Coastal-Cosmo-Model

A cosmogenic nuclide model for coastal erosion of rocky shore platforms.

Best-fit scenarios of cliff retreat, down-wearing and relative sea-level change are determined using an optimised solver to predict measured ^{10}Be concentrations across a platform.

This document provides a user guide for the model, including an outline of the approach, and description of the input data, calculation method and best-fit optimisation framework.

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Contents

1. Approach to modelling cosmogenic nuclides & coastal erosion

Cosmogenic nuclide analysis provides an important tool for evaluating the erosional history of rocky coasts (Trenhaile, 2018). This is due to the tight relationship between the formation of platforms (through cliff retreat and subsequent down-wearing) and nuclide concentrations measured in platform surfaces (e.g. Choi et al., 2012). However, numerical models are necessary to quantify the formation history, including past rates of cliff retreat and down-wearing, owing to the complex interaction between nuclide production, erosion, sea-level history, tidal regime and beach cover.

Forward models have been created and applied to explore how different processes influence nuclide concentrations on shore platforms (e.g. Hurst et al., 2017; Regard et al., 2012), and to reconstruct past coastal erosion rates by validating model predictions against observations (e.g. Hurst et al., 2016; Swirad et al., 2020). Like these forward models, the approach presented here predicts *in-situ*-produced cosmogenic nuclide (e.g. ^{10}Be) concentrations along a platform as a function of cliff retreat, down-wearing and sea-level change over time, incorporating spatially and temporally varying topographic, water and cover shielding effects. However, this model differs in a several key ways:

1) ***Flexible parameter fitting*** – Rather than enforcing predefined relationships between processes, the model prioritizes finding the best combination of discrete parameters. While this contrasts with dynamic process-based models that explicitly incorporate physical drivers of erosion (e.g. Hurst et al., 2017), it allows greater flexibility in capturing potential variations in erosion history.
2) ***Community-standard nuclide calculations*** – The model calculates nuclide concentrations using CRONUSCalc (Marrero et al., 2016) and the global production rate calibration dataset (Borchers et al., 2016). This accurately accounts for both spallation and muon-induced nuclide production at any depth and time in the past using a unified, widely accepted computational framework. This approach allows for the calculation of time-dependent nuclide production at depth, which would occur from a multiphase cliff retreat (Regard et al., 2012), and could be extended to other nuclides (^{26}Al, ^{36}Cl, ^{3}He, ^{14}C, ^{21}Ne), which would further constrain erosion and shielding histories.
3) ***Empirical optimisation model*** – Although formulated as a forward model, it is implemented within an inverse framework through optimisation. Model predictions are compared against measured nuclide concentrations from sampled locations along the shore platform. The misfit is expressed as a least-squares objective function and minimized using a nonlinear optimization algorithm, yielding an optimal exposure and erosion history that best explains the data.

Comment on sea-level change. As with other cosmogenic nuclide models (e.g. Hurst et al., 2017; Regard et al., 2012; Swirad et al., 2020), this model currently uses mean relative sea level to prescribe the sea-level history. However, this variable has uncertainty, and for some study regions it may be unknown. A future version of this model will account for the uncertainties in relative sea-level data derived from proxy-based reconstructions or glacial isostatic adjustment models. For fast-eroding sites, this would provide a more rigorous quantification of cliff retreat and down-wearing rates. For slowly-eroding sites, where cosmogenic nuclide samples record a strong signal of platform emergence (e.g. Bierman et al., 2018), this would provide a test of plausible sea-level histories and enable reconstruction of relative sea-level change.

2. Inputs

2.1 Cosmogenic nuclide samples

As the model is design to find the best-fit coastal erosion scenario for a given cosmogenic nuclide dataset, details of each measured cosmogenic nuclide sample from a platform transect is required as an input. The following information must be included in a Microsoft© Excel© or comma-separated values spreadsheet, or in a tab-delimited text file: sample name; latitude (decimal degrees); longitude (decimal degrees); elevation (m above sea level); atmospheric pressure (hPa; if known); distance along transect from cliff (m); sample thickness (cm); bulk density (g cm^{-3}); topographic shielding factor (unitless, between 0 and 1); 10Be concentration (atoms g^{-1}; mean and 1 sigma uncertainty); year sample was collected.

