NPTEL Week-4 Live Session

on Machine Learning and Deep Learning - Fundamentals and Applications (noc24_ee146)

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A course offered by: Prof. Manas Kamal Bhuyan, IIT Guwahati

NPTEL Quiz Solution: Week-3, Practice problem on Support Vector Machine

By

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The MLE for the data samples X={x1,x2,…,xi,…,xk} with the Bernoulli distribution is

$X \sim$ Bernoulli Dist.

$\rightarrow$ iid (identical and independent distribution).

(1)

$$PMF = p^{x_i}(1-p)^{(1-x_i)}$$

$$\mathcal{L}(p) = \prod_{i=1}^{k} p^{x_i}(1-p)^{(1-x_i)}$$

$$\Rightarrow \ln \mathcal{L}(p) = \sum_{i=1}^{k}\left(x_i \ln p + (1-x_i)\ln(1-p)\right)$$

$$\frac{d}{dp}\left(\ln \mathcal{L}(p)\right) = \sum_{i=1}^{k}\left(x_i \times \frac{1}{p} + (1-x_i) \times \frac{1}{1-p} \times (-1)\right)$$

$$\frac{d}{dp}\left(\ln \mathcal{L}(p)\right) = \sum_{i=1}^{k}\left(\frac{x_i}{p} - \frac{1-x_i}{1-p}\right) = 0.$$

$$\Rightarrow \frac{\sum_{i=1}^{k} x_i}{p} = \frac{\sum_{i=1}^{k}(1-x_i)}{1-p}$$

$$\Rightarrow \sum_{i=1}^{k} x_i - \cancel{p\sum_{i=1}^{k} x_i} = p\sum_{i=1}^{k} 1 - \cancel{p\sum_{i=1}^{k} x_i}$$

$$\Rightarrow \sum_{i=1}^{k} x_i = p \cdot k$$

$$\Rightarrow \boxed{p = \frac{1}{k}\sum_{i=1}^{k} x_i}$$

$$p = \frac{1}{k}\sum_{i=1}^{k} x_i$$

$\rightarrow$ MLE - The sample mean of the distribution

9) Consider single observation X that depends on a random parameter .Suppose θ has a prior distribution

Prior

$$f_\Theta(\theta) = \lambda e^{-\lambda\theta} \text{ for } \theta \geq 0, \lambda > 0$$

$$f_{\frac{x}{\theta}}(x) = \theta e^{-\theta x} |x| > 0$$

Find the MAP estimation of θ

- [x] $\frac{1}{\lambda+X}$
- [ ] $\frac{1}{\lambda-X}$
- [ ] $\frac{\lambda}{X}$
- [ ] X

Prior :- $f_\theta(\theta) = \lambda e^{-\lambda\theta}$; $\theta \geq 0$, $\lambda > 0$

Conditional density :- $f_{x|\theta}(x) = \theta e^{-\theta x}$; $|x| > 0$

$f_{\theta|x}(\theta) = \frac{f_{x|\theta}(x) \cdot f(\theta)}{}$

Total probability

Aposteriori probability $f_{\theta|x}(\theta) \propto f_{x|\theta}(x) \cdot f_\theta(\theta)$.

$\propto \theta e^{-\theta x} \cdot \lambda e^{-\lambda\theta}$

$f_{\theta|x}(\theta) \propto \lambda \theta e^{-(\lambda+x)\theta}$

$\ln f_{\theta|x}(\theta) \propto \ln(\lambda\theta e^{-(\lambda+x)\theta})$

$\ln f_{\theta|x}(\theta) \propto \ln\lambda + \ln\theta - (\lambda+x)\theta$

$\frac{d}{d\theta} \ln f_{\theta|x}(\theta)$.

$\propto \frac{1}{\theta} - (\lambda+x)$

$\frac{d}{d\theta}(\ln f_{\theta|x}(\theta)) = k\left(\frac{1}{\theta} - (\lambda+x)\right)$

$= 0$

$k\left(\frac{1}{\theta} - (\lambda+x)\right) = 0.$

$\boxed{\theta = \frac{1}{\lambda+x}}$.

