Sectioned angle embedding for Quantum Neural Networks*

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Abstract. This paper discusses sectioned angle embedding, an image embedding approach used for classification of the MNIST dataset with a hybrid quantum neural network. The embedding requires several classical preprocessing techniques which allow for high information retention represented via a 10×10 image. The expressivity of sectioned angle embedding is demonstrated by applying it to train a QNN for 10 class multi-classification of the MNIST dataset. The QNN uses 327 parameters and three ladder-like layers on 10 qubits and an AMM-strategy for the readout non-linearity, achieving 85.36% test accuracy averaged over 10 random seeds. The circuit uses the open source framework Tensorflow Quantum and Cirq and shows further potential in improving the accuracy in existing quantum image classification tasks.

Keywords: QNN · Classification · Embedding Technique

1 Introduction

Quantum computing (QC) is acknowledged to have significant speedups in solving intractable problems for classical computation by utilizing quantum mechanical phenomena such as entanglement and superposition[5]. Current development of QCs have allowed for successful manipulation of approximately several dozen to a hundred noisy qubits, placing progress of various quantum architectures to be in the Noisy Intermediate Scale Quantum (NISQ) computing era[**?**]. A promising area of research is Quantum Machine Learning (QML) which is known to provide atypical calculation patterns as well as relatively strong performances with low parameter usage compared to their classical counterparts [6].

In this era, Parameterized Quantum Circuits (PQC), where gates consist of fixed and trained unitary operators which perform rotations on qubits, dominate in the implementation of QML algorithms. More specifically, PQC based QML algorithms currently involve seminal machine learning datasets such as MNIST, fashion-MNIST, CIFAR-10 to understand the performance of various architectures such as TTN, MERA, and ladder-like [7][8][1]. For MNIST classification, several of the aforementioned circuits have produced comparable or

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outperforming results for binary classification compared to their classical counterparts. The performances of the most recent models achieve 95% to 97% for 5 vs 3 classification and are consistently outperforming fair classical models [2]. The strong binary classification results and consistent improvements since Farhi et al's [2] binary classification, which achieved 85%, strongly suggesting that multi-classification is an exciting step in furthering understanding the expressivity of PQCs. Models such as those by Chalumuri et al.[4] which achieves 92.10% on the IRIS dataset Bokhan et al. [3] where their model classified 4 classes to acheive 85.14% (3456) and 90.03% (0123), as well as Zeng et al. [1] have shown exciting potential for QNN models in multiclassification.

2 Method

This project is structured to provide an in depth overview of the performance of section angle embedding on the MNIST dataset for 10 class multi-classification on a preprocessed dataset. The embedding method was used with a ladder-like PQC (parametrized quantum circuit) and an AMM (all-qubit, multi-observable, measurement) strategy[1] to calculate the expectation values for a softmax layer to generate predicted labels. The preprocessing leaves a 10×10 image which is embedded upon 10 qubits using sectioned embedding, thereby improving upon prior pre-processing techniques which used heavily downsampled 4×4 images or 8×8 images which used 16 or more qubit proposed by Farhi et al.[2] and improved by Zeng et al.[1] A table of performances is supplied in Table 3 and an architecture diagram is shown in Figure 3.

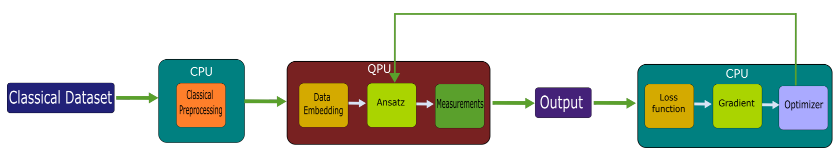


Fig. 1. Diagram for the general process for this model.

