TESTING A SPATIAL INTERACTING-MARKETS HYPOTHESIS

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Abstract—Two models for competitive spatial pricing between retail outlets in a single urban area are presented. The models are distinguished in terms of first, their relationships to underlying supply and demand conditions in the market and second, the spatial forms of inter-site competition described. Properties of the models are identified. The statistical theory appropriate to these models is reviewed and the models then fitted to data for retail gasoline prices during periods of falling and then rising average price levels.

I. Introduction

ECONOMISTS are accustomed to dealing with variate dependency through time (serial correlation) and with the specification and estimation problems associated with the stochastic processes that generate such dependency. Less frequently encountered are problems of variate dependency in space (or spatial autocorrelation). A specific problem occurs in the case of price theory, however, where spatial dependency in a set of prices for a single homogeneous good may arise from the interaction of spatially distributed markets. In such cases it may be of interest to estimate spatial autocorrelation as a measure of inter-market dependence.

A case where such price interaction may occur is in the provision of a single homogeneous good at a number of retail outlets in an urban area. Distances between such retailers may be negligible, retailers may be well aware of their competitors' prices and purchasers are able to obtain relative price information at almost no cost and, moreover, are highly mobile with respect to several retailers.

As a consequence, market areas may be considered to overlap. In those cases where product differentiation is relatively weak, this might lead to a situation in which prices at any individual retail outlet are set, at least in part, in terms of prices charged at competing, neighbouring outlets.

The models to be introduced here, for this situation, involve the estimation of measures of spatial association. Specifically, the paper brings together two models for spatial pricing (at the intra-urban scale) with some quite recent developments in the statistical theory for the analysis of data on irregular spatial lattices. This statistical theory arises from the theory of two dimensional stochastic processes or random field theory (Preston, 1974).

In the following sections the models are introduced, elements of the statistical theory are outlined, and some results given on the fitting of the models to petrol prices at a set of retail outlets with price posting. For comparison two time periods were analysed—the first when prices were falling, the second when they were rising.

II. Two Models of Spatial Pricing

Both models—the pure competition model and the supply-demand model—assume a fixed spatial distribution of point sites representing the locations of the n retail outlets. Subsequently a set of edges will be superimposed on the set of sites to form a graph. The statistical problem is to describe the distribution of site values where the edge system identifies, for each site, its set of neighbours. The edge system may be thought of as an attempt to "order" the set of sites though the ordering is clearly not natural in the same way that time (past and future) orders serial observations. In deriving the two models in this section we shall assume a general edge system, taking up the problem of edge specification in section III.

The two models differ in important respects. The pure competition model emphasizes retailer price aggressiveness. It reflects the importance of the structure of the petrol retailing sector in determining prices and in particular the co-existence within a geographical area of "price-marketers" and "majors" (Allvine and Patterson, 1972). The former are price aggressive, and particularly when prices are falling this group may lead the price fall with other retailers responding through the "sharing agreement" schemes (Schendel and Balestra, 1969) that give site managers freedom in setting their own prices in order to retain market share. Such a process may only be indirectly tied to the real pattern and variability of demand in the market. On the other hand, the supply-demand model emphasizes consumer price sensitivity (be-

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tween sites) and the effect this has on stimulating price adjustment in the market. This second model may be more appropriate for describing competition in a more settled market where the competitive influence of the price marketers is weaker.

A second important distinction between the models concerns the spatial forms of inter-site competition that they describe. The supply-demand model seeks to impose a single average measure of inter-site dependence assuming that the entire study area constitutes a single (though possibly partitionable) interacting market. In this, however, it may underestimate or obscure more subtle spatial interaction effects that may be confined to certain price levels within the overall price distribution. These might arise from the local ripple effects of competition in sub-areas of the market possibly brought about by the operations of price marketers. This element of the pattern, the pure competition model may be better at detecting.

