Sistemas de Controle II

Nas últimas semanas...

- Modelagem de sinais discretos
- Teorema da amostragem e aliasing
- Equações de diferenças
- Projeto por emulação

Dedução do método de Tustin

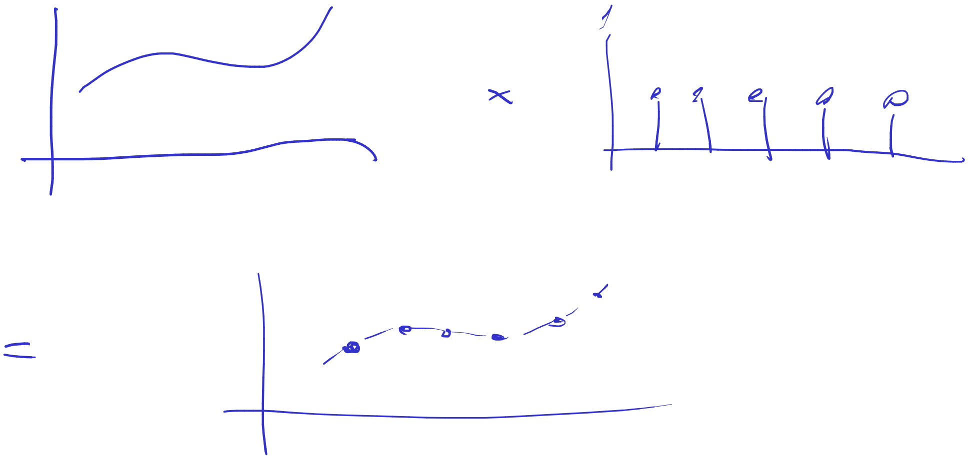
- Por integração
- Por série de Taylor

Equivalentes discretos

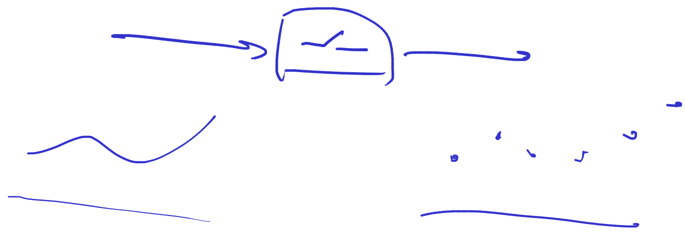
- O que é?
- Outros tipos

Exercícios:

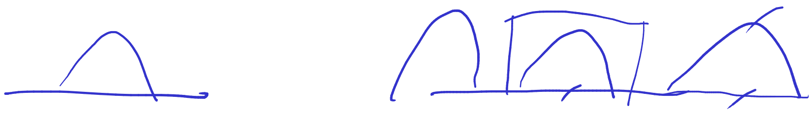
- Equações de diferenças e análise de sistemas
- Emulação



SC SD

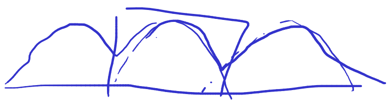


tempo



freq

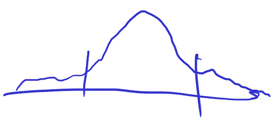
2n banda

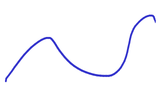


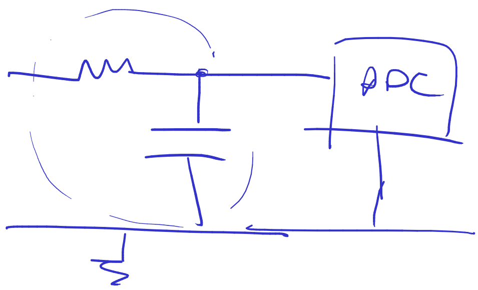
2n aliasing.

filtro anti-aliasing.

c. A/D







$$y[k+1] + y[k] = x[k] \qquad y[0] = 1$$

Analítico $\quad y[k] = 9\,(0,1)^k$

Numérico $\quad y[k] = [\,9 \quad 0,9 \quad 0,09 \quad \cdots\,]$

Qualitativo.

