Master Thesis Round Three

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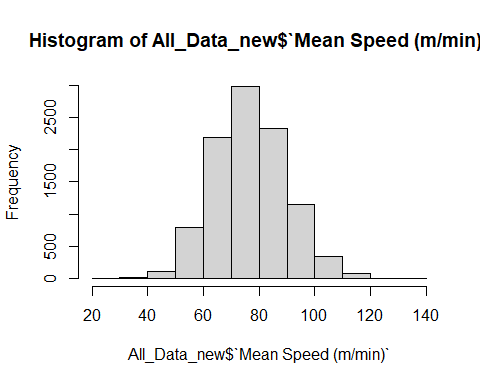
# FLOW Vs SPEED + DENSITY

## Enter Data

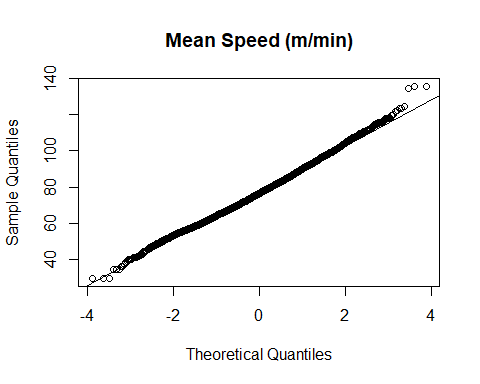
All\_Data\_new = read\_excel("Final\_Working\_Data\_File.xlsx",   
 sheet = "Big\_Data\_SPSS")

## Test normality of data

hist(All\_Data\_new$`Mean Speed (m/min)`)



qqnorm(All\_Data\_new$`Mean Speed (m/min)`, main='Mean Speed (m/min)')  
qqline(All\_Data\_new$`Mean Speed (m/min)`)



##### — One-sample Kolmogorov-Smirnov test

##   
## One-sample Kolmogorov-Smirnov test  
##   
## data: All\_Data\_new$`Mean Speed (m/min)`  
## D = 1, p-value < 2.2e-16  
## alternative hypothesis: two-sided

##   
## One-sample Kolmogorov-Smirnov test  
##   
## data: All\_Data\_new$`Pedestrian Density (Peds/m2)`  
## D = 0.86105, p-value < 2.2e-16  
## alternative hypothesis: two-sided

##   
## One-sample Kolmogorov-Smirnov test  
##   
## data: All\_Data\_new$`Pedestrian Flow (peds/m/min)`  
## D = 0.98761, p-value < 2.2e-16  
## alternative hypothesis: two-sided

#### — Lilliefors (Kolmogorov-Smirnov) test for normality

##   
## Lilliefors (Kolmogorov-Smirnov) normality test  
##   
## data: All\_Data\_new$`Mean Speed (m/min)`  
## D = 0.019794, p-value = 2.033e-09

##   
## Lilliefors (Kolmogorov-Smirnov) normality test  
##   
## data: All\_Data\_new$`Pedestrian Density (Peds/m2)`  
## D = 0.010606, p-value = 0.01325

##   
## Lilliefors (Kolmogorov-Smirnov) normality test  
##   
## data: All\_Data\_new$`Pedestrian Flow (peds/m/min)`  
## D = 0.018775, p-value = 1.999e-08

#### — Anderson-Darling test for normality

##   
## Anderson-Darling normality test  
##   
## data: All\_Data\_new$`Mean Speed (m/min)`  
## A = 6.7262, p-value < 2.2e-16

##   
## Anderson-Darling normality test  
##   
## data: All\_Data\_new$`Pedestrian Density (Peds/m2)`  
## A = 2.2497, p-value = 1.058e-05

##   
## Anderson-Darling normality test  
##   
## data: All\_Data\_new$`Pedestrian Flow (peds/m/min)`  
## A = 5.4227, p-value = 2.256e-13

#### — Cramer-von Mises test for normality

##   
## Cramer-von Mises normality test  
##   
## data: All\_Data\_new$`Mean Speed (m/min)`  
## W = 1.0138, p-value = 1.361e-09

##   
## Cramer-von Mises normality test  
##   
## data: All\_Data\_new$`Pedestrian Density (Peds/m2)`  
## W = 0.29058, p-value = 0.0004282

##   
## Cramer-von Mises normality test  
##   
## data: All\_Data\_new$`Pedestrian Flow (peds/m/min)`  
## W = 0.87956, p-value = 5.171e-09

#### — Pearson chi-square test for normality

nortest::pearson.test(All\_Data\_new$`Mean Speed (m/min)`)

##   
## Pearson chi-square normality test  
##   
## data: All\_Data\_new$`Mean Speed (m/min)`  
## P = 207.49, p-value = 6.405e-14

nortest::pearson.test(All\_Data\_new$`Pedestrian Density (Peds/m2)`)

##   
## Pearson chi-square normality test  
##   
## data: All\_Data\_new$`Pedestrian Density (Peds/m2)`  
## P = 201.6, p-value = 4.201e-13

nortest::pearson.test(All\_Data\_new$`Pedestrian Flow (peds/m/min)`)

##   
## Pearson chi-square normality test  
##   
## data: All\_Data\_new$`Pedestrian Flow (peds/m/min)`  
## P = 171.32, p-value = 3.923e-09