Atmospheric pressure at each sample location (latitude, longitude) is derived from the ERA-40 atmospheric model (Uppala et al., 2005), with an elevation-pressure relationship (Radok et al., 1996) instead used if the sample is from <-60 °S (Balco et al., 2008; Stone, 2000).

Additionally, if available, the nuclide concentration of an inheritance sample (atoms g^{-1}; mean and 1 sigma uncertainty) should be provided. This is specified in the MATLAB© code, separate to the sample data spreadsheet. This cosmogenic inheritance sample should be collected in the near-shore, ideally at the base of a cliff or cave within the cliff, to quantify how much inherited ^{10}Be is in the rock prior to platform exposure (Hurst et al., 2016).

2.2 Topographic shielding across the platform

At any point along a platform, a rock surface can be shielded from cosmic rays by the topographic relief of the surrounding terrain. A shielding factor can be calculated to account for this impact on cosmogenic production, with values ranging from 0 (completely shielded by topography) and 1 (production is unaffected by topography).

A tab-delimited text file (.txt) is required with two columns: distance from cliff (m), and topographic shielding factor (unitless, 0-1). These values can be obtained from measurements (Dunne et al., 1999) at each rock sample collected or at a series of points across the platform, which are then linearly interpolated across the platform. Alternatively, shielding can be calculated from a Digital Elevation Model (Mudd et al., 2016).

Shielding factors can be calculated using the online calculator described by Balco et al. (2008) (http://stoneage.ice-d.org/math/v3/skyline_in.html), or by using the iceTEA (Jones et al., 2019) MATLAB© tool Topographic_shielding (https://github.com/iceTEA-code/Tools/blob/master/Topographic_shielding.m).

2.3 Elevation profile of the platform

To model and visualise nuclide concentrations across the platform, a two-dimensional elevation profile of the platform from the present-day cliff to the seaward edge is required as an input. This should be provided as a tab-delimited text file (.txt) with two columns: distance from cliff (m), and elevation (m AOD).

Elevations can be measured points, interpolated points or idealised values. Nuclide concentrations are predicted in the model at the spatial resolution of the profile provided here. While the predicted concentrations are linearly interpolated in the model for visualisation and to assess model fit, ideally the elevation should be measured as accurately as possible at or close to each measured sample.

2.4 Relative sea-level history

As seawater can partially or fully shield the platform from cosmic rays, a sea-level history is used in the model to account for the effects on nuclide production. It is particularly important for slowly-eroding coastlines, where the effects from sea-level change could dominate that from cliff retreat and down-wearing. Sea level would have varied spatially as well as temporally, and therefore a local (or at least regional) relative sea-level history is required; a global mean sea-level history would likely not provide an accurate estimate of platform shielding. This history can be sourced from a proxy-based reconstruction or glacial isostatic adjustment model.

The sea-level history should be provided as a tab-delimited text file (.txt) with two columns: time (in years before present, ideally with 'present' being the year that the samples were collected), and sea level (mean; m relative to present AOD). The model linearly interpolates between data points to derive a continuous sea-level history, with water shielding calculated at each model time interval (see section 3.4).

2.5 Tidal data and benchmarks

On top of relative sea level, tides determine how much time a platform surface spends submerged by seawater, and how much water shields that surface. The tidal regime therefore determines nuclide production and resulting nuclide concentrations across a platform (Hurst et al., 2017; Regard et al., 2012). This model uses local tidal data to calculate water shielding through time and space (see section 3.4).

