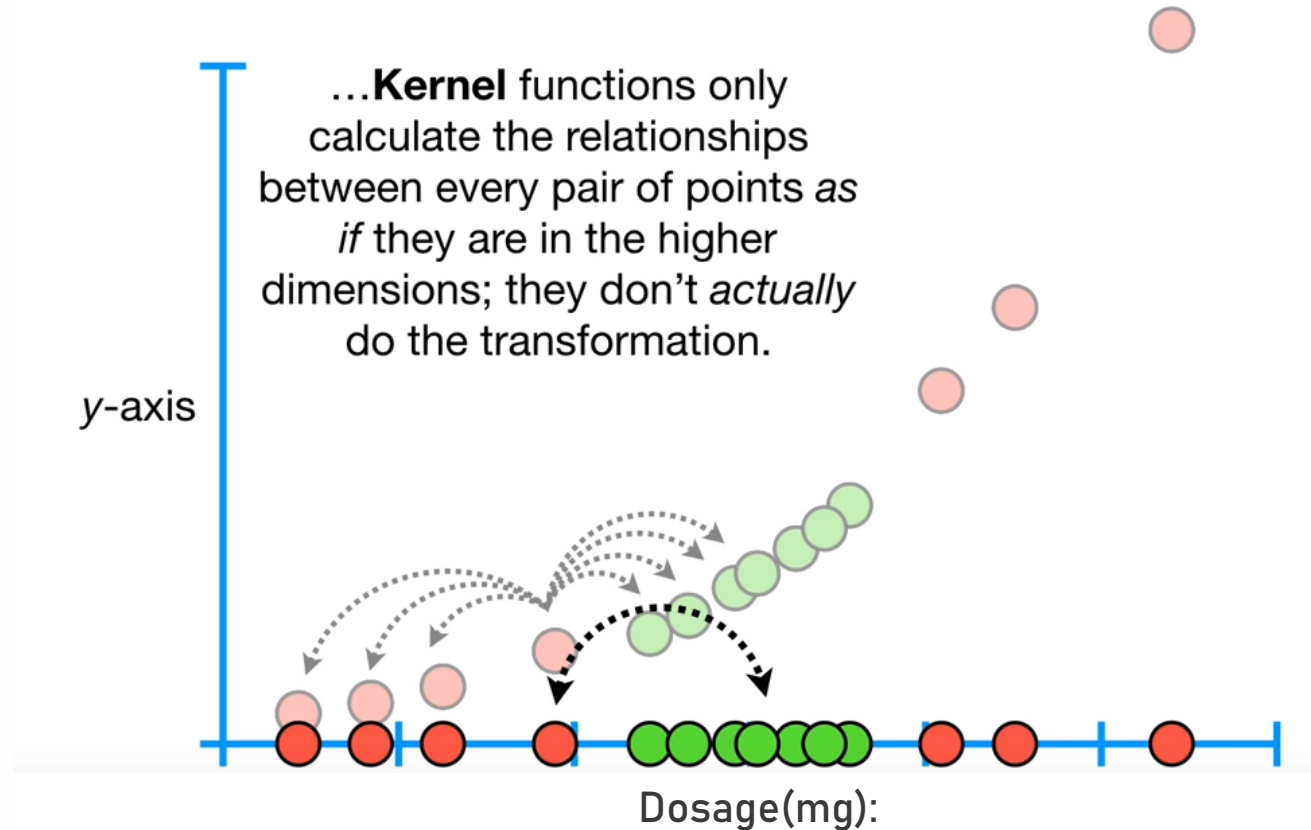


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Maps the original feature space to a higher dimensional feature space





Transforms the quadratic optimization problem

Can be transformed into another optimization problem called “the Lagrangian dual problem”

$$\max_{\alpha} \min_{w, b} \frac{\|w\|^2}{2} + C \sum_{i=1}^N \alpha_i (1 - w^T \phi(x_i) + b)$$

Or

Subject to: $w = \sum_{i=1}^N \alpha_i y_i \phi(x_i)$, $0 \leq \alpha_i \leq C, i = 1, \dots, N$

$$\max_{\alpha} \sum_{i=1}^N \alpha_i - \sum_{i=1}^N \sum_{j=1}^N (y_i \alpha_i \phi(x_i)^T \phi(x_j) y_j \alpha_j)$$

$$\min_{\alpha} \left(\frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N y_i y_j \alpha_i \alpha_j (\phi(x_i) \cdot \phi(x_j) + \lambda \delta_{ij}) - \sum_{i=1}^N \alpha_i \right)$$



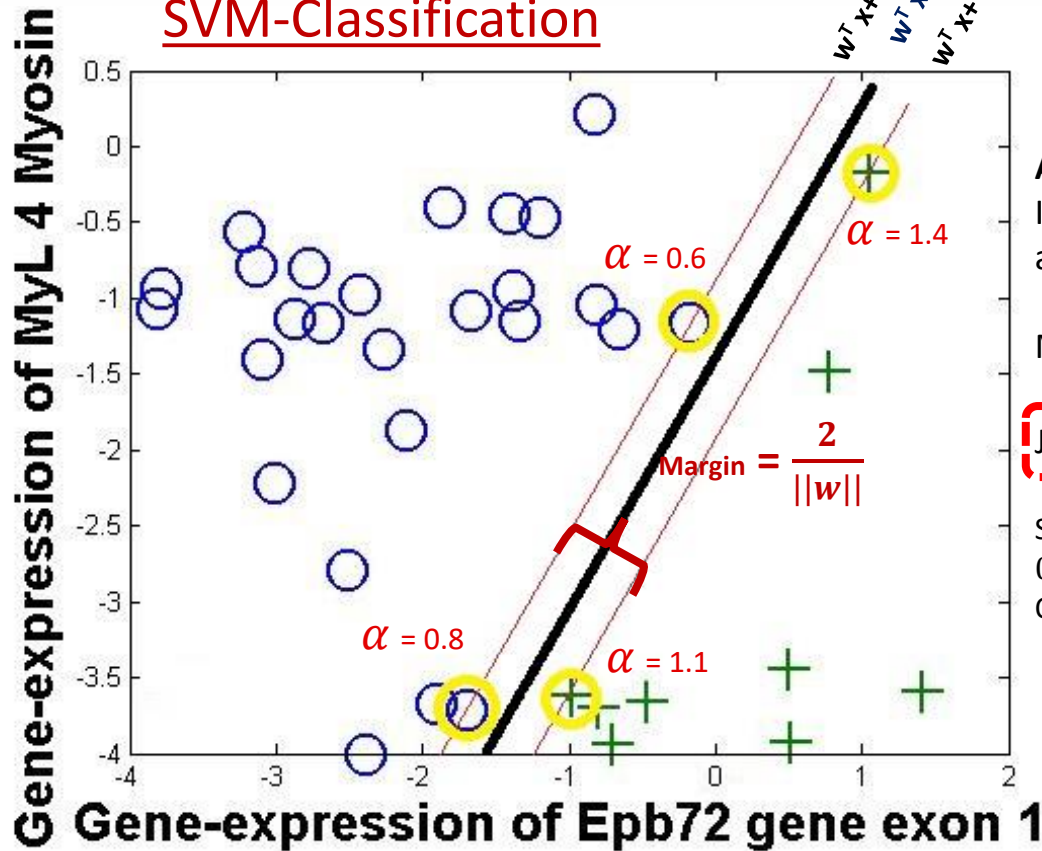
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SVM classify the two labeled-data based on the discriminant line or hyperplane

