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NPTEL Week 2 Live Sessions

on Deep Learning (noc24_ee04)

A course offered by: Prof. Prabir Kumar Biswas, IIT Kharagpur

- Python coding: Feature Engineering
- Week 2 practice questions



PMRF

Prime Minister's Research Fellows
Ministry of Education
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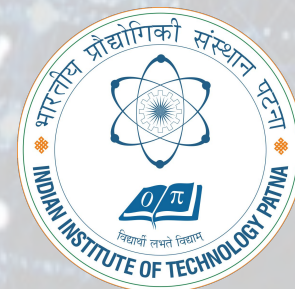
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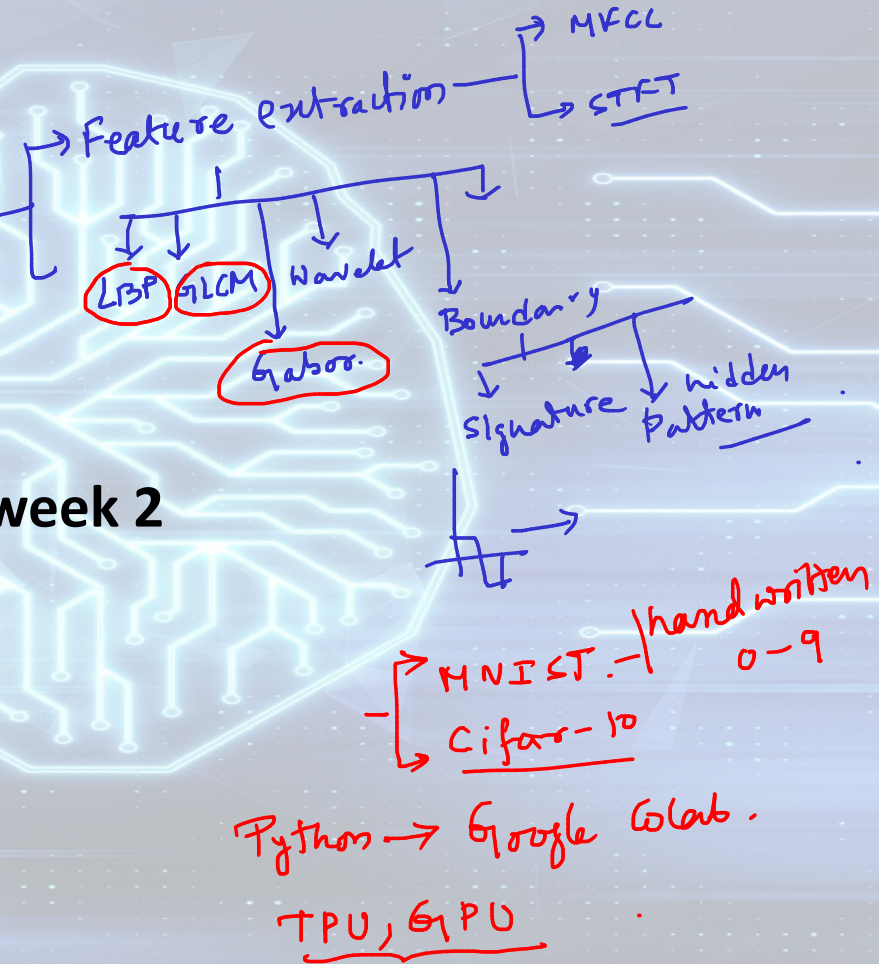
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Content of the live session

1. Python coding for week-1 content
2. Glimpse of week 2 ~ a revisit !!
3. Solving numerical problems from week 2



2D
 $2 \times 2 \rightarrow \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}$

3D
 $32 \times 32 \times 3$

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

Column Axis = 1
 Axis = 0

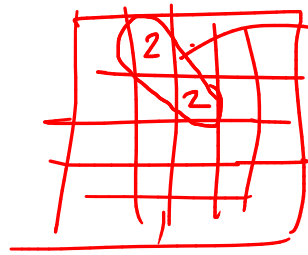
$$e^{-j\omega t} = \cos \omega t - j \sin \omega t$$

Real

$$z = a + jb$$

$A(i, j)$
 $\langle 1, 0 \rangle$
 distance \rightarrow Angle.

