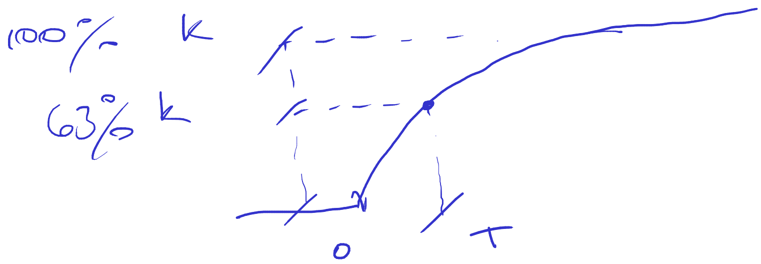
Revisão de controle analógico:

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$H(s) = \frac{K}{Ts+1}$$

K - ganho DC

$T \rightarrow$ cte tempo



$$K\left(1 - e^{-t/T}\right)$$

$t = T$

$$\underbrace{K\left(1 - e^{-1}\right)}_{\approx\, 0{,}63\ldots}$$

2º ordem

polos < 2 reais < $Im(s) = 0$ em cima do eixo real; sobreamortecido; $\zeta > 1$; s/ overshoot.

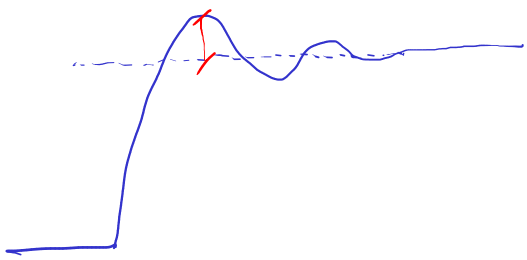
2 comp. conjugados.

$Im(s) \neq 0$

subamortecido

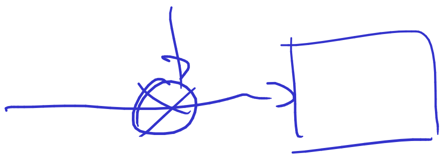
$0 \leq \zeta < 1$

overshoot.



resp. ao degrau

pert. aditiva



$$M_p = |Max(y) - 1|$$

$0 < \zeta < 1$

$$U(s) = \frac{1}{s} \longrightarrow \boxed{\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}} \longrightarrow Y(s)$$

$$y(t) = 1 - e^{-\sigma t}\cos(\omega_d t + \phi)$$

$$s_p = -\sigma \pm \cancel{j\omega_d}$$

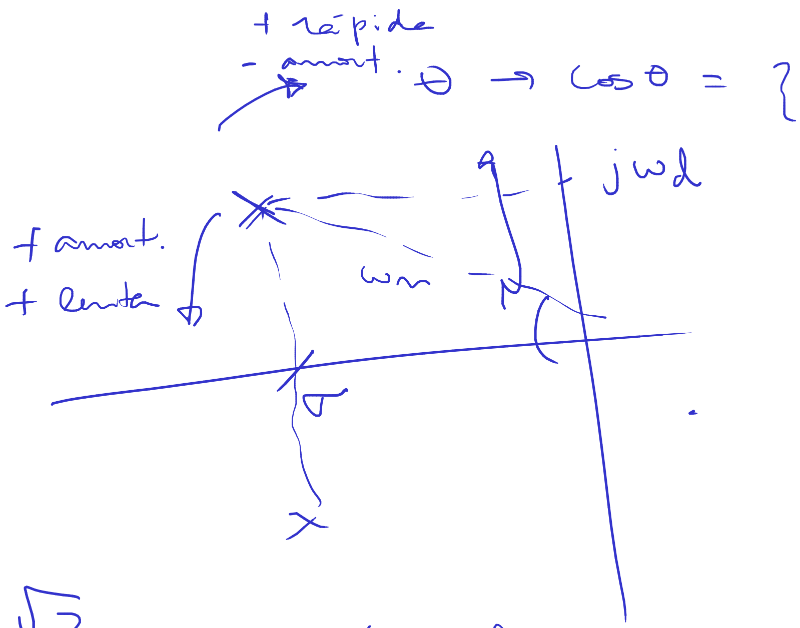
$$\longrightarrow \begin{cases} \sigma = \zeta\omega_n \\ \omega_d = \omega_n\sqrt{1-\zeta^2} \end{cases}$$

