A Nonlinear Bayesian Filtering Framework for ECG Denoising

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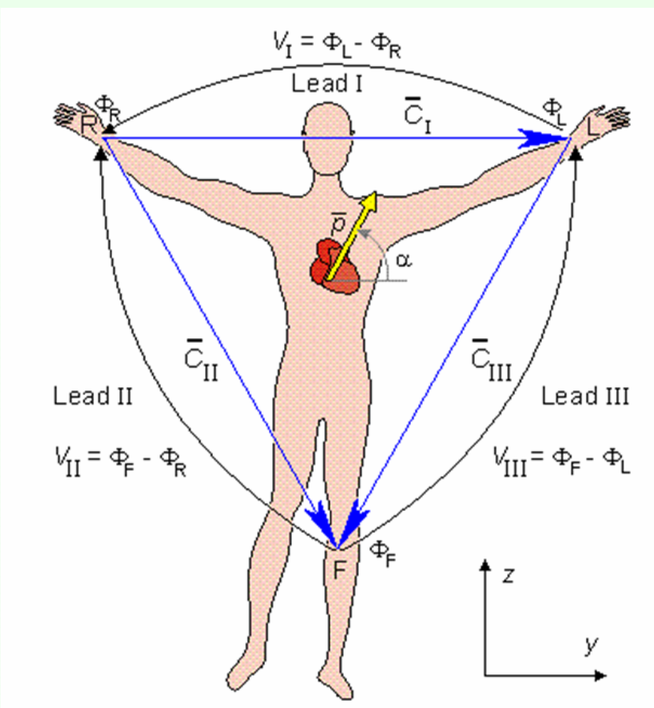
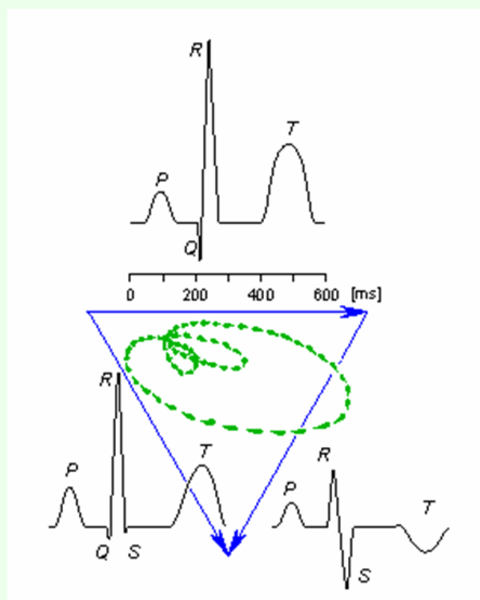
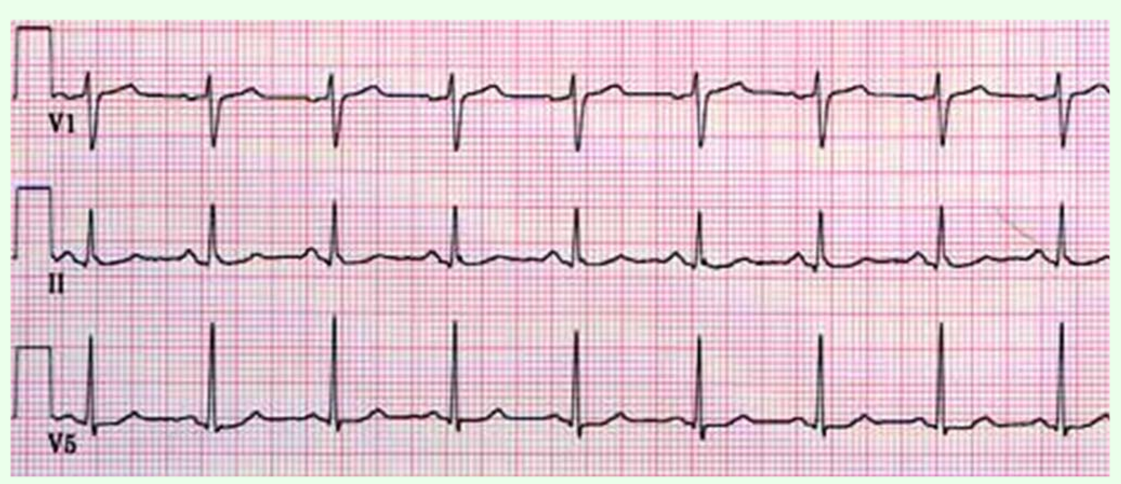
Dipole Model of the Heart

The cardiac potentials may be modeled by a single rotating dipole located at the heart. This model is known as the Single Dipole Model (SDM).





The ECGs can be considered as projections of the cardiac dipole onto the electrode axes.



