Image Registration With Fourier-Based Image Correlation: A Comprehensive Review of Developments and Applications

IMA204

Julián Alvarez

Rafael Benatti

Vitor França

Summary

- Introduction;
- Background;
- Sub-pixel methods;
- Application Examples:
  - Fingerprint matching;
  - Rotation and translation of moving cells;
- Conclusion.

Introduction

- **The goal:**
  - **Detailed review of the literature of Image Registration with Fourier-Based Image Correlation**

- **How they organized it:**
  - **Overview of the fundamentals;**
  - **Developments;**
  - **Some applications.**

Introduction

- **What is Image Registration?**
  - **Image registration is the process of overlaying images of the same scene.**

- **What's Fourier-based image correlation?**
  - **Is an area-based method;**
  - **Utilize information in the frequency domain;**
  - **Theoretical accuracy and high computational efficiency;**

Background

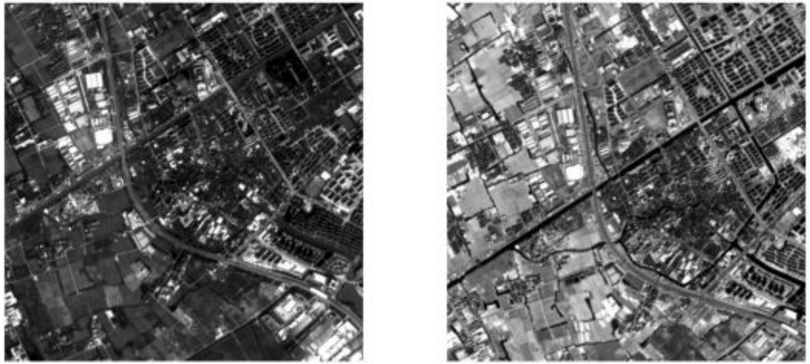
- Cross correlation in the frequency domain (CCF)
- Phase correlation

$$g(x,y) = f(x-\Delta x, y-\Delta y)$$

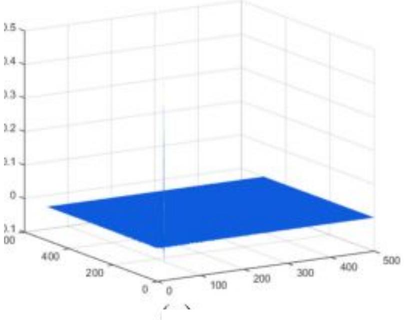
$$CC = F^{-1}(F(u,v)^* G(u,v))$$

$$PC = F^{-1}(Q(u,v)) = F^{-1}\left\{\frac{F(u,v)^* G(u,v)}{|F(u,v)^* G(u,v)|}\right\}$$
$$= F^{-1}\{\exp(-i(u\Delta x + v\Delta y))\}$$

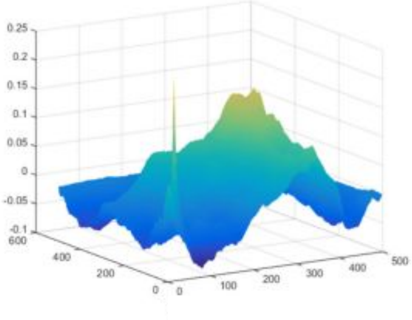
$$\mathrm{PC} = F^{-1}(Q(u,v)) = F^{-1}\{\exp(-i(u\Delta x + v\Delta y))\}$$
$$= \delta(x-\Delta x, y-\Delta y).$$



