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Title Nonparametric Bootstrap and Permutation Tests

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Depends parallel			
Description Robust nonparametric bootstrap and permutation tests for location, correlation, and regression problems, as described in Helwig (2019a) <doi:10.1002 wics.1457=""> and Helwig (2019b) <doi:10.1016 j.neuroimage.2019.116030="">. Univariate and multivariate tests are supported. For each problem, exact tests and Monte Carlo approximations are available. Five different nonparametric bootstrap confidence intervals are implemented. Parallel computing is implemented via the 'parallel' package.</doi:10.1016></doi:10.1002>			
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nptest-package	Nonparametric Bootstrap and Permutation Tests

Description

Robust nonparametric bootstrap and permutation tests for location, correlation, and regression problems, as described in Helwig (2019a) <doi:10.1002/wics.1457> and Helwig (2019b) <doi:10.1016/j.neuroimage.2019.116030 Univariate and multivariate tests are supported. For each problem, exact tests and Monte Carlo approximations are available. Five different nonparametric bootstrap confidence intervals are implemented. Parallel computing is implemented via the 'parallel' package.

Details

The DESCRIPTION file:

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Author: Nathaniel E. Helwig <helwig@umn.edu> Maintainer: Nathaniel E. Helwig <helwig@umn.edu>

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Description: Robust nonparametric bootstrap and permutation tests for location, correlation, and regression problems, as des

License: GPL (>=2)

Index of help topics:

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Nonparametric Tests

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```
# See examples for...
# flipn (generate all sign flip vectors)
# mcse (Monte Carlo standard errors)
# np.boot (nonparametric bootstrap resampling)
# np.cor.test (nonparametric correlation tests)
# np.loc.test (nonparametric location tests)
# np.reg.test (nonparametric regression tests)
# permn (generate all permutation vectors)
```

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flipn

Generate All Sign-Flips of n Elements

Description

Generates all 2^n vectors of length n consisting of the elements -1 and 1.

Usage

flipn(n)

Arguments

n

Number of elements.

Details

Adapted from the "bincombinations" function in the e1071 R package.

Value

Matrix of dimension n by 2^n where each column contains a unique sign-flip vector.

Warning

For large n this function will consume a lot of memory and may even crash R.

Note

Used for exact tests in np.loc.test and np.reg.test.

Author(s)

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References

Meyer, D., Dimitriadou, E., Hornik, K., Weingessel, A., & Leisch, F. (2018). e1071: Misc Functions of the Department of Statistics, Probability Theory Group (Formerly: E1071), TU Wien. R package version 1.7-0. https://CRAN.R-project.org/package=e1071

Examples

flipn(2)

flipn(3)

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mcse

Monte Carlo Standard Errors for Tests

Description

This function calculates Monte Carlo standard errors for (non-exact) nonparametric tests. The MC-SEs can be used to determine (i) the accuracy of a test for a given number of resamples, or (ii) the number of resamples needed to achieve a test with a given accuracy.

Usage

```
mcse(R, delta, conf.level = 0.95, sig.level = 0.05,
    alternative = c("two.sided", "one.sided"))
```

Arguments

R	Number of resamples (positive integer).
delta	Accuracy of the approximation (number between 0 and 1).
conf.level	Confidence level for the approximation (number between 0 and 1).
sig.level	Significance level of the test (number between 0 and 1).
alternative	Alternative hypothesis (two-sided or one-sided).

Details

Note: either R or delta must be provided.

Let F(x) denote the distribution function for the full permutation distribution, and let G(x) denote the approximation obtained from R resamples. The *Monte Carlo standard error* is given by

$$\sigma(x) = \sqrt{F(x)[1 - F(x)]/R}$$

which is the standard deviation of G(x).

A symmetric confidence interval for F(x) can be approximated as

$$G(x) + / - C\sigma(x)$$

where C is some quantile of the standard normal distribution. Note that the critical value C corresponds to the confidence level (conf.level) of the approximation.

Let α denote the significance level (sig.level) for a one-sided test (α is one-half the significance level for two-sided tests). Define a to be the value of the test statistic such that $F(a) = \alpha$.

The parameter δ (delta) quantifies the accuracy of the approximation, such that

$$|G(a) - \alpha| < \alpha \delta$$

with a given confidence, which is controlled by the conf.level argument.

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Value

mcse Monte Carlo standard error.

R Number of resamples.

delta Accuracy of approximation.

conf.level Confidence level.

sig.level Significance level.

alternative Alternative hypothesis.

Note

This function is only relevant for non-exact tests. For exact tests, F(x) = G(x) so the Monte Carlo standard error is zero.

Author(s)

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References

Helwig, N. E. (2019). Statistical nonparametric mapping: Multivariate permutation tests for location, correlation, and regression problems in neuroimaging. WIREs Computational Statistics, 11(2), e1457. doi: 10.1002/wics.1457

See Also