Tidal data is required as a tab-delimited text file (.txt) with three columns: tidal frequency, tidal duration (%), and elevation (central value of 10-cm vertical bins; m AOD).

Additionally, a series of tidal benchmarks need to be specified in the MATLAB© code, separate to the tidal data: highest astronomical tide (HAT), mean high water level of spring tides (MHWS), mean high water level of neap tides (MHWN), mean low water level of spring tides (MLWS), mean low water level of neap tides (MLWN). These are used for visualisation and model calculations (see sections 3 and 4).

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3. Calculating cosmogenic nuclide concentrations

3.1 Nuclide concentration in a rock surface

At the present time, the concentration ($N_{x,k}$) (atoms g^{-1}) of nuclide k at point x along the rock platform (equivalent to a surface sample) is given by:

$$N_{x,k} = \frac{P_k}{\lambda_k + \frac{\rho \cdot \varepsilon}{\Lambda}} \cdot \left(1 - exp\left[-\left(\lambda_k + \frac{\rho \cdot \varepsilon}{\Lambda}\right)\right] \cdot t_{expo}\right) \tag{1}$$

where P_k is the nuclide's production rate (atoms g^{-1} yr^{-1}), λ_k is the nuclide's decay constant (1/years), ρ is rock density (g cm^{-3}), ε is the surface erosion rate (cm yr^{-1}), Λ is the attenuation length (g cm^{-2}), and t_{expo} is the exposure time (year).

For ^{10}Be, a global production rate calibration dataset (Borchers et al., 2016) scaled to the site location (see section 3.2) and decay constant of 4.99 × 10^{-7} corresponding to a half-life of 1.387 ± 0.012 Ma (Chmeleff et al., 2010; Korschinek et al., 2010) is used, with the atmospheric attenuation length calculated dependent on the location of the sample (Sato et al., 2008) and adjusted for lithospheric attenuation (see Marrero et al., 2016).

3.2 Nuclide production

The total nuclide production is calculated from the combination of spallogenic, which dominates near the surface, and muonogenic production, which dominates at depth:

$$P_k(t) = S_{EL,\zeta}(p, R_c, t) \cdot S_T \cdot S_W \cdot S_C \cdot P_{ref,s,\zeta,k} \cdot exp\left(\frac{-z}{\Lambda_s}\right) + S_T \cdot S_W \cdot S_C \cdot P_\mu(p, R_c, z) \tag{2}$$

where $S_{EL,\zeta}$ is the time-dependent elevation-latitude scaling factor for a particular scaling model (ζ), S_T is the shielding factor from topography of the surrounding terrain (see section 3.3Topographic shielding), S_W is the shielding factor from water (see section 3.4), S_C is the shielding factor from surface cover (see section 3.5), $P_{ref,s,\zeta}$ is the reference spallogenic (s) production rate (atom g^{-1} yr^{-1}) at present-day sea-level high-latitude (where atmospheric pressure, $p = 1013.25$) for nuclide k, Λ_s is the effective attenuation length (g cm^{-2}), z is the depth (g cm^{-2}), and P_μ is the production rate (atom g^{-1} a^{-1}) at z due to muons (μ), which is a function of pressure, depth and the cutoff rigidity (R_c).

Three principal scaling models for production by spallation can be used with this model: 1) 'Lm', the time-dependent version of Lal (1991), which uses variations in the dipole magnetic field intensity (Nishiizumi et al., 1989); 2) 'LSD', the time-dependent model of Lifton et al. (2014), which includes dipole and non-dipole magnetic field fluctuations and solar modulation; and 3) 'LSDn', a version of LSD that implements nuclide-specific scaling by incorporating cross-sections for the different reactions (Lifton et al., 2014).

The geomagnetic history used in all of the time-dependent scaling models includes the CALS3k model for 0-3 ka (Korte et al., 2009; Korte and Constable, 2011), the CALS7k model for 3-7 ka (Korte and Constable, 2005), the GLOPIS-75 model for 7-18 ka (Laj et al., 2004), and the PADM2M model for 18-2000 ka (Ziegler et al., 2011).