Translated image with different magnitudes



Phase correlation



Cross correlation

Background

Log-polar coordinates

$$\theta = \arctan \frac{y}{x}$$

$$\lambda = \log \sqrt{x^2 + y^2}$$

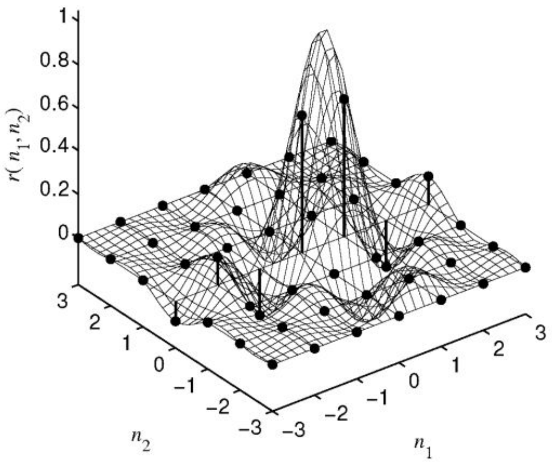
- **Fourier-Mellin transform**

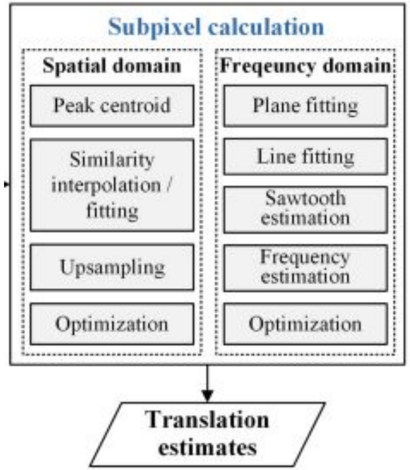
$$g(x, y) = f(s(x \cos\theta_0 + y \sin\theta_0) - \Delta x, s(-x \sin\theta_0 + y \cos\theta_0) - \Delta y).$$

$$M_G(u, v) = s^{-2} M_F[s^{-1}(u \cos\theta_0 + v \sin\theta_0), s^{-1}(-u \sin\theta_0 + v \cos\theta_0)]$$

$$M_{\mathrm{Glp}}(\lambda, \theta) = s^{-2} M_{\mathrm{Flp}}(\lambda - \log s, \theta - \theta_0)$$

Subpixel methods





Subpixel methods

- Spatial domain
  - Peak centroid
  - Interpolation
    - Quadratic, gaussian, sinc derivation, dirichlet, modified sinc, modified mexican hat wavelet
  - Upsampling
  - Optimization
    - Minimizing the gradient of the inverse transform of cross-power spectrum

Subpixel methods

- **Frequency domain**
  - **Plane fitting**

$$\varphi(u,v) = \angle Q(u,v) = -(u\Delta x + v\Delta y)$$

  - **Line fitting**

$$Q(u,v) = \exp(-i(u\Delta x + v\Delta y))$$
$$= \exp(-iu\Delta x)\exp(-iv\Delta y) = q_x(u)q_y(v)$$