Other models: Moving dipole, Multipole, Activation maps, …

Bayesian Filters

The Kalman Filter (KF) is an optimal filter for linear Gaussian processes

$$\begin{cases} \underline{x}_{k+1} = A_k \underline{x}_k + B_k \underline{w}_k \\ \underline{y}_k = H_k \underline{x}_k + \underline{v}_k \end{cases}$$

The Kalman Smoother (KS) is a non-causal version of the KF which uses future measurements to give better estimates of the current state

The Extended Kalman Filter (EKF) is a generalization of the KF for nonlinear systems

$$\begin{cases} \underline{x}_{k+1} = f(\underline{x}_k, \underline{w}_k, k) \\ \underline{y}_k = g(\underline{x}_k, \underline{v}_k, k) \end{cases}$$

The Extended Kalman Smoother (EKS) is the nonlinear extension of the KS

The Unscented Kalman Filter (UKF) is a variant of the EKF for highly nonlinear (non-)Gaussian systems, based on the Unscented Transform (UT)

Particle Filters (PF) are Monte Carlo estimation techniques, based on Sequential Importance Sampling which are useful for nonlinear non-Gaussian systems

Abstract

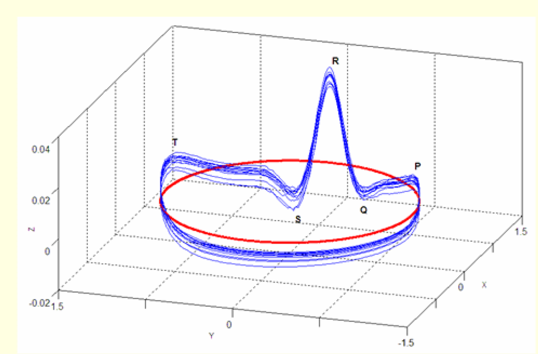
In this work a nonlinear Bayesian filtering framework is proposed for the filtering of noisy Electrocardiogram (ECG) recordings. The ECG is modeled by a state-space nonlinear dynamic model. This model is used for constructing Bayesian filters, such as the Extended Kalman Filter (EKF), Extended Kalman Smoother (EKS), and Unscented Kalman Filter (UKF). The method is evaluated on several ECG recordings, by artificially adding real non-stationary muscle artifacts, white and colored Gaussian noises to clean ECG recordings, and studying the SNRs and morphology of the filter outputs. The results of the study show superior results compared with conventional ECG denoising approaches such as band-pass FIR filtering, adaptive filtering (AF), and wavelet denoising (WD).

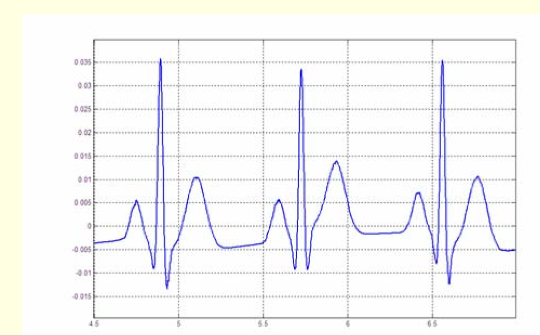
A Synthetic ECG Generator

A modified version of the synthetic ECG generator developed by McSharry *et al.*

$$\begin{cases} \theta[k+1] = \theta[k] + \omega\delta \\ z[k+1] = -\sum_i \delta \frac{\alpha_i \omega}{b_i^2} \Delta\theta_i exp(-\frac{\Delta\theta_i^2}{2b_i^2}) + z[k] + \eta \end{cases}$$

$$\Delta\theta_i = (\theta - \theta_i) mod(2\pi)$$

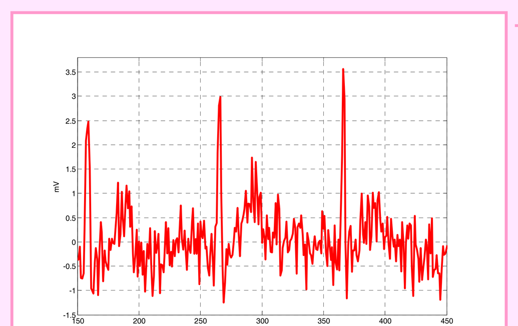




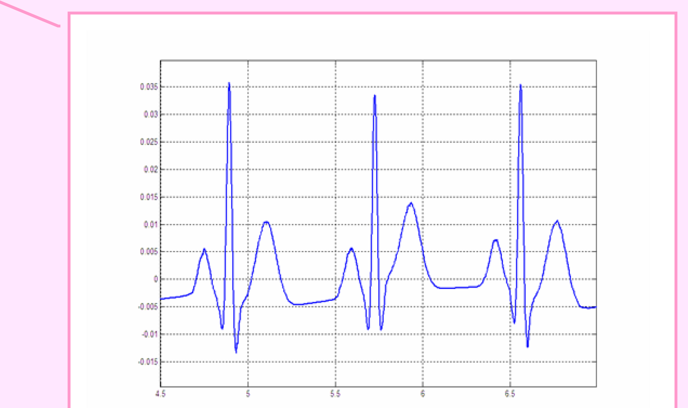
Filtering Scheme

(State equation) (Observation equation)