$A(2, 2)$
 $(1, 1)$



$A = \begin{bmatrix} 5 & 10 \\ 15 & 13 \end{bmatrix}$ 2×2

Norm: $\max(A_{ij}) = 15$

$\begin{bmatrix} 5/15 & 10/15 \\ 15/15 & 13/15 \end{bmatrix}$

$$|z|^2 = (\sqrt{a^2 + b^2})^2$$

$1 \rightarrow 9 \times 1$
 $50000 \rightarrow 50000 \times 9$

$50000 \times 32 \times 32 \times 3$

(4D) → Tensor

scalar → 1.

2D → 2×2

3D → $M \times N \times L$

$M \times N \times L \times D \dots$

Multidimensional array

→ Tensor

↓
TPU



$$P(w_1/x) = \frac{P(x/w_1) \cdot P(w_1)}{\rightarrow P(x)}$$

$$P(w_2/x) = \frac{P(x/w_2) \cdot P(w_2)}{P(x)}$$

$$\rightarrow P(x) = P(w_1) \cdot P(x/w_1) + P(w_2) \cdot P(x/w_2)$$

$$\underline{x \in w_1} :- P(w_1/x) > P(w_2/x)$$

$$\Rightarrow \frac{P(x/w_1) \cdot P(w_1)}{\cancel{P(x)}} > \frac{P(x/w_2) \cdot P(w_2)}{\cancel{P(x)}}$$

The class conditional probability density function for the class ω_i , i.e. $P(x|\omega_i)$ for a multivariate normal (or Gaussian) distribution (where x is a d dimensional feature vector) is given by

a. $p(x|\omega_i) = \frac{1}{(2\pi)^{d/2} |\Sigma_i|^{1/2}} \exp(-\frac{1}{2} (x - \mu_i)^T \Sigma_i^{-1} (x - \mu_i))$

b. $p(x|\omega_i) = \frac{1}{(2\pi)^{d/2}} \exp(-\frac{1}{2} (x - \mu_i)^T \Sigma_i^{-1} (x - \mu_i))$

c. $p(x|\omega_i) = \frac{1}{(2\pi)^{d/2}} \exp(-\frac{1}{2} (x - \mu_i)^T (x - \mu_i))$

d. None of the above

$\Sigma \times$

0 0

$x = \begin{bmatrix} \text{Gender}(x_1) \\ \text{Profession}(x_2) \end{bmatrix}_{2 \times 1}$

$\Sigma = \begin{bmatrix} & \\ & \end{bmatrix}$

$\frac{1}{\sqrt{2\pi}(\sigma)} = \frac{1}{\sqrt{2\pi\sigma^2}}$

$\frac{1}{\sqrt{2\pi}(\Sigma)}$

$\frac{x \in d \times 1}{R}$
 $\Sigma \in R^{d \times d}$

$\Sigma \rightarrow \text{general}$
 $\frac{dx_1}{\Sigma dx_1}$

$\Sigma \rightarrow \Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_{12}^2 \\ \sigma_{23}^2 & \sigma_4^2 \end{bmatrix}$

$\Sigma_{1d} = \sigma_1^2$

For n dim -

$\frac{1}{\sqrt{2\pi} \Sigma_{nd}}$

$\frac{|x|}{|x|}$
 $\begin{vmatrix} a_1 & a_2 \\ a_3 & a_4 \end{vmatrix} = (a_1 a_4 - a_3 a_2)$

$\frac{1}{\sqrt{2\pi} \Sigma_{1d}}$

$\frac{1}{\sqrt{2\pi} \Sigma_{2d}}$

There are some data points for two different classes given below.

