NPTEL Week-1 Live Session

on Machine Learning and Deep Learning - Fundamentals and Applications (noc24_ee146)

A course offered by: Prof. Manas Kamal Bhuyan, IIT Guwahati

Week-0-1 practice questions: Bayes classification, ROC-AUC, Classification metrics, Least mean square fit

Powered by:

FREE ONLINE EDUCATION swayam

शिक्षित भारत, उन्नत भारत

PMRF
Prime Minister's Research Fellows
Ministry of Education
Government of India

By

Arka Roy
NPTEL PMRF TA

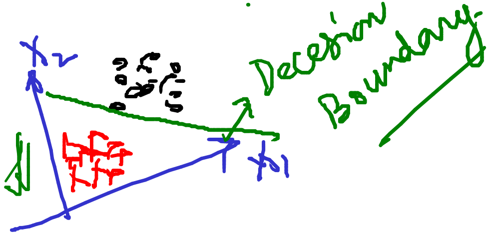
Prime Minister's Research Fellow
Department of Electrical Engineering, IIT Patna
Web: https://sites.google.com/view/arka-roy/home

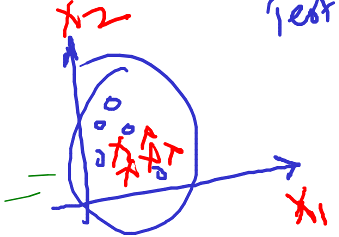
Content of the live session

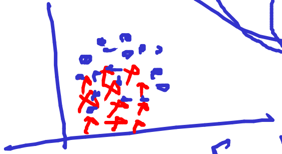
1. Glimpse of week 1 content: a revisit !!

2. Solving numerical problems from week 0 and 1 content

Machine learning vs. deep learning:





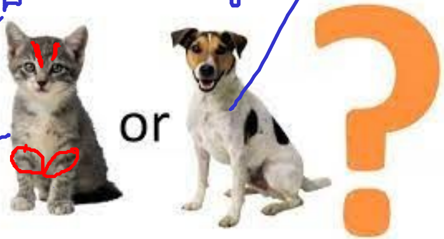


Color based.

Cat

Dog

Texture

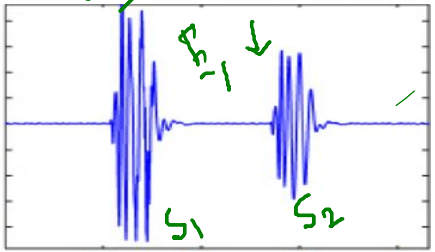


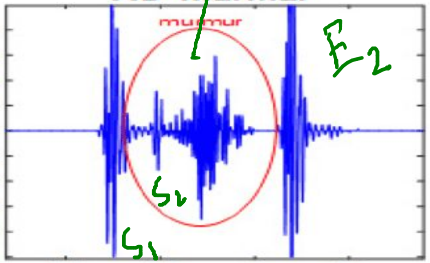
Redbuck.

Eland.

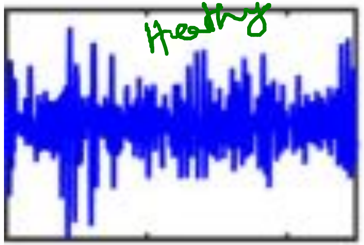
Heart

Murmurs

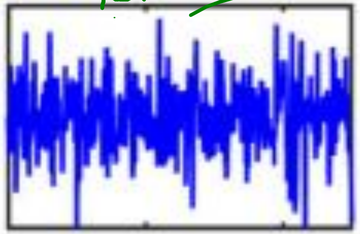




Healthy



Asthma.