SVM-Classification



The primal form of the optimization problem, α and w are related as the dual problem



Algorithm SVM-train:

Inputs: Training examples $\{x_1, x_2, \dots, x_i, \dots, x_l\}$ and class labels $\{y_1, y_2, \dots, y_i, \dots, y_l\}$

Minimize over α_i :

$$J = \left(\frac{1}{2}\right) \sum_{i=1}^N \sum_{j=1}^N y_i y_j \alpha_i \alpha_j (\phi(x_i) \cdot \phi(x_j) + \lambda \delta_{ij}) - \sum_{i=1}^N \alpha_i$$

Subject to:

$0 \leq \alpha_i \leq C$ and $\sum_{i=1}^N \alpha_i y_i = 0$

Outputs: Parameter α_i



$$w = \sum_{i=1}^N \alpha_i y_i \phi(x_i)$$

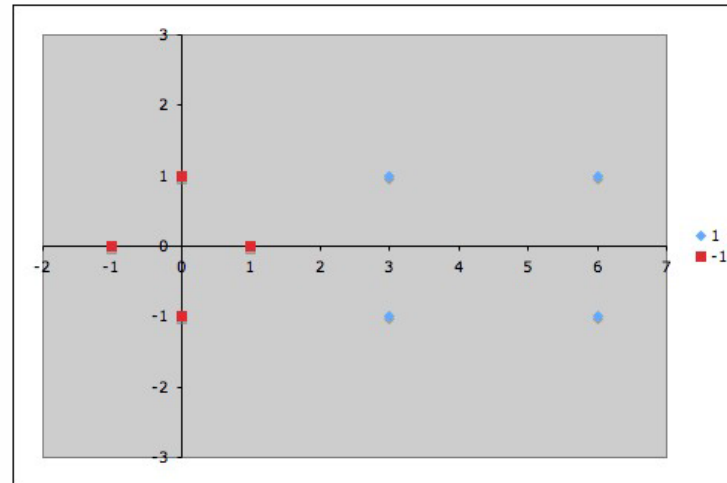


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An example 1, of linear classifiable dataset



A sample data point in \mathbb{R}^2 (Dan Ventura, 2009)

Training Set : $\left\{ \begin{pmatrix} 3 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ -1 \end{pmatrix}, \begin{pmatrix} 6 \\ 1 \end{pmatrix}, \begin{pmatrix} 6 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \end{pmatrix} \right\}$

Training Set Labeled : $\{ 1, 1, 1, 1, -1, -1, -1, -1 \}$

Testing Set: $\left\{ \begin{pmatrix} 4 \\ 1 \end{pmatrix}, \begin{pmatrix} 1.9 \\ -0.8 \end{pmatrix}, \begin{pmatrix} 2.3 \\ 1 \end{pmatrix} \right\}$

Testing Set Labeled : $\{ 1, -1, 1 \}$



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An example of linear classifiable dataset

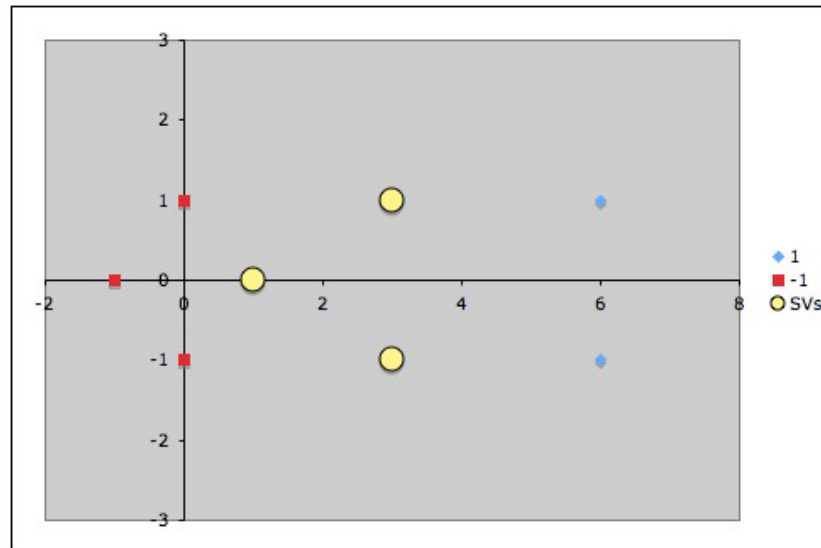
1. *Find the support vectors by random-creating lines or hyperplane based on the decision function $w \cdot x + b = 0$, and the support decision function $w \cdot x + b = 1$ for class $\{+1\}$, and the support decision function $w \cdot x + b = -1$ for class $\{-1\}$.*



An example of linear classifiable dataset

2. By the results from step 1, the 3 support vectors are

$S1 = (1,0)$, $S2 = (3,1)$ and $S3 = (3,-1)$. Next, compute the alphas by using the equations in step 3



(Dan Ventura, 2009)