$$\phi = \angle \sigma + j\omega_d$$

$$M_p \rightarrow \frac{dy(t)}{dt} = 0 \rightarrow t_p \rightarrow M_p = y(t_p)$$

$$M_p = e^{-\pi\zeta / \sqrt{1-\zeta^2}}$$

$$\zeta = \frac{\sqrt{2}}{2}$$



$$\zeta = \frac{\sqrt{2}}{2} \Rightarrow \sigma = \omega_d$$

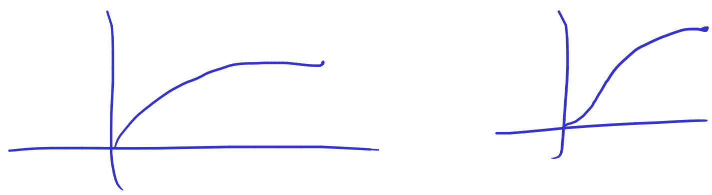
$$G(s) = \frac{\omega_n^2}{(s+\sigma)^2}$$

$$X(s) = \frac{1}{s} \longrightarrow$$

$$y(t) = \underline{R_1 e^{-\sigma_1 t}} + \underline{R_2 e^{-\sigma_2 t}} + 1$$

$$y(t) = 1 - e^{-\sigma t}\cos(\omega_d t + \phi)$$

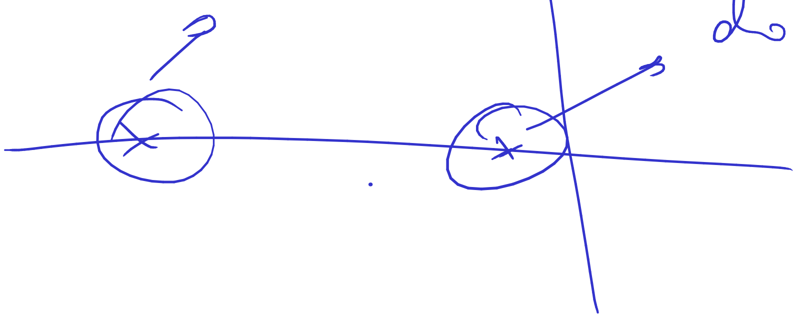
$$y(t) = R_1 e^{-\sigma t} + R_2 t e^{-\sigma t} + 1$$



Mais de 2 polos?

desprezível

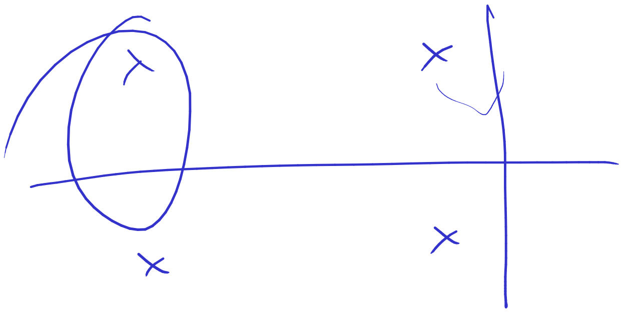
dominantes.



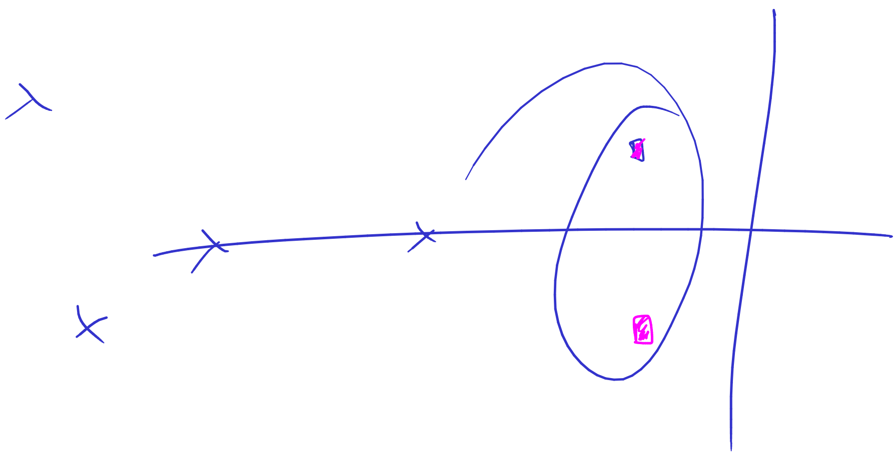
$$1 - 0{,}1e^{-t} - 0{,}01e^{-100t} \approx 1 - e^{-t}$$

