Espaço de estados

$$\dot{x} = Fx + Gu$$
$$y = Hx + Ju$$

F $n \times n$
G $n \times 1$
H $1 \times n$
J 1×1

FCC $\Longleftrightarrow$ Func. transf.

controlabilidade $\mathcal{C} = [G \quad FG \quad F^2G \quad \dots]$

Regulador: $u = -Kx$

K $1 \times n$

Regulador c/ referência

$$u = -Kx + Nr$$

$$N = N_x K + N_u$$

Escolha de polos.

$\underline{n=2}$ ... polos dominantes: $\underline{2}$ (complexos)

$\hookrightarrow M_p$ (ζ)

t_s (ω_n)

K 1×2

$n = 3$ 2 dominantes

$K \Rightarrow 1 \times 3$ +1 extra

$n = 5$ 2 dominantes

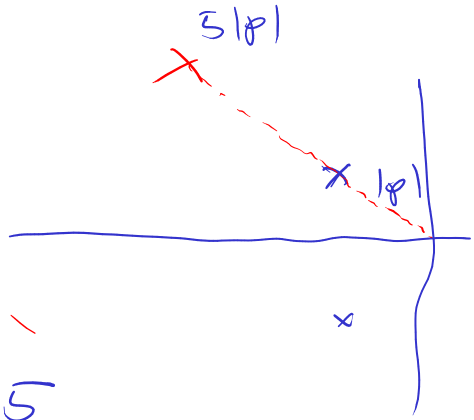
$k \Rightarrow 1 \times 5$ +3 extras. ?

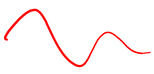
Extras × dominantes.

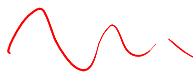
$\Downarrow$

mais longe do eixo imaginário

p. extras = $\begin{Bmatrix} 5 \\ 4 \\ 3 \end{Bmatrix} \times p$







$-1 + j1 \rightarrow -5 + j5$

$\begin{Bmatrix} 5 \\ 4 \\ 3 \end{Bmatrix}$ Kp

$p = a + jb \rightarrow 5a + jb$

$5a$

$p = -3 \pm j4 \rightarrow -12$

-10

-11

-13

-14

+3 extras.

Ex:

p donn ✓

p extras ✓

K ✓

N ✓

$$\begin{pmatrix} F & G \\ H & J \end{pmatrix} v = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix} \Rightarrow v \Rightarrow N_x,\ N_u \Rightarrow N$$

MA

$$\dot{x} = Fx + G\underline{u}$$

$$u = -Kx + N\underline{r}$$

$$\dot{x} = Fx + G(-Kx + Nr)$$

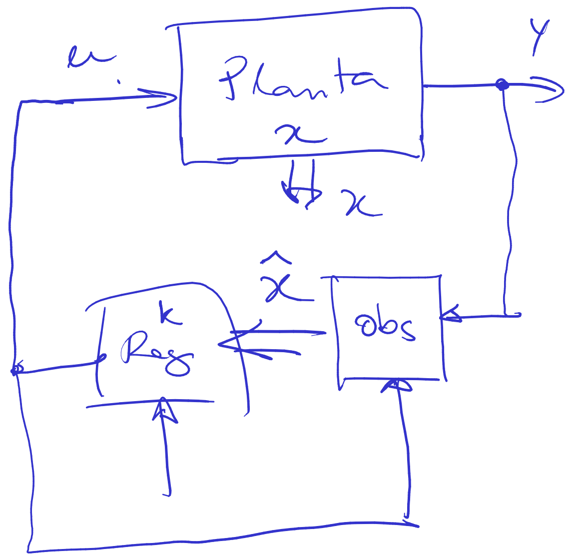
$$= Fx - GKx + GNr$$

$$= \underbrace{(F - GK)}_{F_{mf}}\underline{x} + \underbrace{GN}_{G_{mf}}\underline{r}$$