An example of linear classifiable dataset

3. Use the alphas from step 2 to compute the w and create a hyperplane equation.

$S_1 = (1,0)$ class -1, $S_2 = (3,1)$ class +1 and $S_3 = (3,-1)$ class +1



$$\begin{aligned}\alpha_1 \phi(s_1) \cdot \phi(s_1) + \alpha_2 \phi(s_2) \cdot \phi(s_1) + \alpha_3 \phi(s_3) \cdot \phi(s_1) &= -1 \\ \alpha_1 \phi(s_1) \cdot \phi(s_2) + \alpha_2 \phi(s_2) \cdot \phi(s_2) + \alpha_3 \phi(s_3) \cdot \phi(s_2) &= +1 \\ \alpha_1 \phi(s_1) \cdot \phi(s_3) + \alpha_2 \phi(s_2) \cdot \phi(s_3) + \alpha_3 \phi(s_3) \cdot \phi(s_3) &= +1\end{aligned}$$

let $\phi() = I$, and reduce to



$$\begin{aligned}\alpha_1 \tilde{s}_1 \cdot \tilde{s}_1 + \alpha_2 \tilde{s}_2 \cdot \tilde{s}_1 + \alpha_3 \tilde{s}_3 \cdot \tilde{s}_1 &= -1 \\ \alpha_1 \tilde{s}_1 \cdot \tilde{s}_2 + \alpha_2 \tilde{s}_2 \cdot \tilde{s}_2 + \alpha_3 \tilde{s}_3 \cdot \tilde{s}_2 &= +1 \\ \alpha_1 \tilde{s}_1 \cdot \tilde{s}_3 + \alpha_2 \tilde{s}_2 \cdot \tilde{s}_3 + \alpha_3 \tilde{s}_3 \cdot \tilde{s}_3 &= +1\end{aligned}$$

add the bias inputs = 1 to $\tilde{s}_1, \tilde{s}_2, \tilde{s}_3$



$$\tilde{s}_1 = \{1, 0, 1\}, \tilde{s}_2 = \{3, 1, 1\} \text{ and } \tilde{s}_3 = \{3, -1, 1\}$$



An example of linear classifiable dataset

$$\alpha_1 \tilde{s}_1 \cdot \tilde{s}_1 + \alpha_2 \tilde{s}_2 \cdot \tilde{s}_1 + \alpha_3 \tilde{s}_3 \cdot \tilde{s}_1 = -1$$

$$\alpha_1 \tilde{s}_1 \cdot \tilde{s}_2 + \alpha_2 \tilde{s}_2 \cdot \tilde{s}_2 + \alpha_3 \tilde{s}_3 \cdot \tilde{s}_2 = +1$$

$$\alpha_1 \tilde{s}_1 \cdot \tilde{s}_3 + \alpha_2 \tilde{s}_2 \cdot \tilde{s}_3 + \alpha_3 \tilde{s}_3 \cdot \tilde{s}_3 = +1$$

$$\tilde{s}_1 = \{1, 0, 1\}, \tilde{s}_2 = \{3, 1, 1\} \text{ and } \tilde{s}_3 = \{3, -1, 1\}$$

compute the dot products results



$$\alpha_1((1 \times 1) + (0 \times 0) + (1 \times 1)) + \alpha_2((3 \times 1) + (1 \times 0) + (1 \times 1)) + \alpha_3((3 \times 1) + (-1 \times 0) + (1 \times 1)) = -1$$

$$\alpha_1((1 \times 3) + (0 \times 1) + (1 \times 1)) + \alpha_2((3 \times 3) + (1 \times 1) + (1 \times 1)) + \alpha_3((3 \times 3) + (-1 \times 1) + (1 \times 1)) = +1$$

$$\alpha_1((1 \times 3) + (0 \times -1) + (1 \times 1)) + \alpha_2((3 \times 3) + (1 \times -1) + (1 \times 1)) + \alpha_3((3 \times 3) + (-1 \times -1) + (1 \times 1)) = +1$$



$$2\alpha_1 + 4\alpha_2 + 4\alpha_3 = -1$$

$$4\alpha_1 + 11\alpha_2 + 9\alpha_3 = +1$$

$$4\alpha_1 + 9\alpha_2 + 11\alpha_3 = +1$$



$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\longrightarrow \begin{vmatrix} 2 & 4 & 4 \\ 4 & 11 & 9 \\ 4 & 9 & 11 \end{vmatrix} \begin{matrix} -1 \\ 1 \\ 1 \end{matrix}$$



An example of linear classifiable dataset

$$\left| \begin{array}{ccc|c} 1 & 2 & 2 & -1/2 \\ 4 & 11 & 9 & 1 \\ 4 & 9 & 11 & 1 \end{array} \right| \begin{array}{l} r_1/2 \\ \\ \end{array} \longleftrightarrow \left| \begin{array}{ccc|c} 2 & 4 & 4 & 1 \\ 4 & 11 & 9 & 1 \\ 4 & 9 & 11 & 1 \end{array} \right| \begin{array}{l} 2=1 \quad 4=2 \quad 4=2 \\ \\ \end{array}$$

$$\left| \begin{array}{ccc|c} 1 & 2 & 2 & -1/2 \\ 0 & 3 & 1 & 3 \\ 0 & 1 & 3 & 3 \end{array} \right| \begin{array}{l} \\ r_2 - 4r_1 \\ r_3 - 4r_1 \end{array} \longleftrightarrow \left| \begin{array}{ccc|c} 1 & 2 & 2 & -1/2 \\ 4 - 4(1) = 0 & 11 - 4(2) = 3 & 9 - 4(2) = 1 & 1 - 4(-1/2) = 3 \\ 4 - 4(1) = 0 & 9 - 4(2) = 1 & 11 - 4(2) = 3 & 1 - 4(-1/2) = 3 \end{array} \right| \begin{array}{l} \\ \\ \end{array}$$

$$\left| \begin{array}{ccc|c} 1 & 2 & 2 & -1/2 \\ 0 & 1 & 1/3 & 1 \\ 0 & 1 & 3 & 3 \end{array} \right| \begin{array}{l} \\ r_2/3 \\ \end{array}$$

$$\left| \begin{array}{ccc|c} 1 & 2 & 2 & -1/2 \\ 0 & 1 & 1/3 & 1 \\ 0 & 0 & 8/3 & 2 \end{array} \right| \begin{array}{l} \\ \\ r_3 - r_2 \end{array}$$

$$\left| \begin{array}{ccc|c} 1 & 2 & 2 & -1/2 \\ 0 & 1 & 1/3 & 1 \\ 0 & 0 & 1 & 3/4 \end{array} \right| \begin{array}{l} \\ \\ r_3 \times 3/8 \end{array}$$



An example of linear classifiable dataset

$$\left| \begin{array}{ccc|c} 1 & 2 & 0 & -2 \\ 0 & 1 & 0 & 3/4 \\ 0 & 0 & 1 & 3/4 \end{array} \right| \begin{array}{l} r_1 - 2r_3 \\ r_2 - (1/3)r_3 \\ \end{array}$$



$$\left| \begin{array}{ccc|c} 1 & 0 & 0 & -7/2 \\ 0 & 1 & 0 & 3/4 \\ 0 & 0 & 1 & 3/4 \end{array} \right| \begin{array}{l} r_1 - 2r_2 \\ \\ \end{array}$$



$$\alpha_1 = -3.5, \alpha_2 = 0.75, \alpha_3 = 0.75$$



$$w = \sum_{i=1}^N \alpha_i \tilde{s}_i = -3.5 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + 0.75 \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} + 0.75 \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}$$



