

STAT432 Assignment 2

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Question 1

a)

These are independent binomial processes (despite happening in sequence), so the three trapping occasions can be denoted:

$$n_1 \sim \text{Binomial}(N, \theta) = \binom{N}{n_1} \theta^{n_1} (1 - \theta)^{N - n_1}$$

$$n_2 \sim \text{Binomial}(N - n_1, \theta) = \binom{N - n_1}{n_2} \theta^{n_2} (1 - \theta)^{N - n_1 - n_2}$$

$$n_3 \sim \text{Binomial}(N - n_1 - n_2, \theta) = \binom{N - n_1 - n_2}{n_3} \theta^{n_3} (1 - \theta)^{N - n_1 - n_2 - n_3}$$

So the probability of catching $n_1 + n_2 + n_3$ rats is given by

$$\text{Binomial}(N, \theta) \times \text{Binomial}(N - n_1, \theta) \times \text{Binomial}(N - n_1 - n_2, \theta)$$

b)

$$\ell(N, \theta) = \ln \left[\frac{N!}{n_1(N - n_1)!} \theta^{n_1} (1 - \theta)^{N - n_1} \times \frac{(N - n_1)!}{n_2(N - n_1 - n_2)!} \theta^{n_2} (1 - \theta)^{N - n_1 - n_2} \times \frac{(N - n_1 - n_2)!}{n_3(N - n_1 - n_2 - n_3)!} \theta^{n_3} (1 - \theta)^{N - n_1 - n_2 - n_3} \right]$$

$$\begin{aligned} \ell(N, \theta) = & \ln N! - [\ln n_1 + \ln(N - n_1)] + n_1 \ln \theta + (N - n_1) \ln(1 - \theta) + \\ & \ln(N - n_1)! - [\ln n_2 + \ln(N - n_1 - n_2)] + n_2 \ln \theta + (N - n_1 - n_2) \ln(1 - \theta) + \\ & \ln(N - n_1 - n_2)! - [\ln n_3 + \ln(N - n_1 - n_2 - n_3)] + n_3 \ln \theta + (N - n_1 - n_2 - n_3) \ln(1 - \theta) \end{aligned}$$

finish that bit of algebra somehow

c)

```
llfunc <- function(par, n1, n2, n3) {  
  N <- par[1]  
  theta <- par[2]  
  lfactorial(N) - lfactorial(N - n1 - n2 - n3) +  
    sum(n1, n2, n3)*log(theta) +  
    (3*N - 3*n1 - 2*n2 - n3)*log(1 - theta)  
}  
  
n1 <- 82L  
n2 <- 54L  
n3 <- 23L  
## since N >= n1 + n2 + n3
```

```

N_0 <- n1 + n2 + n3
par_start <- c(N_0, 0.5)

optim_fit <- optim(par = par_start,
                  fn = llfunc,
                  n1 = n1, n2 = n2, n3 = n3,
                  lower = c(N_0, 1e-4),
                  upper = c(Inf, 1),
                  method = "L-BFGS-B",
                  control = list(fnscale = -1),
                  hessian = TRUE)

MLE <- optim_fit$par
SE <- sqrt(diag(solve(-optim_fit$hessian)))
LowerBound <- MLE - qnorm(0.975) * SE
UpperBound <- MLE + qnorm(0.975) * SE
results <- cbind(MLE, SE, LowerBound, UpperBound) %>% round(4)
rownames(results) <- c("N", "theta")
results

```

```

##           MLE           SE LowerBound UpperBound
## N      189.4254 13.2342    163.4868    215.3639
## theta   0.4539  0.0579     0.3404     0.5675

```

189.4253774