$$0 < t < 1$$

s_2 s_1



$$y(t) = 1 - R_1 e^{-\sigma_1 t}\cos(\omega_1 t + \phi_1) - \cancel{R_2 e^{-\sigma_2 t}} \cos(\omega_2 t + \phi_2)$$



$$\frac{1}{s} \longrightarrow \boxed{\frac{5}{s+5}} \longrightarrow$$

$k = 1$

$T = 1/5 = 0{,}25$

1º ordem

$$\boxed{\frac{k}{Ts+1}}$$

$$\frac{5}{s+5}\,\frac{/5}{/5} = \frac{1}{\frac{s}{5}+1}$$

$$y(t) = 1\left(1 - e^{-5t}\right)$$

$$= A - Be^{-5t}$$

$$\frac{1}{s} \longrightarrow \boxed{\frac{20}{s+20}} \longrightarrow$$

$k = 1$ $\quad T = 1/20$

$s = -20$

$$y(t) = A - \underline{Be^{-20t}}$$

$$\cos(2t) \longrightarrow \boxed{\frac{20}{s+20}} \longrightarrow$$

$$y(t) = Be^{-20t} + A\cos(2t + \phi)$$

$$A = 1 \cdot \left|\frac{20}{j\cdot 2 + 20}\right| = 0{,}995 \qquad \phi = \angle \frac{20}{j2+20} = -5{,}7^\circ$$

$$\frac{20}{s+20} \xrightarrow{s = j\omega} \frac{20}{j\omega + 20} \xrightarrow{\omega = 2} \frac{20}{j2+20}$$

$$\frac{2}{(s+2)(0s+1)}$$

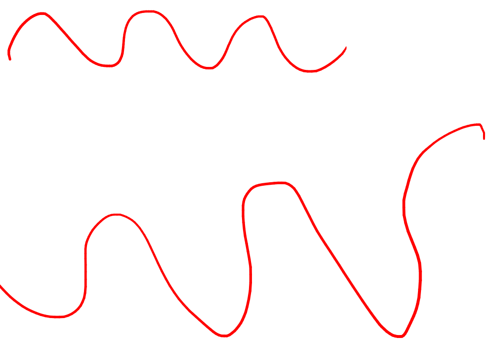
Sobreamort.