From $y = w^\top x_i + b$ then $w = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $b = -2$



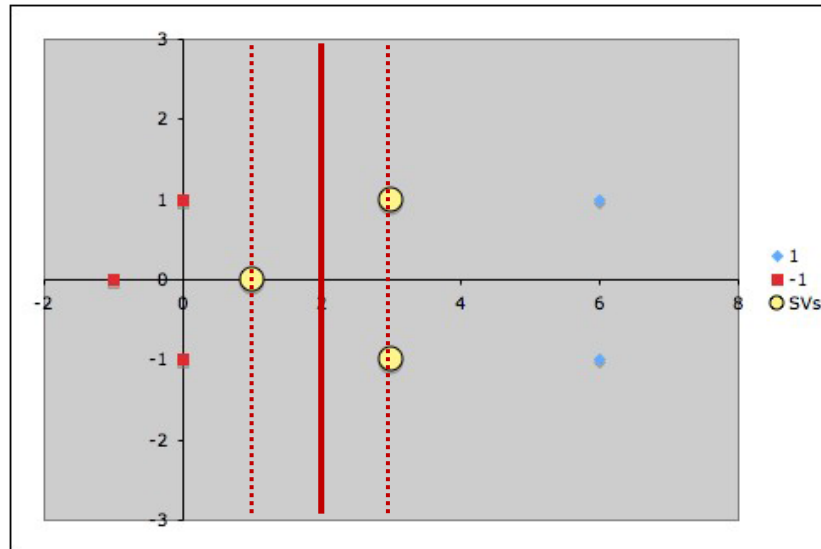
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An example of linear classifiable dataset

$$w = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } b = -2$$





How to apply SVM to categorical data

2 sub-types of categorical features: *Ordinal* and *nominal*

Ordinal features example:

- a patient satisfaction metric {'satisfied', 'neutral', 'dissatisfied'}

is a ordinal variable since we can order it: '*satisfied*' > '*neutral*' > '*dissatisfied*'

we can simply map the 'string' notation into an integer notation, for example

'*satisfied*'=1, '*neutral*'=0, and '*dissatisfied*'=-1.



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How to apply SVM to categorical data

*2 sub-types of categorical features: **Ordinal** and **nominal***

***Nominal** features example:*

- *think of ‘color’; there are some cases in image processing where ordering color values makes sense, but for simplicity, we can’t say ‘**red** > **blue** > **yellow**’*
- *To deal with such variables in SVM classification, we typically do a “one-hot” encoding*



A “one-hot” encoding for SVM

Nominal features: ‘red > blue > yellow’

- *Create one dummy variable for each possible value of that nominal feature variable*
- *Our color variable can have one of the three values: ‘red,’ ‘blue,’ ‘yellow.’*

	blue	red	yellow
sample 1	1	0	0
sample 2	0	0	1
sample 3	0	1	0
sample 4	0	0	1



Note:

- For numerical data

no consider about create more dimension

- For nominal category data – sex = [male, female]

need to create more 2-dimensions male {1,0} female {0,1}

- For ordinal category data salary = [low, medium, high]

no need to create another 3-dimensions

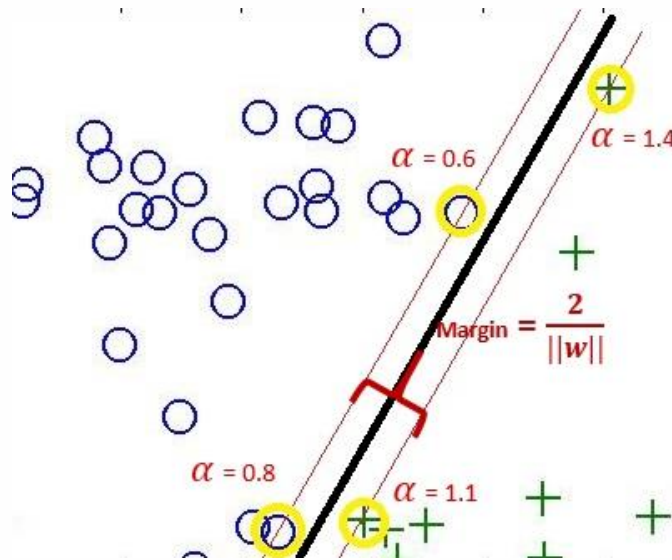
can apply dummy data = 0, 1, 2



Pros and Cons of SVM

Pros:

1. SVM accurate in high dimensional spaces.
2. SVM uses a subset of training points in the decision function (called support vectors), so it's also memory efficient.



Minimize α_i to find w

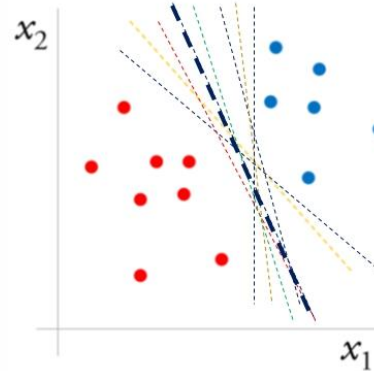
$$w = \sum_{i=1}^N \alpha_i y_i \phi(x_i)$$



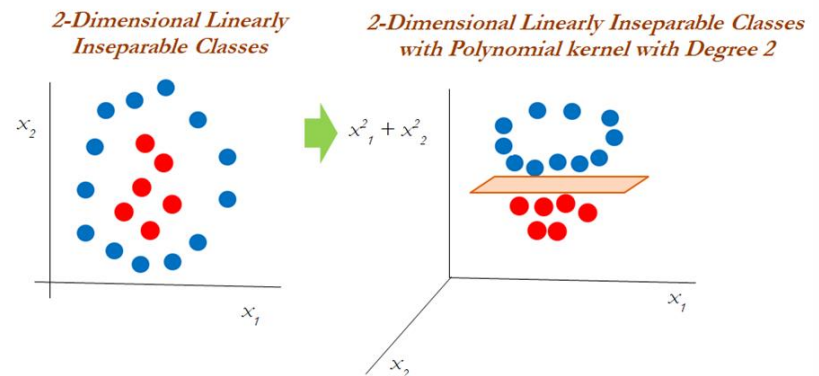
Pros and Cons of SVM

Pros:

3. SVM can guarantee optimality.



4. SVM is useful for both Linearly Separable(hard margin) and Non-linearly Separable(soft margin) data



<https://hub.packtpub.com/what-is-a-support-vector-machine/>



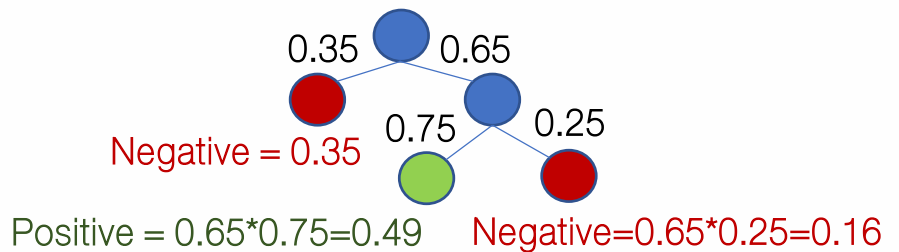
Pros and Cons of SVM

Cons:

1. SVM is prone for over-fitting, if the number of features is much greater than the number of samples.
2. SVM do not directly provide probability estimates, which are desirable in most classification problems.

$$\mathbf{w}^T \mathbf{x} + \mathbf{b} = \begin{cases} -1 \rightarrow \text{class} - 1 \\ 0 \rightarrow \text{class} + 1 \\ 1 \rightarrow \text{class} + 1 \end{cases}$$

Decision Tree



3. SVM is not very efficient computationally, if your dataset is very big, such as when you have more than one thousand rows.



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Implementing SVM with Python

<https://stackabuse.com/implementing-svm-and-kernel-svm-with-pythons-scikit-learn/>



Implementing SVM with Python

```
import pandas as pd
```

```
import numpy as np
```

```
import matplotlib.pyplot as plt
```

```
df_colon = pd.read_csv(r'H:\Coding_python\colon.csv')
```

```
df_colon.shape
```

(62, 2001)

```
df_colon.head()
```

	H55933	R39465	R39465.1	R85482	...	H40891	R77780	T49647	Class
0	3.62	3.31	2.986154	2.71	...	-0.315	-1.764190	-2.75	1
1	3.47	3.68	3.425553	3.05	...	-1.210	-1.062064	-2.13	1
2	3.02	2.78	2.569772	3.21	...	-1.010	-2.260031	-1.50	1
3	3.10	2.86	2.772942	3.19	...	-1.610	-1.223450	-1.07	1
4	3.01	2.91	2.560548	3.25	...	-1.210	-1.232686	-1.62	1



Data Preprocessing

Data preprocessing involves

(1) Dividing the data into attributes and labels

```
X = df_colon.drop('Class', axis=1)
```

```
y = df_colon['Class']
```

`X.shape`

(62, 2000)

`X.head()`

```
H55933 R39465 R39465.1 R85482 ... H40891 R77780 T49647
0  3.62  3.31  2.986154  2.71  ... -0.315 -1.764190 -2.75
1  3.47  3.68  3.425553  3.05  ... -1.210 -1.062064 -2.13
2  3.02  2.78  2.569772  3.21  ... -1.010 -2.260031 -1.50
3  3.10  2.86  2.772942  3.19  ... -1.610 -1.223450 -1.07
4  3.01  2.91  2.560548  3.25  ... -1.210 -1.232686 -1.62
```



Data Preprocessing

(2) dividing the data into training and testing sets.

```
from sklearn.model_selection import train_test_split
```

```
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size = 0.20)
```

<i>X_train.shape</i>	(49, 2000)
<i>X_test.shape</i>	(13, 2000)
<i>y_train.shape</i>	(49,)
<i>y_test.shape</i>	(13,)



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Training the Algorithm

```
from sklearn.svm import SVC
```

support vector classifier class

```
svclassifier = SVC(kernel='linear')
```

Linear classifier

```
svclassifier.fit(X_train, y_train)
```



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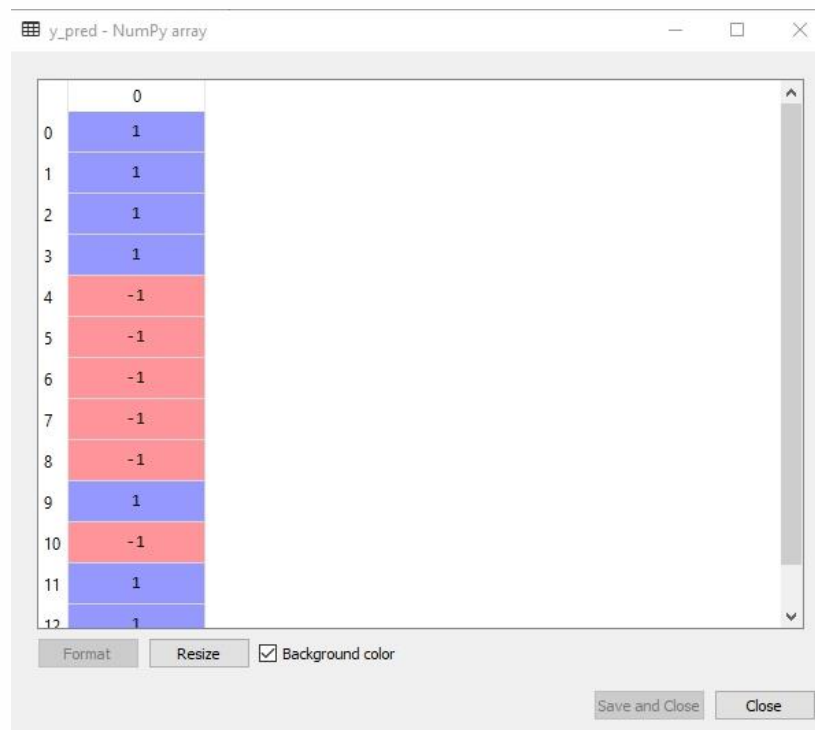
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Making Predictions

`y_pred = svcclassifier.predict(X_test)`

Predict y by using X_test





Evaluating the Algorithm

```
from sklearn.metrics import classification_report, confusion_matrix
```

```
confusion = confusion_matrix(y_test, y_pred)
```

```
print(confusion)
```

[6 3]

[0 4]

```
print(classification_report(y_test, y_pred))
```

	precision	recall	f1-score	support
-1	1.00	0.67	0.80	9
1	0.57	1.00	0.73	4
micro avg	0.77	0.77	0.77	13
macro avg	0.79	0.83	0.76	13
weighted avg	0.87	0.77	0.78	13



Evaluating the Algorithm

#edit target name

```
target_names = ['Cancer', 'Healthy']
```

```
print(classification_report(y_test, y_pred, target_names=target_names))
```

	precision	recall	f1-score	support
Cancer	1.00	0.67	0.80	9
Healthy	0.57	1.00	0.73	4
micro avg	0.77	0.77	0.77	13
macro avg	0.79	0.83	0.76	13
weighted avg	0.87	0.77	0.78	13



Evaluating the Algorithm

True Positives

TP = confusion[1, 1]

