Robotic Vision
Assignment No 4:-

A1. The given system is holonomic & the motion is characterized by the following model.

$$\dot{x} = v_x$$
$$\dot{y} = v_y$$
$$\dot{\theta} = \omega$$

Considering the initial actual position to be given by $\begin{bmatrix} x_{actual} \\ y_{actual} \\ \theta_{actual} \end{bmatrix}$ & the final point to be $\begin{bmatrix} x_{desired} \\ y_{desired} \\ \theta_{desired} \end{bmatrix}$

Now

Error; $E = X_{actual} - X_{desired}$

$$e(t) = \begin{bmatrix} x_{actual}(t) - x_{desired}(t) \\ y_{actual}(t) - y_{desired}(t) \\ \theta_{actual}(t) - \theta_{desired}(t) \end{bmatrix}$$

$$\dot{e}(t) = \begin{bmatrix} \dot{x}_{actual} - \dot{x}_{desired} \\ \dot{y}_{actual} - \dot{y}_{desired} \\ \dot{\theta}_{actual} - \dot{\theta}_{desired} \end{bmatrix}$$

Now we know,

$$\dot{e}(t) = -\lambda e(t).$$

& substituting values for $\dot{x}$, $\dot{y}$ & $\dot{\theta}$, we get,

$$\begin{bmatrix} v_x - \dot{x}_{desired} \\ v_y - \dot{y}_{desired} \\ \omega - \dot{\theta}_{desired} \end{bmatrix} = -\lambda \begin{bmatrix} x_{actual} - x_{desired} \\ y_{actual} - y_{desired} \\ \theta_{actual} - \theta_{desired} \end{bmatrix}$$

$$\begin{bmatrix} v_x \\ v_y \\ \omega \end{bmatrix} = -\lambda \begin{bmatrix} x_{actual} - x_{desired} \\ y_{actual} - y_{desired} \\ \theta_{actual} - \theta_{desired} \end{bmatrix} + \begin{bmatrix} \dot{x}_{desired} \\ \dot{y}_{desired} \\ \dot{\theta}_{desired} \end{bmatrix}$$

$$\therefore \begin{bmatrix} x(s) \\ y(s) \end{bmatrix} = \begin{cases} \begin{bmatrix} x_i \\ y_i \end{bmatrix} & \text{if } s = 1. \\ \begin{bmatrix} x_f \\ y_f \end{bmatrix} & \text{if } s = 0. \end{cases}$$

$$\therefore x(s) = s^3 x_f - (s-1)^3 x_i + \alpha_x s^2 (s-1) + \beta_x s (s-1)^2$$

$$y(s) = s^3 y_f - (s-1)^3 y_i + \alpha_y s^2 (s-1) + \beta_y s (s-1)^2.$$

differentiating wrt s,

$$\dot{x}(s) = 3s^2 x_f - 3(s-1)^2 x_i + \alpha_x [2s(s-1) + s^2] + \beta_x [(s-1)^2 + 2s(s-1)].$$

$$\dot{y}(s) = 3s^2 y_f - 3(s-1)^2 y_i + \alpha_y [2s(s-1) + s^2] + \beta_y [(s-1)^2 + 2s(s-1)]$$

Take $s = 0$,

$$\dot{x}(0) = -3x_i + \beta_x = k_i \cos\theta_i$$

$$\dot{y}(0) = -3y_i + \beta_y = k_i \sin\theta_i$$

$x(1) = 3x_f + \alpha_x = k_f \cos\theta_f$

$y(1) = 3y_f + \alpha_y = k_f \sin\theta_f$

$\therefore \quad \alpha_x = k_f \cos\theta_f + 3x_f.$

$\alpha_y = k_f \sin\theta_f + 3y_f.$

$\beta_x = k_i \cos\theta_i + 3x_i$

$\beta_y = k_i \sin\theta_i + 3y_i$

$\therefore \quad \alpha_\theta = -3\theta_f$ & $\beta_f = -3\theta_i$