6) Suppose that X is a discrete random variable with the following probability mass function: where is a parameter.
$(0 \leq \theta \leq 1)$

X	0	1	2	3
P(X)	$2\theta/3$	$\theta/3$	$2(1-\theta)/3$	$(1-\theta)/3$

Probability is multiplicative in nature.

$x_i = \{3, 0, 2, 1, 3, 2, 1, 0, 2, 1\}$

The following 10 independent observations were taken from such a distribution: $(3, 0, 2, 1, 3, 2, 1, 0, 2, 1)$. What is the maximum likelihood estimate of θ ?

- 2
- 1
- 0.5
- 0

$$\mathcal{L}(\theta) = \prod_{i=1}^{10} P(X = x_i)$$

$$= P(X=3) \cdot P(X=0) \cdot P(X=2) \cdot P(X=1) \cdot P(X=3)\, P(X=2) \cdot P(X=1) \cdot P(X=0) \cdot P(X=2) \cdot P(X=1)$$

$$= [P(X=3)]^2 \, [P(X=0)]^2 \, (P(X=2))^3 \, (P(X=1))^3$$

$$\boxed{\mathcal{L}(\theta) = \left(\frac{1-\theta}{3}\right)^2 \cdot \left(\frac{2\theta}{3}\right)^2 \cdot \left(\frac{2}{3}(1-\theta)\right)^3 \cdot \left(\frac{\theta}{3}\right)^3}$$

6) Suppose that X is a discrete random variable with the following probability mass function: where is a parameter.
$(0 \le \theta \le 1)$

$P(X=1|\theta)$

X	0	1	2	3
P(X)	$2\theta/3$	$\theta/3$	$2(1-\theta)/3$	$(1-\theta)/3$

The following 10 independent observations were taken from such a distribution: $(3,0,2,1,3,2,1,0,2,1)$. What is the maximum likelihood estimate of θ?

- 2
- 1
- 0.5
- 0

$$\mathcal{L}(\theta) = \left(\frac{1-\theta}{3}\right)^2 \cdot \left(\frac{2\theta}{3}\right)^2 \cdot \left(\frac{2}{3}(1-\theta)\right)^3 \cdot \left(\frac{\theta}{3}\right)^3$$

iid

$$\mathcal{L}(\theta) = \prod_{i=1}^{10} P(X=x_i|\theta)$$

$P(x|\theta)$ class conditional density

$P(X=1|\theta)$ $\theta/3$

$$\ln \mathcal{L}(\theta) = 2\left[\ln\left(\frac{1}{3}\right) + \ln(1-\theta)\right] + 3\left[\ln\frac{2}{3} + \ln(1-\theta)\right]$$
$$+ 2\left[\ln\frac{2}{3} + \ln\theta\right] + 3\left[\ln\frac{1}{3} + \ln\theta\right]$$

$$\ln \mathcal{L}(\theta) = 5\ln\frac{1}{3} + 5\ln\frac{2}{3} + 5\ln(1-\theta) + 5\ln\theta.$$

$$\frac{d}{d\theta}\left(\ln \mathcal{L}(\theta)\right) = 0 + \frac{-5}{1-\theta} + \frac{5}{\theta} = 0$$

$$\Rightarrow \frac{5}{\theta} = \frac{5}{1-\theta}$$

$$\Rightarrow 1-\theta = \theta$$

$$\boxed{\theta = 1/2 = 0.5}$$

4) There are 18 points in an axis plane namely -

$[(0.8, 0.8)^t, (1, 1)^t, (1.2, 0.8)^t, (0.8, 1.2)^t, (1.2, 1.2)^t]$, → 1

belong to class 1;

$[(4, 3)^t, (3.8, 2.8)^t, (4.2, 2.8)^t, (3.8, 3.2)^t (4.2, 3.2)^t, (4.4, 2.8)^t, (4.4, 4.4)^t]$,