True Negatives

TN = confusion[0, 0]

False Positives

FP = confusion[0, 1]

False Negatives

FN = confusion[1, 0]

print('accuracy: ', (TP + TN) / float(TP + TN + FP + FN))

print('sensitivity: ', TP / float(TP + FN))

print('specificity: ', TN / float(TN + FP))

accuracy: 0.7692307692307693
sensitivity: 1.0
specificity: 0.6666666666666666



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Implementing Kernel SVM with Scikit-Learn

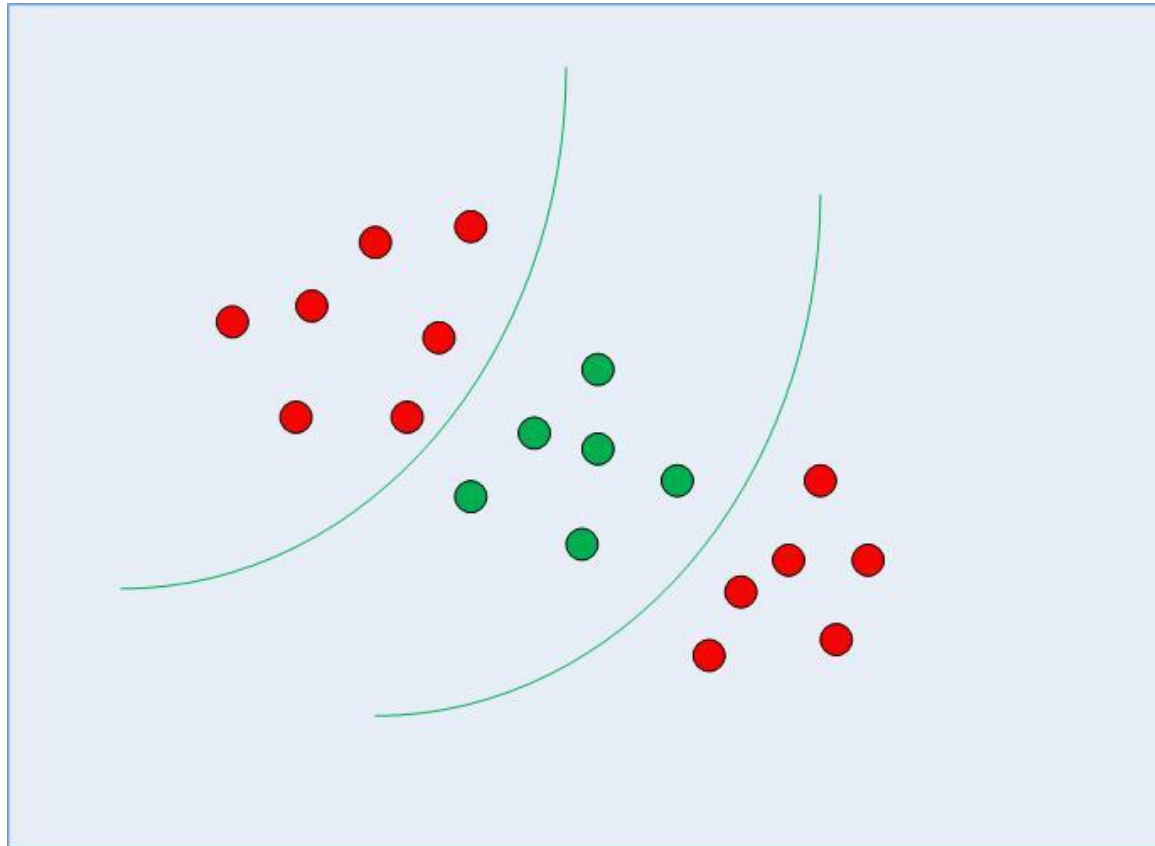


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Non-linearly Separable Data



<https://stackabuse.com/implementing-svm-and-kernel-svm-with-pythons-scikit-learn/>



Kernel SVM with Scikit-Learn

```
import numpy as np
```

```
import matplotlib.pyplot as plt
```

```
import pandas as pd
```

URL for downloading iris.data

```
url = "https://archive.ics.uci.edu/ml/machine-learning-databases/iris/iris.data"
```

```
# Assign column names to the dataset
```

```
colnames = ['sepal-length', 'sepal-width', 'petal-length', 'petal-width', 'Class']
```

```
# Read dataset to pandas dataframe
```

```
irisdata = pd.read_csv(url, names=colnames)
```



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Kernel SVM with Scikit-Learn

#Preprocessing

```
X = irisdata.drop('Class', axis=1)
```

```
y = irisdata['Class']
```

#Train Test Split

```
from sklearn.model_selection import train_test_split
```

```
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size = 0.20)
```

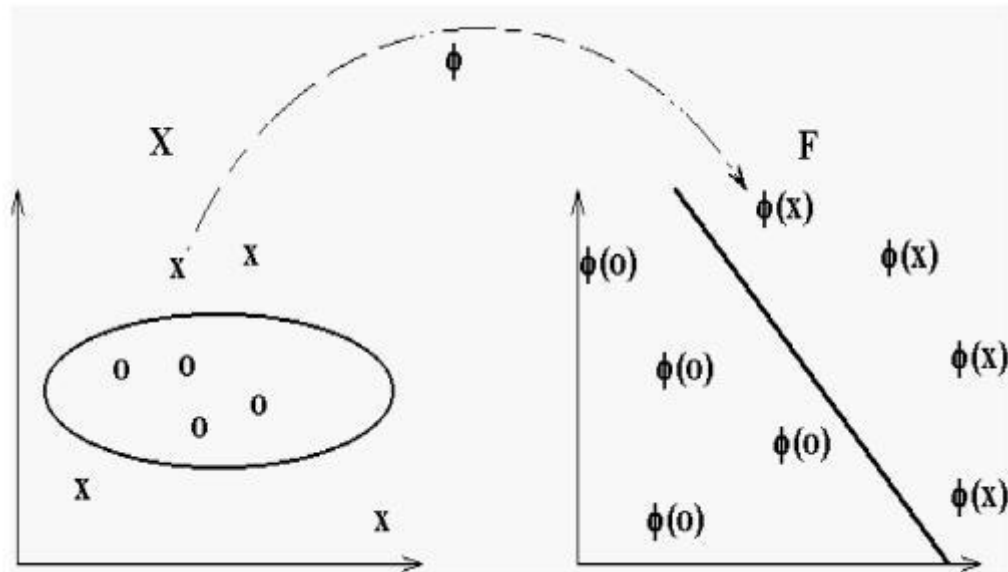


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Polynomial Kernel SVM with Scikit-Learn



https://en.wikipedia.org/wiki/Polynomial_kernel#/media/File:Svm_8_polynomial.JPG



Polynomial Kernel SVM with Scikit-Learn

```
from sklearn.svm import SVC
```

```
svclassifier = SVC(kernel='poly', degree=8)
```

```
svclassifier.fit(X_train, y_train)
```

```
y_pred = svclassifier.predict(X_test)
```

```
from sklearn.metrics import classification_report, confusion_matrix
```

```
print(confusion_matrix(y_test, y_pred))
```

```
print(classification_report(y_test, y_pred))
```