```
np.cor.test, np.loc.test, np.reg.test
```

```
###***###
           EXAMPLE 1
                       ###***###
# get the Monte Carlo standard error and the
# accuracy (i.e., delta) for given R = 10000
# using the default two-sided alternative hypothesis,
# the default confidence level (conf.level = 0.95),
# and the default significance level (sig.level = 0.05)
mcse(R = 10000)
# se = 0.0016
# delta = 0.1224
###***###
           EXAMPLE 2
                       ###***###
# get the Monte Carlo standard error and the
# number of resamples (i.e., R) for given delta = 0.01
# using a one-sided alternative hypothesis,
```

```
# the default confidence level (conf.level = 0.95),
# and the default significance level (sig.level = 0.05)
mcse(delta = 0.1, alternative = "one.sided")
# se = 0.0026
# R = 7299
```

np.boot

Nonparametric Bootstrap Resampling

Description

Nonparametric bootstrap resampling for univariate and multivariate statistics. Computes bootstrap estimates of the standard error, bias, and covariance. Also computes five different types of bootstrap confidence intervals: normal approximation interval, basic (reverse percentile) interval, percentile interval, studentized (bootstrap-t) interval, and bias-corrected and accelerated (BCa) interval.

Usage

```
np.boot(x, statistic, ..., R = 9999, level = c(0.9, 0.95, 0.99),
    method = c("norm", "basic", "perc", "stud", "bca")[-4],
    sdfun = NULL, sdrep = 99, jackknife = NULL,
    parallel = FALSE, cl = NULL, boot.dist = TRUE)
```

Arguments

х	vector of data (for univariate data) or vector of row indices (for multivariate data). See examples for bootstrapping multivariate data.
statistic	function that takes in x (and possibly additional arguments passed using) and returns a vector containing the statistic(s). See examples.
	additional named arguments for the statistic function.
R	number of bootstrap replicates
level	desired confidence level(s) for the computed intervals. Default computes 90%, 95%, and 99% confidence intervals.
method	method(s) for computing confidence intervals. Partial matching is allowed. Any subset of allowable methods is permitted (default computes all intervals except studentized). Set method = NULL to produce no confidence intervals.
sdfun	function for computing the standard deviation of statistic. Should produce a vector the same length as the output of statistic. Only applicable if "stud" %in% method. If NULL, an inner bootstrap is used to estimate the standard deviation.
sdrep	number of bootstrap replicates for the inner bootstrap used to estimate the standard deviation of statistic. Only applicable if "stud" %in% method and sdfun

= NULL. Larger values produce more accurate estimates (see Note).

jackknife function that takes in x (and possibly additional arguments passed using ...)
and returns a vector containing the jackknife statistic(s). Should produce a vector the same length as the output of statistic. Only applicable if "bca" %in% method. If NULL, the jackknife function is defined as the statistic function (default). See the last example for a case when statistic and jackknife are different.

parallel Logical indicating if the parallel package should be used for parallel computing (of the bootstrap distribution). Defaults to FALSE, which implements sequential computing.

Cluster for parallel computing, which is used when parallel = TRUE. Note that if parallel = TRUE and cl = NULL, then the cluster is defined as makeCluster(detectCores()).

,

boot.dist Logical indicating if the bootstrap distribution should be returned (see Note).

Details

cl

The first three intervals (normal, basic, and percentile) are only first-order accurate, whereas the last two intervals (studentized and BCa) are both second-order accurate. Thus, the results from the studentized and BCa intervals tend to provide more accurate coverage rates.

Unless the standard deviation function for the studentized interval is input via the sdfun argument, the studentized interval can be quite computationally costly. This is because an inner bootstrap is needed to estimate the standard deviation of the statistic for each (outer) bootstrap replicate—and you may want to increase the default number of inner bootstrap replicates (see Note).

The efficiency of the BCa interval will depend on the sample size n and the computational complexity of the (jackknife) statistic estimate. Assuming that n is not too large and the jackknife statistic is not too difficult to compute, the BCa interval can be computed reasonably quickly—especially in comparison the studentized interval with an inner bootstrap.

Computational details of the various confidence intervals are described in Efron and Tibshirani (1994) and in Davison and Hinkley (1997). For a useful and concise discussion of the various intervals, see Carpenter and Bithell (2000).

Value

t0	Observed statistic, computed using statistic(x,)
se	Bootstrap estimate of the standard error.
bias	Bootstrap estimate of the bias.
cov	Bootstrap estimate of the covariance (for multivariate statistics).
normal	Normal approximation confidence interval(s).
basic	Basic (reverse percentile) confidence interval(s).
percent	Percentile confidence interval(s).
student	Studentized (bootstrap-t) confidence interval(s).
bca	Bias-corrected and accelerated (BCa) confidence interval(s).
z0	Bias-correction factor(s). Only provided if bca %in% method.
acc	Acceleration factor(s). Only provided if bca %in% method.
boot.dist	Bootstrap distribution of statistic(s). Only provided if boot.dist = TRUE.

R	Number	of bootstrap	replicates ((same as input).

level Confidence level (same as input).

sdfun Standard deviation function for statistic (same as input).

Sdrep Number of inner bootstrap replicates (same as input).

jackknife Jackknife function (same as input).

Note

If boot.dist = TRUE, the output boot.dist will be a matrix of dimension R by length(statistic(x,...)) if the statistic is multivariate. Otherwise the bootstrap distribution will be a vector of length R.

For the "stud" method, the default of sdrep = 99 may produce a crude estimate of the standard deviation of the statistic(s). For more accurate estimates, the value of sdrep may need to be set substantially larger, e.g., sdrep = 999.

Author(s)

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References

Carpenter, J., & Bithell, J. (2000). Bootstrap confidence intervals: when, which, what? A practical guide for medical statisticians. *Statistics in Medicine*, *19*(9), 1141-1164. doi: 10.1002/(SICI)1097-0258(20000515)19:9%3C1141::AID-SIM479%3E3.0.CO;2-F

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Efron, B., & Tibshirani, R. J. (1994). *An Introduction to the Boostrap*. Chapman & Hall/CRC. doi: 10.1201/9780429246593