Class 1 points: $\{(2, 6), (3, 4), (3, 8), (4, 6)\}$

Class 2 points: $\{(3, 0), (1, -2), (5, -2), (3, -4)\}$

Compute the mean vectors μ_1 and μ_2 for these two classes and choose the correct option.

a. $\mu_1 = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$ and $\mu_2 = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

b. $\mu_1 = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$ and $\mu_2 = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$

☒ c. $\mu_1 = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$ and $\mu_2 = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$

d. $\mu_1 = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$ and $\mu_2 = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$

$$\mu_2 = \frac{\begin{bmatrix} 3 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \end{bmatrix} + \begin{bmatrix} 5 \\ -2 \end{bmatrix} + \begin{bmatrix} 3 \\ -4 \end{bmatrix}}{4} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

$$\mu_1 = \frac{\begin{bmatrix} 2 \\ 6 \end{bmatrix} + \begin{bmatrix} 3 \\ 4 \end{bmatrix} + \begin{bmatrix} 3 \\ 8 \end{bmatrix} + \begin{bmatrix} 4 \\ 6 \end{bmatrix}}{4}$$

$$= \frac{\begin{bmatrix} 12 \\ 24 \end{bmatrix}}{4} = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$$

$$\underline{\underline{\text{Avg}(D_1, D_2, \dots, D_n)}}$$

$$\mu = E(\cdot)$$

There are some data points for two different classes given below.

Class 1 points: $\{(2, 6), (3, 4), (3, 8), (4, 6)\}$

Class 2 points: $\{(3, 0), (1, -2), (5, -2), (3, -4)\}$

Compute the covariance matrices Σ_1 and Σ_2 and choose the correct option.

a. $\Sigma_1 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ and $\Sigma_2 = \begin{bmatrix} 2 & 0 \\ 0 & 0.5 \end{bmatrix}$

b. $\Sigma_1 = \begin{bmatrix} 0.5 & 0 \\ 0 & 2 \end{bmatrix}$ and $\Sigma_2 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

c. $\Sigma_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $\Sigma_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

d. $\Sigma_1 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$ and $\Sigma_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$\mu_1 = \begin{bmatrix} 3 \\ 6 \end{bmatrix}_{2 \times 1} \quad \left(\sum_{i=1}^n \frac{1}{n} \text{Exp} \left((x - \mu) (x - \mu)^T \right) \right)_{2 \times 2}$$

for first data point
 $(x - \mu) (x - \mu)^T = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \begin{bmatrix} -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

$$\begin{bmatrix} 0 \\ -2 \end{bmatrix} \begin{bmatrix} 0 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 4 \end{bmatrix} \quad \checkmark$$

$$\begin{bmatrix} 0 \\ 2 \end{bmatrix} \begin{bmatrix} 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 4 \end{bmatrix} \quad \checkmark$$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad \checkmark$$

off diag = 0 \rightarrow They are statistically independent.

Exp \rightarrow Avg

$$= \begin{bmatrix} 0.5 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

4

There are some data points for two different classes given below.

Class 1 points: $\{(2, 6), (3, 4), (3, 8), (4, 6)\}$

Class 2 points: $\{(3, 0), (1, -2), (5, -2), (3, -4)\}$

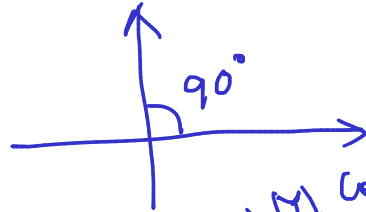
Compute the covariance matrices Σ_1 and Σ_2 and choose the correct option.

a. $\Sigma_1 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ and $\Sigma_2 = \begin{bmatrix} 2 & 0 \\ 0 & 0.5 \end{bmatrix}$

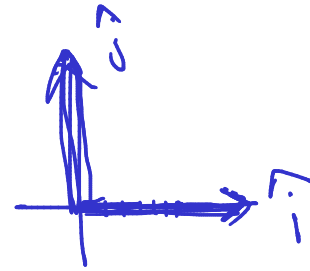
b. $\Sigma_1 = \begin{bmatrix} 0.5 & 0 \\ 0 & 2 \end{bmatrix}$ and $\Sigma_2 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

c. $\Sigma_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $\Sigma_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

d. $\Sigma_1 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$ and $\Sigma_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