Bayes classifier:

$P(W), P(X|W)$

Tr

$X \rightarrow$ Model $\rightarrow P(w|X)$

X

$P(W_1)$ = prior

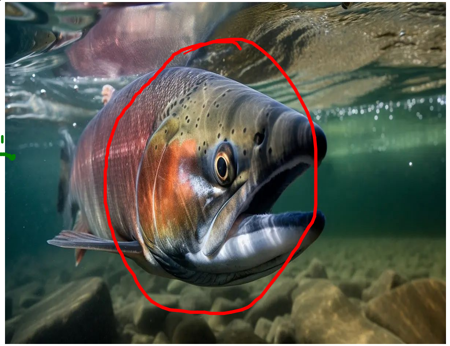
$P(W_2)$ =

class based prior knowledge

class-Prior probabilities

Feature

$\underbrace{p(X|W_1)}_{\text{class conditional prob.}}$



$$\underbrace{P(W_j|X)}_{\text{aposteriorie prob.}} = \frac{P(X|W_j) \cdot P(W_j)}{\sum_{j=1} P(W_j) \cdot P(X|W_j)}$$

$P(X|W_2)$

$$P(W_1|X) = \frac{P(X|W_1)\, P(W_1)}{P(W_1)\, P(X|W_1) + P(W_2)\, P(X|W_2)}$$

$$P(W_2|X) = \frac{P(X|W_2) \cdot P(W_2)}{}$$

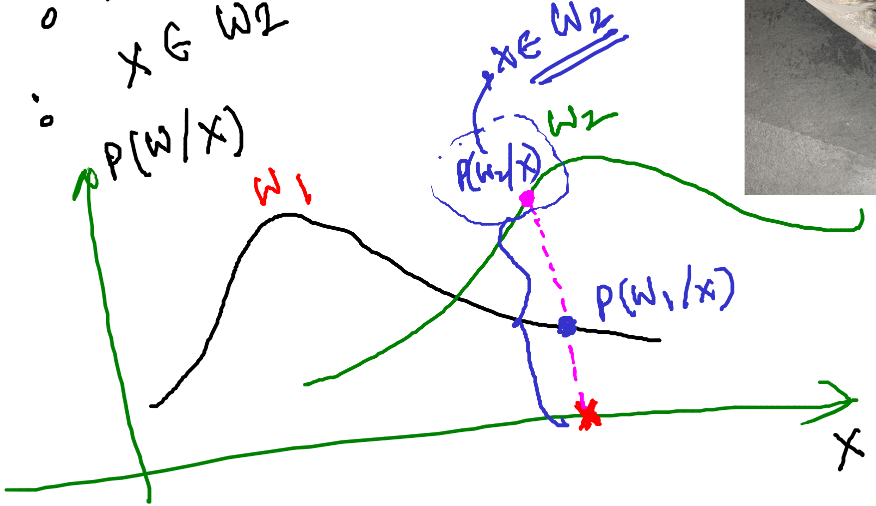
Bayes classifier:

$$P(\omega_1|x) = P(x|\omega_1) \cdot P(\omega_1) = \lambda_1$$

$$P(\omega_2|x) = P(x|\omega_2) \cdot P(\omega_2) = \lambda_2$$

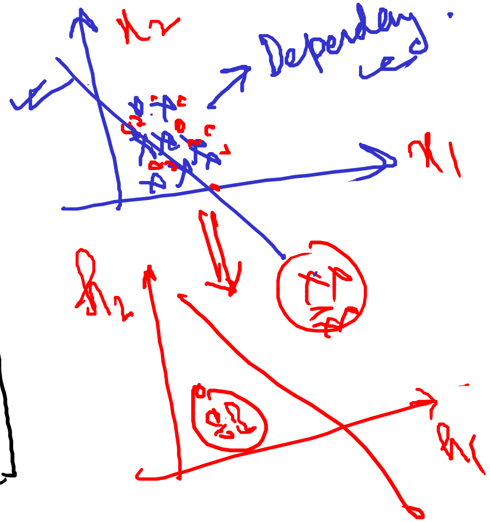
$\lambda_1 > \lambda_2$: $x \in \omega_1$