A. A Pure Competition Model

Let X_i denote the random variable describing the price charged at the i^{th} retail outlet, where the price has been transformed to a binary valued variable, the value depending on whether the original price lies inside or outside a specified interval. That is,

$$X_i = \begin{cases} 1 & \text{if the } i^{\text{th}} \text{ retail outlet charges in the price range } [s_1, s_2] \\ 0 & \text{otherwise.} \end{cases}$$

Let $\underline{x} = (x_1, \ldots, x_n)$ denote the vector describing the configuration of n transformed or binary prices, where n is the number of retail outlets. Further let $\underline{x}_i = (x_1, \ldots, x_{i-1}, 0, x_{i+1}, \ldots, x_n)$. Now define "birth" and "death" rates as follows:

$\gamma(x_i = 1, \underline{x}_i)\,dt$ = probability that site i charges in the price range $[s_1, s_2]$ at time $t + dt$ given that it did not at time t.

$\delta(x_i = 0, \underline{x})\,dt$ = probability that site i does not charge in the price range $[s_1, s_2]$ at time $t + dt$ given that it did at time t.

All other transitions have negligible probabilities.

Assume now that these birth and death rates are functions of prices elsewhere and let

$$\gamma(x_i = 1, \underline{x}_i) = \exp\left[\alpha' + \sum_{\substack{j=1 \\ j \neq i}}^{n} \beta'_{ij} x_j\right]$$

$$\delta(x_i = 0, \underline{x}) = \exp\left[\alpha'' + \sum_{\substack{j=1 \\ j \neq i}}^{n} \beta''_{ij} x_j\right]$$

(Bartlett, 1975, p. 43).

If the birth-death process is reversible and irreducible the price distribution is a spatial Markov field (Preston, 1974, p. 13). Then using the definition of reversibility where $P(\underline{x})$ is the probability of the vector $\underline{x}$:

$$\frac{P(\underline{x})}{P(\underline{x}_i)} = \frac{\gamma(x_i = 1, \underline{x}_i)}{\delta(x_i = 0, \underline{x})} = \exp\left[(\alpha' - \alpha'') + \sum_{\substack{j=1 \\ j \neq i}}^{n} \left(\beta'_{ij} - \beta''_{ij}\right) x_j\right].$$

But

$$\frac{P(\underline{x})}{P(\underline{x}_i)} = \frac{P(X_i = x_i \mid \text{all other site values})}{P(X_i = 0 \mid \text{all other site values})} \equiv \frac{P(X_i = x_i \mid \cdot)}{P(X_i = 0 \mid \cdot)}.$$

Setting $(\alpha' - \alpha'') = \alpha$ and $(\beta'_{ij} - \beta''_{ij}) = \beta_{ij}$ and noting that $P(X_i = 1 \mid \cdot) + P(X_i = 0 \mid \cdot) = 1$ it follows that

$$P(X_i = x_i \mid \cdot) = \exp\left[x_i \left\{\alpha + \sum_{\substack{j=1 \\ j \neq i}}^{n} \beta_{ij} x_j\right\}\right] \Bigg/ \left[1 + \exp\left\{\alpha + \sum_{\substack{j=1 \\ j \neq i}}^{n} \beta_{ij} x_j\right\}\right] \tag{1}$$

which is called the autologistic model (Besag, 1974). We shall refer to (1), here, as the pure competition model. It describes the equilibrium distribution of the transformed set of prices in terms of conditional probabilities.

This is more widely known as the Ising model. An interesting property of this model is that the variance of the ΣX_i decreases for increasing values of the interaction parameter $\beta - a$ property that is associated with the existence of phase transitions in this class of models (see, for example, Kindermann and Snell (1980)). The model suggests therefore that during a price war, when inter-site competition effects may become more intense, total price variance may decline.

B. A Supply and Demand Model

Let $\underline{D}_t$ and $\underline{S}_t$ denote n-dimensional demand and supply vectors at time t. Let $\underline{p}_t$ and $\underline{p}_e$ denote n-dimensional price vectors at time t and at equilibrium, respectively, and let $\underline{c}$ and $\underline{e}$ denote vectors of constants. Let $\underline{u}$ be a vector random variable where $\underline{u}$ is IN $(\underline{0}, \sigma^2 \underline{I})$. Finally, let $\underline{A} = \{a_{ij}\}$ be an $n \times n$ matrix with diagonal entries less than zero and off-diagonal entries greater than or equal to zero. Let $\underline{B}$ be an $n \times n$ diagonal matrix with positive entries in the diagonal.