```
x <- rnorm(n)</pre>
# nonparametric bootstrap
npbs \leftarrow np.boot(x = x, statistic = quantile,
                 probs = c(0.25, 0.5, 0.75)
npbs
######***##### MULTIVARIATE DATA ######***#####
### Example 1: univariate statistic (correlation)
# correlation matrix square root (with rho = 0.5)
rho <- 0.5
val \leftarrow c(sqrt(1 + rho), sqrt(1 - rho))
corsqrt <- matrix(c(val[1], -val[2], val), 2, 2) / sqrt(2)</pre>
# generate 100 bivariate observations (with rho = 0.5)
n <- 100
set.seed(1)
data <- cbind(rnorm(n), rnorm(n)) %*% corsqrt</pre>
# define statistic function
tatfun \leftarrow function(x, data) cor(data[x,1], data[x,2])
# nonparametric bootstrap
npbs <- np.boot(x = 1:n, statistic = statfun, data = data)</pre>
npbs
### Example 2: multivariate statistic (variances and covariance)
# correlation matrix square root (with rho = 0.5)
rho <- 0.5
val \leftarrow c(sqrt(1 + rho), sqrt(1 - rho))
corsqrt <- matrix(c(val[1], -val[2], val), 2, 2) / sqrt(2)</pre>
# generate 100 bivariate observations (with rho = 0.5)
n <- 100
set.seed(1)
data <- cbind(rnorm(n), rnorm(n)) %*% corsqrt</pre>
# define statistic function
statfun <- function(x, data) {</pre>
  cmat <- cov(data[x,])</pre>
 ltri <- lower.tri(cmat, diag = TRUE)</pre>
  cvec <- cmat[ltri]</pre>
  names(cvec) <- c("var(x1)", "cov(x1,x2)", "var(x2)")
  cvec
}
# nonparametric bootstrap
```