Subpixel methods

- **Frequency domain**
  - **Sawtooth estimation**

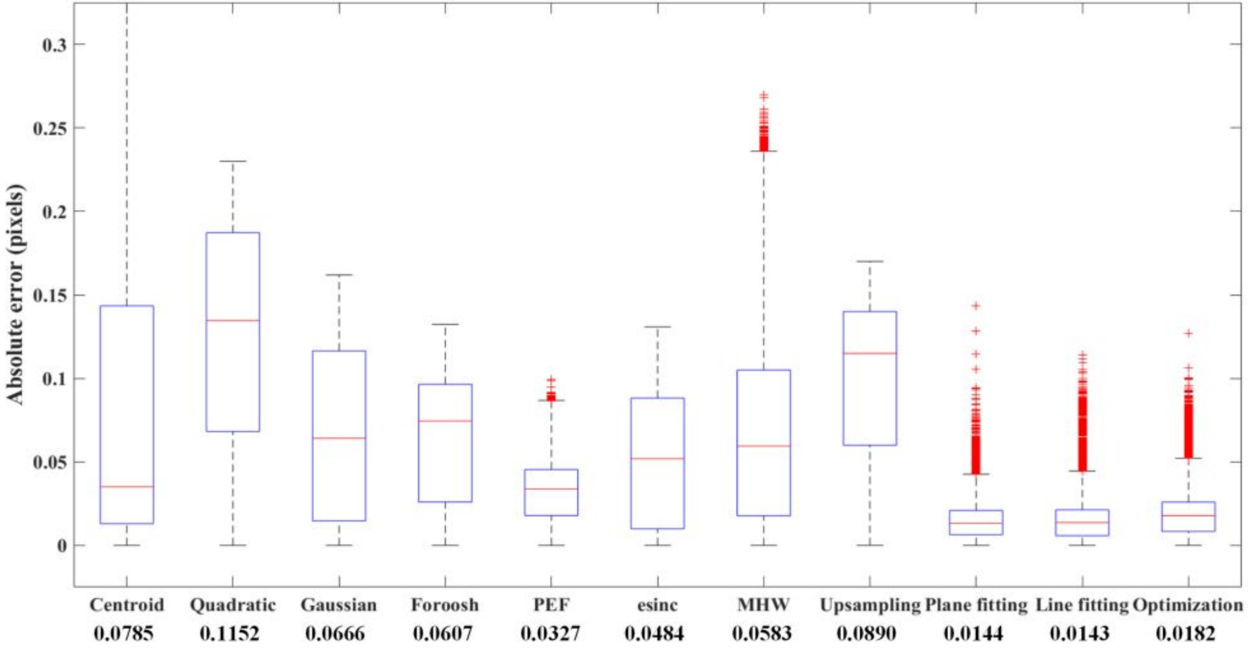
$$\hat{\varphi}_{mn} = \begin{cases} 2\pi\left(x_o\frac{n}{N} + y_o\frac{m}{M}\right) & \\ \hat{\varphi}_{m'n}, & \text{if } m'\frac{2\pi}{M} = m\frac{2\pi}{M} + j\frac{2\pi}{x_o} \\ \hat{\varphi}_{mn'}, & \text{if } n'\frac{2\pi}{N} = n\frac{2\pi}{N} + k\frac{2\pi}{y_o} \end{cases}$$

  - **Frequency estimation**
  - **Optimization**

$$\phi(\Delta x, \Delta y) = \sum_{u=-\pi}^{\pi} \sum_{v=-\pi}^{\pi} W(u,v) \cdot \left|Q(u,v) - \exp(-i(u\Delta x + v\Delta y))\right|^2$$

Subpixel methods

- **Some results**



Application Examples

- K. Ito, H. Nakajima, K. Kobayashi, and T. Higuchi, "A fingerprint matching algorithm using phase-only correlation," IEICE Trans. Fundam. Electron. Commun. Comput. Sci., vol. 87, no. 3, pp. 682–691, 2004

- Wilson, C.A.; Theriot, J.A. (2006). A correlation-based approach to calculate rotation and translation of moving cells. IEEE Transactions on Image Processing, 15(7), 1939–1951. doi:10.1109/tip.2006.873434

Fingerprint matching.

- **POC (PC):**

$$\hat{r}_{fg}(n_1, n_2) = \frac{1}{N_1 N_2} \sum_{k_1, k_2} \hat{R}_{FG}(k_1, k_2) W_{N_1}^{-k_1 n_1} W_{N_2}^{-k_2 n_2}.$$

- **Band-Limited POC (PC):**

$$\hat{r}_{fg}^{K_1 K_2}(n_1, n_2) = \frac{1}{L_1 L_2} \sum_{k_1=-K_1}^{K_1} \sum_{k_2=-K_2}^{K_2} \hat{R}_{FG}(k_1, k_2) \times W_{L_1}^{-k_1 n_1} W_{L_2}^{-k_2 n_2},$$