We define the following spatial interacting-markets model:

$$\begin{aligned} \underline{D}_t &= \underline{A}\underline{p}_t + \underline{c} + \underline{u} \\ \underline{S}_t &= \underline{B}\underline{p}_{t-1} + \underline{e}. \end{aligned} \tag{2}$$

Interaction is introduced through the off-diagonal elements of the demand matrix, $\underline{A}$.

Assuming market clearance at each of the n outlets and using standard procedures it can be shown that

$$\underline{p}_e = (\underline{A} - \underline{B})^{-1}(\underline{e} - \underline{c} - \underline{u})$$

and the equilibrium is stable providing the eigenvalues of $(\underline{A}^{-1}\underline{B})$ are all less than unity in absolute value. Further, exploiting the diagonal nature of $\underline{B}$ it can be shown (Haining, 1983) that the equilibrium price for the i^{th} retail outlet is

$$p_{i,e} = \sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{a_{ij}}{b_{ii} - a_{ii}} p_{j,e} + \frac{c_i - e_i}{b_{ii} - a_{ii}} + \frac{u_i}{b_{ii} - a_{ii}} \qquad i = 1, \cdots, n. \tag{3}$$

This equation describes an autoregressive spatial scheme with positive coefficient $(a_{ij}/(b_{ii} - a_{ii}) \geq 0)$ and hence positive spatial autocorrelation at all lags. However, since $(c_i - e_i) \neq 0$ the model also implies the presence of intra-site effects.

Implicit in this model is the assumption that interaction effects arise from the price awareness and price sensitivity of consumers. Uncertainty is introduced into the model via the demand function reflecting not only variation in consumer behaviour but also variation in the total level of demand (which is then apportioned over the various retail outlets). It can be shown from (3) that when the coefficients in $\underline{A}$ increase in absolute value (as they might do during a price war when consumers are more aware of the gains to be had from comparing prices) the variance in equilibrium prices decreases (Haining, 1983). The model also implies serial correlation in prices at a particular station and between stations (space-time correlation). Because of data limitations these were not analysed.

III. Statistical Theory for Irregular Lattice Models

There are two broad classes of models for describing lattice distributions—those arising from the conditional specification (Besag, 1974) and those arising from the simultaneous or joint specification (Whittle, 1954). They are not, as in the time series case, equivalent approaches (Brook, 1964) and each presents distinctive statistical problems (Besag, 1975). Whereas the pure competition model, (1), arises from the conditional specification, the supply and demand model, (3), arises from the joint specification.

The difference between these two approaches is best illustrated by example. Let Y_i denote the variate (price) at location i. The first order (spatial) autoregressive scheme is a joint probability model. This is defined as

$$Y_i = \alpha_i + \tau \sum w_{ij} Y_j + u_i \qquad (i = 1, \ldots, n)$$

where $\{w_{ij}\}$ are a set of given non-negative weights representing site interactions (of j on i) and with $w_{ii} \equiv 0$. The sum is over all sites, j, that are neighbours of i, these being the sites that are assumed to interact directly with i. The $\{u_i\}$ are IN$(0, \sigma^2)$ error variates and $\{\alpha_i\}$ and τ are parameters. Taking expectations it follows that

$$E[Y_i] = \alpha_i + \tau \sum w_{ij} E[Y_j]. \tag{4}$$

This may be contrasted with the first order autonormal scheme (Besag, 1974), which is a conditional model. Here we define

$$E[Y_i | \text{all other site values}] = \alpha_i + \tau \sum w_{ij} y_j \tag{5}$$

where y_j denotes the observed value at site j. In fact only values at neighbouring sites of site i need be included in the conditioning.

Now (4) and (5) are equivalent only if u_i and $\Sigma w_{ij} Y_j$ are uncorrelated, that is,

$$E[u_i | \text{all other site values}] = 0$$

(Ord, 1975). In time series modeling the unilateral nature of dependence makes this a natural restriction so that joint and conditional models are equivalent. In the case of the joint spatial model the bilateral nature of spatial dependence means that this condition is not satisfied, as can be verified from the above scheme.