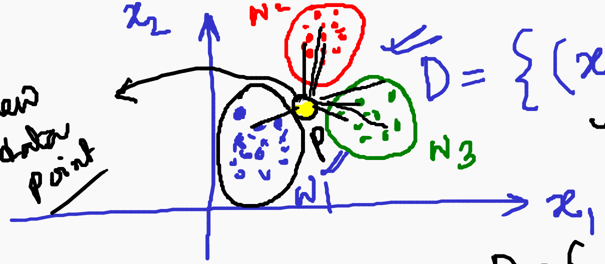
belong to class 2; → 2

$[(3.2, 0.4)^t, (3.2, 0.7)^t, (3.8, 0.5)^t, (3.5, 1)^t, (4, 1)^t, (4, 0.7)^t]$,

belong to class 3. → 3

KNN

K=5



$D = \{(x_1, x_2)_i\}_{i=1}^{18}$

$D = \{X_i, Y_i\}$

Data

Label of class

A new point

$P = (4.2, 1.8)^t$ introduces into the map. The point P belongs to which class? Use k -nearest neighbor technique with $k = 5$ to calculate the result

- Class 1
- Class 2 ✓
- Class 3
- None of the above

Euclidean distance → $d(X_1, P) =$

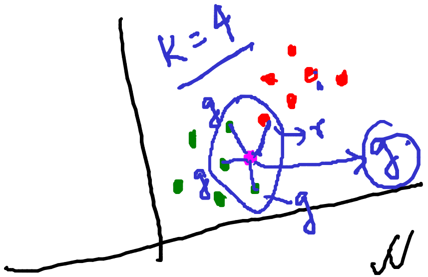
$d(X_2, P) =$

$d(X_3, P) =$

$0^2 + (-1 \times -1) \Leftarrow \begin{bmatrix} 0 \\ -1 \end{bmatrix} \otimes \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

$= 1$

a



4 nearest points from the new data point

$\text{eucdist}\left(\begin{bmatrix} 4.2 \\ 1.8 \end{bmatrix}, \begin{bmatrix} 4.2 \\ 2.8 \end{bmatrix}\right) = \begin{bmatrix} 4.2-4.2 \\ 1.8-2.8 \end{bmatrix}$

$[4.2, 1.8]^t \Leftarrow [4.2, 2.8]^t$

$d(X_8, P) = \sqrt{(4.2-4.2)^2 + (2.8-1.8)^2}$

$= 1.$

$d(x_1,P) = 3.54$

$d(x_2,P) = 3.29$

$d(x_3,P) = 3.16$

$d(x_4,P) = 3.45$

$d(x_5,P) = 3.06$

$d(x_6,P) = 1.22$

$\boxed{d(x_7,P) = 1.08} \rightarrow 2$

$\boxed{d(x_8,P) = 1} \rightarrow 2$

$d(x_9,P) = 1.46$

$d(x_{10},P) = 1.4$

$\boxed{d(x_{11},P) = 1.01} \rightarrow 2$

$d(x_{12},P) = 2.6$

$d(x_{13},P) = 1.72$

$d(x_{14},P) = 1.49$

$d(x_{15},P) = 1.36$

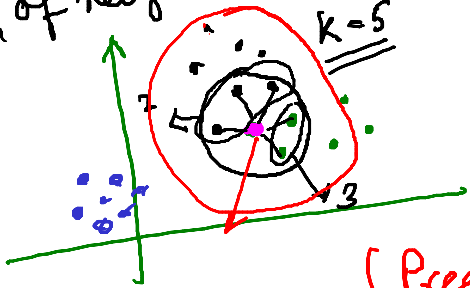
$\boxed{d(x_{16},P) = 1.06} \rightarrow 3$

$\boxed{d(x_{17},P) = 0.82} \rightarrow 3$

$d(x_{18},P) = 1.12$

K=5

No of neighbors.



$P \in$ Class 2 (Prediction)

Acc Test Sample → original lable

→ Radius : 2.6

The bandwidth parameter in the Parzen Window method determines:

The number of neighbors to consider for classification

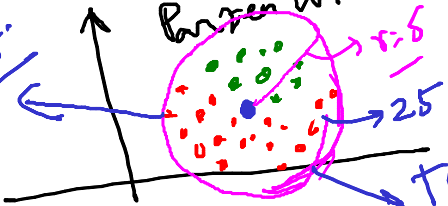
The size of the neighborhood around a test instance

The dimensionality of the feature space.

