

$$\text{var}(X) = E(X - E(X))^2 = E(X^2) - E(X)^2$$

$$y = XB + \epsilon$$

$$\text{var}(y) = \text{var}(\epsilon) = \sigma^2$$

$$\min ||y - XB||_2^2 = \min \text{sum of squared errors}$$

$$\hat{B} = (X^T X)^{-1} X^T y$$

=

If X is not full rank - one eigenvalue of X is 0

<https://math.stackexchange.com/questions/2131803/why-can-a-matrix-without-a-full-rank-not-be-invertible>

If X is not full rank

Then there exists some non-zero c st $X.c = 0$ (from definition of linear dependence)

so 0 is an eigenvalue of X

but we also know $X.0 = 0 \Rightarrow 0 = \text{inv}(X).0 \neq c!$ so $\text{inv}(x)$ cannot exist.

Also, $X(B + c) = XB + 0 = XB$

So there are infinitely many solutions B - i.e. there is more than one way to express the projection of y into the column space of X

Lasso

$$\min ||y - XB||_2^2 + \lambda ||B||$$