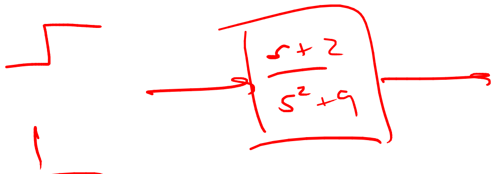
-2 $\qquad s = -\frac{1}{0} = -\infty$

$-3 \pm j\,11,6$

$s^2 + 9 = 0$

$s = \pm\sqrt{-9} = \pm j3$





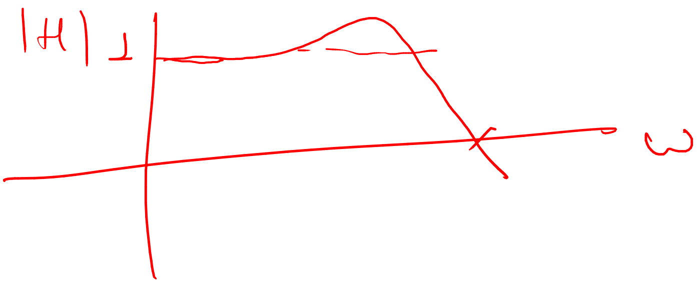
$$\frac{1}{s^2+9}$$

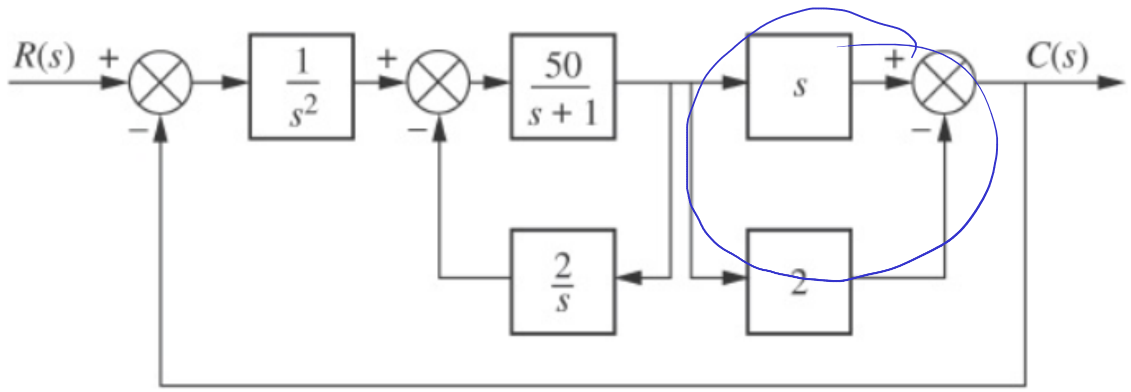
$$Y(s) = \frac{s+2}{(s^2+9)^2} \to y(t) = A_1 t \cos(3t+\phi)$$

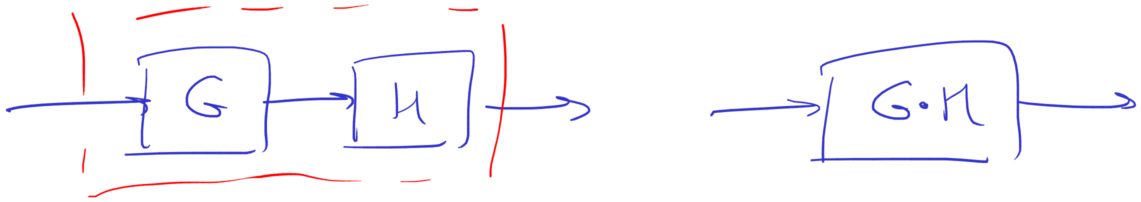
$$Y(s) = \frac{s+2}{(s^2+9)(s^2+36)} \to y(t) = A\cos(3t+\phi)$$