Besag (1974, 1975) has suggested two ways of estimating the parameters of conditional models. The first approach involves coding the lattice, which means dividing the set of sites up into two groups: one group a set of "dependents" the other of "conditioners." The "dependents" are sites chosen in such a way that no two are neighbours. Hence they are mutually independent observations, given the values at the set of conditioner sites. Estimates of α and β in (1) (where $\beta_{ij} = \beta w_{ij}$) may be obtained by maximizing the conditional log likelihood function. Providing the set of dependent sites is not too small, classical maximum likelihood theory applies (Besag, 1975). However, the approach is clearly not unique (many different codings can be imposed and hence many different estimates may be obtained for the same edge system) and the method is wasteful of data particularly for irregular lattices.

Besag (1975) also discussed a maximum pseudo-likelihood estimation procedure which uses all the data. It is less wasteful of data and does provide a consistent estimator; however, no sampling properties for the estimates have been given.

The results given in this paper for the pure competition model are maximum pseudo-likelihood estimates, and goodness-of-fit tests for the significance of the interaction coefficient (β) are based on treating the difference in the log likelihoods ($\ln(\tilde{\alpha}, \tilde{\beta}) - \ln(\tilde{\alpha}, 0)$) as $1/2\ \chi^2$ distributed with one degree of freedom. This is not exact, since the result assumes independent observations, but does provide an approximate measure of significance.

Spatial autoregressive schemes of the form (3) cannot be estimated without reducing the number of unknown parameters. In the next section, for estimation purposes, the supply-demand model has been re-expressed in the form

$$p_{i,e} = \sum_{\substack{j=1 \\ j \neq i}}^{n} \rho w_{ij} p_{j,e} + \mu_i + u_i \qquad i = 1, \dots, n \tag{6}$$

where w_{ij} is an a priori weighting measure which is positive valued if site j is joined by an edge to site i, and is zero otherwise. The site effects term is now represented by μ_i and interaction effects are estimated by the single coefficient ρ.

Unlike equivalent time series models, the parameters of spatial autoregressions cannot be estimated by least squares procedures because of the correlation between the dependent variable and the errors, arising from the bilateral nature of spatial dependence (Ord, 1975). In the case of pure spatial autoregressions ($\mu_i = 0$ for all i), ρ is estimated by maximum likelihood. This yields, in the case of (6) for normal errors (u_i), a "weighted" least squares function to be minimized with respect to ρ of the form:

$$\left\{\left[\prod_{i=1}^{n}(1 - \rho\eta_i)\right]^{-2/n}\right\}\left\{\underline{p}_e^T(\underline{I} - \rho\underline{W})^T \times (\underline{I} - \rho\underline{W})\underline{p}_e\right\} \tag{7}$$

where the first term in brackets is the "weighting" or Jacobian. This term emerges because of the bilateral nature of spatial dependence. The matrix $\underline{I}$ is the identity matrix and $\underline{W}$ is the $n \times n$ matrix (n = lattice size) of w_{ij}'s. The η_i are the n eigenvalues of $\underline{W}$. Row sums of $\underline{W}$ are usually scaled to unity. Properties of this estimator and the modifications needed to simultaneously estimate a non-zero intra-site effects term, μ_i, are discussed in Ord (1975), Haining (1978) and Anselin (1982).

Goodness-of-fit tests can be constructed for these models by comparing variances (Whittle, 1954). Let $(kU)_p$ be the minimized value of the weighted least squares function for a model with p parameters (k represents the weighting term and U the least squares term). If $(kU)_{p+q}$ is the minimized value of the function for a model with q additional parameters then $(n - p - q)\ln((kU)_p/(kU)_{p+q})$ is asymptotically χ^2 distributed with q degrees of freedom. This test statistic is a likelihood ratio test, using the ratio of the minimized error variances. Further details with examples on the statistical analysis of lattice models can be obtained from Whittle (1954), Besag (1974), Bartlett (1975) and Ripley (1981).

In the next section results are reported on the estimation of spatial interaction effects for petrol price data using (1), with $\beta_{ij} = \beta w_{ij}$, and (6). Both models use the linkage structure given in Haining (1983). In an earlier study, Robinson and Hebden (1973) tried, unsuccessfully, to show the importance of competitor price. They remained convinced, however, that competition, with those other sites closest along the road carrying the main traffic flow past the site, was an important factor in price determination. The same conjecture was made by Allvine and Patterson (1972, p. 103). Hence the results reported here are based on the rule that each site on a principal routeway is linked to its two nearest neighbours that are either side of it on the routeway. In addition, sites clustering together, for example, near major intersections, are also connected. Sites off the major routeways were of less interest and were either left unlinked if they were isolated, or joined together if they formed clusters, or linked to near neighbours on the major routeways if they were deemed to be sufficiently large and close enough.