$0,8^k \rightarrow$ decrescente $\qquad |base| < 1$

$7^k \rightarrow$ crescente $\qquad |base| > 1$

$(-0,8)^k \rightarrow$ decrescente alterna $\qquad |base| < 1$, base < 0

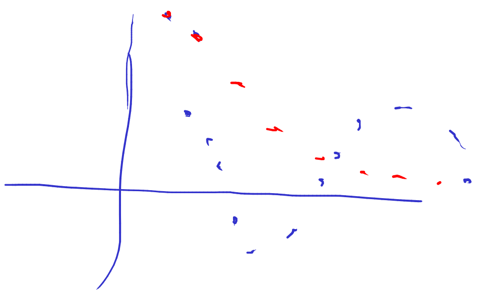
$$9\,(0,8\,e^{j2})^k + 9\,(0,8\,e^{-j2})^k$$

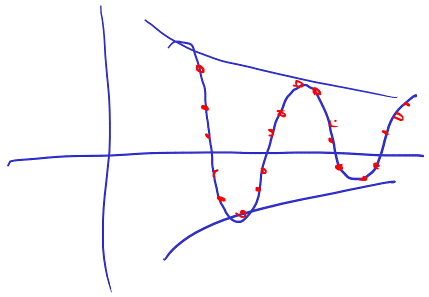
$$(0,8\,e^{j2})^k = 0,8^k\left(\cos 2k + j\,\mathrm{sen}\,2k\right) \qquad \times 9$$

$$(0,8\,e^{-j2})^k = 0,8^k\left(\cos(-2k) + j\,\mathrm{sen}(-2k)\right)$$

$$= 0,8^k\left(\cos 2k - j\,\mathrm{sen}(2k)\right) \qquad \times 9$$

$$9\,(0,8\,e^{j2})^k + 9\,(0,8\,e^{-j2})^k = 18 \cdot 0,8^k \cos 2k$$





$$y[k] = (1+j2)(0{,}8e^{j2})^k + (1-j2)(0{,}8e^{-j2})^k$$

$$= ?$$

Transformada $\mathcal{Z}$

$$\mathcal{Z}\{x[k]\} = \sum_{k=0}^{\infty} x[k]\, z^{-k}$$

$$z = e^{sT}$$ $T \rightarrow$ período de amostragem

$$\mathcal{Z}\{\delta[k]\} = 1$$

$$\mathcal{Z}\{a^n u[n]\} = \frac{z}{z-a}$$

$$y[k+2] + 9y[k] = x[k] \qquad \mathcal{Z}$$

$$\mathcal{Z}\{y[k+2] + 9y[k]\} = \mathcal{Z}\{x[k]\}$$

$$\mathcal{Z}\{y[k+2]\} + 9\,\mathcal{Z}\{y[k]\} = \mathcal{Z}\{x[k]\}$$

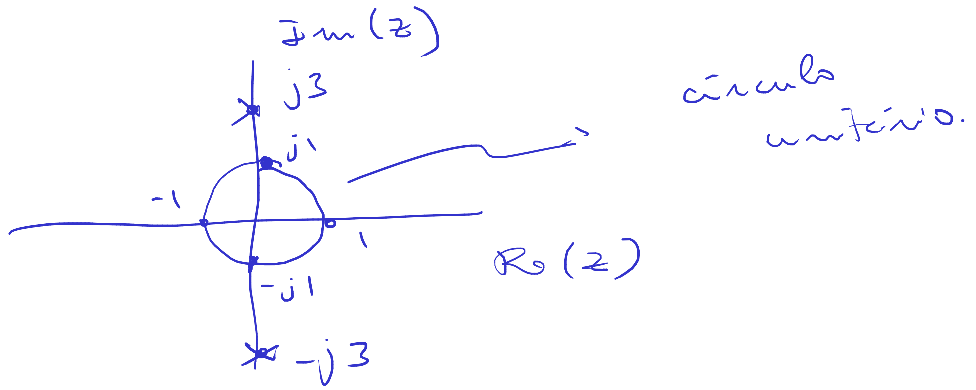
$$\mathcal{Z}\{y[k+2]\} = z^2 Y(z)$$

$$z^2 Y(z) + 9Y(z) = X(z)$$

$$(z^2+9)Y(z) = X(z)$$

$$\frac{Y(z)}{X(z)} = \frac{1}{z^2+9}$$ Função de transferencia

Polos: $z^2+9=0 \Rightarrow z = \pm j3$



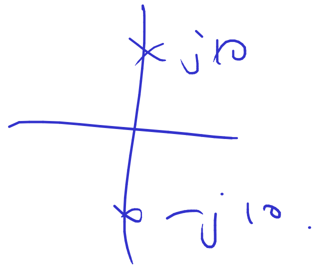
estabilidade: $|\text{todos os polos}| < 1$