$\lambda_2 > \lambda_1$: $x \in \omega_2$



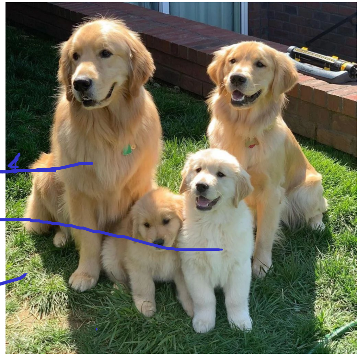
Why cover's theorem is important in ML:

X_1	X_2	Y	h_1	h_2
0	0	0	0	1
0	1	1	1	1
1	0	1	1	1
1	1	0	1	0



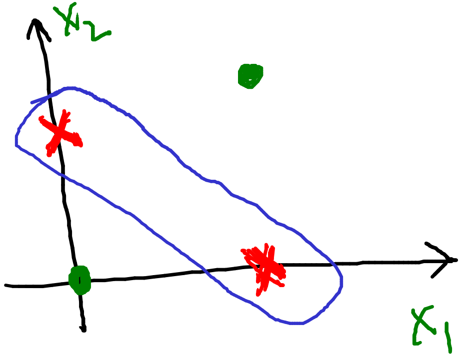
$p(X|w_1)$

$p(X|w_2)$



$p(w_1)$

$p(w_2)$

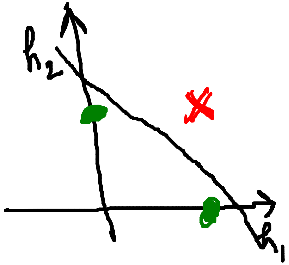


$$X_1 \oplus X_2 = X_1\bar{X}_2 + X_2\bar{X}_1 + X_1\bar{X}_1 + X_2\bar{X}_2$$

$$= (X_1 + X_2)\cdot(\bar{X}_1 + \bar{X}_2)$$

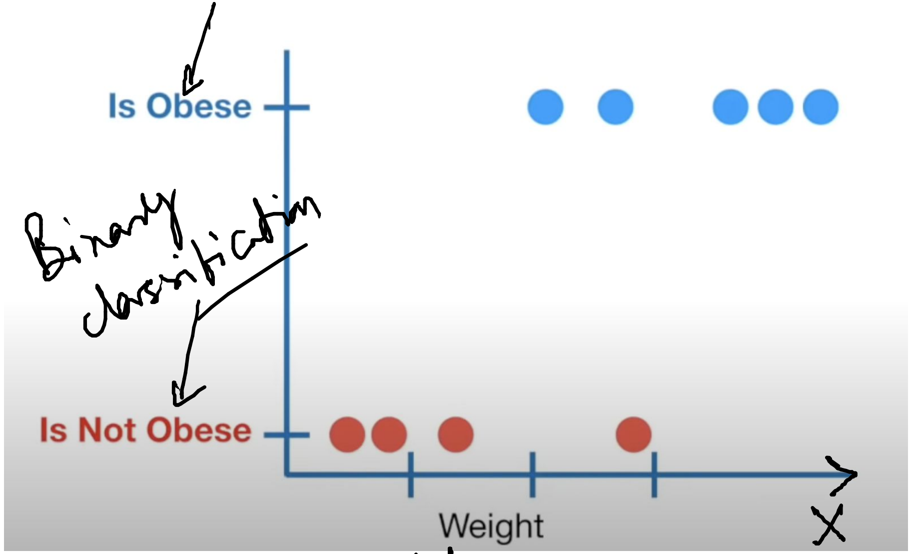
$$= h_1 \cdot h_2 .$$

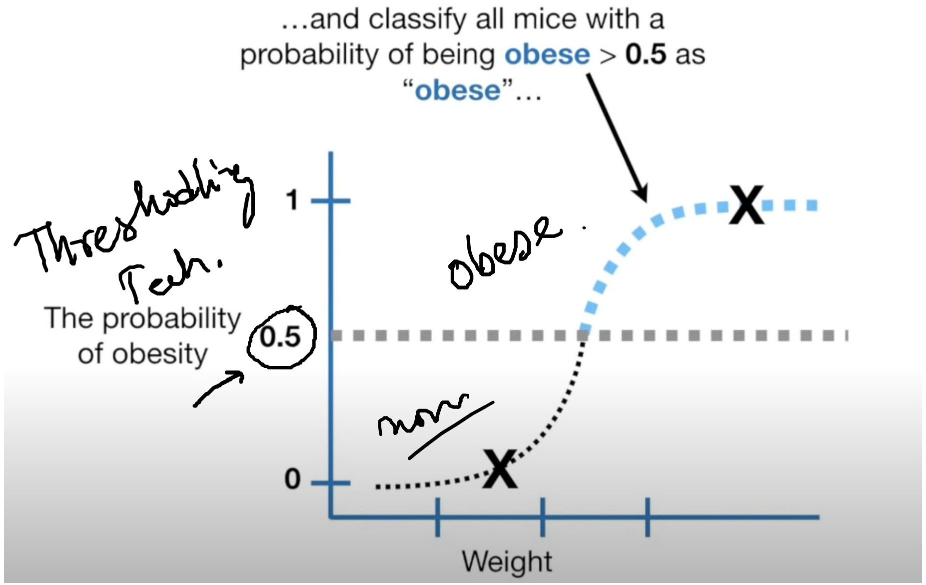
$x = x + y$ Bad.



ROCs:

$X \rightarrow \boxed{M_{tr}} \rightarrow \hat{Y} \Leftarrow R\ Y \quad (2,2)$





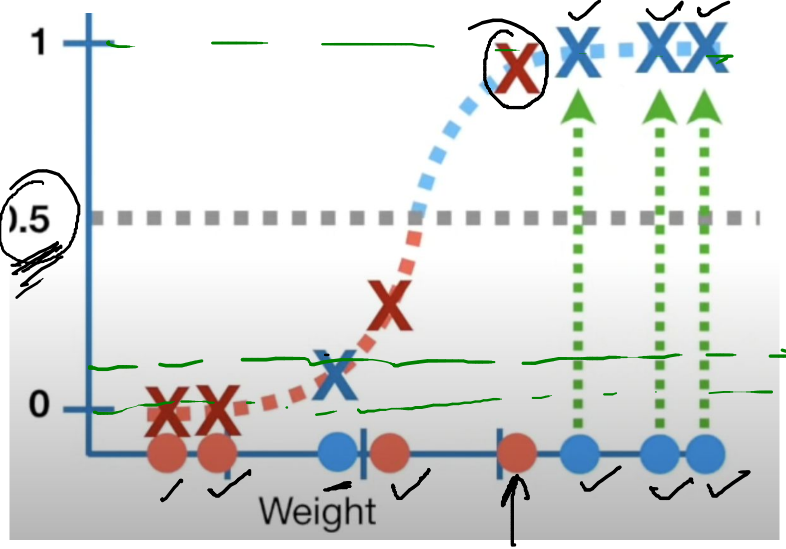
rat

Predicted →

Actual ↓		
obese		
not obese		

$$\text{Sigmoid}(x) = \frac{1}{1+e^{-x}}$$

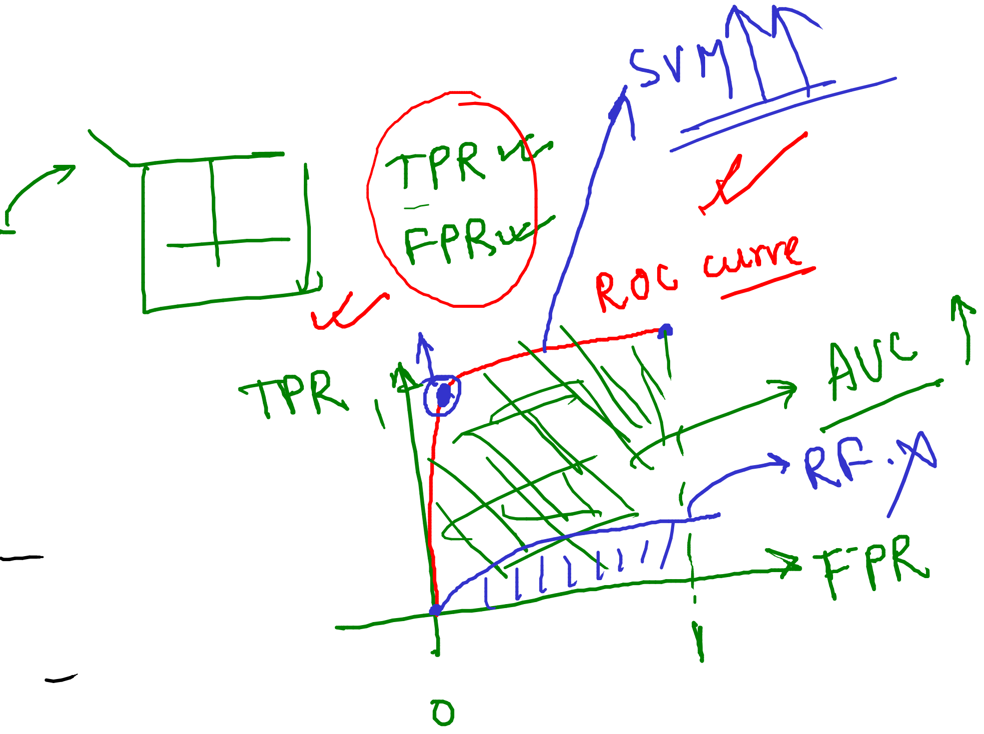
$\in (0,1)$



Conf. Th = 0.5.

TPR = _

FPR = _



1) For what value of x will the matrix given below become singular?

$$A = \begin{bmatrix} 8 & x & 0 \\ 4 & 0 & 2 \\ 12 & 6 & 0 \end{bmatrix}$$

- ○ 2
- ○ 3
- ○ 4
- ○ 5

$|A| = 0.$

$$\begin{vmatrix} 8 & x & 0 \\ 4 & 0 & 2 \\ 12 & 6 & 0 \end{vmatrix} = 0.$$

$\Rightarrow 8(0-12) - x(0-24) = 0.$

$-96 - 24x = 0.$

$\Rightarrow -96$

$24x = 96$

$x = 4$

If A is a real square matrix then $\underbrace{AA^T}$ is

- ◯ Unsymmetric
- ☑ Always symmetric
- ◯ Skew-Symmetric
- ◯ Sometimes symmetric

$B^T = B \rightarrow$ Symmetric matrix.

$B = (AA^T)$

$B^T = (AA^T)^T = (A^T)^T A^T$

$= \underline{AA^T} = B.$

$\underbrace{(AB)^T = B^T A^T}$

$\underline{B^T = B.} \rightarrow$ symmetric $\underline{\text{matrix}}$

Let A be 3X3 matrix with rank 2. Then AX=0 has

- [ ] Only trivial solution X=0
- [ ] One independent solution
- [ ] Two independent solution
- [ ] Three independent solution

There are two non zero rows.

$$A = \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}$$

non zero rows

all zeros

Rank = 2.

Nullity = Dimension of matrix − Rank.

$= 3 - 2 = 1$

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ -1 & -2 & 0 \end{bmatrix}_{3\times 3} \xrightarrow{R_2 - 2R_1} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & 0 \end{bmatrix} \xrightarrow{R_3 + R_1} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Rank = 2

Nullity = 1.