2.1 Preprocessing

The proposal is inspired by the model determined by Zeng et al.[1] which uses basis encoding where 4×4 images are embedded upon 16 qubits. However, the limitation of NISQ based devices requires for better feature retention on fewer qubits to be legitimately applicable in the near term[1]. The preprocessing method for this paper achieves a reduction of qubits to 10 qubits, which is 6 less than qubits Zeng et al.[1], whilst using 10×10 images compared to 4×4 images. To produce the 10×10 images, the original images are normalized from $[0, 255]$ to $[0, 1]$ and are cropped from 28×28 to 20×20, thus removing empty pixels.

After, the images are deskewed to straighten and reduce possible discrepancies amongst equivalently labeled images. Then an average pooling over a 2×2 block is used to downsample the images to 10×10. Finally, a threshold to binarize the pixels to a value of 1 or 0 (see Figure 2) is applied.

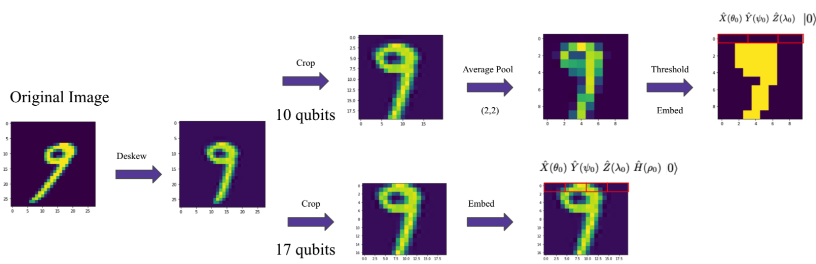


Fig. 2. Sectioned angle embedding for 10 and 17 qubits. Each section of the image is broken into thirds which results in 10 qubits with X, Y, and Z gates applied per section.

2.2 Sectioned Angle Embedding

The sectioned embedding method takes inspiration from sevens segment displays by sectioning portions of the images and angle embedding each section. To embed the first row of an image of width L into k sections, L is divided by k and a floor function is applied to get the section length l. In the case of remaining pixels, they are appended to the last section. The equation is as follows for $k = 3$ and $L = 10$ for the first section, up to a global phase:

$$U_0 = R_X\left(\frac{\sum_{j=0}^{l-1} n_{0,j}}{\pi l}\right), \tag{1}$$

where n is each image row and the choice of single qubit gate is X. This is repeated for each section where different pauli gates such as Y and Z gates are applied to the remaining sections respectively. For each section, the pixels are averaged and the n pixel values are divide by π. The resulting array of three values are then used as angles for single qubit gates, which can be considered equivalent to rotations of a single unitary gate

$$U(\phi_{f(x_0 \ldots x_{l-1})}, \theta_{f(x_l \ldots x_{2l-1})}, \psi_{f(x_2 l \ldots x_L)}) \left|n_i\right\rangle = X^{\frac{\mathrm{mean}(x_0 \cdots x_{l-1})}{\pi}} Y^{\frac{\mathrm{mean}(x_l \cdots x_{2l-1})}{\pi}} Z^{\frac{\mathrm{mean}(x_{2l} \cdots x_L)}{\pi}} \left|n_i\right\rangle \tag{2}$$

2.3 The HQNN's proposal

The quantum neural network (QNN) for this proposal is a modification of the QNN model by Zeng et al.[1] which uses a layer-like circuit of Pauli gates and two qubit gates on N-1 qubits. More importantly, take advantage of the AMM strategy for readout, which has produced significant advantages over single qubit or multi qubit readout strategies (see Figure 3). The PQC can be written as,

$$U_{PQC}(\theta)\left|l_i\right\rangle\left|\psi_{enc}\right\rangle = \Pi_{i=0}^{m-1} U_i(\theta_i)\left|\psi_{in}\right\rangle, \tag{3}$$

where $U_i(\theta_i)$ are parameterized two qubit gates with the form $exp(-i\theta_i Z_j Z_k)$[1]. For the 10 qubit model, two additional layers are added where each layer uses a Y and Z gate and the CNOT gate for entanglement results to produce strong disentanglement of the superpositioned input state $\left|\psi_{in}\right\rangle$. The additional layers as well as the sectioned angle embedding increase the depth of the circuit to 33 which is an inherent trade off in the reduction of qubits from 16 to 10.