For comparison, two other linkage rules were also tried. One rule paired all sites but with a weighting that decreased with increasing distance separation. Another rule linked each site to its six closest neighbours. Neither of these orderings produced any significant or theoretically consistent results. (For a discussion of rules for generating edge systems see, for example, Besag, 1975).

IV. Spatial Pricing

Data on posted prices for a gallon of four star (regular grade) petrol were collected for petrol retail outlets in S. W. Sheffield for seven months in 1982. From January to March prices were falling quite sharply (the minimum fell from 153.9 to 141.8), although in stages rather than continuously. From September to December prices were rising slowly but again in stages. All prices were recorded on a single day at times when prices appeared to be steady.

The pure competition model, (1), was fitted to the price data for different price ranges for each of the seven months. The main results are recorded in table 1. The 95% significance level for the χ^2 test is approximately 1.92. Using this yardstick the results for the period January to March show evidence of interaction effects at several levels. Since all the β estimates are positive (except one that is not significant) there is strong evidence of price clustering, that is, neighbouring outlets charging within the same price band. However, in January not only are significant interaction effects detected at the low end of the price level (< 156.3), there is also evidence of interaction effects well above the area minimum (158.7, 160.0). In February interaction effects are more closely associated with the minimum price band (148.7, 152.0) although this includes more than 50% of all retail outlets. By March, the period of lowest prices, all interaction effects are exclusively associated with the lowest end of the price range. Throughout this period prices at the low end of the scale became increasingly bunched. In January the price range of the 50% of cheapest outlets was 2 pence. By March the price range of the 50% of cheapest outlets was 0.2 pence. This seems to be consistent with the reduction in price variance hypothesised in part II and due to intensifying spatial competition. By contrast, during the months September to December interaction effects are not as evident except within certain price bands and not associated particularly with the lower price levels. This may reflect the weakening, during this period, of this type of very localized competition.

The supply and demand model includes intrasite effects as well as interaction effects but gives no explicit information on the sorts of variables that might be important. It was decided to estimate and test for site effects in terms of four variables: whether the outlet carried out car repairs and servicing, whether the outlet sold cars and whether the outlet was located on one of the principal radial routeways in the city. These were measured as dummy variables. In order to detect possible trends in pricing away from the city centre, each outlet's position was digitized and the two coordinates included in the regression. (Most of these variables have been found to be of importance in earlier studies; see, for example, Livingston and Levitt, 1959). The results are summarized in table 2, from which it is clear that site location was the only consistently significant factor and this irrespective of whether prices were rising or falling. In all cases prices were higher off the principal radial routeways.

TABLE 1.—INTERACTION EFFECTS WITHIN PRICE LEVELS: FITTING THE PURE COMPETITION MODEL

	Price Range		$\bar{\alpha}$	$\bar{\beta}$	$\ln(\bar{\alpha}, \bar{\beta})$	$\ln(\bar{\alpha}, \bar{\beta}) - \ln(\bar{\alpha}, \beta = 0)$
			Parameter Estimates			
January						
	153.9	156.3	−0.50	0.80	−42.52	4.06
	155.0	156.0	−0.87	0.40	−45.39	1.19
	156.9	157.0	−1.49	−0.35	−28.08	0.47
	157.8	158.0	−2.35	0.97	−23.64	1.11
	158.7	160.0	−2.15	1.52	−23.65	4.90
February						
	148.7	151.0	−0.39	0.72	−47.61	3.24
	148.7	152.0	−0.15	0.97	−34.56	5.31
	151.9	152.3	−1.12	0.40	−40.25	0.79
	149.5	150.0	−0.79	0.77	−40.20	5.66
March						
	140.9	142.1	−1.07	1.10	−42.65	10.00
	142.0	142.0[a]	−2.02	1.14	−33.51	2.90
	145.9	146.0	−2.03	0.11	−27.63	0.01
September						
	169.4	170.1	−0.91	0.58	−49.00	1.27
	170.4	170.6	−1.14	0.55	−47.73	2.36
	171.9	172.0	−2.32	1.81	−29.45	4.23
October						
	169.9	171.6	−1.76	0.50	−30.62	0.31
	172.7	173.1	−1.74	−0.33	−30.74	0.20
November						
	173.6	174.7	−1.29	0.57	−36.30	3.05
	175.4	175.6	−1.56	0.58	−33.67	1.79
	177.7	178.3	−1.28	0.15	−38.08	0.05
December						
	172.9	174.7	−1.24	0.55	−41.64	0.89
	172.9	174.2	−1.57	0.21	−33.94	0.05