$R_2 = 2R_1$

$a_1R_1 + a_2R_2 = 0.$

$a_1 = 2$

$a_2 = 1$

Are $X = 0$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$\rightarrow x_1 + 2x_2 + x_3 = 0$

$\rightarrow x_3 = 0.$

$x_3 = 0.$

$x_1 + 2x_2 = 0.$

$x_1 = -2x_2.$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Let A be 3X3 matrix with rank 2. Then AX=0 has

- ◯ Only trivial solution X=0
- ◯ One independent solution ✓
- ◯ Two independent solution
- ◯ Three independent solution

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2x_2 \\ x_2 \\ 0 \end{bmatrix} = x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$$

$$= \lambda \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} v_1$$

$x_3 = 0$

$x_1 = -2x_2$

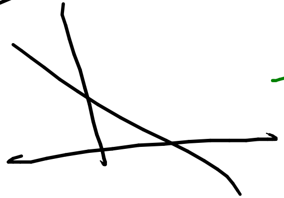
$x_1 + 2x_2 + x_3 = 0$

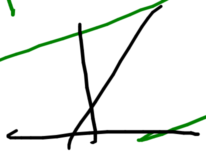
$x_3 = 0$

OR

NAND

M2P





dependent

$\lambda_2 v_1 \quad (-2, 1, 0) \quad v_1 \quad \lambda_1 v_1$

The eigen value of $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ are

- 0,0,0
- 0,0,1
- ✓ 0,0,3
- 1,1,1

$|A - \lambda I| = 0.$

$\begin{vmatrix} 1-\lambda & 1 & 1 \\ 1 & 1-\lambda & 1 \\ 1 & 1 & 1-\lambda \end{vmatrix} = 0.$

$\Rightarrow (1-\lambda)\left[(1-\lambda)^2 - 1\right] - 1\left(1-\lambda-1\right] + 1\left[1+\lambda-1\right] = 0.$

$\Rightarrow (1-\lambda)\left[1+\lambda^2-2\lambda-1\right] + \lambda + \lambda = 0.$

$(1-\lambda)(\lambda^2 - 2\lambda) + 2\lambda = 0.$

$\Rightarrow \lambda^2 - 2\lambda - \lambda^3 + 2\lambda^2 + 2\lambda = 0$

$\Rightarrow \lambda^2(\lambda - 3) = 0.$

$\underline{\lambda = 0, 0, 3.}$

Four fair coins are tossed simultaneously. The probability that at least one head and one tails turn up is

- $\frac{1}{16}$
- $\frac{1}{8}$
- $\frac{7}{8}$
- $\frac{15}{16}$

Sample space $= 2^4 = 16$.

A

$\left[\{HHHH, \underbrace{HHHT, \ldots\ldots}_{}, TTTT\}\right]$.

$A = 16 - 2 = 14$

$P(A) = \frac{14}{16} = 7/8$

A bag I contains 4 white balls and 6 black balls. Another bag II contains 4 white balls and 3 black balls. One ball is selected at random from one of the bags and it is found to be black. Find the probability that it was drawn from bag I?

- $\frac{2}{3}$
- $\frac{4}{9}$
- $\frac{3}{5}$
- $\frac{7}{12}$

$P(b_1) = 1/2$

$P(b_2) = 1/2.$

Black ball taken from Bag 1

$P(A/b_1) = 6/10.$

$3/7.$

$P(A/b_2)$

black ball

$$P(b_1/A) = \frac{P(A/b_1)\,P(b_1)}{P(A/b_1)\cdot P(b_1) + P(A/b_2)\cdot P(b_2)}$$

$= 7/12$

The standard deviation of a uniformly distributed random variable between 0 and 1 is

- ✓ $\frac{1}{\sqrt{12}}$
- $\frac{1}{\sqrt{3}}$
- $\frac{5}{\sqrt{12}}$
- $\frac{7}{\sqrt{12}}$

$f(x) = \frac{1}{b-a}$; $a \leq x \leq b$

$= 0$; elsewhere.