The complexity of the classifier

Size of neighbor

Parzen window



KNN

K

Total no of data points for neighbors 'bw' specific

The disadvantage of using k -NN as classifier:

- Fails while handling large dataset
- Fails while handling small dataset
- Sensitive to outliers
- Training is required

Large dataset → KNN will have computational problem

Inference Time/Test Time taken by algo. ↑↑

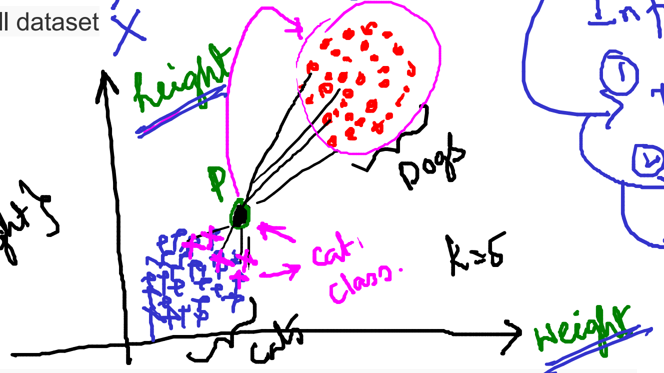
① There is no need of training.

② because this is a non parametric method.

③ And it just has to calculate the distance value from the new point to all other point

W={cat, Dogs}

x={height, weight}

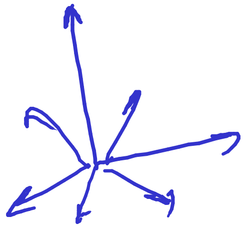


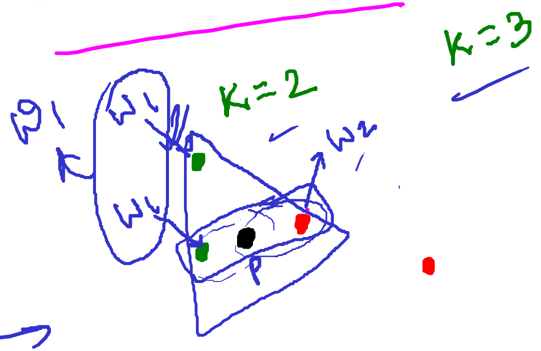
True label of (P)= Dog

Prediction is: Cat → mis classification.

Which of the following statements are true about k - nearest neighbor (KNN)-

- Odd value of "K" preferred over even values.
- Does more computation on test time rather than train time.
- Work well with high dimension. → All points become almost equally distant in higher dimension
- The optimum value of K for KNN is highly independent on the data.





If the number of data samples becomes very large.

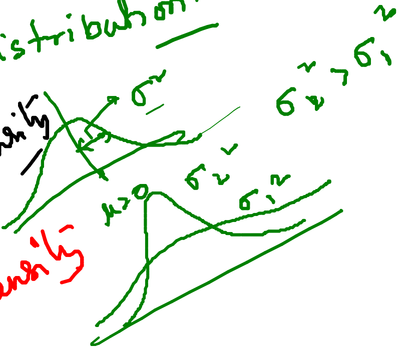
- Bayesian Estimation is worse than MLE
- Maximum Likelihood estimates are slightly bad
- Bayesian Estimation performs same as MLE
- None

$f(\theta|x) = f(x|\theta) \cdot \cancel{f(\theta)}$

aposteriori — class conditional prob. density — Prior

Data

Parameter of distribution.