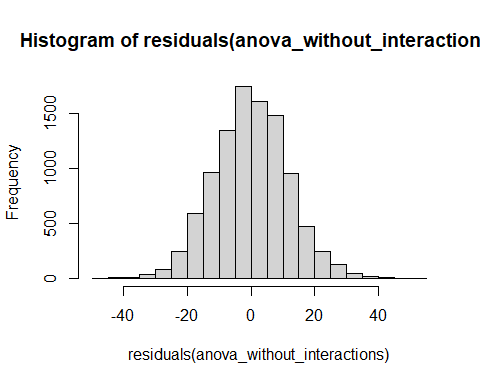
## —- Run ANOVA or non-parametric alternative

### TWO WAY ANOVA WITHOUT INTERACTION EFFECTS

anova\_without\_interactions <- aov(`Mean Speed (m/min)` ~ Location +  
 Time + Sex,   
 data = All\_Data\_new)  
summary(anova\_without\_interactions)

## Df Sum Sq Mean Sq F value Pr(>F)   
## Location 3 21242 7081 52.77 < 2e-16 \*\*\*  
## Time 1 3023 3023 22.53 2.1e-06 \*\*\*  
## Sex 1 303614 303614 2262.81 < 2e-16 \*\*\*  
## Residuals 9991 1340548 134   
## ---  
## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

hist(residuals(anova\_without\_interactions))



nortest::ad.test(residuals(anova\_without\_interactions))

##   
## Anderson-Darling normality test  
##   
## data: residuals(anova\_without\_interactions)  
## A = 1.6106, p-value = 0.0003883

We can see clearly that the data is **NOT normally distributed.**

### TWO WAY ANOVA WITH INTERACTION EFFECTS

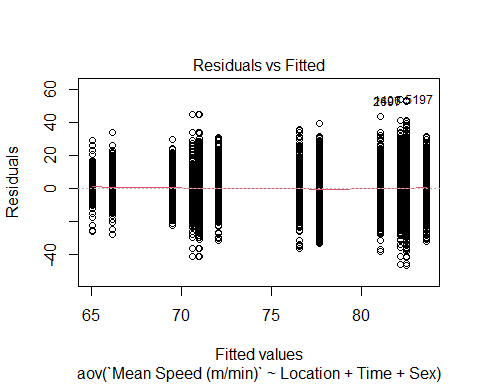
anova\_with\_interactions <- aov(`Mean Speed (m/min)` ~ Location + Time   
 + Sex + Location:Time + Location:Sex +  
 Time:Sex,   
 data = All\_Data\_new)  
  
summary(anova\_with\_interactions)

## Df Sum Sq Mean Sq F value Pr(>F)   
## Location 3 21242 7081 52.874 < 2e-16 \*\*\*  
## Time 1 3023 3023 22.571 2.05e-06 \*\*\*  
## Sex 1 303614 303614 2267.239 < 2e-16 \*\*\*  
## Location:Time 2 546 273 2.039 0.130195   
## Location:Sex 3 2549 850 6.344 0.000271 \*\*\*  
## Time:Sex 1 328 328 2.453 0.117349   
## Residuals 9985 1337125 134   
## ---  
## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

### —- Check ANOVA assumptions: test validity?

#### 1. Check the homogeneity of variance assumption

plot(anova\_without\_interactions, 1)



#### Use the Levene’s test to check the homogeneity of variances.

library(car)

## Loading required package: carData

##   
## Attaching package: 'car'

## The following object is masked from 'package:dplyr':  
##   
## recode

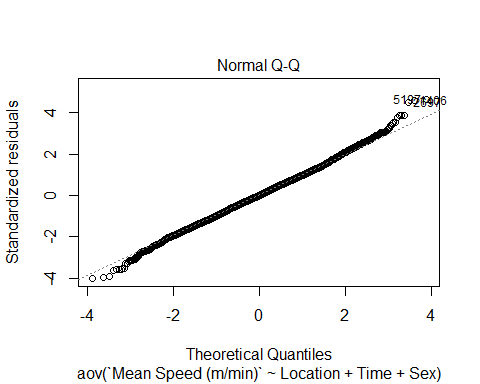
car::leveneTest(`Mean Speed (m/min)` ~ Location \* Time \* Sex,   
 data = All\_Data\_new)

## Levene's Test for Homogeneity of Variance (center = median)  
## Df F value Pr(>F)   
## group 13 19.589 < 2.2e-16 \*\*\*  
## 9983   
## ---  
## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

### 2. Check the normality assumption

### Normality plot of the residuals Q-Q plot

plot(anova\_without\_interactions, 2)



### cross-check with Anderson-Darling test test about normality

nortest::ad.test(residuals(anova\_without\_interactions) )

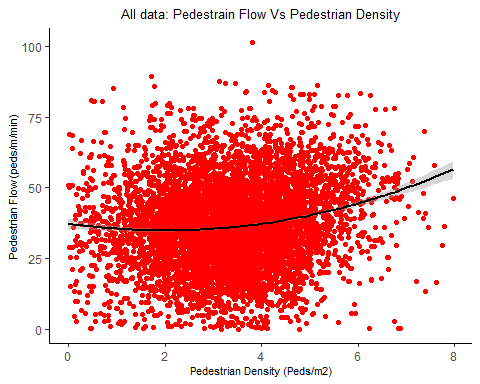
##   
## Anderson-Darling normality test  
##   
## data: residuals(anova\_without\_interactions)  
## A = 1.6106, p-value = 0.0003883

## Split data into training data set and validation data set

n=dim(All\_Data\_new)[1]  
set.seed(12345)  
id=sample(1:n, floor(n\*0.8))  
All\_Data\_new\_train = All\_Data\_new[id,]  
All\_Data\_new\_validation = All\_Data\_new[-id,]