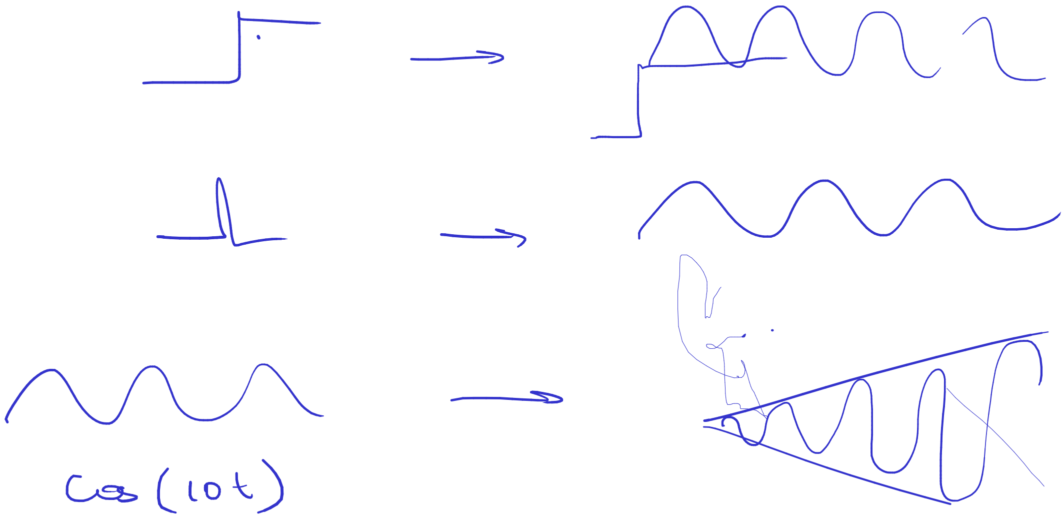
polos estritamente reais > 0 → exponencial não oscilatória

" " " < 0 → exponencial oscilatória.

polos imaginários conjugados → oscilatório.

$$\longrightarrow \boxed{\frac{1}{s^2+100}} \longrightarrow$$

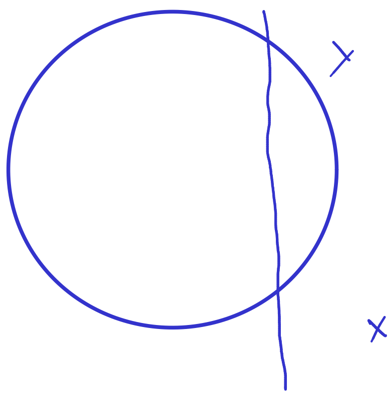




cos(10t)

$$\frac{1}{s^2+10} \cdot \frac{1}{s^2+10} \Rightarrow Y(s) = \frac{1}{(s^2+10)^2}$$

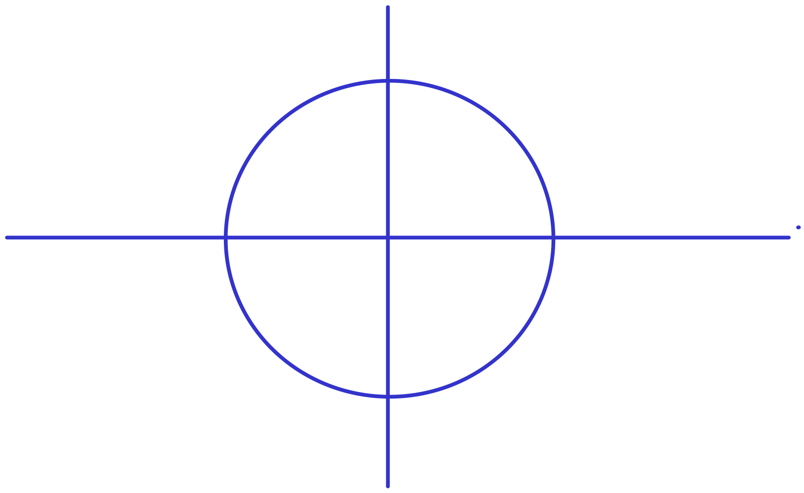
$$y(t) = A\, t \cos(10t)$$

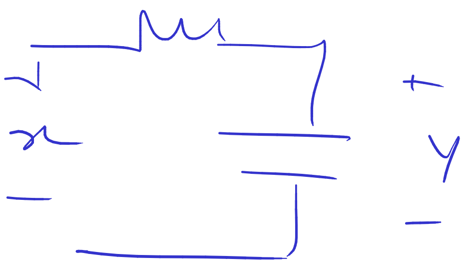


$$z = e^{sT}$$

$$\frac{1}{s+5} \longrightarrow \boxed{\frac{1}{s+5}} \longrightarrow \frac{1}{(s+5)^2}$$