$$y = (1\ 0\ 0\ 0\ 0)x + 0r$$

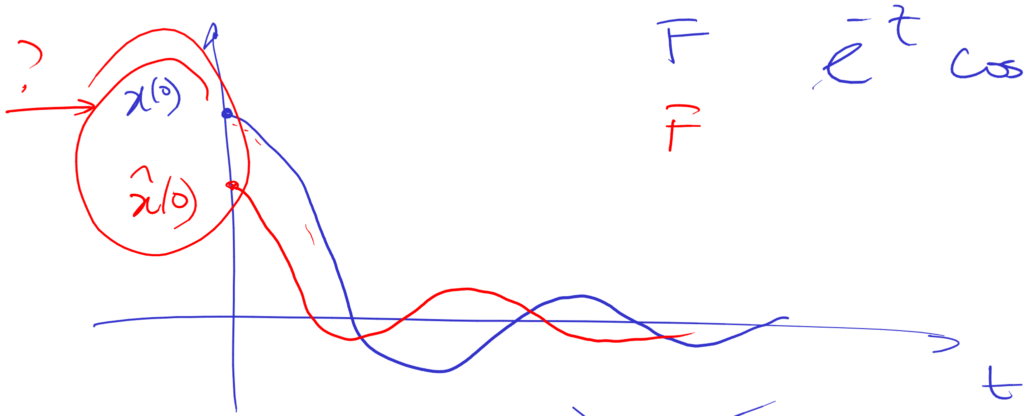
$$\underline{u} = -Kx + Nr$$

$$\underline{\text{Saída}} = \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ & & -K & & \end{pmatrix}}_{H_{mf}} x + \underbrace{\begin{pmatrix} 0 \\ N \end{pmatrix}}_{J_{mf}} r$$



$u = -K\hat{x} + Nr$

$\hat{x} \approx x$



F $\quad e^{-t}\cos(2t)$

F

verd: $y = Hx \quad \cancel{+ Ju}$

imit: $\hat{y} = H\hat{x}$

$y - \hat{y}$ = erro. de estimação.

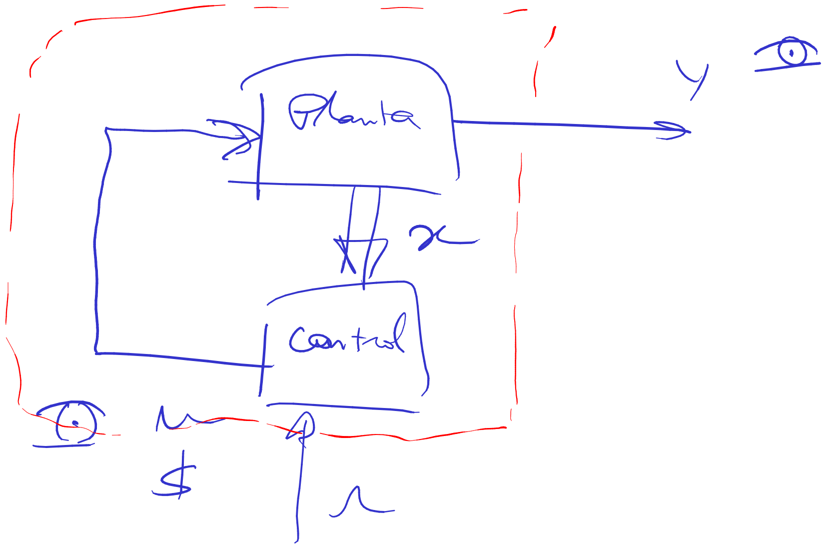
verd - imit.

$y = 1\times 1$

$H\hat{x} = 1\times 1$

$$L$$

$$(\;) = (\;) + (\;) + (\;)\, 0_{1\times 1}$$



Sist: $\dot{x} = Fx + Gu \quad ; \quad y = Hx$

obsv: $\dot{\hat{x}} = F\hat{x} + Gu + L(y - H\hat{x})$

erro: $e = x - \hat{x} \Rightarrow \dot{e} = \dot{x} - \dot{\hat{x}}$

$$\dot{\hat{x}} = F\hat{x} + Gu + L(Hx - H\hat{x})$$
$$= F\hat{x} + Gu + LH(x - \hat{x})$$

$$\dot{x} = Fx + Gu \qquad -$$
$$\dot{\hat{x}} = F\hat{x} + Gu + LH(x - \hat{x})$$