For production by muons, the muon flux is scaled using the energy-dependent model of Lifton et al. (2014), within the CRONUScalc numerical framework (see Marrero et al., 2016).

3.3 Topographic shielding

As the production rate is dependent on any shielding of the rock surface (Dunne et al., 1999; Gosse and Phillips, 2001), a time-dependent topographic shielding factor (S_T) is used with values ranging from 0 and 1. A typical present-day coastal platform would typically have relatively low values close to a cliff, increasing towards the sea.

In the model, topographic shielding factors are taken from the input data (see sections 2.1 and 2.2) for cases where the cliff position is the same as today. However, where cliff retreat occurs, more seaward points on the platform would have had higher shielding in the past. In this case, topographic shielding is calculated through time as a function of cliff retreat following Swirad et al. (2020):

$$S_T(x) = \frac{\sum_0^{t_{max}} f(S_{T,pres,x}, X_{pres,x}, X_{cliff,t}, t_{expo,x})}{t_{expo,x}} \quad (3)$$

where $S_{T,pres}$ is the present-day topographic shielding factor at point x along the rock platform, X_{pres} is the distance along the platform from the present-day cliff position (m), X_{cliff} is the cliff position at time t in the model run (m), and t_{expo} is the time of exposure at point x (years ago), with the resulting shielding factor integrated between present day and maximum modelled time (t_{max}). In effect, the shielding factor is calculated from the present-day shielding based on the relative distance to the cliff at each model time, assuming that the cliff retains a constant morphology. Shielding is only calculated for times after the cliff has retreated past each respective point along the platform ($X_x > X_{cliff}$).

3.4 Water shielding

Cosmic rays attenuate through water, reducing nuclide production in underlaying rock. This shielding by seawater therefore needs to be calculated through time and across a platform for accurate nuclide concentration estimates. A water-shielding model is used here (after Swirad et al., 2020), which combines the effects of tidal regime and relative sea-level (RSL) change.

A time-averaged water shielding factor (S_W, value of 0-1) is calculated at each point along the platform (x):

$$S_W(\mathrm{x}) = \frac{\sum_0^{t_{max}} exp\left(-\frac{z_W\,(Y_{pres,x}, h_W, RSL_t) \cdot \rho_W}{\Lambda}\right)}{t_{expo,i}} \quad (4)$$

where z_W is the water depth (cm), which is a function of the present-day elevation of the platform (Y_{pres}) at point x, the tidal height (h_W), and the relative sea level at time t (years ago), with ρ_W as the average density of seawater (1.024 g cm^{-3}) and Λ as the attenuation length (160 g cm^{-2}). For each point along the platform (x), tidal height is determined from the given tidal elevations and durations (see section 2.5), with platform elevations above HAT + RSL assumed to have no shielding from water. As with topographic shielding, water shielding is only calculated for times after the cliff has retreated past each respective point along the platform.

3.5 Surface cover shielding

Much like water, any surface material such as beach sand, talus or soil that is covering the platform partially shields the rock from cosmic rays. This can limit nuclide production, reducing ^{10}Be concentrations in upper portions of platforms. For platforms with fast-retreating cliffs, the reduction in concentration is likely minimal. However, the effect is more significant on slowly-eroding coastlines where high beach widths (>50 m) can absorb wave energy and protect the cliff (Hurst et al., 2017), and likely during periods of prolonged lower relative sea level when the platform could be more of a depositional than erosional environment.