Kernel = Polynomial

Degree of the polynomial kernel function ('poly'). Ignored by all other kernels.

```
[[ 7  0  0]
```

```
 [ 0 11  0]
```

```
 [ 0  3  9]]
```

	precision	recall	f1-score	support
Iris-setosa	1.00	1.00	1.00	7
Iris-versicolor	0.79	1.00	0.88	11
Iris-virginica	1.00	0.75	0.86	12
micro avg	0.90	0.90	0.90	30
macro avg	0.93	0.92	0.91	30
weighted avg	0.92	0.90	0.90	30



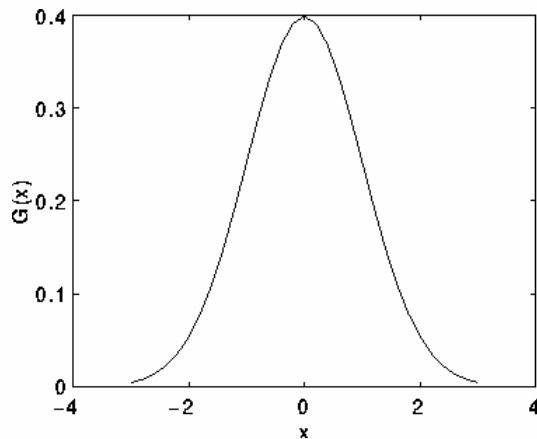
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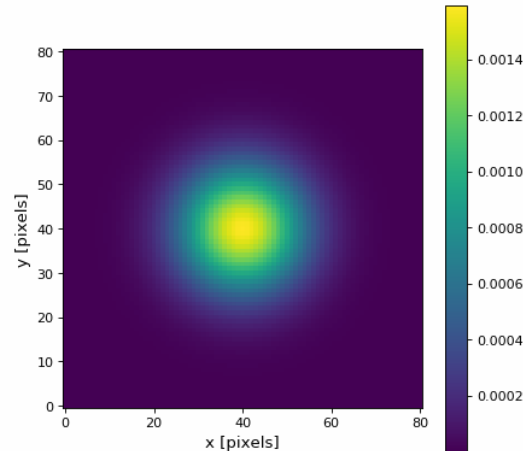
Gaussian Kernel SVM with Scikit-Learn

1 Dimension



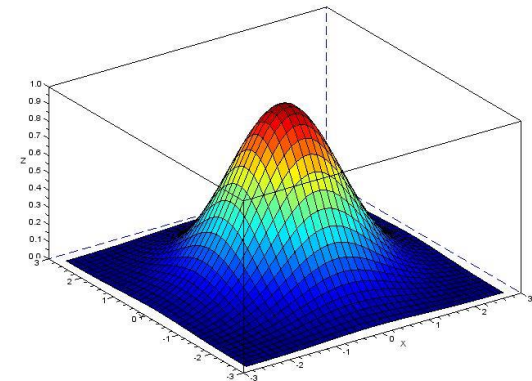
<https://homepages.inf.ed.ac.uk/rbf/HIPR2/gsmooth.htm>

2 Dimensions



<https://docs.astropy.org/en/stable/api/astropy.convolution.Gaussian2DKernel.html>

3 Dimensions



<https://jamesmccaffrey.files.wordpress.com/2014/01/gaussiankernel.jpg>



Gaussian Kernel SVM with Scikit-Learn

```
from sklearn.svm import SVC
```

```
svclassifier = SVC(kernel='rbf')
```

Kernel = the radial basis function

```
svclassifier.fit(X_train, y_train)
```

```
y_pred = svclassifier.predict(X_test)
```

```
from sklearn.metrics import classification_report, confusion_matrix
```

```
print(confusion_matrix(y_test, y_pred))
```

```
print(classification_report(y_test, y_pred))
```

```
[[ 7  0  0]
```

```
 [ 0 10  1]
```

```
 [ 0  0 12]]
```

	precision	recall	f1-score	support
Iris-setosa	1.00	1.00	1.00	7
Iris-versicolor	1.00	0.91	0.95	11
Iris-virginica	0.92	1.00	0.96	12
micro avg	0.97	0.97	0.97	30
macro avg	0.97	0.97	0.97	30
weighted avg	0.97	0.97	0.97	30



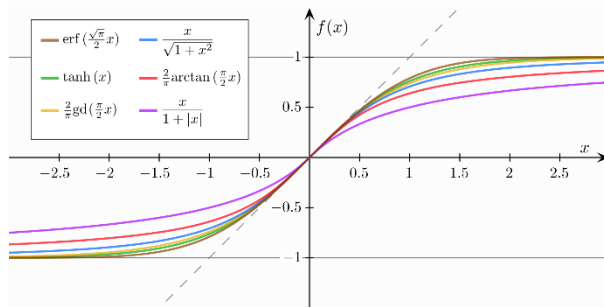
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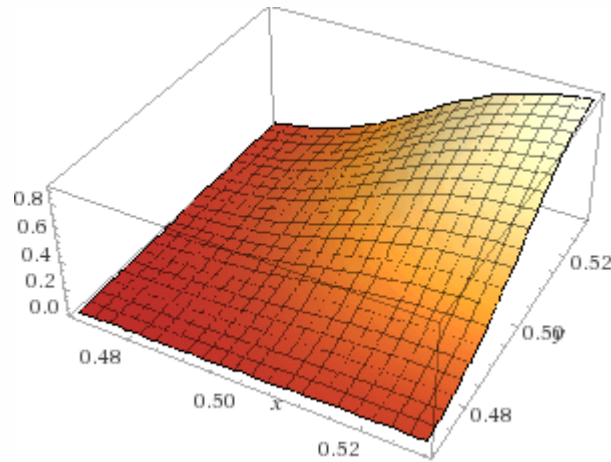
Sigmoid Kernel SVM with Scikit-Learn