```
npbs <- np.boot(x = 1:n, statistic = statfun, data = data)</pre>
npbs
## Not run:
#####***#####
                   REGRESSION
                                 ######***######
### Example 1: bootstrap cases
# generate 100 observations
n <- 100
set.seed(1)
x \leftarrow seq(0, 1, length.out = n)
y < -1 + 2 * x + rnorm(n)
data <- data.frame(x = x, y = y)
# define statistic function
statfun <- function(x, data) {</pre>
  xmat <- cbind(1, data$x[x])</pre>
  xinv <- solve(crossprod(xmat)) %*% t(xmat)</pre>
  coef <- as.numeric(xinv %*% data$y[x])</pre>
  names(coef) \leftarrow c("(Intercept)", "x")
  coef
}
# nonparametric bootstrap
npbs \leftarrow np.boot(x = 1:n, statistic = statfun, data = data)
npbs
### Example 2: bootstrap residuals
# generate 100 observations
n <- 100
set.seed(1)
x \leftarrow seq(0, 1, length.out = n)
y < -1 + 2 * x + rnorm(n)
# prepare data
xmat <- cbind(1, x)
xinv <- solve(crossprod(xmat)) %*% t(xmat)</pre>
fit <- xmat %*% xinv %*% y
data <- list(fit = fit, resid = y - fit, xinv = xinv, x = x)</pre>
# define statistic function
statfun <- function(x, data) {</pre>
  ynew <- data$fit + data$resid[x]</pre>
  coef <- as.numeric(data$xinv %*% ynew)</pre>
  names(coef) \leftarrow c("(Intercept)", "x")
  coef
}
```

np.cor.test 13

np.cor.test

Nonparametric Tests of Correlation Coefficients

Description

Denoting the Pearson product-moment correlation coefficient as

$$\rho = Cov(X, Y) / \sqrt{Var(X)Var(Y)}$$

this function implements permutation tests of H_0 : $\rho = \rho_0$ where ρ_0 is the user-specified null value. Can also implement tests of partial correlations, semi-partial (or part) correlations, and independence.

Usage

Arguments

```
x X vector (n by 1).

y Y vector (n by 1).

z Optional Z matrix (n by q). If provided, the partial (or semi-partial if partial = FALSE) correlation is calculated between x and y controlling for z.

alternative Alternative hypothesis. Must be either "two.sided" (H_1: \rho \neq \rho_0), "less" (H_1: \rho < \rho_0), or "greater" (H_1: \rho > \rho_0).
```

np.cor.test

rho Null hypothesis value ρ_0 . Defaults to zero.

independent If FALSE (default), the null hypothesis is $H_0: \rho = \rho_0$. Otherwise, the null hyth-

pothesis is that X and Y are independent, i.e., $H_0: F_{XY}(x,y) = F_X(x)F_Y(y)$.

partial Only applicable if z is provided. If TRUE (default), the partial correlation be-

tween x and y controlling for z is tested. Otherwise the semi-partial correlation

is tested. See Details.

R Number of resamples for the permutation test (positive integer).

parallel Logical indicating if the parallel package should be used for parallel com-

puting (of the permutation distribution). Defaults to FALSE, which implements

sequential computing.

cl Cluster for parallel computing, which is used when parallel = TRUE. Note that

if parallel = TRUE and cl = NULL, then the cluster is defined as makeCluster(detectCores()).

perm. dist Logical indicating if the permutation distribution should be returned.

Details

Default use of this function tests the Pearson correlation between X and Y using the studentized test statistic proposed by DiCiccio and Romano (2017). If independent = TRUE, the classic (unstudentized) test statistic is used to test the null hypothesis of independence.

If Z is provided, the partial or semi-partial correlation between X and Y controlling for Z is tested. For the semi-partial correlation, the effect of Z is partialled out of X.

Value

statistic Test statistic value.

p.value p-value for testing $H_0: \rho = \rho_0$ or $H_0: F_{XY}(x,y) = F_X(x)F_Y(y)$.

perm.dist Permutation distribution of statistic.

alternative Alternative hypothesis.

null.value Null hypothesis value for ρ .

independent Independence test?

R Number of resamples.

exact Exact permutation test? See Note.

estimate Sample estimate of correlation coefficient ρ .

Note

The permutation test will be exact when the requested number of resamples R is greater than factorial(n) minus one. In this case, the permutation distribution perm. dist contains all factorial(n) possible values of the test statistic.

If z = NULL, the result will be the same as using np.reg.test with method = "perm".

If z is supplied and partial = TRUE, the result will be the same as using np.reg.test with method = "KC" and homosced = FALSE.

Author(s)

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References

DiCiccio, C. J., & Romano, J. P. (2017). Robust permutation tests for correlation and regression coefficients. Journal of the American Statistical Association, 112(519), 1211-1220. doi: 10.1080/01621459.2016.1202117

Helwig, N. E. (2019). Statistical nonparametric mapping: Multivariate permutation tests for location, correlation, and regression problems in neuroimaging. WIREs Computational Statistics, 11(2), e1457. doi: 10.1002/wics.1457

Pitman, E. J. G. (1937b). Significance tests which may be applied to samples from any populations. ii. the correlation coefficient test. Supplement to the Journal of the Royal Statistical Society, 4(2), 225-232. doi: 10.2307/2983647

See Also

plot.np.cor.test S3 plotting method for visualizing the results

