Projeto de controlador usando “emulação”

1 Problema 1

Sistema:

$$G(s) = \frac{10}{2s+3} \tag{1}$$

Especificações: $t_s \leq 1$, $M_p \leq 12\%$.

$$M_p = \exp\left(\frac{-\pi\xi}{\sqrt{1-\xi^2}}\right) \Leftrightarrow \xi = \frac{-\log M_p}{\sqrt{\pi^2 + (\log M_p)^2}} \tag{2}$$

Para $M_p = 0.12$, $\xi \approx 0.56$.
algum texto

$$t_s = \frac{4}{\xi\omega_n} \Rightarrow \omega_n = \frac{4}{\xi t_s} \tag{3}$$

$$\omega_n \approx 7.15 \text{ rad/s} \tag{4}$$

Polos e polinômio desejados:

$$p_{1,2} = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2} \approx -4 \pm j5.93 \tag{5}$$

$$P(s) = (s+4-j5.93)(s+4+j5.93) = s^2 + 8s + 51.12 \tag{6}$$

Controlador PI:

$$C(s) = k_p + \frac{k_i}{s} = \frac{k_p s + k_i}{s} \tag{7}$$

$$1 + C(s)G(s) = 1 + \frac{k_p s + k_i}{s}\frac{10}{2s+3} \tag{8}$$

$$= \frac{2s^2 + 3s + 10k_p s + 10k_i}{s(2s+3)} \tag{9}$$

$$= \frac{2s^2 + (3+10k_p)s + 10k_i}{s(2s+3)} \tag{10}$$

Logo, o polinômio de malha fechada é:

$$2s^2 + (3 + 10k_p)s + 10k_i = 2\left(s^2 + \frac{3 + 10k_p}{2}s + 5k_i\right) \tag{11}$$

$$s^2 + \frac{3 + 10k_p}{2}s + 5k_i = s^2 + 8s + 51.12 \tag{12}$$

Logo:

$$\frac{3 + 10k_p}{2} = 8 \Rightarrow k_p = 1.3 \tag{13}$$

$$5k_i = 51.12 \Rightarrow k_i = 10.224 \tag{14}$$

$$\Rightarrow C(s) = 1.3 + \frac{10.224}{s} \tag{15}$$

Frequência de amostragem: $\omega_s = 2\pi f_s$

$$\omega_s \geq 10\omega_n \tag{16}$$

$$\omega_s \geq 71.5 \tag{17}$$

$$f_s \geq \frac{71.5}{2\pi} \approx 11.38 \text{ Hz} \tag{18}$$

Podemos escolher então $f_s = 16$ Hz. Logo $T = 1/f_s = 62.5$ milisegundos.

Método trapezoidal, bilinear ou de Tustin:

$$s \leftarrow \frac{2}{T}\,\frac{z-1}{z+1} \tag{19}$$

$$\begin{align}
\frac{1}{T} &= 16 \tag{20}\\
s &\leftarrow 32\frac{z-1}{z+1} \tag{21}\\
\frac{1}{s} &\leftarrow \frac{z+1}{32(z-1)} \tag{22}
\end{align}$$

Logo:

$$\begin{align}
C(z) &= 1.3 + \frac{10.22}{s} \tag{23}\\
&= 1.3 + 10.22\frac{z+1}{32(z-1)} \tag{24}\\
&= \frac{32 \cdot 1.3(z-1) + 10.22(z+1)}{32(z-1)} \tag{25}\\
&= \frac{51.82z - 31.38}{32(z-1)} \tag{26}\\
&= \frac{1.62z - 0.98}{z-1} \cdot \frac{z^{-1}}{z^{-1}} \tag{27}\\
&= \frac{1.62 - 0.98z^{-1}}{1 - z^{-1}} \tag{28}
\end{align}$$

Implementação:

$$C(z) = \frac{U(z)}{E(z)} \tag{29}$$

onde:

$$U(z) = \mathcal{Z}\left\{u[k]\right\} \tag{30}$$
$$E(z) = \mathcal{Z}\left\{e[k]\right\} \tag{31}$$

$$\frac{U(z)}{E(z)} = \frac{1.62 - 0.98z^{-1}}{1 - z^{-1}} \tag{32}$$
$$(1 - z^{-1})U(z) = (1.62 - 0.98z^{-1})E(z) \tag{33}$$
$$U(z) - z^{-1}U(z) = 1.62E(z) - 0.98z^{-1}E(z) \tag{34}$$
$$\mathcal{Z}^{-1}\left\{U(z) - z^{-1}U(z)\right\} = \mathcal{Z}^{-1}\left\{1.62\,E(z) - 0.98z^{-1}E(z)\right\} \tag{35}$$
$$u[k] - u[k-1] = 1.62e[k] - 0.98e[k-1] \tag{36}$$
$$u[k] = u[k-1] + 1.62e[k] - 0.98e[k-1] \tag{37}$$

Código de implementação:

```
float uk=0;
float uk1 = 0;
float ek=0;
float ek1=0;
float y;
int k=0;
float r=1;
float T = 0.0625;
float Tm = 1000*T;
while {
    y = readAnalog(A1);
    ek = r-y;
    uk = uk1 +1.62*ek-0.98*ek1;
    analogWrite(uk);
    ek1 = ek;
    uk1 = uk;
    k = k+1;
    delay(Tm)
}
```