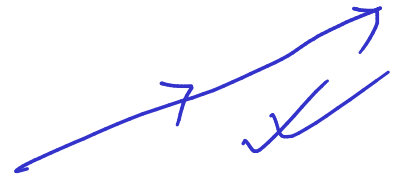
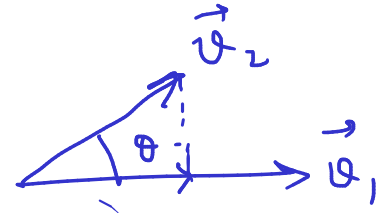
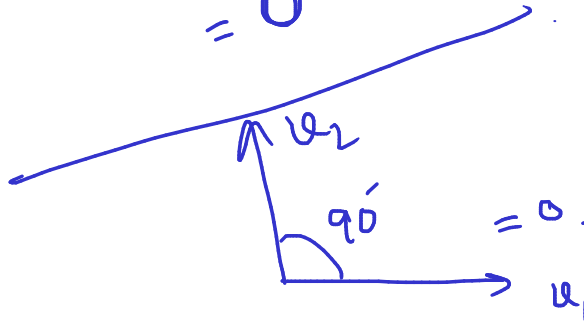
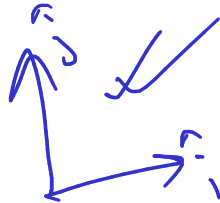


$\langle x, y \rangle = |x||y| \cos 90^\circ$ independence Means \rightarrow
 $= |\hat{i}| |\hat{j}| \cos 90^\circ$
 $= 0$



$3\hat{i} + 0\hat{j}$
 \swarrow
 No dependence
 in the
 axis.

$\langle x, y \rangle$



Let Σ_i represents the covariance matrix for i^{th} class. Assume that the classes have the same co-variance matrix. Also assume that the features are statistically independent and have same co-variance. Which of the following is true?

- a. $\Sigma_i = \Sigma_j$ (diagonal elements of Σ are zero) ~~X~~
- b. $\Sigma_i = \Sigma_j$ (diagonal elements of Σ are non-zero and different from each other, rest of the elements are zero) ~~X~~
- c. $\Sigma_i = \Sigma_j$ (diagonal elements of Σ are non-zero and equal to each other, rest of the elements are zero) ✓
- d. None of these

$$\begin{aligned}\Sigma_i &= \Sigma_j \\ &= \begin{bmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{bmatrix}_{2 \times 2} \\ &= \sigma^2 I\end{aligned}$$

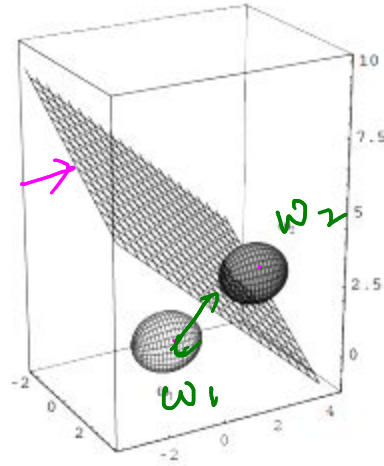
$$\Sigma_i = \Sigma_j = \sigma^2 I$$

$$\sigma^2 I = \sigma^2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\sigma^2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The decision surface between two normally distributed class ω_1 and ω_2 is shown on the figure.

Can you comment which of the following is true?



a. $p(\omega_1) = p(\omega_2)$

b. $p(\omega_2) > p(\omega_1)$

☒ c. $p(\omega_1) > p(\omega_2)$

d. None of the above.

$$\Sigma_i = \Sigma_j = \sigma^2 I.$$

$$= \begin{bmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{bmatrix}$$

what is the decision surface=!