```
# generate data
rho <- 0.5
val <- c(sqrt(1 + rho), sqrt(1 - rho))
corsqrt <- matrix(c(val[1], -val[2], val), 2, 2) / sqrt(2)
set.seed(1)
n <- 10
z <- cbind(rnorm(n), rnorm(n)) %*% corsqrt
x <- z[,1]
y <- z[,2]
# test H0: rho = 0
set.seed(0)
np.cor.test(x, y)
# test H0: X and Y are independent
set.seed(0)
np.cor.test(x, y, independent = TRUE)</pre>
```

Description

Performs one and two sample nonparametric (randomization) tests of location parameters, i.e., means and medians. Implements univariate and multivariate tests using eight different test statistics: Student's one-sample t-test, Johnson's modified t-test, Wilcoxon's Signed Rank test, Fisher's Sign test, Student's two-sample t-test, Welch's t-test, Wilcoxon's Rank Sum test (i.e., Mann-Whitney's U test), and a studentized Wilcoxon test for unequal variances.

Usage

Arguments

٠	8	
	x	Numeric vector (or matrix) of data values.
	у	Optional numeric vector (or matrix) of data values.
	alternative	Alternative hypothesis. Must be either "two.sided" $(H_1: \mu \neq \mu_0)$, "less" $(H_1: \mu < \mu_0)$, or "greater" $(H_1: \mu > \mu_0)$.
	mu	Null hypothesis value μ_0 . Defaults to zero.
	paired	Logical indicating whether you want a paired location test.
	var.equal	Logical indicating whether to treat the two variances as being equal.
	median.test	Logical indicating whether the location test is for the median. Default is FALSE, i.e., μ is the mean.
	symmetric	Logical indicating if the distribution of x should be assumed to be symmetric around μ . Only used for one (or paired) sample tests.
	R	Number of resamples for the permutation test (positive integer).
	parallel	Logical indicating if the parallel package should be used for parallel computing (of the permutation distribution). Defaults to FALSE, which implements sequential computing.
	cl	Cluster for parallel computing, which is used when parallel = TRUE. Note that if parallel = TRUE and cl = NULL, then the cluster is defined as makeCluster(detectCores()).

Details

perm.dist

```
One sample \mu is the mean (or median) of X
Paired \mu is the mean (or median) of X-Y
Two sample \mu is the mean difference E(X)-E(Y)
or the median of the differences X-Y
```

Logical indicating if the permutation distribution should be returned.

For one (or paired) sample tests, the different test statistics can be obtained using

For two sample tests, the different test statistics can be obtained using

Value

statistic Test statistic value.

p. value p-value for testing $H_0: \mu = \mu_0$.

perm.dist Permutation distribution of statistic.

alternative Alternative hypothesis. null.value Null hypothesis value for μ .

var.equal Assuming equal variances? Only for two sample tests.

median.test Testing the median?

symmetric Assuming symmetry? Only for one sample and paired tests.

R Number of resamples.

exact Exact permutation test? See Note.

estimate Estimate of parameter μ .

univariate Univariate test statistic value for *j*-th variable (for multivariate input).

adj.p.value Adjusted p-value for testing significance of *j*-th variable (for multivariate input).

method Method used for permutation test. See Details.

Multivariate Tests

If the input x (and possibly y) is a matrix with m>1 columns, the multivariate test statistic is defined as

alternative statistic

two.sided max(abs(univariate))
less min(univariate)
greater max(univariate)

The global null hypothesis (across all m variables) is tested by comparing the observed statistic to the permutation distribution perm. dist. This produces the p.value for testing the global null

hypothesis.

The local null hypothesis (separately for each variable) is tested by comparing the univariate test statistic to perm. dist. This produces the adjusted p-values (adj.p.values), which control the familywise Type I error rate across the m tests.

Note

For one sample (or paired) tests, the permutation test will be exact when the requested number of resamples R is greater than 2ⁿ minus one. In this case, the permutation distribution perm.dist contains all 2ⁿ possible values of the test statistic.

For two sample tests, the permutation test will be exact when the requested number of resamples R is greater than choose(N,n) minus one, where m = length(x), n = length(y), and N = m + n. In this case, the permutation distribution perm.dist contains all choose(N,n) possible values of the test statistic.

Author(s)

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See Also

plot.np.loc.test S3 plotting method for visualizing the results

