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Robert Haining

The Spatial Structure of Competition and Equilibrium Price Dispersion

1. INTRODUCTION

In an earlier paper (Haining 1982), we discussed the appropriateness of interaction models for certain types of geographical diffusion processes. In this paper we consider in more detail the case of spatial price variation arising from a process of intermarket competition. There are two objectives. The first is to consider properties of interaction models in relation to certain reported empirical findings on spatial price variation. These empirical findings, which will be briefly summarized in the next section, might appear at first sight to be unrelated. However, as the discussion here seeks to demonstrate, these findings are compatible with the properties of interaction models. We establish the nature of these links by reviewing a number of significant theoretical properties of interaction models which also serve to extend the arguments in the earlier paper.

The second aim is to focus attention on the conditional approach to the specification of spatial models. Drawing on analogies with the specification of temporal (process) models, it has become rather common to specify spatial models using the simultaneous approach. Because of the mathematical equivalence between conditional and simultaneous models in time-series analysis, the distinction is not, there, an issue. But such equivalence is not true for spatial models. (For a fuller discussion of the mathematical and statistical distinctions, see Haining (1979).) In the pricing problem there are some theoretical issues relating to the process of spatial price competition that make a conditional specification attractive, so that this problem also provides the opportunity for developing this approach to spatial modeling.

A simultaneous specification specifies the properties of the regional price set in terms of a joint probability distribution. The model for intraurban pricing in Haining (1983b) is of this type. A conditional specification on the other hand starts by specifying individual market price probabilities conditional on given prices at a set of neighboring markets. The joint probability distribution for the regional price set is then derived from the set of individual conditional probabilities. Some of the general issues that distinguish the two specifications appear in the discussion to Besag (1974) (see the contribution by Whittle). There it is noted that the choice of

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specification will depend crucially on the nature of the assumed underlying space-time process. Appropriate space-time competition processes for the conditional models discussed here will be defined in section 4. These may be contrasted with the process for the simultaneous models given in Haining (1983b, sec. 2). It is worth emphasizing, however, that whereas the latter make assumptions about market supply and demand conditions, the former disregard these and emphasize instead the ripple effects of local price comparison. But both appear to generate valid models for different aspects of spatial price variation arising from intense short-term price competition (Haining 1984).

2. DESCRIPTION OF THE PRICE PROBLEM

We consider the geographical distribution of markets as a factor in the study of regional price variation and short-term price dynamics. In particular, we consider a situation in which there are many sellers, and where each is close enough to some subset of the remainder so as to lead to a sense of "local price awareness." In such cases, it is argued, interseller price interaction may be an important element in price setting at least over the short term. It is this geographically localized competition and its regional effects that we investigate.

A stochastic model of prices is developed for a single homogeneous good that can be purchased from each of a set of sellers located within a geographical region. Because of proximity, interaction may take place between sellers in determining price, and this interaction becomes a central feature of the model. So, for each seller the price of his good depends upon his environment, that is, the price levels charged by neighboring sellers. Such interaction might occur, for example, because market areas overlap over subareas of the region so that each seller is competing, with a limited number of other sellers, for specific subsets of purchasers. Alternatively, groups of buyers and sellers may only have market price data for limited geographical areas so that the decision making of actors is based on incomplete, localized information. We do not concentrate on the supply and demand characteristics of sellers and purchasers, and these do not enter explicitly into market price setting.

The assumed neighborhood effect underlies the choice of a microeconomic specification for the theory. Price probabilities are defined for each seller in turn using local conditional price probabilities. These are then used to derive joint probability distributions that describe the macroeconomic or regional price system at equilibrium. It turns out that such a system has a number of interesting properties. Of particular interest are the conditions under which the microeconomic probabilities do not uniquely determine the macroeconomic distribution so that subregional or micro-market-level price adjustments or behavioral changes on the part of sellers or buyers become an important source of uncertainty in the regional pricing system. This condition is known as a phase transition, and we explore the implications of this property for price variation and short-term price dynamics. We focus on three attributes of the price-interaction process in relation to the presence or absence of phase transitions. These are (i) the sensitivity of each seller to the reference group comprising his local environment; (ii) the configuration and complexity of the local environment monitored by each seller for purposes of determining his own price; (iii) the influence of external pricing agents (such as parent companies).

