STAT432 Assignment 2

Rory Sarten 301005654

21 August 2020

Question 1

a)

These are independent binomial processes (despite happening in sequence), so the three trapping occasions can be denoted:

$$
\begin{aligned}
n_1 \sim Binomial(N, \theta) &= \binom{N}{n_1}\theta^{n_1}(1-\theta)^{N-n_1} \\
n_2 \sim Binomial(N-n_1, \theta) &= \binom{N-n_1}{n_2}\theta^{n_2}(1-\theta)^{N-n_1-n_2} \\
n_3 \sim Binomial(N-n_1-n_2, \theta) &= \binom{N-n_1-n_2}{n_3}\theta^{n_3}(1-\theta)^{N-n_1-n_2-n_3}
\end{aligned}
$$

So the probability of catching $n_1 + n_2 + n_3$ rats is given by

$$Binomial(N, \theta) \times Binomial(N-n_1, \theta) \times Binomial(N-n_1-N-2, \theta)$$

b)

$$\ell(N, \theta) = \ln \Big[\Big] \frac{N!}{n_1(N-n_1)!}\theta^{n_1}(1-\theta)^{N-n_1} \times \frac{(N-n_1)!}{n_2(N-n_1-n_2)!}\theta^{n_2}(1-\theta)^{N-n_1-n_2} \times \frac{(N-n_1-n_2)!}{n_3(N-n_1-n_2-n_3)!}\theta^{n_3}(1-\theta)^{N-n_1-}$$

$$
\begin{aligned}
\ell(N, \theta) = \ln N! - [\ln n_1 + \ln(N-n_1)] + n_1 \ln\theta + (N-n_1)\ln(1-\theta) + \\
\ln(N-n_1)! - [\ln n_2 + \ln(N-n_1-n_2)] + n_2 \ln\theta + (N-n_1-n_2)\ln(1-\theta) + \\
\ln(N-n_1-n_2)! - [\ln n_3 + \ln(N-n_1-n_2-n_3)] + n_3 \ln\theta + (N-n_1-n_2-n_3)\ln(1-\theta)
\end{aligned}
$$

finish that bit of algebra somehow

c)

```r
llfunc <- function(par, n1, n2, n3) {
  N <- par[1]
  theta <- par[2]
  lfactorial(N) - lfactorial(N - n1 - n2 - n3) +
    sum(n1, n2, n3)*log(theta) +
    (3*N - 3*n1 - 2*n2 - n3)*log(1 - theta)
}

n1 <- 82L
n2 <- 54L
n3 <- 23L
## since N >= n1 + n2 + n3
```

```r
N_0 <- n1 + n2 + n3
par_start <- c(N_0, 0.5)

optim_fit <- optim(par = par_start,
                   fn = llfunc,
                   n1 = n1, n2 = n2, n3 = n3,
                   lower = c(N_0, 1e-4),
                   upper = c(Inf, 1),
                   method = "L-BFGS-B",
                   control = list(fnscale = -1),
                   hessian = TRUE)

optim_fit$par
```

```
## [1] 189.4253774   0.4539277
```

```r
optim_fit$value
```

```
## [1] 490.043
```