```
######*****##### UNIVARIATE #####*****
###***### ONE SAMPLE ###***###
# generate data
set.seed(1)
n <- 10
x <- rnorm(n, mean = 0.5)
# one sample t-test
set.seed(0)
np.loc.test(x)
# Johnson t-test
set.seed(0)
np.loc.test(x, symmetric = FALSE)
# Wilcoxon signed rank test
set.seed(0)
np.loc.test(x, median.test = TRUE)
# Fisher sign test
set.seed(0)
np.loc.test(x, median.test = TRUE, symmetric = FALSE)
###***### PAIRED SAMPLE ###***###
# generate data
set.seed(1)
n <- 10
x <- rnorm(n, mean = 0.5)
y <- rnorm(n)</pre>
# paired t-test
set.seed(0)
np.loc.test(x, y, paired = TRUE)
# paired Johnson t-test
set.seed(0)
np.loc.test(x, y, paired = TRUE, symmetric = FALSE)
# paired Wilcoxon signed rank test
set.seed(0)
np.loc.test(x, y, paired = TRUE, median.test = TRUE)
```

```
# paired Fisher sign test
set.seed(0)
np.loc.test(x, y, paired = TRUE, median.test = TRUE, symmetric = FALSE)
###***### TWO SAMPLE ###***###
# generate data
set.seed(1)
m <- 7
n <- 8
x <- rnorm(m, mean = 0.5)
y <- rnorm(n)
# Welch t-test
set.seed(0)
np.loc.test(x, y)
# Student t-test
set.seed(0)
np.loc.test(x, y, var.equal = TRUE)
# Studentized Wilcoxon test
set.seed(0)
np.loc.test(x, y, median.test = TRUE)
# Wilcoxon rank sum test
set.seed(0)
np.loc.test(x, y, var.equal = TRUE, median.test = TRUE)
## Not run:
######*****##### MULTIVARIATE #####*****#####
###***### ONE SAMPLE ###***###
# generate data
set.seed(1)
n <- 10
x \leftarrow cbind(rnorm(n, mean = 0.5),
          rnorm(n, mean = 1),
           rnorm(n, mean = 1.5))
# multivariate one sample t-test
set.seed(0)
ptest <- np.loc.test(x)</pre>
ptest
ptest$univariate
ptest$adj.p.values
```