$$\begin{cases} \underline{x}_{k+1} = f(\underline{x}_k, \underline{w}_k, k) \\ \underline{y}_k = g(\underline{x}_k, \underline{v}_k, k) \end{cases}$$



Noisy observations



Dynamical ECG model

Dynamical ECG model

$$\begin{cases} \underline{x}_{k+1} = f(\underline{x}_k, \underline{w}_k, k) \\ \underline{y}_k = g(\underline{x}_k, \underline{v}_k, k) \end{cases}$$

Noisy observations

Linearization

(Process noise covariance matrix)

$$Q_k = E\{\underline{w}_k \underline{w}_k^T\}$$

$$\begin{cases} \underline{x}_{k+1} = A_k \underline{x}_k + B_k \underline{w}_k \\ \underline{y}_k = H_k \underline{x}_k + D_k \underline{v}_k \end{cases}$$

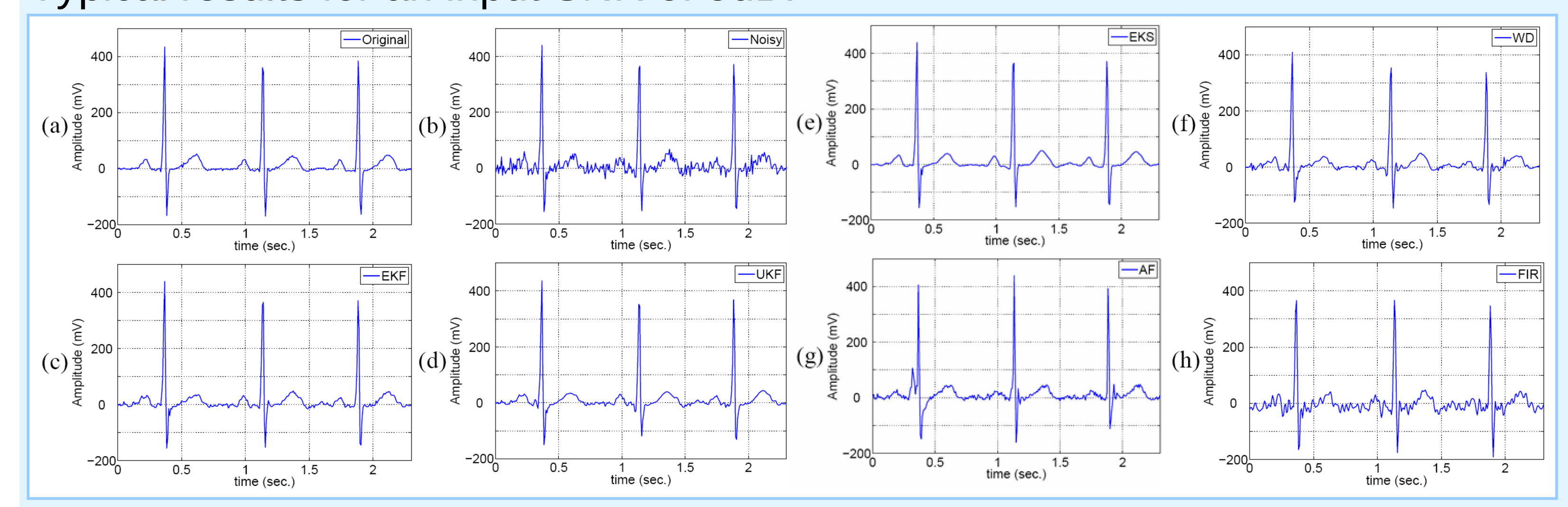
$$R_k = E\{\underline{v}_k \underline{v}_k^T\}$$

(Observation noise covariance matrix)