MLE :- $\mathcal{L} \approx f(x|\theta)$

likelyhood estimate

for large data size

Bayesian estimate $f(\theta|x) \propto f(x|\theta)$

Similar to

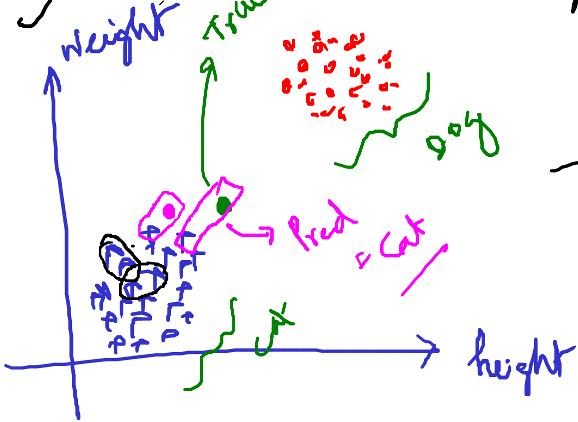
likelyhood function/density

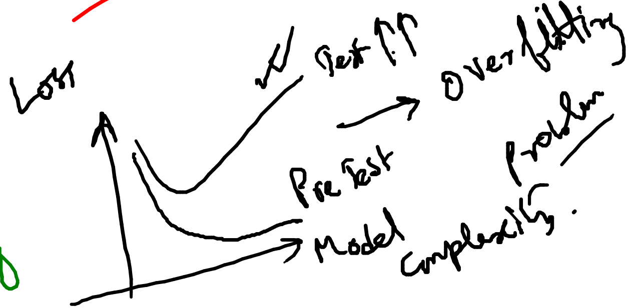
class conditional prob-density

Posterior prob.

What happens when k=1 in k -Nearest Neighbor algorithm:

- Underfitting
- Overfitting
- High testing accuracy
- All the above





Find the scalar projection of vector b = <-3, 2> onto vector a = <1,1>? SVM.

a. 0

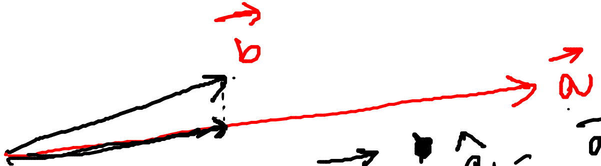
b. $\frac{1}{\sqrt{2}}$

c. $\frac{-1}{\sqrt{2}}$

d. $\frac{-1}{2}$

Dot product = Dot product of $\vec{b}$ and $\hat{a}$

$= \vec{b} \cdot \frac{\vec{a}}{\|\vec{a}\|}$



$\rightarrow \hat{a} = \frac{\vec{a}}{\|\vec{a}\|}$

$= \frac{\langle -3, 2\rangle \langle 1, 1\rangle}{\sqrt{1^2 + 1^2}}$

$= \frac{-3+2}{\sqrt{2}} = \frac{-1}{\sqrt{2}}$

Suppose there is a feature vector represented as [1, 4, 3]. What is the distance of this feature vector from the separating plane $x_1 + 2x_2 - 2x_3 + 3 = 0$. Choose the correct option.

a. 1

b. 5

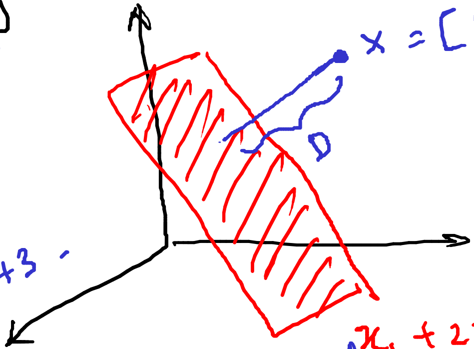
c. 3

d. 2

$X = [1, 4, 3]$

$D = \frac{W \cdot X + W_0}{\|W\|}$

$W^t X + W_0 = [1\ 2\ -2]\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + 3 = x_1 + 2x_2 - 2x_3 + 3$



$x = [1, 4, 3]$

$W^t X + W_0 = 0$ Bias

$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$W^t = [1, 2, -2]$

$W_0 = 3$

$x_1 + 2x_2 - 2x_3 + 3 = 0$

Find the scalar projection of vector b = <-3, 2> onto vector a = <1,1>?

a. 0

b. $\frac{1}{\sqrt{2}}$

c. $\frac{-1}{\sqrt{2}}$

d. $\frac{-1}{2}$

Suppose there is a feature vector represented as [1, 4, 3]. What is the distance of this feature vector from the separating plane $x_1 + 2x_2 - 2x_3 + 3 = 0$. Choose the correct option.