[a]Analysis of sites charging exactly 142.0.

TABLE 2.—RESULTS OF REGRESSION ANALYSES OF SITE PRICES

	Independent Variables				
Month	Car Repair/ Servicing	Car Sales	Outlet Loc.	Trend Variables	R^2
January	95% (1.96p)	NS	95% (3.7p)	NS	19%
February	NS	NS	95% (4.3p)	NS	15%
March	NS	NS	95% (2.4p)	NS	8%
September	99% (0.48p)	NS	99% (0.81p)	NS	42%
October	NS	NS	99% (1.26p)	NS	9%
November	NS	NS	95% (0.87p)	NS	10%
December	NS	NS	NS	NS	0%

Note: NS denotes not significant. Figures in brackets denote partial regression coefficients.

The model given by (6) was then fitted to the price data with the mean assumed given by the results of the regression analysis. (This still leaves a constant, non-zero, mean which is extracted by taking the mean of the regression-adjusted price data and subtracting this value from each of the adjusted price values). The autoregressive model was then fitted to all the price data and, subsequently, to prices on two of the major radial routeways in Sheffield. (In the latter two cases all regression effects were ignored and the models fitted to the original price data with constant mean extracted.)

The results are given in table 3. The critical χ^2 value for a 90% significance level is 2.706. The evidence for this form of interaction effect is much weaker but the poor results for the individual routeways reflect in part the small sample sizes. It is noticeable, for example, that during the period when prices were falling all the autocorrelation parameter estimates for both routeways were positive which is consistent with the original model.

The results from tables 1 and 3 suggest the following interpretation. As prices fell in the first three months, interaction effects due to local ripple effects (table 1) became stronger but increasingly restricted just to the low end of the price range. This might reflect leadership by certain price aggressive retailers and the creation of two classes of outlets—those charging low prices and sensitive to local competition and those charging higher prices

TABLE 3.—ESTIMATES OF THE INTERACTION COEFFICIENT IN THE SUPPLY AND DEMAND MODEL

	Month	ρ	Likelihood Function H_0: $\rho = 0$	Likelihood Function H_a: $\rho \neq 0$	χ^2 Value	Significance
(a) All Stations						
	Jan	0.33	141.15	121.07	9.36	99%
	Feb	−0.21	212.90	212.79	0.03	NS
	Mar	0.03	338.66	338.09	0.11	NS
	Sept	−0.00	57.73	57.73	0.00	NS
	Oct	0.08	108.69	107.90	0.47	NS
	Nov	0.17	120.64	116.55	2.38	NS (≈ 90%)
	Dec	0.20	155.38	147.73	3.42	90%
(b) Infirmary Rd./Langsett Rd. (sites 30–47)[a]						
	Jan	0.29	23.02	21.41	1.01	NS
	Feb	0.14	13.89	13.64	0.25	NS
	Mar	0.52	32.93	21.56	5.92	95%
	Sept	0.40	5.42	4.73	1.89	NS
	Oct	−0.40	17.72	15.75	1.64	NS
	Nov	−0.10	20.87	20.68	0.12	NS
	Dec	0.37	11.29	9.82	1.94	NS
(c) City Rd./Ring Rd./Chesterfield Rd. (sites 26; 61–64; 66–70; 72–74; 77–79, 86)[a]						
	Jan	0.43	15.91	13.34	2.46	NS (≈ 90%)
	Feb	0.15	36.08	35.27	0.31	NS
	Mar	0.14	36.84	36.01	0.31	NS
	Sept	−0.38	5.64	4.97	1.77	NS
	Oct	0.07	16.76	16.69	0.05	NS
	Nov	−0.10	18.35	18.18	0.12	NS
	Dec	0.02	25.10	25.09	0.00	NS

[a] See Haining (1983).

and insensitive to local competition. To some extent this competition was organized along the principal routeways but was in any case quite localized (table 3).