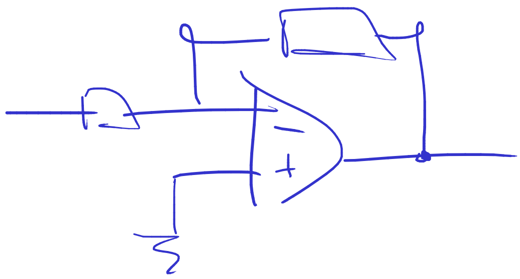
$$y(t) = A\, t\, e^{-5t}$$



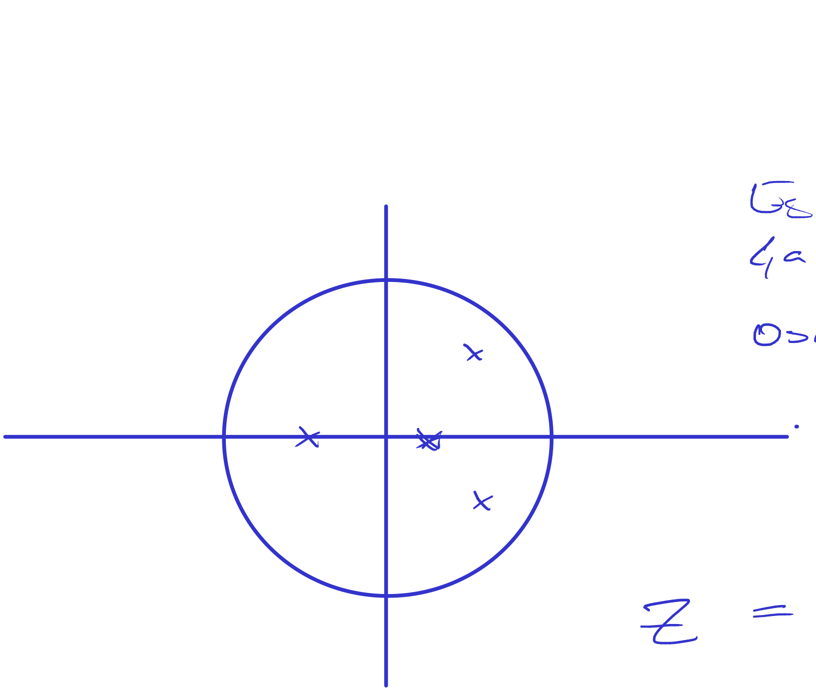


$$\frac{Y}{X} = \frac{1}{RCs+1}$$

$$s = -\,{}^{1}/_{RC}$$



Estável
4a ordem
Oscilatória.