a. 1

b. 5

c. 3

d. 2 ✓

$$D = \frac{W \cdot X + W_0}{\|W\|}$$

$$= \frac{1 + 8 - 6 + 3}{\sqrt{1^2 + 2^2 + 2^2}}$$

$W = [1, 2, -2]$

$X = [1 \;\; 4 \;\; 3]$

$w_0 = 3$

$$= \frac{6}{\sqrt{9}} = \frac{6}{3} = 2$$ ✓

Find the distance of the 3D point, $P = (-2, 4, 1)$ from the plane defined by $2x + 3y + 6z + 7 = 0$?

a. 3

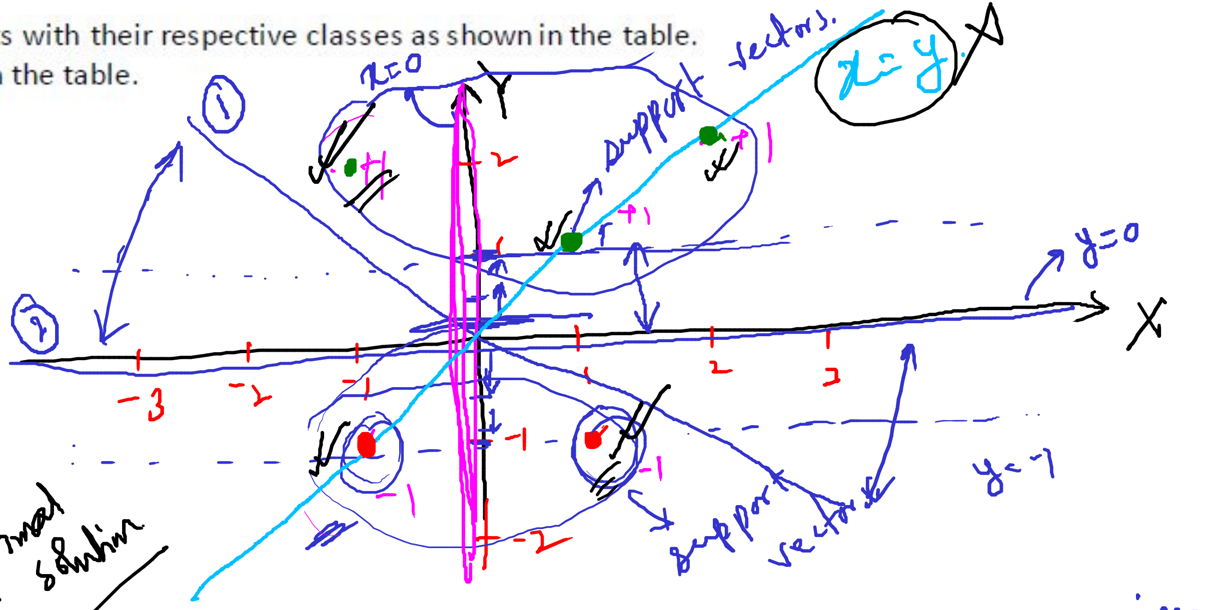
b. 4

c. 0

d. ∞ (infinity)

Suppose we have the below set of points with their respective classes as shown in the table. Answer the following question based on the table.

X	Y	Class Label
1	1	+1
-1	-1	-1
2	2	+1
-1	2	+1
1	-1	-1



optimal solution

SVM Corresponds to decision boundary which gives maximum margin.