With sectioned embedding, the most optimal results were achieved with a learning rate of 0.002 using the Adam optimizer. With the given configuration ran until 120 epochs, the validation accuracy comfortably averaged around 85.36%. To be noted, however, is that the convergence of the model could potentially be further improved as seen in 3 which was ran for a longer 240 epochs.

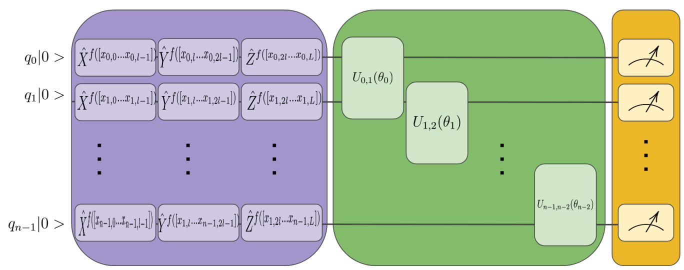


Fig. 3. The quantum circuit for the HQNN's proposal uses an AMM strategy for readout which is then passed on to a softmax layer

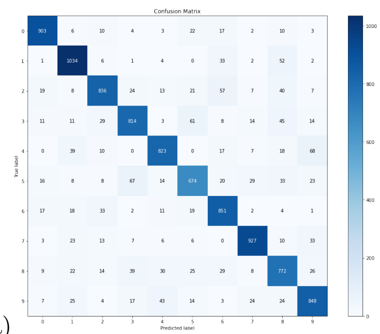
3 Results

The results illustrate sectioned angle embedding as a promising method in multiclass classification of MNIST using an amount of qubits which are reasonable for the NISQ era[1]. More specifically, applying the embedding technique to ladder-like PQCs with the AMM readout strategy results in performances that outperform the likes of Oh [8]and Farhi[2]. The model also beats the fair classical model by 3% using 337 parameters. While the full classical model easily out competes the HQNN model, its usage of 93,692 parameters is significantly more than the HQNN.

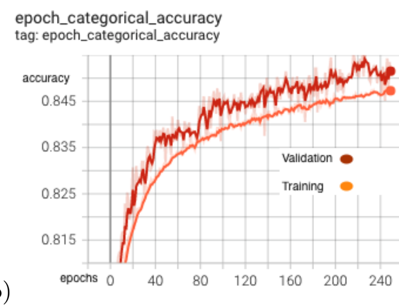
The model does struggle with 3 vs 5 and 4 vs 9 [9] according to the confusion matrix, similar to other works [1][10], which inspecting MNIST suggests images with nearly identical embeddings.

Table 1. Various performance of multi-classification

Model	Performance ($0 \sim 9$)	Number of parameters	Number of qubits
Full Classical Model	97%	93,962	0 qubits
AMM by Zeng et al.	89.09%	505	16 qubits
10 qubits model	85.36%	327	10 qubits
Fair Classical Model	82%	330	0 qubits



a)



b)

Fig. 4. Result figures for the experiments, a) is the confusion matrix of the 10 multi-class classification; b) the training accuracy (orange) and validation accuracy (dark orange) of the model

4 Conclusion

In this paper, the sectional angle embedding technique which allows for minimal downsampling of classical MNIST image dataset using 10 qubits and averaging to 85.32% over 10 trials is demonstrated. In doing so, the HQNN results in comparable performances of 89.09% by Zeng et al.[1] for 10 class multi-classification. While optimistic performances were achieved, sectioned angle embedding allows for a desired number of sections embedded on single qubit gates suggesting further improvements to be explored to reduce the qubit count and improve the image granularity.

5 Future Work

The embedding of sectioned angles will continue to be investigated, and different strategies will be applied to improve the classification of 3 vs 5 as well as 4 vs 9. In addition, sectioned angle embedding will be implemented for the IRIS, FASHION MNIST, and SEMEION dataset to further understand its performance.

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