$$z = e^{sT}$$

$$s = j\omega$$

$z = e^{sT}$

$s = j\omega \Rightarrow z = e^{j\omega T}$

$\omega = \frac{2\pi}{7} \Rightarrow e^{j\frac{2\pi T}{7}}$

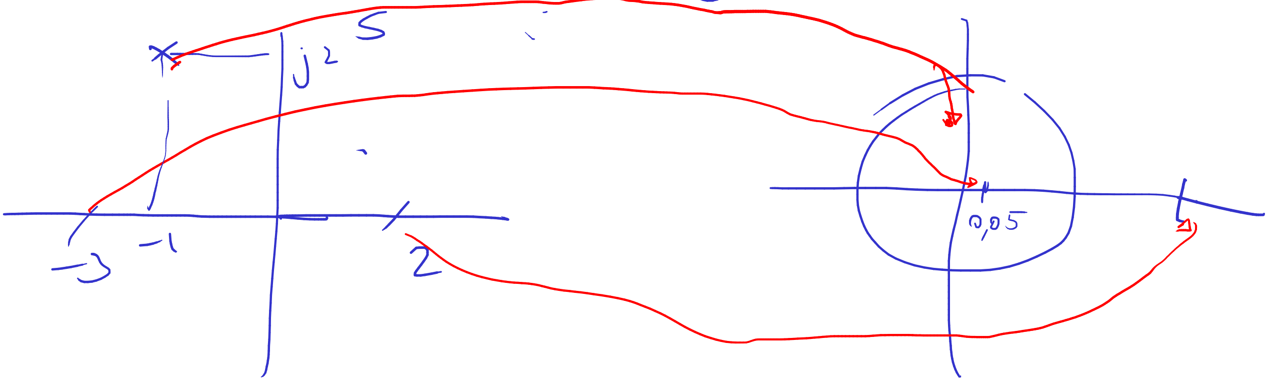
$a + jb \rightarrow |\,| e^{j\angle}$

$3 + j4 \rightarrow 5 e^{j---}$

$T = 1$

$s = -3 \longrightarrow z = e^{-3} =$

$s = 2 \longrightarrow z = e^{2}$



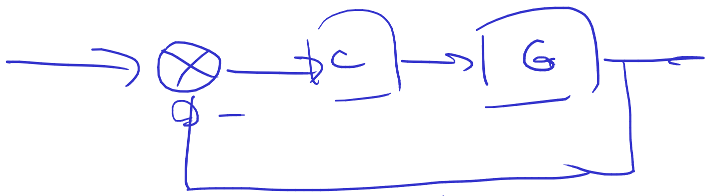
$s = -1 + j2 \longrightarrow z = e^{-1+j2} = e^{-1}(\cos 2 + j \sin 2)$

$= -0,15 + j\,0,33$

Método de Tustin:

Projeto por emulação:

- Projeta o analógico normal
- Escolhe o período de amostragem
- Discretiza o controlador



$$\underline{C}(s) \to \overset{rad/s}{\omega_n} \to \overset{Hz}{F_s} \to T \to s = \frac{2}{T}\,\frac{z-1}{z+1}$$

$\to \underline{C}(z) \to$ E. Dif. ($u[k] \to e[k]$) $\to$ código.

$$z = e^{sT}$$

$$= e^{sT/2} \cdot e^{sT/2}$$

$$= \frac{e^{sT/2}}{e^{sT/2}}$$

$$\cong \frac{1 + \frac{sT}{2}}{\left(1 - \frac{sT}{2}\right)} = z$$

$$1 + \frac{sT}{2} = z - z\,\frac{sT}{2}$$

$$x^2 = y$$

$$x = \pm\sqrt{y}$$

$$e^x \cong 1 + x + \cancel{\frac{x^2}{2} + \frac{x^3}{3!}}$$

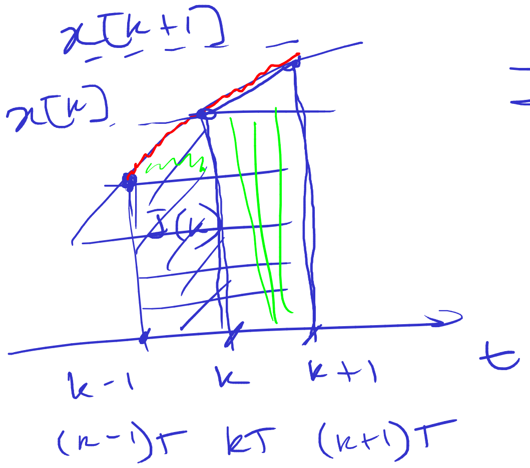
$$f(x) \approx f(0) + f'(0)\,x + \frac{f''(0)\,x^2}{2!}$$