## 2D graph

ggplot(data = All\_Data\_new\_train,   
 aes(x = `Pedestrian Density (Peds/m2)`,   
 y = `Pedestrian Flow (peds/m/min)`)) +   
 geom\_point(color='red') +  
 geom\_smooth(method = "lm", se = TRUE, color="black",   
 formula = y ~ poly(x, 2, raw = T))+  
 labs(title="All data: Pedestrain Flow Vs Pedestrian Density")+  
 theme(plot.title = element\_text(hjust = 0.5, size = 10))+  
 theme(axis.title.x = element\_text(size = 8))+  
 theme(axis.title.y = element\_text(size = 8))+  
 theme(panel.grid.major = element\_blank(),   
 panel.grid.minor = element\_blank(),  
 panel.background = element\_blank(),   
 axis.line = element\_line(colour = "black"))



## – three dimensional models

##### with intercept included and linear model

summary(All\_Data\_new\_unknown <-   
 lm(data = All\_Data\_new\_train,   
 formula = `Pedestrian Flow (peds/m/min)` ~   
 `Pedestrian Density (Peds/m2)` +  
 `Mean Speed (m/min)` ))

##   
## Call:  
## lm(formula = `Pedestrian Flow (peds/m/min)` ~ `Pedestrian Density (Peds/m2)` +   
## `Mean Speed (m/min)`, data = All\_Data\_new\_train)  
##   
## Residuals:  
## Min 1Q Median 3Q Max   
## -43.513 -10.686 -0.561 10.232 61.585   
##   
## Coefficients:  
## Estimate Std. Error t value Pr(>|t|)   
## (Intercept) 9.68243 2.72957 3.547 0.000392 \*\*\*  
## `Pedestrian Density (Peds/m2)` 3.44153 0.25445 13.525 < 2e-16 \*\*\*  
## `Mean Speed (m/min)` 0.20375 0.02539 8.023 1.18e-15 \*\*\*  
## ---  
## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
##   
## Residual standard error: 15.38 on 7994 degrees of freedom  
## Multiple R-squared: 0.02762, Adjusted R-squared: 0.02738   
## F-statistic: 113.5 on 2 and 7994 DF, p-value: < 2.2e-16

##### with intercept removed and linear model

summary(All\_Data\_new\_unknown <-   
 lm(data = All\_Data\_new\_train,   
 formula = `Pedestrian Flow (peds/m/min)` ~   
 0 + `Pedestrian Density (Peds/m2)` +  
 `Mean Speed (m/min)` ))

##   
## Call:  
## lm(formula = `Pedestrian Flow (peds/m/min)` ~ 0 + `Pedestrian Density (Peds/m2)` +   
## `Mean Speed (m/min)`, data = All\_Data\_new\_train)  
##   
## Residuals:  
## Min 1Q Median 3Q Max   
## -44.12 -10.51 -0.46 10.31 60.72   
##   
## Coefficients:  
## Estimate Std. Error t value Pr(>|t|)   
## `Pedestrian Density (Peds/m2)` 4.276297 0.096850 44.15 <2e-16 \*\*\*  
## `Mean Speed (m/min)` 0.292391 0.004518 64.71 <2e-16 \*\*\*  
## ---  
## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
##   
## Residual standard error: 15.39 on 7995 degrees of freedom  
## Multiple R-squared: 0.8532, Adjusted R-squared: 0.8532   
## F-statistic: 2.324e+04 on 2 and 7995 DF, p-value: < 2.2e-16

##### with intercept removed and equation of an ellipse

summary(All\_Data\_new\_unknown <-   
 lm(data = All\_Data\_new\_train,   
 formula = `Pedestrian Flow (peds/m/min)` ~   
 0 + (`Pedestrian Density (Peds/m2)`)^2 +  
 (`Mean Speed (m/min)`)^2 ))

##   
## Call:  
## lm(formula = `Pedestrian Flow (peds/m/min)` ~ 0 + (`Pedestrian Density (Peds/m2)`)^2 +   
## (`Mean Speed (m/min)`)^2, data = All\_Data\_new\_train)  
##   
## Residuals:  
## Min 1Q Median 3Q Max   
## -44.12 -10.51 -0.46 10.31 60.72   
##   
## Coefficients:  
## Estimate Std. Error t value Pr(>|t|)   
## `Pedestrian Density (Peds/m2)` 4.276297 0.096850 44.15 <2e-16 \*\*\*  
## `Mean Speed (m/min)` 0.292391 0.004518 64.71 <2e-16 \*\*\*  
## ---  
## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
##   
## Residual standard error: 15.39 on 7995 degrees of freedom  
## Multiple R-squared: 0.8532, Adjusted R-squared: 0.8532   
## F-statistic: 2.324e+04 on 2 and 7995 DF, p-value: < 2.2e-16

##### with intercept removed and polynomial of degree 2

summary(All\_Data\_new\_unknown <-   
 lm(data = All\_Data\_new\_train,   
 formula = `Pedestrian Flow (peds/m/min)` ~   
 0 + poly(`Pedestrian Density (Peds/m2)`,2, raw = T) +  
 poly(`Mean Speed (m/min)`,2, raw = T)))