- L1 = 2K1 + 1
- L2 = 2K2 +1

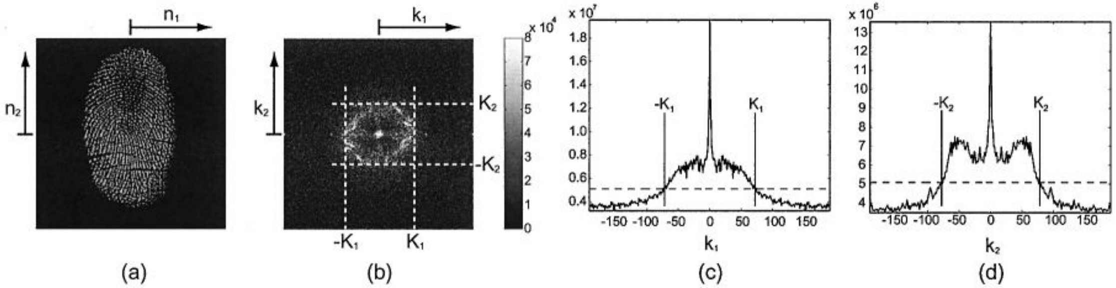


Fig. 3 Fingerprint image in space domain (a) and in frequency domain (b) (amplitude spectrum). (c) and (d) are k_2-axis and k_1-axis projection of the amplitude spectrum. The dashed lines denote the mean value for each projection.

Fingerprint matching.

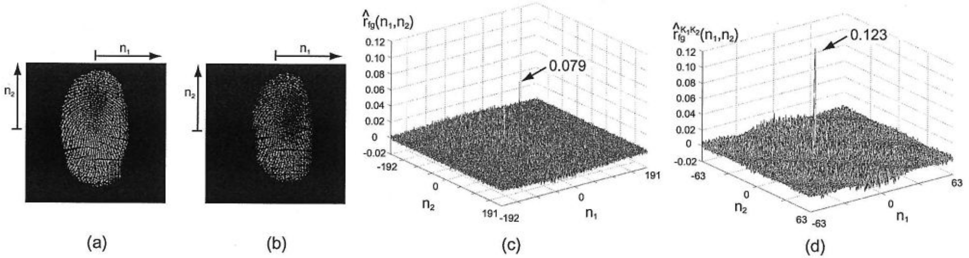


Fig. 4 Example of genuine matching using the original POC function and the band-limited POC function: (a) registered fingerprint image $f(n_1, n_2)$, (b) input fingerprint image $g(n_1, n_2)$ captured from the same fingertip, (c) original POC function $\hat{r}_{fg}(n_1, n_2)$ and (d) band-limited POC function $\hat{r}_{fg}^{K_1 K_2}(n_1, n_2)$ where $K_1 = 63$ and $K_2 = 63$.

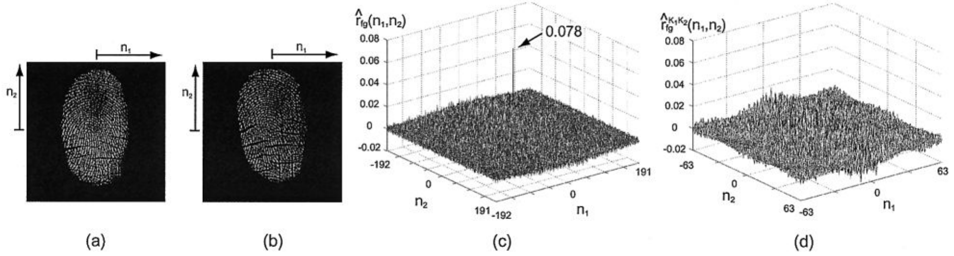


Fig. 5 Example of impostor matching using the original POC function and the band-limited POC function: (a) registered fingerprint image $f(n_1, n_2)$, (b) input fingerprint image $g(n_1, n_2)$ captured from the different fingertip, (c) original POC function $\hat{r}_{fg}(n_1, n_2)$ and (d) band-limited POC function $\hat{r}_{fg}^{K_1 K_2}(n_1, n_2)$ where $K_1 = 63$ and $K_2 = 63$.

Fingerprint matching.