$E(x) = \int_0^1 x f(x)\, dx$

$= \int_0^1 x \, dx = \left.\frac{x^2}{2}\right|_0^1 = \frac{1}{2}$.

$E(x^2) = \int_0^1 x^2 f(x)\, dx = \int_0^1 x^2 dx = \left.\frac{x^3}{3}\right|_0^1 = 1/3$.

$Var(x) = E(x^2) - (E(x))^2$

$= \frac{1}{3} - \frac{1}{4}$

$Var(x) = \frac{1}{12}$

$\sigma = \frac{1}{\sqrt{12}}$

If a random variable X satisfies the Poisson's distribution with a mean value of 2, then the probability that X>2 is

- $2e^{-2}$
- $1-2e^{-2}$
- $3e^{-2}$
- $1-3e^{-2}$

$$P(X=k) = \frac{e^{-\lambda}\lambda^{k}}{k!}$$

Mean ⇒ λ

$1-3e^{-2}$

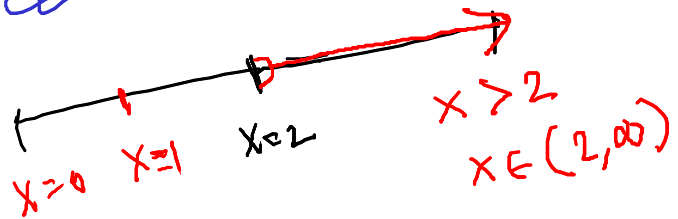
$$P(X>2) = \boxed{1 - P(X=0) - P(X=1)} - P(X=2)$$

$$= 1 - P(X \le 2)$$

$$= 1 - e^{-2} - \frac{e^{-2}\cdot 2}{1} - \frac{e^{-2}\, 2^{2}}{2}$$

$$= 1 - e^{-2} - 2e^{-2} - 2e^{-2}$$

$$= 1 - 5e^{-2}$$



$P(X \geq 2)$?

Compute the derivative $f'(x)$ of the logistic sigmoid

- $\frac{1}{1+e^{-x}}$
- [
- $\frac{1}{e^{-x}}$

$$\sigma(x) = \frac{1}{1+e^{-x}}$$

$$\sigma'(x) = \frac{(1+e^{-x})\cdot 0 - 1\cdot(0 - e^{-x})}{(1+e^{-x})^2}$$

$$= \boxed{\frac{e^{-x}}{(1+e^{-x})^2}} = \left(\frac{1}{1+e^{-x}}\right)\cdot\left(\frac{e^{-x}}{1+e^{-x}}\right)$$

$$= \underbrace{\left(\frac{1}{1+e^{-x}}\right)\left(1 - \frac{1}{1+e^{-x}}\right)}_{}$$

$$= \sigma(x)\cdot(1-\sigma(x))$$

$$\boxed{= f(x)\cdot(1-f(x))}$$

$$\boxed{\sigma'(x) = \sigma(x)(1-\sigma(x))}$$

3) How many solution does the following system of linear equation have

$-x + 5y = -1$

$x - y = 2$

$x + 3y = 3$

- ○ Infinitely many
- ○ Two distinct solution
- ○ Unique
- ○ None

$-x + 5y = -1$

$x - y = 2$

$x + 3y = 3.$

$A = \begin{bmatrix} -1 & 5 \\ 1 & 3 \end{bmatrix}$

$A_3 = \begin{bmatrix} -1 & 5 \\ 1 & -1 \end{bmatrix}$

$A_2 = \begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix}$

$$x + 2y - z = 1 \quad \cdots\cdots\cdots\cdots (1)$$
$$-2x - 4y + 2z = -2 \quad \cdots\cdots\cdots\cdots (2)$$
$$z = 2 \quad \cdots\cdots\cdots\cdots (3)$$

What are the values of x, y, z?

a. $x = 0, y = 0, z = 2$
b. $z = 2$ and infinitely possible x, y
c. $z = 2$ and no possible x, y
d. None of the above

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \lambda y \\ y \\ 2 \end{pmatrix}$$

Rank = 1

nullity = 1

$$A = \begin{bmatrix} 1 & 2 \\ -2 & -4 \end{bmatrix}$$

Dependency.