To account for surface cover, a time-averaged shielding factor (S_C, value of 0-1) is calculated at each point along the platform (x) based on code from iceTEA (Jones et al., 2019):

$$S_C(\mathrm{x}) = \frac{\sum_0^{t_{max}} exp\left(-\frac{z_C\,(h_C, X_{pres,x}, Y_{pres,x}, RSL_t)\,\cdot\,\rho_C}{\Lambda}\right)}{t_{expo,i}} \tag{5}$$

where ρ_C is the average density of cover (g cm^{-3}) and z_C is the average depth of that cover (cm), which is a function of the given cover thickness (h_C), present-day distance along the platform (X_{pres}) and elevation of the platform (Y_{pres}) at point x, and the relative sea level at time t (years ago). As the existence of surface cover is dependent on relative sea level and the tide, h_C is applied to platform elevations above HAT + RSL and which linearly reduces to zero cover thickness at MHWN + RSL, producing a surface cover profile similar to other models (Hurst et al., 2017). The density (ρ_C) should be specified, with sand being 1.6 g cm^{-3} and soil approx. 1.3 g cm^{-3}. As with topographic shielding, cover shielding is only calculated for times after the cliff has retreated past each respective point along the platform.

While more complex mass-shielding approaches more accurately account for production from thermal neutron capture (Delunel et al., 2014; Dunai et al., 2014; Zweck et al., 2013) and variations in cover density with depth (Jonas et al., 2009), a simpler approach is preferred here for computational efficiency, and because spatial and temporal variations in cover depth are unknown.

4. Calculating erosion of the platform

4.1 Down-wearing

Nuclides produced in the platform surface can be lost from down-wearing (i.e. vertical erosion). This model calculates nuclide concentrations resulting from a constant, increasing or decreasing rate of down-wearing through time, similar to other models (e.g. Hurst et al., 2017; Swirad et al., 2020). However, unlike other models (e.g. Hurst et al., 2017; Regard et al., 2012), down-wearing is not coupled to the rate cliff retreat, ignoring a possible relationship between the two processes, but enabling down-wearing to be explored in the absence of cliff retreat. The rate of surface erosion (down-wearing) over time is calculated from:

$$z_{down}(t) = \rho \cdot \left(\varepsilon_{pres} + \frac{t}{t_{max}-1} \cdot \left(\left\{ \begin{matrix} \varepsilon_{pres} \cdot |\theta_{down}|, & if\ \theta_{down} < 0 \\ \varepsilon_{pres} \cdot \left(1 + (\theta_{down} - 1)\right), & if\ \theta_{down} > 0 \\ 0, & if\ \theta_{down} = 0 \end{matrix} \right\} - \varepsilon_{pres} \right) \right) \tag{6}$$

where $z_{down}(t)$ is erosion in the form of mass depth removed (g cm^{-2} yr^{-1}), ρ is rock density (g cm^{-3}), ε_{pres} is the present-day erosion rate (cm yr^{-1}), and θ_{down} is an erosion multiplier parameter determining the amount of down-wearing in the past relative to present, which is applied linearly between the maximum modelled time (t_{max}) and present day.

Down-wearing is only applied for times after the cliff has retreated past each respective point along the platform, when and where the platform has surface cover (e.g. beach), and when and where the platform is above MLWN + RSL (i.e. not when a platform surface is submerged by water throughout a year). Episodic, instantaneous jumps in vertical erosion of the platform (i.e. removal of blocks) is not included due to the complex and somewhat random processes involved; however, such an event could be inferred from a cosmogenic nuclide dataset where an outlier exists below an across-platform trend in concentrations, due to the increased loss of nuclides, or in cases of stepped platforms from back-wearing, a series of outliers above and below an across-platform trend (Hurst et al., 2017).

4.2 Cliff retreat

Landward retreat of the sea cliff provides the first order control on platform development and, therefore, the timing of platform exposure. Once the cliff has retreated to expose the platform to cosmic rays, it is assumed that nuclide production then initiates in that surface (after accounting for potential cosmogenic inheritance (see Section 2.1; Hurst et al., 2017; Regard et al., 2012; Swirad et al., 2020).