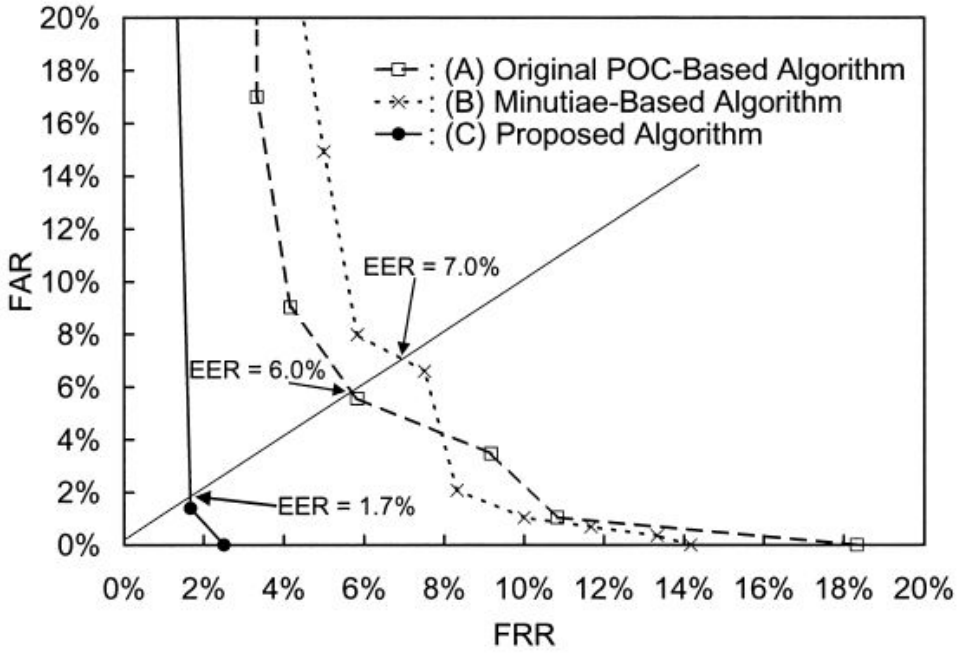
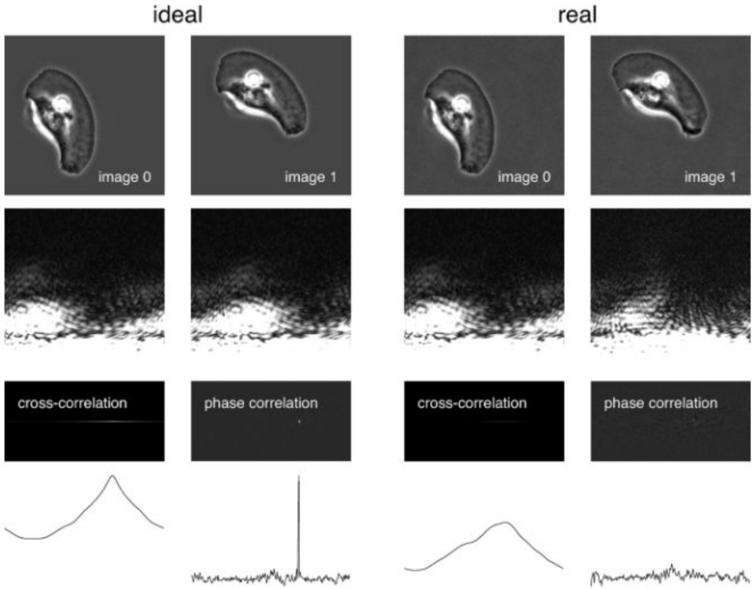
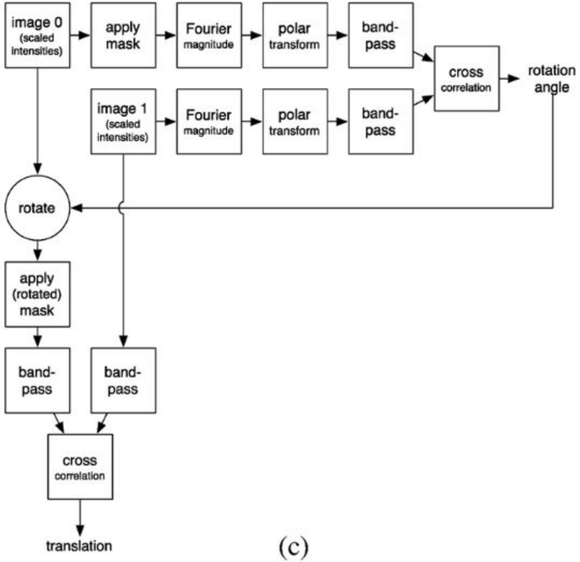