##   
## Call:  
## lm(formula = `Pedestrian Flow (peds/m/min)` ~ 0 + poly(`Pedestrian Density (Peds/m2)`,   
## 2, raw = T) + poly(`Mean Speed (m/min)`, 2, raw = T), data = All\_Data\_new\_train)  
##   
## Residuals:  
## Min 1Q Median 3Q Max   
## -49.77 -10.52 -0.47 10.02 63.17   
##   
## Coefficients:  
## Estimate Std. Error t value  
## poly(`Pedestrian Density (Peds/m2)`, 2, raw = T)1 -3.621382 0.746758 -4.849  
## poly(`Pedestrian Density (Peds/m2)`, 2, raw = T)2 0.916497 0.091047 10.066  
## poly(`Mean Speed (m/min)`, 2, raw = T)1 0.864846 0.058004 14.910  
## poly(`Mean Speed (m/min)`, 2, raw = T)2 -0.004810 0.000511 -9.414  
## Pr(>|t|)   
## poly(`Pedestrian Density (Peds/m2)`, 2, raw = T)1 1.26e-06 \*\*\*  
## poly(`Pedestrian Density (Peds/m2)`, 2, raw = T)2 < 2e-16 \*\*\*  
## poly(`Mean Speed (m/min)`, 2, raw = T)1 < 2e-16 \*\*\*  
## poly(`Mean Speed (m/min)`, 2, raw = T)2 < 2e-16 \*\*\*  
## ---  
## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
##   
## Residual standard error: 15.29 on 7993 degrees of freedom  
## Multiple R-squared: 0.8553, Adjusted R-squared: 0.8552   
## F-statistic: 1.181e+04 on 4 and 7993 DF, p-value: < 2.2e-16

##### with intercept removed and polynomial of degree 2 and transformation

summary(All\_Data\_new\_unknown <-   
 lm(data = All\_Data\_new\_train,   
 formula = `Pedestrian Flow (peds/m/min)` ~   
 0 + poly(`Pedestrian Density (Peds/m2)`,2, raw = T) +  
 poly(1/(log10(`Mean Speed (m/min)`)),2, raw = T)))

##   
## Call:  
## lm(formula = `Pedestrian Flow (peds/m/min)` ~ 0 + poly(`Pedestrian Density (Peds/m2)`,   
## 2, raw = T) + poly(1/(log10(`Mean Speed (m/min)`)), 2, raw = T),   
## data = All\_Data\_new\_train)  
##   
## Residuals:  
## Min 1Q Median 3Q Max   
## -49.079 -10.354 -0.423 10.083 62.238   
##   
## Coefficients:  
## Estimate Std. Error  
## poly(`Pedestrian Density (Peds/m2)`, 2, raw = T)1 -1.46000 0.53742  
## poly(`Pedestrian Density (Peds/m2)`, 2, raw = T)2 0.73529 0.07423  
## poly(1/(log10(`Mean Speed (m/min)`)), 2, raw = T)1 252.48129 12.27323  
## poly(1/(log10(`Mean Speed (m/min)`)), 2, raw = T)2 -359.48243 24.36066  
## t value Pr(>|t|)   
## poly(`Pedestrian Density (Peds/m2)`, 2, raw = T)1 -2.717 0.00661 \*\*   
## poly(`Pedestrian Density (Peds/m2)`, 2, raw = T)2 9.906 < 2e-16 \*\*\*  
## poly(1/(log10(`Mean Speed (m/min)`)), 2, raw = T)1 20.572 < 2e-16 \*\*\*  
## poly(1/(log10(`Mean Speed (m/min)`)), 2, raw = T)2 -14.757 < 2e-16 \*\*\*  
## ---  
## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
##   
## Residual standard error: 15.28 on 7993 degrees of freedom  
## Multiple R-squared: 0.8553, Adjusted R-squared: 0.8553   
## F-statistic: 1.181e+04 on 4 and 7993 DF, p-value: < 2.2e-16

summary(All\_Data\_new\_unknown <-   
 lm(data = All\_Data\_new\_train,   
 formula = `Pedestrian Flow (peds/m/min)` ~   
 0 + poly(`Pedestrian Density (Peds/m2)`,2, raw = T) +  
 poly(1/(sqrt(`Mean Speed (m/min)`)),2, raw = T)))

##   
## Call:  
## lm(formula = `Pedestrian Flow (peds/m/min)` ~ 0 + poly(`Pedestrian Density (Peds/m2)`,   
## 2, raw = T) + poly(1/(sqrt(`Mean Speed (m/min)`)), 2, raw = T),   
## data = All\_Data\_new\_train)  
##   
## Residuals:  
## Min 1Q Median 3Q Max   
## -49.363 -10.424 -0.465 9.980 62.504   
##   
## Coefficients:  
## Estimate Std. Error t value  
## poly(`Pedestrian Density (Peds/m2)`, 2, raw = T)1 -2.625e+00 5.447e-01 -4.818  
## poly(`Pedestrian Density (Peds/m2)`, 2, raw = T)2 8.820e-01 7.648e-02 11.533  
## poly(1/(sqrt(`Mean Speed (m/min)`)), 2, raw = T)1 8.485e+02 2.500e+01 33.942  
## poly(1/(sqrt(`Mean Speed (m/min)`)), 2, raw = T)2 -4.741e+03 2.176e+02 -21.788  
## Pr(>|t|)   
## poly(`Pedestrian Density (Peds/m2)`, 2, raw = T)1 1.47e-06 \*\*\*  
## poly(`Pedestrian Density (Peds/m2)`, 2, raw = T)2 < 2e-16 \*\*\*  
## poly(1/(sqrt(`Mean Speed (m/min)`)), 2, raw = T)1 < 2e-16 \*\*\*  
## poly(1/(sqrt(`Mean Speed (m/min)`)), 2, raw = T)2 < 2e-16 \*\*\*  
## ---  
## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
##   
## Residual standard error: 15.26 on 7993 degrees of freedom  
## Multiple R-squared: 0.8558, Adjusted R-squared: 0.8557   
## F-statistic: 1.186e+04 on 4 and 7993 DF, p-value: < 2.2e-16

summary(All\_Data\_new\_unknown <-   
 lm(data = All\_Data\_new\_train,   
 formula = `Pedestrian Flow (peds/m/min)` ~   
 0 + poly(1/(log10(`Pedestrian Density (Peds/m2)`)),2, raw = T) +  
 poly(1/(log10(`Mean Speed (m/min)`)),2, raw = T)))