Calculation of cliff retreat in the model follows the approach of Swirad et al. (2020), where each new cliff position (X_{cliff}) is based on the previous position and a retreat rate, which can be a constant, increasing or decreasing rate through time (similar to down-wearing; section 4.1):

$$X_{cliff}(t) = t \cdot \beta_{pres} + \frac{t(t+1)}{2(t_{max}-1)} \cdot \left(\begin{Bmatrix} \beta_{pres} \cdot |\theta_{cliff}|, & if\ \theta_{cliff} < 0 \\ \beta_{pres} \cdot (1 + (\theta_{cliff} - 1)), & if\ \theta_{cliff} > 0 \\ 0, & if\ \theta_{cliff} = 0 \end{Bmatrix} - \beta_{pres} \right) \quad (7)$$

where β_{pres} is the present-day cliff retreat rate (m yr^{-1}), and θ_{cliff} is a retreat multiplier parameter determining the rate of retreat in the past relative to present, which is applied linearly between the maximum modelled time (t_{max}) and present day. The cliff retreat rate is assumed to be zero when the lowest point of the platform is above HAT + RSL (i.e. no retreat occurs when the platform is fully elevated above the water throughout a year).

The exposure time (t_{expo}) at each point along the platform (x) is then calculated as:

$$t_{expo}(x) = \begin{cases} t_{max}, & if\ X_{cliff}(t_{max}) < X_x \\ t^*, & if\ X_{cliff}(t^*) > X_x \end{cases} \quad (8)$$

where X_x is distance along the platform of point x, and t^* is the first time when a retreating cliff passes that position, defined as:

$$t^* = min\{t \mid X_{cliff}(t) > X_x\}. \quad (9)$$

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5. Model structure

5.1 Suite of sub-models to evaluate a cosmogenic nuclide dataset

The model consists of four sub-models to be used for different purposes, requiring different inputs and assessing different variables (Table 1). A cosmogenic inheritance model finds the best-fit surface erosion rate for a measured inheritance sample assuming steady-state production (Hurst et al., 2016), which can be used to evaluate whether the sample represents a feasible inheritance value for the site; if a best-fit erosion rate is found, the inheritance can be justifiably used to correct platform nuclide concentrations prior to modelling the platform erosion history.

A series of platform erosion models are designed to then test hypotheses of increasing complexity. The simplest hypothesis is that there is no relationship between erosion and the nuclide concentrations (i.e. the platform is a stable feature and no erosion has occurred), whereas the most complex hypothesis is that the nuclide concentrations are the result of cliff retreat, down-wearing and other factors over time. The simplest hypotheses should be ruled out before advancing to a more complex hypothesis to explain the data. A zero erosion model, which includes no cliff retreat or down-wearing, finds the best-fit total exposure time from a given sea-level history, with or without surface cover (e.g. beach). A down-wearing model, with no cliff retreat, finds the best-fit total exposure time from a given sea-level history and present-day down-wearing rate, with or without surface cover, keeping the present-day rate constant through time or finding a best-fit past rate relative to present (accelerating, decelerating or constant). Finally, a cliff retreat model (including down-wearing) finds the best-fit total exposure time and cliff retreat rate (accelerating, decelerating or constant), from a given sea-level history, present-day cliff retreat and down-wearing rates, with or without surface cover.