1 Dimension



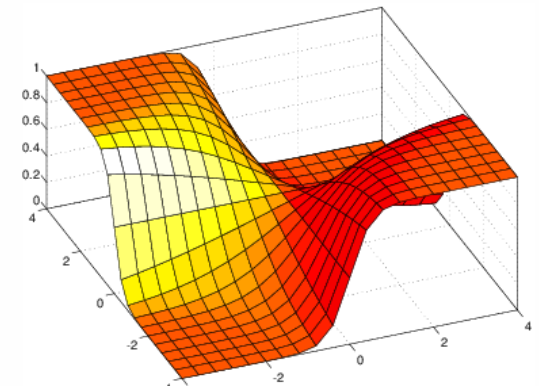
[https://en.wikipedia.org/wiki/Sigmoid_function#/media/File:Gjl-t\(x\).svg](https://en.wikipedia.org/wiki/Sigmoid_function#/media/File:Gjl-t(x).svg)

2 Dimensions



<https://math.stackexchange.com/questions/863662/need-function-for-2d-sigmoid-shaped-monotonic-surface>

3 Dimensions



https://www.researchgate.net/figure/3D-classical-sigmoid-function-f-x-T_fig1_221165140



Sigmoid Kernel SVM with Scikit-Learn

```
from sklearn.svm import SVC
```

```
svclassifier = SVC(kernel='sigmoid')
```

Kernel = sigmoid

```
svclassifier.fit(X_train, y_train)
```

```
y_pred = svclassifier.predict(X_test)
```

```
from sklearn.metrics import classification_report, confusion_matrix
```

```
print(confusion_matrix(y_test, y_pred))
```

```
print(classification_report(y_test, y_pred))
```

```
[[ 7  0  0]
```

```
 [11  0  0]
```

```
 [12  0  0]]
```

	precision	recall	f1-score	support
Iris-setosa	0.23	1.00	0.38	7
Iris-versicolor	0.00	0.00	0.00	11
Iris-virginica	0.00	0.00	0.00	12
micro avg	0.23	0.23	0.23	30
macro avg	0.08	0.33	0.13	30
weighted avg	0.05	0.23	0.09	30



Comparison of Kernel Performance

[[7 0 0]

[0 11 0]

[0 3 9]]

Kernel = Polynomial

	precision	recall	f1-score	support
Iris-setosa	1.00	1.00	1.00	7
Iris-versicolor	0.79	1.00	0.88	11
Iris-virginica	1.00	0.75	0.86	12
micro avg	0.90	0.90	0.90	30
macro avg	0.93	0.92	0.91	30
weighted avg	0.92	0.90	0.90	30

[[7 0 0]

[0 10 1]

[0 0 12]]

Kernel = Gaussian

	precision	recall	f1-score	support
Iris-setosa	1.00	1.00	1.00	7
Iris-versicolor	1.00	0.91	0.95	11
Iris-virginica	0.92	1.00	0.96	12
micro avg	0.97	0.97	0.97	30
macro avg	0.97	0.97	0.97	30
weighted avg	0.97	0.97	0.97	30

[[7 0 0]

[11 0 0]

[12 0 0]]

Kernel = sigmoid

	precision	recall	f1-score	support
Iris-setosa	0.23	1.00	0.38	7
Iris-versicolor	0.00	0.00	0.00	11
Iris-virginica	0.00	0.00	0.00	12
micro avg	0.23	0.23	0.23	30
macro avg	0.08	0.33	0.13	30
weighted avg	0.05	0.23	0.09	30



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Tuning Hyper Parameters



Tuning Hyper Parameters

```
from sklearn.model_selection import GridSearchCV
```

```
# Set the parameters by cross-validation
```

```
tuned_parameters = [{'kernel': ['rbf'], 'gamma': [1e-2, 1e-3, 1e-4, 1e-5],  
                    'C': [0.001, 0.10, 0.1, 10, 25, 50, 100, 1000]},  
                    {'kernel': ['sigmoid'], 'gamma': [1e-2, 1e-3, 1e-4, 1e-5],  
                    'C': [0.001, 0.10, 0.1, 10, 25, 50, 100, 1000]},  
                    {'kernel': ['linear'], 'C': [0.001, 0.10, 0.1, 10, 25, 50, 100, 1000]}  
]
```

```
scores = ['precision', 'recall']
```



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Tuning Hyper Parameters

for score in scores:

```
print("# Tuning hyper-parameters for %s" % score)
```

```
print()
```

```
clf = GridSearchCV(SVC(C=1), tuned_parameters, cv=5,
```

```
scoring='%s_macro' % score)
```

```
clf.fit(X_train, y_train)
```



Tuning Hyper Parameters

```
print("Best parameters set found on development set:")  
print()  
print(clf.best_params_)  
print()  
print("Grid scores on development set:")  
print()  
means = clf.cv_results_['mean_test_score']  
stds = clf.cv_results_['std_test_score']  
for mean, std, params in zip(means, stds, clf.cv_results_['params']):  
    print("%0.3f (+/-%0.03f) for %r"  
          % (mean, std * 2, params))  
print()
```




Tuning Hyper Parameters

Best parameters set found on development set:

{'C': 10, 'gamma': 0.0001, 'kernel': 'rbf'}

Grid scores on development set:

0.306 (+/-0.026) for {'C': 0.001, 'gamma': 0.01, 'kernel': 'rbf'}

0.306 (+/-0.026) for {'C': 0.001, 'gamma': 0.001, 'kernel': 'rbf'}

0.306 (+/-0.026) for {'C': 0.001, 'gamma': 0.0001, 'kernel': 'rbf'}

.....



Tuning Hyper Parameters

Tuning hyper-parameters for recall

Best parameters set found on development set:

{'C': 10, 'gamma': 0.0001, 'kernel': 'rbf'}

Grid scores on development set:

0.500 (+/-0.000) for {'C': 0.001, 'gamma': 0.01, 'kernel': 'rbf'}

0.500 (+/-0.000) for {'C': 0.001, 'gamma': 0.001, 'kernel': 'rbf'}

0.500 (+/-0.000) for {'C': 0.001, 'gamma': 0.0001, 'kernel': 'rbf'}

...



Assignment:

SVM - due on 11 November, 2022 (10 points)

1. (3 points)

From data: $(-2, 1)$ class 1, $(-2, -1)$ class -1, $(-1, -1.5)$ class -1,
 $(1, 1)$ class 1, $(1.5, -0.5)$ class 1, $(2, -2)$ class -1

Find a vector w and bias b , please show the calculation step by step as same as example 1

If the support vectors are $(1.5, -0.5)$ and $(2, -2)$

2. (3 points)

Create a SVM-model and plot a 2D-SVM classification by using Python and colon data set (use only two genes, T62947 and H64807), and find your best hyper-parameters for precision, recall, and accuracy. (Training:Testing = 80:20)

3. (4 points)

Train a SVM-model by using colon-data set and tuning the hyper-parameters, and select the best model. (Training:Testing = 80:20) and give your comments.