```
###***###
            PAIRED SAMPLE
                              ###***###
# generate data
set.seed(1)
n <- 10
x \leftarrow cbind(rnorm(n, mean = 0.5),
            rnorm(n, mean = 1),
           rnorm(n, mean = 1.5))
y \leftarrow matrix(rnorm(n * 3), nrow = n, ncol = 3)
# multivariate paired t-test
set.seed(0)
ptest <- np.loc.test(x, y, paired = TRUE)</pre>
ptest
ptest$univariate
ptest$adj.p.values
###***###
            TWO SAMPLE
                          ###***###
# generate data
set.seed(1)
m <- 7
n <- 8
x \leftarrow cbind(rnorm(m, mean = 0.5),
            rnorm(m, mean = 1),
            rnorm(m, mean = 1.5))
y \leftarrow matrix(rnorm(n * 3), nrow = n, ncol = 3)
# multivariate Welch t-test
set.seed(0)
ptest <- np.loc.test(x, y)</pre>
ptest$univariate
ptest$adj.p.values
## End(Not run)
```

np.reg.test

Nonparametric Tests of Regression Coefficients

Description

Assuming a linear model of the form

$$Y = \alpha + X\beta + \epsilon$$

or

$$Y = \alpha + X\beta + Z\gamma + \epsilon$$

this function implements permutation tests of H_0 : $\beta=\beta_0$ where β_0 is the user-specified null vector.

Usage

Arguments

X	Matrix of predictor variables (n by p).
у	Response vector or matrix (n by m).
z	Optional matrix of nuisance variables (n by q).
method	Permutation method. See Details.
beta	Null hypothesis value for β (p by m). Defaults to matrix of zeros.
homosced	Are the ϵ terms homoscedastic? If FALSE (default), a robust Wald test statistic is used. Otherwise the classic F test statistic is used.
lambda	Scalar or vector of ridge parameter(s). Defaults to vector of zeros.
R	Number of resamples for the permutation test (positive integer).
parallel	Logical indicating if the parallel package should be used for parallel computing (of the permutation distribution). Defaults to FALSE, which implements sequential computing.
cl	Cluster for parallel computing, which is used when parallel = TRUE. Note that if parallel = TRUE and cl = NULL, then the cluster is defined as makeCluster(detectCores()).
perm.dist	Logical indicating if the permutation distribution should be returned.

Details

With no nuisance variables in the model (i.e., z = NULL), there are three possible options for the method argument:

Method	Model
perm	$PY = \alpha + X\beta + \epsilon$
flip	$SY = \alpha + X\beta + \epsilon$
both	$PSY = \alpha + X\beta + \epsilon$

where ${\cal P}$ is a permutation matrix and ${\cal S}$ is a sign-flipping matrix.

With nuisance variables in the model, there are eight possible options for the method argument:

Method	Name	Model
HJ	Huh-Jhun	$PQ'R_zY = \alpha + Q'R_zX\beta + \epsilon$
KC	Kennedy-Cade	$PR_zY = \alpha + R_zX\beta + \epsilon$
SW	Still-White	$PR_zY = \alpha + X\beta + \epsilon$
TB	ter Braak	$(PR_m + H_m)Y = \alpha + X\beta + Z\gamma + \epsilon$

FL	Freedman-Lane	$(PR_z + H_z)Y = \alpha + X\beta + Z\gamma + \epsilon$
MA	Manly	$PY = \alpha + X\beta + Z\gamma + \epsilon$
OS	O'Gorman-Smith	$Y = \alpha + PR_z X\beta + Z\gamma + \epsilon$
DS	Draper-Stoneman	$Y = \alpha + PX\beta + Z\gamma + \epsilon$

where P is permutation matrix and Q is defined as $R_z = QQ'$ with Q'Q = I.

Note that H_z is the hat matrix for the nuisance variable design matrix, and $R_z = I - H_z$ is the corresponding residual forming matrix. Similarly, H_m and R_m are the hat and residual forming matrices for the full model including the predictor and nuisance variables.

Value

statistic Test statistic value.

p. value p-value for testing $H_0: \beta = \beta_0$.

perm.dist Permutation distribution of statistic.

method Permutation method.

null.value Null hypothesis value for β . homosced Homoscedastic errors? R Number of resamples.

exact Exact permutation test? See Note.

coefficients Least squares estimates of α , β , and γ (if applicable).

univariate Univariate test statistic value for j-th variable (for multivariate inputs).

adj.p.value Adjusted p-value for testing significance of j-th variable (for multivariate in-

puts).

Multivariate Tests

If the input y is a matrix with m>1 columns, the multivariate test statistic is defined as statistic = max(univariate) given that the univariate test statistics are non-negative.

The global null hypothesis (across all m variables) is tested by comparing the observed statistic to the permutation distribution perm.dist. This produces the p.value for testing the global null hypothesis.

The local null hypothesis (separately for each variable) is tested by comparing the univariate test statistic to perm.dist. This produces the adjusted p-values (adj.p.values), which control the familywise Type I error rate across the m tests.

Note

If method = "flip", the permutation test will be exact when the requested number of resamples R is greater than 2^n minus one. In this case, the permutation distribution perm. dist contains all 2^n possible values of the test statistic.

If method = "both", the permutation test will be exact when the requested number of resamples R is greater than factorial(n) * (2^n) minus one. In this case, the permutation distribution perm.dist contains all factorial(n) * (2^n) possible values of the test statistic.

If method = "HJ", the permutation test will be exact when the requested number of resamples R is greater than factorial(n-q-1) minus one. In this case, the permutation distribution perm.dist contains all factorial(n-q-1) possible values of the test statistic.

Otherwise the permutation test will be exact when the requested number of resamples R is greater than factorial(n) minus one. In this case, the permutation distribution perm.dist contains all factorial(n) possible values of the test statistic.

Author(s)

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See Also

```
plot.np.reg.test S3 plotting method for visualizing the results
```

```
######****
                    UNIVARIATE
                                ######****
###***###
          TEST ALL COEFFICIENTS ###***###
# generate data
set.seed(1)
n <- 10
x <- cbind(rnorm(n), rnorm(n))</pre>
y <- rnorm(n)</pre>
# Wald test (method = "perm")
set.seed(0)
np.reg.test(x, y)
# F test (method = "perm")
set.seed(0)
np.reg.test(x, y, homosced = TRUE)
###***###
           TEST SUBSET OF COEFFICIENTS ###***###
# generate data
set.seed(1)
n <- 10
x <- rnorm(n)
z <- rnorm(n)</pre>
y < -3 + 2 * z + rnorm(n)
# Wald test (method = "HJ")
set.seed(0)
np.reg.test(x, y, z)
# F test (method = "HJ")
set.seed(0)
np.reg.test(x, y, z, homosced = TRUE)
## Not run:
######****
                   MULTIVARIATE
                                   ######****
###***###
          TEST ALL COEFFICIENTS
                                   ###***###
```

26 permn