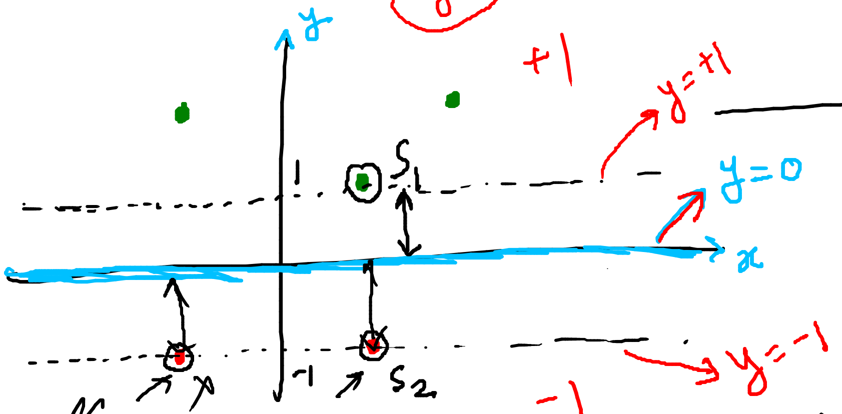
What can be a possible decision boundary of the SVM for the given points?

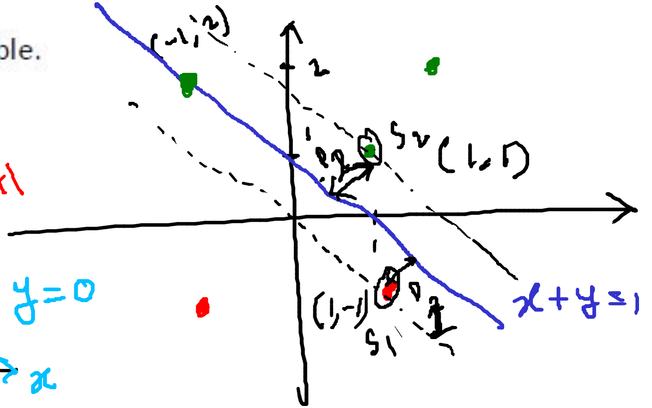
a. $y = 0$
b. $x = 0$
c. $x = y$
d. $x + y = 1$

Suppose we have the below set of points with their respective classes as shown in the table. Answer the following question based on the table.

X	Y	Class Label
1	1	+1
-1	-1	-1
2	2	+1
-1	2	+1
1	-1	-1

$y = K$





$\text{Margin}_{(y=0)} = \sum$ Distance between Decision Surface and unique set of Support vectors, residing at different location.

$= D(S_2 \text{ to Dec}) + D(S_1 \text{ to Dec})$

$= 2.$

What can be a possible decision boundary of the SVM for the given points?

a. $y = 0$ → $M = 2$.

b. $x = 0$

c. $x = y$

d. $x + y = 1$

$-1+2<1$

$\text{Margin}_{(x+y=1)} = D_1 + D_2 = \dfrac{|1-1-1|}{\sqrt{1^2+1^2}} + \dfrac{|1+1-1|}{\sqrt{1^2+1^2}} = \dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{2}}$

$= \dfrac{2}{\sqrt{2}} = \sqrt{2} = 1.414$

$W = [1,1];\ w_0 = -1$

$x + y - 1 < 0$

Suppose we have the below set of points with their respective classes as shown in the table. Answer the following question based on the table.

X	Y	Class Label
1	1	+1
-1	-1	-1
2	2	+1
-1	2	+1
1	-1	-1

D_1

$$D = \frac{W \cdot X + W_0}{\|W\|} = \frac{|1 - 1 - 1|}{\sqrt{1^2 + 1^2}}$$

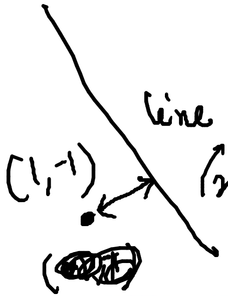
Point $(1,-1)$

$X = (1,-1)$

$w^t = (1, 1)$ $\quad x = [1, -1]$

$W_0 = -1$

$w^t X + W_0 = 0$.



$x + y - 1 = 0$

$X = \begin{bmatrix} x \\ y \end{bmatrix}$

$w^t x = [w_1 \ \ w_2] \begin{bmatrix} x \\ y \end{bmatrix}$

$= [1 \ \ 1] \begin{bmatrix} x \\ y \end{bmatrix}$

$\Rightarrow x + y - 1 = 0$

What can be a possible decision boundary of the SVM for the given points?

a. $y = 0$
b. $x = 0$
c. $x = y$
d. $x + y = 1$