As prices rose towards the end of the year the ripple effects of localized competition became weaker and what interaction effects there were, were not linked with any particular price level (table 1). However, the results of table 3, and the increasing χ^2 values and ρ estimates for part (a) in particular, suggest that spatial competition in the market became increasingly more organized. Localized price aggressiveness, characteristic of a price war, was being replaced by a more stable, area wide, pattern of competition (as had obtained in January). This might reflect the fact that the major brand retailers were once more in the ascendency, concerned more with price stability and market share than price aggressiveness.

V. Conclusion

This paper has introduced models for analysing different types of interaction effects in spatial pricing. In addition to the specific findings of this paper, however, there is a further interesting question concerning the dynamics of local retailer competition and its effect on spatial pricing and in particular the nature and severity of price wars. The evidence here suggests that the relative location of markets together with the dynamics of inter-market interaction may influence the spatial pattern of prices especially when prices are falling. These factors may also influence other aspects of the space-time macro-distribution of prices, such as price variance, even the severity of the price fall itself. Price wars evolve in part through local spatial competition. Ising models (which the pure competition model is an example of) possess phase transition properties which arise from strengthening inter-site interaction effects in the absence of an external field. A manifestation of this is that at any one time almost all sites, or large areal clusters of sites, possess the same binary value (Kindermann and Snell, 1980). This property suggests that some of the observed properties of the spatial and temporal distribution of prices (particularly during a price war) may be closely related to the nature and pattern of local retailer competition. This may be particularly evident when the external control of the market by parent companies is such as not to suppress this form of competition (as would be the case, for example, if the "sharing agreement" scheme was suspended).

REFERENCES

Allvine, F. C., and J. M. Patterson, *Competition Ltd.: The Marketing of Gasoline* (Bloomington: Indiana University Press, 1972).

Anselin, Luc, "A Note on Small Sample Properties of Estimators in a First Order Spatial Autoregressive Model," *Environment and Planning*, A, vol. 14 (1982), 1023–1030.

Bartlett, Maurice S., *The Statistical Analysis of Spatial Pattern* (London: Chapman and Hall, 1975).

Besag, Julian E., "Spatial Interaction and the Statistical Analysis of Lattice Systems," *Journal of the Royal Statistical Society*, B, vol. 36 (1974), 192–236.

______, "The Statistical Analysis of Non-Lattice Data," *The Statistician* 24 (3) (1975), 179–195.

Brook, D., "On the Distinction between the Conditional Probability and the Joint Probability Approaches in the Specification of Nearest Neighbour Systems," *Biometrika* 51 (1964), 481–483.

Haining, Robert P., "Estimating Spatial Interaction Models," *Environment and Planning* A vol. 10 (1978), 305–320.

______, "Modeling Intra-Urban Price Competition: An Example of Gasoline Retailing," *Journal of Regional Science* 23 (4) (1983), 517–528.

Kindermann, Ross, and J. Laurie Snell, "*Markov Random Fields and Their Applications* (Providence, RI: American Mathematical Society, 1980).

Livingston, S. M., and T. Levitt, "Competition and Retail Gasoline Prices," this REVIEW 41 (1959), 119–132.

Ord, J. Keith, "Estimation Methods for Models of Spatial Interaction," *Journal of the American Statistical Association* 70 (1975), 120–126.

Preston, Christopher J., *Gibbs States on Countable Sets* (Cambridge: Cambridge University Press, 1974).

Ripley, Brian D., *Spatial Statistics* (New York: John Wiley, 1981).

Robinson, R. V. F., and J. Hebden, "The Influence of Price and Trading Stamps on Retail Petrol Sales," *Journal of Industrial Economics* 22 (1973), 37–50.

Schendel, D. E., and P. Balestra, "Retail Behavior and Gasoline Price Wars," *Applied Economics* (1969), 89–101.

Whittle, Peter, "On Stationary Processes in the Plane," *Biometrika* 41 (1954), 434–449.