```
# generate data
set.seed(1)
n <- 10
x <- cbind(rnorm(n), rnorm(n))</pre>
y \leftarrow matrix(rnorm(n * 3), nrow = n, ncol = 3)
# multivariate Wald test (method = "perm")
set.seed(0)
np.reg.test(x, y)
# multivariate F test (method = "perm")
set.seed(0)
np.reg.test(x, y, homosced = TRUE)
###***###
           TEST SUBSET OF COEFFICIENTS ###***###
# generate data
set.seed(1)
n <- 10
x <- rnorm(n)
z <- rnorm(n)</pre>
y \leftarrow cbind(1 + 3 * z + rnorm(n),
           2 + 2 * z + rnorm(n),
           3 + 1 * z + rnorm(n)
# multivariate Wald test (method = "HJ")
set.seed(0)
np.reg.test(x, y, z)
# multivariate F test (method = "HJ")
set.seed(0)
np.reg.test(x, y, z, homosced = TRUE)
## End(Not run)
```

permn

Generate All Permutations of n Elements

Description

Generates all n! vectors of length n consisting of permutations of the integers 1 to n.

Usage

```
permn(n)
```

Arguments

n

Number of elements.

Details

Adapted from the "permutations" function in the e1071 R package.

Value

Matrix of dimension n by n! where each column contains a unique permutation vector.

Warning

For large n this function will consume a lot of memory and may even crash R.

Note

Used for exact tests in np.cor.test and np.reg.test.

Author(s)

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References

Meyer, D., Dimitriadou, E., Hornik, K., Weingessel, A., & Leisch, F. (2018). e1071: Misc Functions of the Department of Statistics, Probability Theory Group (Formerly: E1071), TU Wien. R package version 1.7-0. https://CRAN.R-project.org/package=e1071

Examples

permn(2)

permn(3)

plot

Plots Permutation Distribution for Nonparametric Tests

Description

plot methods for object classes "np.cor.test", "np.loc.test", and "np.reg.test"

Usage

Arguments

х	an object of class "np.cor.test" output by the $np.cor.test$ function, "np.loc.test" output by the $np.loc.test$ function, or "np.reg.test" output by the $np.reg.test$ function
alpha	significance level of the nonparametric test
col	color for plotting the non-rejection region
col.rr	color for plotting the rejection region
col.stat	color for plotting the observed test statistic
lty.stat	line type for plotting the observed test statistic
lwd.stat	line width for plotting the observed test statistic
xlab	x-axis label for the plot
main	title for the plot
breaks	defines the breaks of the histogram (see hist)
border	color of the border around the bars
box	should a box be drawn around the plot?
SQRT	for regression tests, should the permutation distribution (and test statistic) be plotted on the square-root scale?
	additional arguments to be passed to hist

Details

Plots a histogram of the permutation distribution and the observed test statistic. The argument 'alpha' controls the rejection region of the nonparametric test, which is plotted using a separate color (default is red).

Author(s)

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References

Helwig, N. E. (2019). Statistical nonparametric mapping: Multivariate permutation tests for location, correlation, and regression problems in neuroimaging. WIREs Computational Statistics, 11(2), e1457. doi: 10.1002/wics.1457

See Also

```
np.cor.test for information on nonparametric correlation tests
np.loc.test for information on nonparametric location tests
np.reg.test for information on nonparametric regression tests
```

```
np.cor.test #####*****#####
######**
# generate data
rho <- 0.5
val \leftarrow c(sqrt(1 + rho), sqrt(1 - rho))
corsqrt <- matrix(c(val[1], -val[2], val), 2, 2) / sqrt(2)</pre>
set.seed(1)
n <- 50
z <- cbind(rnorm(n), rnorm(n)) %*% corsqrt</pre>
x < -z[,1]
y < -z[,2]
# test H0: rho = 0
set.seed(0)
test <- np.cor.test(x, y)</pre>
# plot results
plot(test)
######***
                     np.loc.test #####*****#####
# generate data
set.seed(1)
n <- 50
x <- rnorm(n, mean = 0.5)
# one sample t-test
set.seed(0)
test <- np.loc.test(x)</pre>
# plot results
plot(test)
```

```
#####******##### np.reg.test #####******#####

# generate data
set.seed(1)
n <- 50
x <- cbind(rnorm(n), rnorm(n))
beta <- c(0.25, 0.5)
y <- x %*% beta + rnorm(n)

# Wald test (method = "perm")
set.seed(0)
test <- np.reg.test(x, y)

# plot results
plot(test)</pre>
```

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