$$1 - z = -\frac{sT}{2} - z\frac{sT}{2}$$

$$1 - z = -\frac{sT}{2}(1+z)$$

$$-\frac{sT}{2} = \frac{1-z}{1+z}$$

$$s = \frac{2}{T}\,\frac{z-1}{1+z}$$



$$I(k+1) = I(k) + \Delta$$

$$\Delta = \left(x[k+1] + x[k]\right)\frac{T}{2}$$

$$(\cancel{k}+1)T - \cancel{kT} = T$$

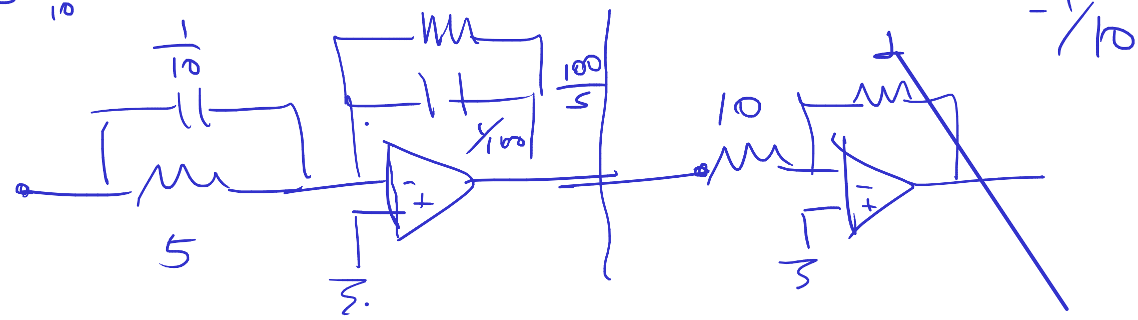
$$I[k+1] = I[k] + \frac{T}{2}\left(x[k+1] + x[k]\right) \quad \mathcal{Z}$$

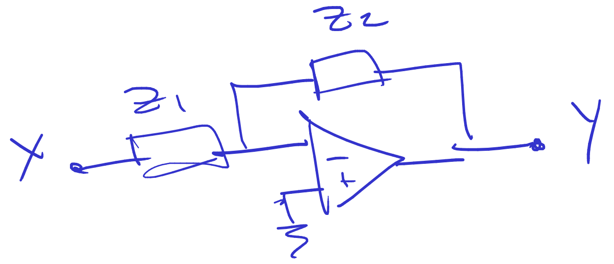
$$zI(z) = I(z) + \frac{T}{2}\left(zX(z) + X(z)\right)$$

$$(z-1)I(z) = \frac{T}{2}(z+1)X(z)$$

$$\frac{I(z)}{X(z)} = \frac{T}{2}\,\frac{(z+1)}{z-1} \longleftrightarrow \frac{1}{s}$$

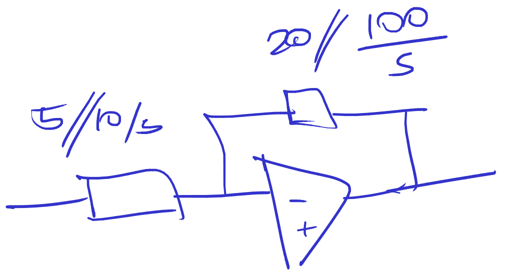
$$\frac{1}{s\cdot\frac{1}{10}} = \frac{10}{s}$$





$$\frac{Y}{X} = \frac{-Z_2}{Z_1}$$

$$20 // \frac{100}{s} = \frac{\cancel{20}\cdot\frac{100}{s}}{\cancel{20}+\frac{\cancel{100}}{s}}$$



$$\Rightarrow \quad = \frac{200/\cancel{s}}{\frac{2s+10}{\cancel{s}}} = \frac{200}{2s+10} = \frac{100}{s+5}$$

$$5//10/s = \frac{5\cdot\frac{10}{s}}{5+\frac{10}{s}} = \frac{50}{5s+10} = \frac{10}{s+2}$$

$$G_1(s) = -\frac{\frac{\cancel{100}}{s+5}}{\frac{\cancel{10}}{s+2}} = -10\,\frac{s+2}{s+5}$$

$$G_2(s) = \frac{-1}{10}$$

$$G_1(s)\,G_2(s) = \frac{s+2}{s+5}$$

$$s = -5$$

$$s = \frac{\alpha}{T}\,\frac{z-1}{z+1}$$

$$\tau = \frac{1}{5} = 0{,}2s.$$

$$T = \frac{\tau}{10} = 0{,}02s$$

$$s = \frac{\alpha}{0{,}02}\,\frac{z-1}{z+1} = 100\,\frac{z-1}{z+1}$$

$$G(s) = \frac{s+2}{s+5}.$$

$$= \frac{100\frac{z-1}{z+1} + 2}{100\frac{z-1}{z+1} + 5} = \frac{100(z-1) + 2(z+1)}{100(z-1) + 5(z+1)}$$

$$= \frac{102z - 98}{105z - 95} \cdot \frac{z^{-1}}{z^{-1}} = \frac{102 - 98z^{-1}}{105 - 95z^{-1}}$$

$$\frac{Y(z)}{X(z)} = \frac{102 - 98z^{-1}}{105 - 95z^{-1}} \Rightarrow (105 - 95z^{-1})Y = (102 - 98z^{-1})X$$

$$105Y - 95z^{-1}Y = 102X - 98z^{-1}X$$

$$105y[k] - 95y[k-1] = 102x[k] - 98x[k-1]$$