##   
## Call:  
## lm(formula = `Pedestrian Flow (peds/m/min)` ~ 0 + poly(1/(log10(`Pedestrian Density (Peds/m2)`)),   
## 2, raw = T) + poly(1/(log10(`Mean Speed (m/min)`)), 2, raw = T),   
## data = All\_Data\_new\_train)  
##   
## Residuals:  
## Min 1Q Median 3Q Max   
## -41.008 -10.851 -0.597 10.252 64.841   
##   
## Coefficients:  
## Estimate  
## poly(1/(log10(`Pedestrian Density (Peds/m2)`)), 2, raw = T)1 -6.064e-03  
## poly(1/(log10(`Pedestrian Density (Peds/m2)`)), 2, raw = T)2 -1.718e-05  
## poly(1/(log10(`Mean Speed (m/min)`)), 2, raw = T)1 9.214e+01  
## poly(1/(log10(`Mean Speed (m/min)`)), 2, raw = T)2 -4.240e+01  
## Std. Error t value  
## poly(1/(log10(`Pedestrian Density (Peds/m2)`)), 2, raw = T)1 1.047e-02 -0.579  
## poly(1/(log10(`Pedestrian Density (Peds/m2)`)), 2, raw = T)2 1.342e-05 -1.281  
## poly(1/(log10(`Mean Speed (m/min)`)), 2, raw = T)1 7.938e+00 11.607  
## poly(1/(log10(`Mean Speed (m/min)`)), 2, raw = T)2 1.483e+01 -2.860  
## Pr(>|t|)   
## poly(1/(log10(`Pedestrian Density (Peds/m2)`)), 2, raw = T)1 0.56267   
## poly(1/(log10(`Pedestrian Density (Peds/m2)`)), 2, raw = T)2 0.20039   
## poly(1/(log10(`Mean Speed (m/min)`)), 2, raw = T)1 < 2e-16 \*\*\*  
## poly(1/(log10(`Mean Speed (m/min)`)), 2, raw = T)2 0.00424 \*\*   
## ---  
## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
##   
## Residual standard error: 15.56 on 7993 degrees of freedom  
## Multiple R-squared: 0.85, Adjusted R-squared: 0.85   
## F-statistic: 1.133e+04 on 4 and 7993 DF, p-value: < 2.2e-16

coefficients(All\_Data\_new\_unknown)

## poly(1/(log10(`Pedestrian Density (Peds/m2)`)), 2, raw = T)1   
## -6.063789e-03   
## poly(1/(log10(`Pedestrian Density (Peds/m2)`)), 2, raw = T)2   
## -1.718206e-05   
## poly(1/(log10(`Mean Speed (m/min)`)), 2, raw = T)1   
## 9.213576e+01   
## poly(1/(log10(`Mean Speed (m/min)`)), 2, raw = T)2   
## -4.240319e+01

# Sunday March 27th 2022

Masters\_Research\_Final\_for\_Modelling =   
 read\_excel("Masters Research Final for Modelling.xls",   
 sheet = "Only\_Data")  
  
LOS\_A = subset(Masters\_Research\_Final\_for\_Modelling,  
 subset = (LOS == "A") )  
  
  
summary(flow\_model <-   
 lm(data = LOS\_A,   
 formula = log(Flow) ~   
 0 + log(Speed) + log(Density) ) )

##   
## Call:  
## lm(formula = log(Flow) ~ 0 + log(Speed) + log(Density), data = LOS\_A)  
##   
## Residuals:  
## Min 1Q Median 3Q Max   
## -5.4152 -0.2565 0.3354 0.5969 1.1138   
##   
## Coefficients:  
## Estimate Std. Error t value Pr(>|t|)   
## log(Speed) 0.43660 0.01307 33.413 <2e-16 \*\*\*  
## log(Density) 0.12082 0.04870 2.481 0.0133 \*   
## ---  
## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
##   
## Residual standard error: 0.9189 on 812 degrees of freedom  
## Multiple R-squared: 0.8298, Adjusted R-squared: 0.8293   
## F-statistic: 1979 on 2 and 812 DF, p-value: < 2.2e-16

summary(flow\_model <-   
 lm(data = LOS\_A,   
 formula = log(Flow) ~   
 0 + log(Speed^(-1)) + log( Density^(-1) ) ) )

##   
## Call:  
## lm(formula = log(Flow) ~ 0 + log(Speed^(-1)) + log(Density^(-1)),   
## data = LOS\_A)  
##   
## Residuals:  
## Min 1Q Median 3Q Max   
## -5.4152 -0.2565 0.3354 0.5969 1.1138   
##   
## Coefficients:  
## Estimate Std. Error t value Pr(>|t|)   
## log(Speed^(-1)) -0.43660 0.01307 -33.413 <2e-16 \*\*\*  
## log(Density^(-1)) -0.12082 0.04870 -2.481 0.0133 \*   
## ---  
## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
##   
## Residual standard error: 0.9189 on 812 degrees of freedom  
## Multiple R-squared: 0.8298, Adjusted R-squared: 0.8293   
## F-statistic: 1979 on 2 and 812 DF, p-value: < 2.2e-16

summary(flow\_model <-   
 lm(data = LOS\_A,   
 formula = log(Flow) ~   
 0 + log(Speed^2) + log( Density^2 ) ) )