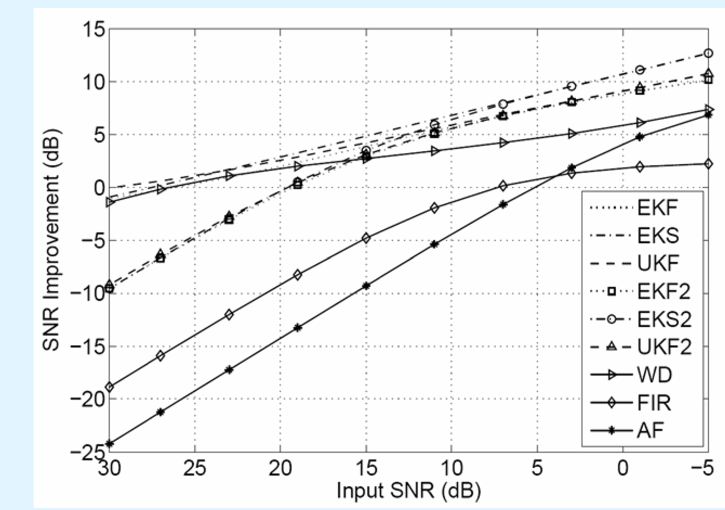
The dynamic model parameters are trained for a given set of noisy ECG recordings

Results

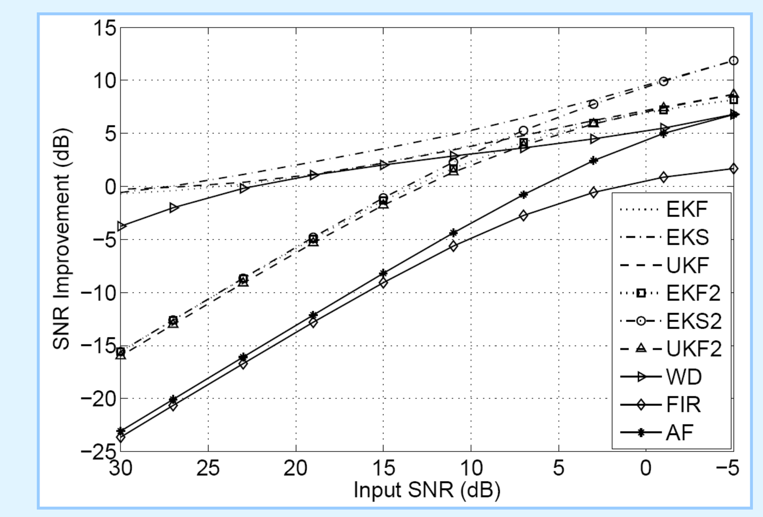
Typical results for an input SNR of 5dB:



Quantitative results for a sweep of input SNR with white additive Gaussian noise:



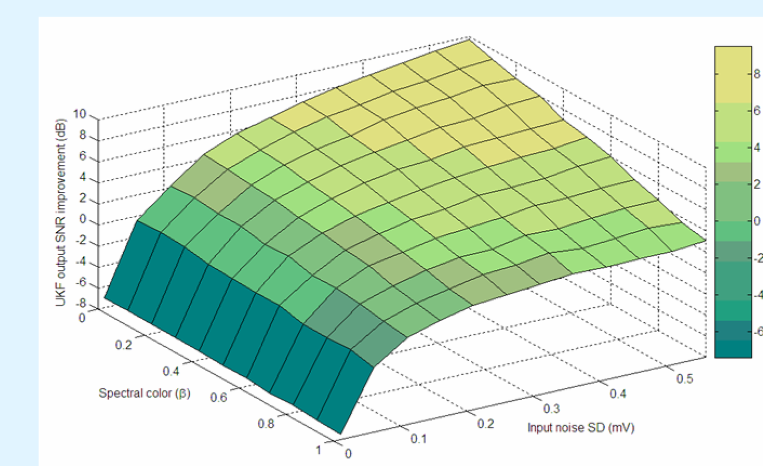
SNR calculated over the whole ECG



SNR calculated over the ST-segment

Results with colored noise:

$$S(f) \propto \frac{1}{f^{\beta}}$$



Conclusions

The proposed method can be assumed as a generalization of conventional ECG filtering approaches, with superior performance. It is hence believed to be of great interest, especially for low SNR applications such as the cancellation of maternal ECG from noninvasive fetal ECG recordings, or the removal of ECG artifacts from Electroencephalogram (EEG) recordings.

References

R. Sameni, M. B. Shamsollahi, C. Jutten, and G. D. Clifford, **A Nonlinear Bayesian Filtering Framework for ECG Denoising**, *Submitted to the IEEE Trans. Biomed. Eng.*, Feb. 2006.

R. Sameni, M. B. Shamsollahi, C. Jutten, **Filtering Noisy ECG Signals Using the Extended Kalman Filter Based on a Dynamic ECG Model**, *Computers in Cardiology*, 32:1017-1020, 2005.

P. E. McSharry, G. D. Clifford, L. Tarassenko, and L. A. Smith. **A Dynamic Model for Generating Synthetic Electrocardiogram Signals**. *IEEE Trans. Biomed. Eng.*, 50:289-294, march 2003.

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