Sujet de Travaux Dirigés / Pratiques - TP MRF - IMA203

NAME : Rafael Senna Benatti

NAME :

Introduction to Markov Random Fields for image processing

Objectifs de la séance :

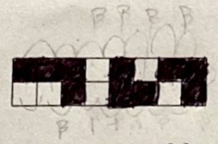
The aim of this session is to program the Gibbs sampler algorithm and study it in the binary case. This program will then be used to do image classification in a bayesian framework (next practical work).
You have to fill by hand-writing the printed version of the practical work (this document) and upload the filled jupyter notebook on e-campus.

This report should be given on the 2nd of december during the course. You can do it in pair (2 students), put both names on the document. The filled notebook should be also uplodaded on e-campus for the 2nd of december.

1 Ising model

In this section we consider a binary Markov random field (taking values in $E = 0, 1$. The neighborhood is defined in 4-connexity and the potential of a clique of order 2 is defined by : $V_c(0,1) = V_c(1,0) = +\beta$ and $V_c(1,1) = V_c(0,0) = 0$ (the potential for singleton clique is 0).

— Draw in the grid the imaU generated with the notebook (fill in black the pixels with value 0) :



13 β

FIGURE 1 – Image generated by the notebook

— Q1 For the Ising model defined above, and the imaU generated in the previous cell, give the formula of the global energy and give its value as a function of β for the generated imaU :

$U(x) = \sum_{c \in C} V_c(x) = 13\beta$

— Draw in the grid the local configuration generated with the notebook (fill in black the pixels with value 0) :

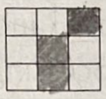


FIGURE 2 – Configuration ImaVois of the local neighborhood (the pixel s to be considered is in the center of the 3x3 window).

— Q2 Write the general form of the local conditional probability in a pixel s. For the neighborhood configuration ImaVois generated with the notebook and represented in figure 2, compute the 2 local conditional energies (for the value 0 and for the value 1 of the central pixel), then the local conditional probabilities (as a function of β). What is the most probable class ? (NB : do the calculation for an 8-neighborhood).

$P_r(X_s = x_s / V_s) = \frac{1}{Z_s} \exp(-U_s(x_s, V_s))$

$U_s(x_s = 0, V_s) = 6\beta$

$U_s(x_s = 1, V_s) = 2\beta$

$P_r(x_s = 0 / V_s) = \frac{e^{-6\beta}}{Z_s}$

$P_r(x_s = 1 / V_s) = \frac{e^{-2\beta}}{Z_s}$

$\boxed{P_r(x_s = 1 | V_s) > P_r(x_s = 0 | V_s), \forall \beta > 0}$

Program the Gibbs sampler on the notebook.

— Q3 Run the program several times. Do you still get the same image ? Comment on this.

No, it's not because we use a random variable to define the whether the pixel is 1 or 0

— Q4 Vary β from 0.5 to 20. Comment on the results.

As β increases the images changes the less.

Also, WE CAN SEE THAT THE BIGGER THE β, WE HAVE BIGGER "SEGMENTED" SPACES

— Q5 Which image minimizes the overall energy for this model ?

THE IMAGE WITH THE SMALLEST β, I.E. β = 0.5, THE FIRST IMAGE

— Q6 Change β and give it a negative value. Describe the result and justify it.

AS RESULT WE HAVE BIGGER LOCAL SEPARATIONS BETWEEN WHITE AND BLACK. THIS HAPPENS BECAUSE WITH β<0 THE VALUE IN THE EXPONENTIAL IS POSITIVE, SO THE CONDITIONAL PROBABILITIES TEND TO A LESS SPARSE DISTRIBUTION

We now work in 8-neighborhood, but still with cliques of order 2 (non-isotropic this time).

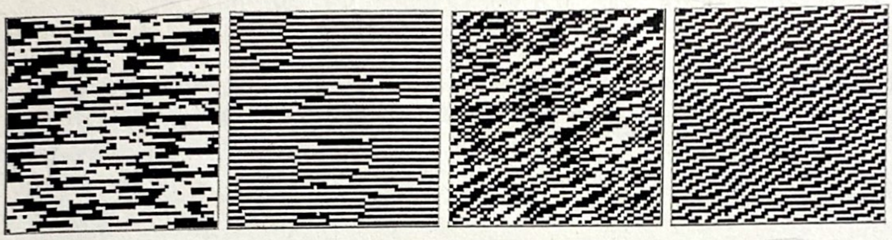


FIGURE 3 – Image A, B, C, D (de gauche à droite)

For each of these images, propose the clique potentials that allow us to obtain these realizations. Initially all clique potentials are zero.

— Image A : there is only one clique potential of order 2 which is -1.

— Image B : in addition to the previous one, there is a clique potential of order 2 which is 1. Indicate which one.

— Image C : in addition to the 2 previous ones, there is a clique potential of order 2 which is -1. Indicate which one.

— Image D : in addition to the 3 previous ones, there is a second order clique potential which is +1. Indicate which one.

— Q8 Propose the clique potentials that allow us to obtain these realizations

Potential	horiz. $V_c(0,1)$ $V_c(1,0)$	horiz. $V_c(0,0)$ $V_c(1,1)$	vertical $V_c(0,1)$ $V_c(1,0)$	vertical $V_c(0,0)$ $V_c(1,1)$	diagonal $(+\frac{\pi}{4})$ $V_c(0,1)$ $V_c(1,0)$	diagonal $(+\frac{\pi}{4})$ $V_c(0,0)$ $V_c(1,1)$	diagonal $(+\frac{3\pi}{4})$ $V_c(0,1)$ $V_c(1,0)$	diagonal $(+\frac{3\pi}{4})$ $V_c(0,0)$ $V_c(1,1)$
Image A	-1	0	0	0	0	0	0	0
Image B	-1	0	1	0	0	0	0	0
Image C	-1	0	1	0	-1	0	0	1
Image D	-1	0	1	0	-1	0	0	1

Modify your program to obtain these results (you can copy and paste the previous cells).

Q9 Modify your program to define an Ising model with diagonal attractive potentials only (the other potentials are zero). Describe and comment on the result.