##   
## Call:  
## lm(formula = log(Flow) ~ 0 + log(Speed^2) + log(Density^2), data = LOS\_A)  
##   
## Residuals:  
## Min 1Q Median 3Q Max   
## -5.4152 -0.2565 0.3354 0.5969 1.1138   
##   
## Coefficients:  
## Estimate Std. Error t value Pr(>|t|)   
## log(Speed^2) 0.218298 0.006533 33.413 <2e-16 \*\*\*  
## log(Density^2) 0.060412 0.024350 2.481 0.0133 \*   
## ---  
## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
##   
## Residual standard error: 0.9189 on 812 degrees of freedom  
## Multiple R-squared: 0.8298, Adjusted R-squared: 0.8293   
## F-statistic: 1979 on 2 and 812 DF, p-value: < 2.2e-16

LOS\_B = subset(Masters\_Research\_Final\_for\_Modelling,  
 subset = (LOS == "B") )  
  
  
summary(flow\_model <-   
 lm(data = LOS\_B,   
 formula = log(Flow) ~   
 0 + log(Speed) + log(Density) ) )

##   
## Call:  
## lm(formula = log(Flow) ~ 0 + log(Speed) + log(Density), data = LOS\_B)  
##   
## Residuals:  
## Min 1Q Median 3Q Max   
## -0.32787 -0.08753 0.00278 0.09487 0.55277   
##   
## Coefficients:  
## Estimate Std. Error t value Pr(>|t|)   
## log(Speed) 0.638710 0.002152 296.86 <2e-16 \*\*\*  
## log(Density) 0.193500 0.007808 24.78 <2e-16 \*\*\*  
## ---  
## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
##   
## Residual standard error: 0.1273 on 980 degrees of freedom  
## Multiple R-squared: 0.9982, Adjusted R-squared: 0.9982   
## F-statistic: 2.685e+05 on 2 and 980 DF, p-value: < 2.2e-16

summary(flow\_model <-   
 lm(data = LOS\_B,   
 formula = log(Flow) ~   
 0 + log(Speed^(-1)) + log( Density^(-1) ) ) )

##   
## Call:  
## lm(formula = log(Flow) ~ 0 + log(Speed^(-1)) + log(Density^(-1)),   
## data = LOS\_B)  
##   
## Residuals:  
## Min 1Q Median 3Q Max   
## -0.32787 -0.08753 0.00278 0.09487 0.55277   
##   
## Coefficients:  
## Estimate Std. Error t value Pr(>|t|)   
## log(Speed^(-1)) -0.638710 0.002152 -296.86 <2e-16 \*\*\*  
## log(Density^(-1)) -0.193500 0.007808 -24.78 <2e-16 \*\*\*  
## ---  
## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
##   
## Residual standard error: 0.1273 on 980 degrees of freedom  
## Multiple R-squared: 0.9982, Adjusted R-squared: 0.9982   
## F-statistic: 2.685e+05 on 2 and 980 DF, p-value: < 2.2e-16

summary(flow\_model <-   
 lm(data = LOS\_B,   
 formula = log(Flow) ~   
 0 + log(Speed^2) + log( Density^2 ) ) )

##   
## Call:  
## lm(formula = log(Flow) ~ 0 + log(Speed^2) + log(Density^2), data = LOS\_B)  
##   
## Residuals:  
## Min 1Q Median 3Q Max   
## -0.32787 -0.08753 0.00278 0.09487 0.55277   
##   
## Coefficients:  
## Estimate Std. Error t value Pr(>|t|)   
## log(Speed^2) 0.319355 0.001076 296.86 <2e-16 \*\*\*  
## log(Density^2) 0.096750 0.003904 24.78 <2e-16 \*\*\*  
## ---  
## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
##   
## Residual standard error: 0.1273 on 980 degrees of freedom  
## Multiple R-squared: 0.9982, Adjusted R-squared: 0.9982   
## F-statistic: 2.685e+05 on 2 and 980 DF, p-value: < 2.2e-16

LOS\_C = subset(Masters\_Research\_Final\_for\_Modelling,  
 subset = (LOS == "C") )  
  
  
summary(flow\_model <-   
 lm(data = LOS\_C,   
 formula = log(Flow) ~   
 0 + log(Speed) + log(Density) ) )

##   
## Call:  
## lm(formula = log(Flow) ~ 0 + log(Speed) + log(Density), data = LOS\_C)  
##   
## Residuals:  
## Min 1Q Median 3Q Max   
## -0.36281 -0.08837 0.00353 0.09593 0.80893   
##   
## Coefficients:  
## Estimate Std. Error t value Pr(>|t|)   
## log(Speed) 0.723906 0.001373 527.3 <2e-16 \*\*\*  
## log(Density) 0.180601 0.004921 36.7 <2e-16 \*\*\*  
## ---  
## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
##   
## Residual standard error: 0.1304 on 2301 degrees of freedom  
## Multiple R-squared: 0.9985, Adjusted R-squared: 0.9985   
## F-statistic: 7.528e+05 on 2 and 2301 DF, p-value: < 2.2e-16

summary(flow\_model <-   
 lm(data = LOS\_C,   
 formula = log(Flow) ~   
 0 + log(Speed^(-1)) + log( Density^(-1) ) ) )