TABLE 1. SUITE OF MODELS

Model	**Required inputs**	**Free parameters (require initial value)**	**Outputs (best-fit result and variables)**
Cosmogenic inheritance	Cosmogenic nuclide samples; Depth of inheritance sample from cliff top	Cliff surface erosion rate	Surface erosion rate; nuclide production profile (from spallation, muons and total)
Zero platform erosion	Cosmogenic nuclide samples; Topographic shielding across the platform; Elevation profile of the platform; Relative sea-level history; Tidal data and benchmarks; Measurements to use for misfit calculation; *Density of surface cover (optional)*	Total model time; *Surface cover depth (optional)*	Total exposure (model time); nuclide concentrations across platform; platform submergence time; cumulative water shielding and topographic shielding; *optionally, mean surface cover depth and cumulative cover shielding*
Down-wearing only	Cosmogenic nuclide samples; Topographic shielding across the platform; Elevation profile of the platform; Relative sea-level history; Tidal data and benchmarks; Present-day down-wearing rate; Measurements to use for misfit calculation; *Density of surface cover (optional)*	Total model time; *Past rate of down-wearing (multiplier relative to present) (optional); Surface cover depth (optional)*	Total exposure (model time); platform profile prior to down-wearing; nuclide concentrations across platform; platform submergence time; cumulative rate of down-wearing across profile; cumulative water shielding and topographic shielding; *optionally, past rate of down-wearing relative to present, mean surface cover depth and cumulative cover shielding*
Cliff retreat and down-wearing	Cosmogenic nuclide samples; Topographic shielding across the platform; Elevation profile of the platform; Relative sea-level history; Tidal data and benchmarks; Present-day cliff retreat rate; Present-day down-wearing rate; Measurements to use for misfit calculation; *Density of surface cover (optional)*	Total model time; Past rate of cliff retreat (multiplier relative to present); *Past rate of down-wearing (multiplier relative to present) (optional); Surface cover depth (optional)*	Total exposure (model time); past rate of cliff retreat relative to present; past rate of down-wearing relative to present; platform profile and cliff position prior to down-wearing and cliff retreat; nuclide concentrations across platform; exposure time across platform; platform submergence time; cumulative rate of down-wearing across profile; cumulative water shielding and topographic shielding; *optionally, mean surface cover depth and cumulative cover shielding*

5.2 Optimisation framework to determine best-fit scenario

Optimisation is designed to efficiently find a solution, and therefore well-suited to testing hypotheses. Optimisation has proven effective for cosmogenic nuclide applications (Schaefer et al., 2016) as nuclide concentrations in a rock surface can be accurately solved (see section 3), allowing for lesser-known environmental variables (e.g. down-wearing and cliff retreat rates) to be numerically explored. An optimisation solver can therefore be used to predict nuclide concentrations from a given model and variables, assess those concentrations against known observations (i.e. measured samples), and then iteratively change the unknown variables to improve the fit with observations. This optimisation can be defined as:

$$\Phi_{opt} = \arg\min_{\Phi} \chi_R^2(\Phi) \tag{10}$$

where Φ is the set of free parameters (unconstrained variables) being optimised (see Table 1), and χ_R^2 is the misfit term (see section 5.2.3). The final minimised misfit (i.e. best-fit result) has the smallest misfit between model-predicted and measured nuclide concentrations. Similar to Schaefer et al. (2016), this model uses MATLAB's © fminsearch optimisation function, which performs multidimensional unconstrained nonlinear minimization using the Nelder-Mead simplex method (Lagarias et al., 1998); note, this requires installation of the Optimization Toolbox.

Optimisation solvers find a local minimum (or *optimum*), which is the smallest misfit value compared to nearby possibilities. However, this result may not be the *global* minimum, which is the smallest misfit value compared to all feasible possibilities. Therefore, it is advised to use all sub-models and model options (e.g. with surface cover and no surface cover) to explore different possible minimums for the data. Additionally, if the conclusions are based on relatively small differences between scenarios (e.g. rates of past down-wearing or cliff retreat), the sensitively to the choice of optimisation initial values should be explored; an effective approach would be to run the sub-model within a Monte Carlo simulation, evaluating the probability distribution of best-fit outputs from an ensemble of randomised initial values.