$$\dot{x} - \dot{\hat{x}} = \underline{Fx} + \cancel{Gu} - \underline{F\hat{x}} - \cancel{Gu} - LH(x - \hat{x})$$

$$\dot{e} = F(x - \hat{x}) - LH(x - \hat{x})$$

$$\dot{e} = Fe - LHe$$
$$= (F - LH)e$$

$$F = \begin{pmatrix} \quad \end{pmatrix}$$

$$L = \begin{pmatrix} \quad \end{pmatrix} \qquad H = \begin{pmatrix} \quad \end{pmatrix}$$

$$LH = \begin{pmatrix} \quad \end{pmatrix} \begin{pmatrix} \quad \end{pmatrix} = \begin{pmatrix} \quad \end{pmatrix}$$

$$F - LH = \begin{pmatrix} \quad \end{pmatrix} - \begin{pmatrix} \quad \end{pmatrix}$$

$$= \begin{pmatrix} \quad \end{pmatrix}$$

$$\dot{e} = \begin{pmatrix} F - L H \end{pmatrix} e$$

Eq. estados

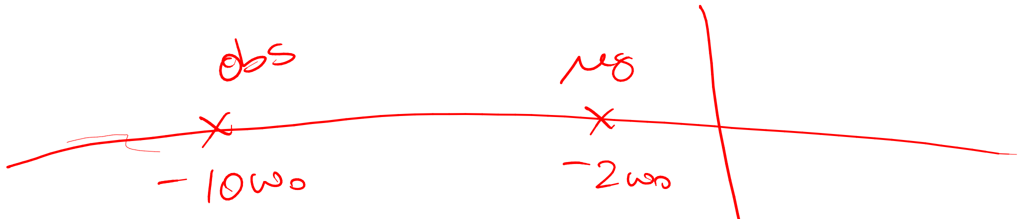
$\boxed{e \to 0}$ $t \to \infty$

Auto valores = rápidos estáveis.

$$\hat{x}(t) \to x(t)$$

$F - GK \rightarrow$ regulador

$F - LH \rightarrow$ (observador / estimador.)



$\hat{x}(t) \longrightarrow x(t)$

rápido.

$$F = \begin{pmatrix} 0 & 1 \\ -\omega_0^2 & 0 \end{pmatrix} \qquad H = \begin{pmatrix} 1 & 0 \end{pmatrix}$$

$$L = \begin{pmatrix} \ell_1 \\ \ell_2 \end{pmatrix} = ?$$

$$F - LH = \begin{pmatrix} 0 & 1 \\ -\omega_0^2 & 0 \end{pmatrix} - \underline{\begin{pmatrix} \ell_1 \\ \ell_2 \end{pmatrix} \begin{pmatrix} 1 & 0 \end{pmatrix}}$$

$$= \begin{pmatrix} 0 & 1 \\ -\omega_0^2 & 0 \end{pmatrix} - \begin{pmatrix} \ell_1 & 0 \\ \ell_2 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} -\ell_1 & 1 \\ -\omega_0^2 - \ell_2 & 0 \end{pmatrix} = M$$

$$\text{poli}(M) = \left| s I_2 - M \right|$$

$$= \left| s \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} -\ell_1 & 1 \\ -\omega_0^2 - \ell_2 & 0 \end{pmatrix} \right|$$

$$= \begin{vmatrix} s+\ell_1 & -1 \\ \omega_0^2 + \ell_2 & s \end{vmatrix} = s(s+\ell_1) - (-1)(\omega_0^2 + \ell_2)$$

$$= \underline{s^2} + \underline{\ell_1 s} + \underline{(\omega_0^2 + \ell_2)}$$

poli desejado: $(s + 10\omega_0)(s + 10\omega_0)$

$$= \underline{s^2} + \underline{20\omega_0 s} + \underline{100\,\omega_0^2}$$

$$\ell_1 = 20\,\omega_0$$

$$\omega_0^2 + \ell_2 = 100\,\omega_0^2 \Rightarrow \ell_2 = 99\,\omega_0^2$$