##   
## Call:  
## lm(formula = log(Flow) ~ 0 + log(Speed^(-1)) + log(Density^(-1)),   
## data = LOS\_C)  
##   
## Residuals:  
## Min 1Q Median 3Q Max   
## -0.36281 -0.08837 0.00353 0.09593 0.80893   
##   
## Coefficients:  
## Estimate Std. Error t value Pr(>|t|)   
## log(Speed^(-1)) -0.723906 0.001373 -527.3 <2e-16 \*\*\*  
## log(Density^(-1)) -0.180601 0.004921 -36.7 <2e-16 \*\*\*  
## ---  
## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
##   
## Residual standard error: 0.1304 on 2301 degrees of freedom  
## Multiple R-squared: 0.9985, Adjusted R-squared: 0.9985   
## F-statistic: 7.528e+05 on 2 and 2301 DF, p-value: < 2.2e-16

summary(flow\_model <-   
 lm(data = LOS\_C,   
 formula = log(Flow) ~   
 0 + log(Speed^2) + log( Density^2 ) ) )

##   
## Call:  
## lm(formula = log(Flow) ~ 0 + log(Speed^2) + log(Density^2), data = LOS\_C)  
##   
## Residuals:  
## Min 1Q Median 3Q Max   
## -0.36281 -0.08837 0.00353 0.09593 0.80893   
##   
## Coefficients:  
## Estimate Std. Error t value Pr(>|t|)   
## log(Speed^2) 0.3619530 0.0006864 527.3 <2e-16 \*\*\*  
## log(Density^2) 0.0903007 0.0024605 36.7 <2e-16 \*\*\*  
## ---  
## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
##   
## Residual standard error: 0.1304 on 2301 degrees of freedom  
## Multiple R-squared: 0.9985, Adjusted R-squared: 0.9985   
## F-statistic: 7.528e+05 on 2 and 2301 DF, p-value: < 2.2e-16

LOS\_D = subset(Masters\_Research\_Final\_for\_Modelling,  
 subset = (LOS == "D") )  
  
  
summary(flow\_model <-   
 lm(data = LOS\_D,   
 formula = log(Flow) ~   
 0 + log(Speed) + log(Density) ) )

##   
## Call:  
## lm(formula = log(Flow) ~ 0 + log(Speed) + log(Density), data = LOS\_D)  
##   
## Residuals:  
## Min 1Q Median 3Q Max   
## -0.40416 -0.10115 0.00344 0.10031 0.59147   
##   
## Coefficients:  
## Estimate Std. Error t value Pr(>|t|)   
## log(Speed) 0.794091 0.001240 640.34 <2e-16 \*\*\*  
## log(Density) 0.228102 0.004389 51.97 <2e-16 \*\*\*  
## ---  
## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
##   
## Residual standard error: 0.1382 on 3679 degrees of freedom  
## Multiple R-squared: 0.9986, Adjusted R-squared: 0.9986   
## F-statistic: 1.315e+06 on 2 and 3679 DF, p-value: < 2.2e-16

summary(flow\_model <-   
 lm(data = LOS\_D,   
 formula = log(Flow) ~   
 0 + log(Speed^(-1)) + log( Density^(-1) ) ) )

##   
## Call:  
## lm(formula = log(Flow) ~ 0 + log(Speed^(-1)) + log(Density^(-1)),   
## data = LOS\_D)  
##   
## Residuals:  
## Min 1Q Median 3Q Max   
## -0.40416 -0.10115 0.00344 0.10031 0.59147   
##   
## Coefficients:  
## Estimate Std. Error t value Pr(>|t|)   
## log(Speed^(-1)) -0.794091 0.001240 -640.34 <2e-16 \*\*\*  
## log(Density^(-1)) -0.228102 0.004389 -51.97 <2e-16 \*\*\*  
## ---  
## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
##   
## Residual standard error: 0.1382 on 3679 degrees of freedom  
## Multiple R-squared: 0.9986, Adjusted R-squared: 0.9986   
## F-statistic: 1.315e+06 on 2 and 3679 DF, p-value: < 2.2e-16

summary(flow\_model <-   
 lm(data = LOS\_D,   
 formula = log(Flow) ~   
 0 + log(Speed^2) + log( Density^2 ) ) )

##   
## Call:  
## lm(formula = log(Flow) ~ 0 + log(Speed^2) + log(Density^2), data = LOS\_D)  
##   
## Residuals:  
## Min 1Q Median 3Q Max   
## -0.40416 -0.10115 0.00344 0.10031 0.59147   
##   
## Coefficients:  
## Estimate Std. Error t value Pr(>|t|)   
## log(Speed^2) 0.3970455 0.0006201 640.34 <2e-16 \*\*\*  
## log(Density^2) 0.1140510 0.0021944 51.97 <2e-16 \*\*\*  
## ---  
## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
##   
## Residual standard error: 0.1382 on 3679 degrees of freedom  
## Multiple R-squared: 0.9986, Adjusted R-squared: 0.9986   
## F-statistic: 1.315e+06 on 2 and 3679 DF, p-value: < 2.2e-16

LOS\_E = subset(Masters\_Research\_Final\_for\_Modelling,  
 subset = (LOS == "E") )  
  
  
summary(flow\_model <-   
 lm(data = LOS\_E,   
 formula = log(Flow) ~   
 0 + log(Speed) + log(Density) ) )