$x = \lambda y.$

x & y are related with each.

x_1, x_2, x_3 are the linearly independent vectors. If $x_1 = \begin{bmatrix}1\\3\\0\end{bmatrix}, x_2 = \begin{bmatrix}-2\\4\\-5\end{bmatrix}$, what is the possible value of x_3?

a. $\begin{bmatrix}-1\\7\\-5\end{bmatrix}$

b. $\begin{bmatrix}0\\10\\-5\end{bmatrix}$

~~c.~~ $\begin{bmatrix}3\\4\\5\end{bmatrix}$

d. $\begin{bmatrix}5\\-5\\10\end{bmatrix}$

$x_1, x_2, x_3 \rightarrow$ linearly independent

$\rightarrow a_1 x_1 + a_2 x_2 + a_3 x_3 \rightarrow 0 \rightarrow$ Any linear Comb. exists then dependent set of vectors

$a_1 = a_2 = a_3 = 0$.

$x_1 + x_2 = x_3$.

$x_1 + x_2 - x_3 = 0$

$\rightarrow 2x_1 + x_2 = x_3$.

$\rightarrow x_1 - 2x_2 = x_3$

which of following is strictly true for a two-class problem Bayes minimum error classifier?

(The two different classes are ω_1 and ω_2, and input feature vector is x)

a. Choose ω_1 if $P(x/\omega_1) > P(x/\omega_2)$

b. Choose ω1 if $P(\omega_1) > P(\omega_2)$

c. Choose ω2 if $P(\omega_1/x) > P(\omega_2/x)$ ✗

d. Choose ω1 if $P(\omega_1/x) > P(\omega_2/x)$ ✓

$P(w/x) = \underbrace{P(x/w) \cdot P(w)}$

$x \in w_1 : \quad P(w_1/x) > P(w_2/x)$

$x \in w_2 : \quad P(w_2/x) > P(w_1/x).$

Given the following confusion matrix for a classification task with three classes (A, B and C), which of the following is correct:

Multi-class classification

Prediction.

Conf. mat

Actual

Ground truth / Predicted	A	B	C
A	0	0	2
B	1	3	1
C	0	0	3

2x2

NxN N>2

- Precision(B) = 1.0
- Precision(C) = 0.5
- Recall(B) = 1.0
- Recall(C) = 0.5

class Dependent measures

Precision → A, B, C. → Avg.

Rec → A, B, C → Avg.

PRC(B) PRC(C) TP = 3

True/Pos class = B; Neg/False class = {A, C}

Given the following confusion matrix for a classification task with three classes (A, B and C), which of the following is correct:

Predict →

(A, A)

(C, C)

Actual →

Ground truth / Predicted	A	B	C
A	0	0 FP	2
B	FN 1	3 TP	FN 1
C	0	0 FP	3

B :→

Total no of samples in class B $= 5 = 3+1+1$

Total no of correctly predicted samples from $B = 3$.

C

- Precision(B) = 1.0
- Precision(C) = 0.5
- Recall(B) = 1.0
- Recall(C) = 0.5

B → True class.

{A, C} → Neg class

A → True

{B, C} → neg.

$TN = (A,C) + (C,A) + (A,A) + (C,C)$

$TP = 3.$

$FN = 2.$ $FP = 0.$

$Prc(B) = 1$

$TN = 0+2+0+3$

$= 5.$

	True	Neg.
True	TP	FN
Neg	FP	TN

2×2