Fig. 11 ROC curve and EER.

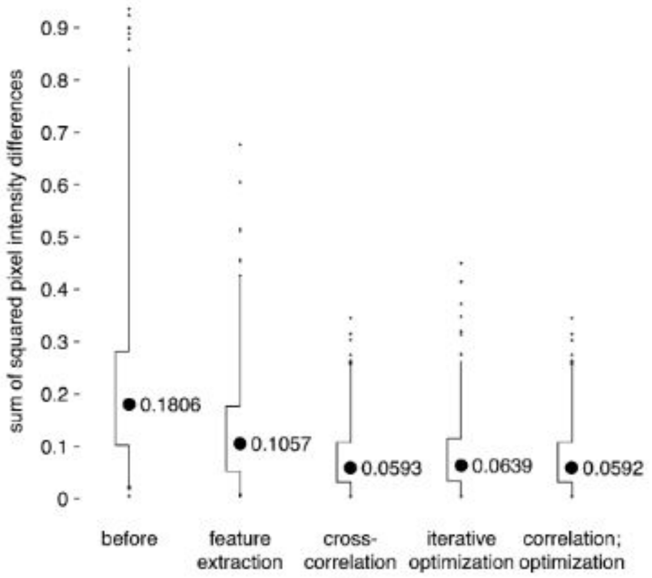
Rotation and translation of moving cells



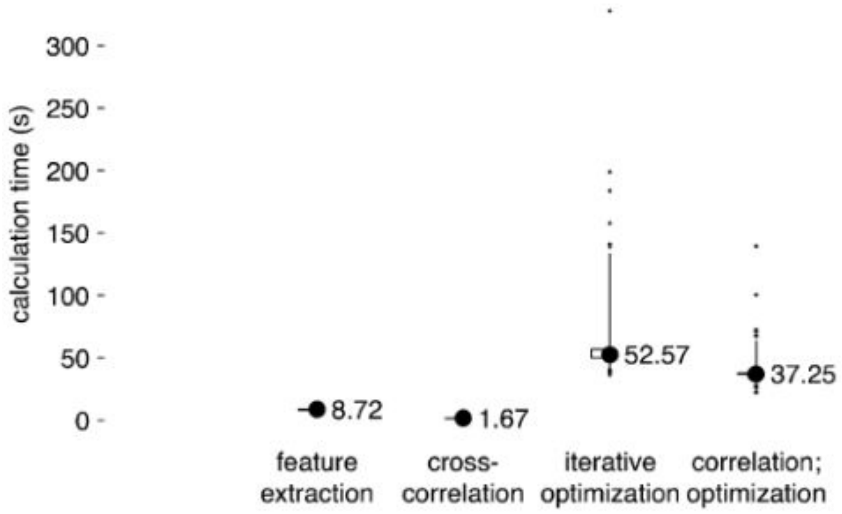


(c)

Rotation and translation of moving cells



(a)



(b)

Potential improvements and applications.

CHALLENGES AND FUTURE TRENDS:

- High-accuracy and high-efficiency subpixel methods.
- Problems with small windows size in frequency domain.
- Quantitative comparison in algorithms, implementations, and progressing frameworks

Improvements:

- Moving cells:
  Peak centroid method → PEF (Peak evaluation formula)

Applications fields:

- Autonomous vehicles: Image align to better understanding of the environment .
- Medicine: video stabilizer in long-distance surgeries or laparoscopy.