##   
## Call:  
## lm(formula = log(Flow) ~ 0 + log(Speed) + log(Density), data = LOS\_E)  
##   
## Residuals:  
## Min 1Q Median 3Q Max   
## -0.31854 -0.10226 -0.01443 0.09951 0.90090   
##   
## Coefficients:  
## Estimate Std. Error t value Pr(>|t|)   
## log(Speed) 0.884396 0.001516 583.35 <2e-16 \*\*\*  
## log(Density) 0.190563 0.004893 38.95 <2e-16 \*\*\*  
## ---  
## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
##   
## Residual standard error: 0.1436 on 2093 degrees of freedom  
## Multiple R-squared: 0.9987, Adjusted R-squared: 0.9987   
## F-statistic: 8.293e+05 on 2 and 2093 DF, p-value: < 2.2e-16

summary(flow\_model <-   
 lm(data = LOS\_E,   
 formula = log(Flow) ~   
 0 + log(Speed^(-1)) + log( Density^(-1) ) ) )

##   
## Call:  
## lm(formula = log(Flow) ~ 0 + log(Speed^(-1)) + log(Density^(-1)),   
## data = LOS\_E)  
##   
## Residuals:  
## Min 1Q Median 3Q Max   
## -0.31854 -0.10226 -0.01443 0.09951 0.90090   
##   
## Coefficients:  
## Estimate Std. Error t value Pr(>|t|)   
## log(Speed^(-1)) -0.884396 0.001516 -583.35 <2e-16 \*\*\*  
## log(Density^(-1)) -0.190563 0.004893 -38.95 <2e-16 \*\*\*  
## ---  
## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
##   
## Residual standard error: 0.1436 on 2093 degrees of freedom  
## Multiple R-squared: 0.9987, Adjusted R-squared: 0.9987   
## F-statistic: 8.293e+05 on 2 and 2093 DF, p-value: < 2.2e-16

summary(flow\_model <-   
 lm(data = LOS\_E,   
 formula = log(Flow) ~   
 0 + log(Speed^2) + log( Density^2 ) ) )

##   
## Call:  
## lm(formula = log(Flow) ~ 0 + log(Speed^2) + log(Density^2), data = LOS\_E)  
##   
## Residuals:  
## Min 1Q Median 3Q Max   
## -0.31854 -0.10226 -0.01443 0.09951 0.90090   
##   
## Coefficients:  
## Estimate Std. Error t value Pr(>|t|)   
## log(Speed^2) 0.442198 0.000758 583.35 <2e-16 \*\*\*  
## log(Density^2) 0.095281 0.002446 38.95 <2e-16 \*\*\*  
## ---  
## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
##   
## Residual standard error: 0.1436 on 2093 degrees of freedom  
## Multiple R-squared: 0.9987, Adjusted R-squared: 0.9987   
## F-statistic: 8.293e+05 on 2 and 2093 DF, p-value: < 2.2e-16

LOS\_F = subset(Masters\_Research\_Final\_for\_Modelling,  
 subset = (LOS == "F") )  
  
  
summary(flow\_model <-   
 lm(data = LOS\_F,   
 formula = log(Flow) ~   
 0 + log(Speed) + log(Density) ) )

##   
## Call:  
## lm(formula = log(Flow) ~ 0 + log(Speed) + log(Density), data = LOS\_F)  
##   
## Residuals:  
## Min 1Q Median 3Q Max   
## -0.27429 -0.08946 0.00480 0.10601 0.39052   
##   
## Coefficients:  
## Estimate Std. Error t value Pr(>|t|)   
## log(Speed) 0.952154 0.005897 161.46 <2e-16 \*\*\*  
## log(Density) 0.256514 0.018960 13.53 <2e-16 \*\*\*  
## ---  
## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
##   
## Residual standard error: 0.13 on 120 degrees of freedom  
## Multiple R-squared: 0.9991, Adjusted R-squared: 0.9991   
## F-statistic: 6.943e+04 on 2 and 120 DF, p-value: < 2.2e-16

summary(flow\_model <-   
 lm(data = LOS\_F,   
 formula = log(Flow) ~   
 0 + log(Speed^(-1)) + log( Density^(-1) ) ) )

##   
## Call:  
## lm(formula = log(Flow) ~ 0 + log(Speed^(-1)) + log(Density^(-1)),   
## data = LOS\_F)  
##   
## Residuals:  
## Min 1Q Median 3Q Max   
## -0.27429 -0.08946 0.00480 0.10601 0.39052   
##   
## Coefficients:  
## Estimate Std. Error t value Pr(>|t|)   
## log(Speed^(-1)) -0.952154 0.005897 -161.46 <2e-16 \*\*\*  
## log(Density^(-1)) -0.256514 0.018960 -13.53 <2e-16 \*\*\*  
## ---  
## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
##   
## Residual standard error: 0.13 on 120 degrees of freedom  
## Multiple R-squared: 0.9991, Adjusted R-squared: 0.9991   
## F-statistic: 6.943e+04 on 2 and 120 DF, p-value: < 2.2e-16

summary(flow\_model <-   
 lm(data = LOS\_F,   
 formula = log(Flow) ~   
 0 + log(Speed^2) + log( Density^2 ) ) )

##   
## Call:  
## lm(formula = log(Flow) ~ 0 + log(Speed^2) + log(Density^2), data = LOS\_F)  
##   
## Residuals:  
## Min 1Q Median 3Q Max   
## -0.27429 -0.08946 0.00480 0.10601 0.39052   
##   
## Coefficients:  
## Estimate Std. Error t value Pr(>|t|)   
## log(Speed^2) 0.476077 0.002949 161.46 <2e-16 \*\*\*  
## log(Density^2) 0.128257 0.009480 13.53 <2e-16 \*\*\*  
## ---  
## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
##   
## Residual standard error: 0.13 on 120 degrees of freedom  
## Multiple R-squared: 0.9991, Adjusted R-squared: 0.9991   
## F-statistic: 6.943e+04 on 2 and 120 DF, p-value: < 2.2e-16