5.2.1 Cosmogenic inheritance

For the cosmogenic inheritance model (see Table 1), calculation of nuclide production (section 3) are used with the assumption of steady-state erosion (denudation) of the cliff-top surface (Hurst et al., 2016). The model-predicted concentration $N_{k,inh}$ (atoms g^{-1}) of nuclide k at depth of the inheritance sample (z_{inh}) is given by the integral:

$$N_{k,inh}(z_{inh}) = \frac{P_{k,z_{inh}}}{\lambda_k + (\frac{\varepsilon \cdot \rho}{\rho/\Lambda})} \tag{11}$$

where P_k is the combined spallogenic and muonogenic production rate (atoms g^{-1} yr^{-1}) of the inheritance sample, λ_k is the nuclide's decay constant, ρ is rock density (g cm^{-3}), ε is the erosion rate of the cliff-top surface (cm yr^{-1}), Λ is the attenuation length (g cm^{-2}).

5.2.2 Platform erosion

For the platform erosion models, calculations of nuclide production (section 3) and platform erosion (section 4) are combined to predict nuclide concentrations. The model-predicted average concentration N_k (atoms g^{-1}) of nuclide k in the platform surface (x) at the present time is given by the integral:

$$N_k(\mathrm{x}) = \frac{1}{(z_{x,bottom} - z_{x,top})} \int_0^{t_{expo}(x)} \int_{z_{x,top}}^{z_{x,bottom}} P_k(z + z_{down}(x) \cdot t, t, S_T, S_W, S_C) \cdot \exp(-\lambda_k \mathrm{t})\, dz\, dt \tag{12}$$

where $z_{x,top}$ and $z_{x,bottom}$ are the top and bottom mass depths (g cm^{-2}) below the platform surface, P_k is the combined spallogenic and muonogenic production rate (atoms g^{-1} yr^{-1}) as function of mass depth (z), time (t) and shielding factors S_T, S_W and S_C, z_{down} is the down-wearing rate (g cm^{-2} yr^{-1}), and λ_k is the decay constant (yr^{-1}) of nuclide k. N_k is zero when the exposure history starts, increasing towards present, with t zero at the present time and positive for past times; the duration of this exposure history is defined by t_{expo}, which is equivalent to the maximum model time (t_{max}) for cases with no cliff retreat.

5.2.3 Assessing misfit

The model uses a least-squares misfit statistic to compare nuclide concentrations predicted by the model to measured concentrations, using the measurement uncertainty as the weighting. It is computed as the mean squared residuals, equivalent to a reduced chi-squared (χ_R^2):

$$\chi_R^2 = \frac{1}{n} \sum_{m=1}^{n} \left(\frac{N_{k,pred}(x_m) - N_{k,meas,m}}{\sigma_{meas,m}}\right)^2 \tag{13}$$

where n is the total number of samples, $N_{k,pred}$ and $N_{k,meas}$ are the predicted and mean measured nuclide concentrations for sample m, and σ_{meas} is the corresponding nuclide measurement uncertainty. $N_{k,pred}$ is taken from the point along the platform (x) where the sample was collected. $N_{k,meas}$ uses the raw measured concentration minus the concentration of the inheritance sample (section 2.1).

The optimisation solver requires a single misfit value, yet multiple samples are typically measured across a platform. The platform models (zero erosion, down-wearing, cliff retreat and down-wearing; Table 1) therefore have options for how to combine misfit values calculated for each measured sample:

- All samples ('all'), which takes a mean of misfits from all measured samples.
- Minimum only ('min'), which uses the minimum measured mean nuclide concentration.
- Maximum only ('max'), which uses the maximum measured mean nuclide concentration.
- Minimum and maximum ('minmax' or 'gradient'), which takes a mean of the misfits from the minimum and maximum measured concentrations, intended for situations where the endmembers are more important than other samples.

The user should choose the most appropriate misfit method for the hypothesis and sample data. The best-fit result is reported as the χ_R^2 with degrees of freedom ($\mathrm{DOF} = \mathrm{n} - 1$), and it is the user's choice about what value is considered acceptable. The critical value can be taken from a chi-squared distribution table using the right-sided tail probability (equal to the significance level, e.g., 0.05 for 95% confidence) and the DOF; for example, the critical value is approximately 12.592 for 6 DOF at 95% confidence.

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