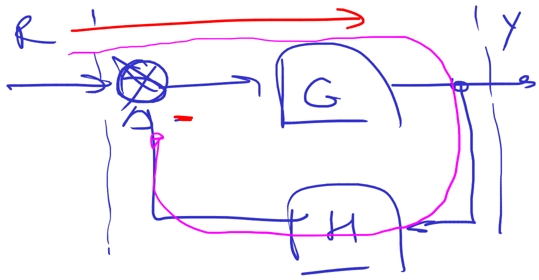
$$B\cos(6t+\theta)$$

$H(s) \to H(j\omega) \to |H(j\omega)| \qquad 0 < \omega < \infty$

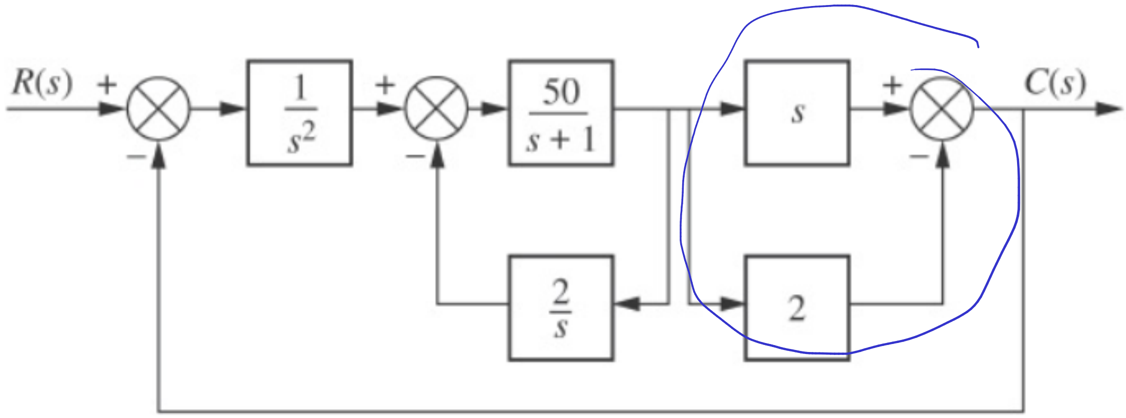


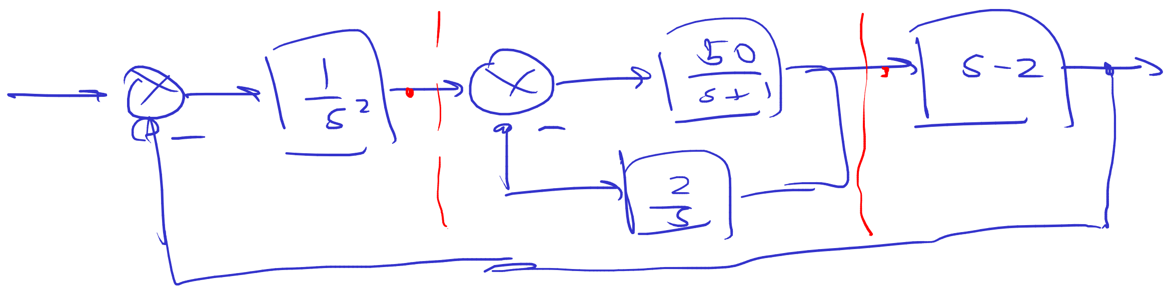






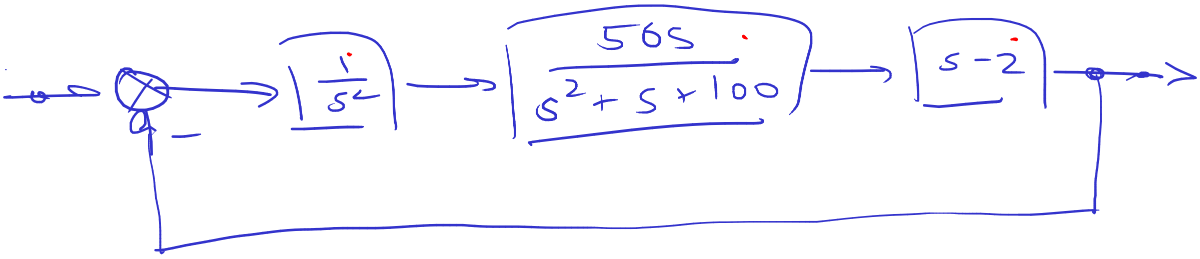
$$M = \frac{G}{1 + GH}$$





$$M_1(s) = \frac{\frac{50}{s+1}}{1+\frac{50}{s+1}\cdot\frac{2}{s}} = \frac{50/s+1}{\frac{s(s+1)+100}{s(s+1)}}$$

$$= \frac{50}{\cancel{s+1}} \cdot \frac{s\cancel{(s+1)}}{s(s+1)+100} = \frac{50s}{s^2+s+100}$$



$$M_2(s) = \frac{\frac{1}{s^2}\,\frac{50s}{s^2+s+100}\cdot(s+2)}{1+\frac{50s(s-2)}{s^2(s^2+s+100)}}$$

$$= \frac{\dfrac{50s(s-2)}{\cancel{s^2(s^2+s+100)}}}{\dfrac{s^2(s^2+s+100)+50s(s-2)}{\cancel{s^2(s^2+s+100)}}}$$

$$= \frac{50s(s-2)}{s^4+s^3+100s^2+50s^2-100s}$$

$$= \frac{50\cancel{s}(s-2)}{s^{\cancel{4}}+s^{\cancel{3}}+150s^{\cancel{2}}-100\cancel{s}} = \frac{50(s-2)}{s^3+s^2+150s-100}$$