The empirical motivation for this paper and the reason for the interest in phase-transition properties is contained in a number of earlier papers that have analyzed data from a gasoline price war. We draw on two sets of findings from these papers to justify the theoretical arguments developed here: (i) spatial interaction effects are evident and the "pure competition" model (see sec. 4) can be used to describe attributes of the spatial structure of equilibrium price distributions (Haining 1983b, 1984); (ii) static regional price variability varies through time and appears to decline with increasing intermarket competition (Haining 1983a). The discussion in this paper develops and provides a theoretical link between both these findings. At this stage, however, we anticipate two important findings. First, under certain conditions a process of increasing intermarket price competition is sufficient to account for changes in spatial price variability. Second, one of the critical conditions that influences whether increasing market interaction results in declining price variability is the particular spatial structure of intermarket competition, that is, the structure of the reference group adopted by each seller in fixing his own price. We are not arguing here that interaction effects alone were responsible for the observed changes in price variability, merely that declining price variability as an observed property of the data is compatible (as we shall show) with that specification of the process so that it is not necessary to look further afield or specify other processes at work. (It should be noted, e.g., that declining price variation is also explicable within the supply and demand model in Haining (1983b). Changes in price variability in that model are brought about by changes in the slope of the demand function relative to the supply function.)

In the next section we discuss the derivation of the regional price distribution from the local conditional probabilities that specify individual seller price interactions and identify general properties. In section 4 we specify two space-time Markov processes for competitive spatial pricing. These lead to equilibrium distributions of the type described in section 3. Properties of one of these models—the pure competition model—are examined in detail in section 5 with particular emphasis on the structure of the reference group attached to each seller. This is particularly important for it is the contention here that the spatial organization of sellers has important implications for regional price behavior (at least over the short term and under conditions of intense competition) and that explanations of interregional differences in price distributions may need to accommodate the spatial organization of intermarket competition in the different regions. The theory presented here may also be of importance for the study of other competition effects, where elements also interact locally but in more abstract spaces (Follmer 1974; Curry 1982).

3. REGIONAL PRICE DISTRIBUTIONS FROM LOCAL CONDITIONAL PROBABILITIES

Let V denote a (countable) set of vertices and let E denote the set of edges between the vertices. Then $G = \{V, E\}$ denotes an arbitrary graph. The vertices of G, of which there are n, denote the region's sellers and if $i \in V$ then $N(i) = \{v\colon v \in V$ is a neighbor of $i\}$. Two sellers are said to be neighbors if an edge $e \in E$ joins them. We consider $N(i)$ to be the reference group of seller i, the set of sellers that influence price setting at i. We assume that the graph is connected so that a path exists between all $i \neq j \in V$, and that $i \notin N(i)$.

Assigned to each $v \in V$ is a value from a countable set S. Such an assignment will be called a regional price configuration $\mathbf{x} = (x_1, \ldots, x_n)$, where each $x_i \in S$. We now introduce probability theory by defining X_i as the (price) random variable associated with site i so that x_i is a particular value of X_i. X_i takes values in S. We denote the set of all possible price configurations as Ω. Now, $\pi(x_i|\{x_j\}_{j\neq i})$ is the probability that seller i charges the price x_i given his price environment. (We shall assume that π is translation invariant so that all sellers react to their environments in the same way.) The collection of conditional probabilities, $\{\pi(x_i|\{x_j\}_{j\neq i})\}_i$ is denoted Π and can be interpreted as the microeconomic characterization of the regional economy. As Follmer has noted, this use of conditional rather than joint probabilities for model specification means that "this conception of the economy is indeed a microeconomic one, in the sense that no prior information on aggregates is required" (1974, p. 54).

The macroeconomic or regional price distribution is defined by any probability measure μ (defined on Ω) which is compatible with Π in the sense that

$$\mu\left(X_i = x_i | \{x_j\}_{j \neq i}\right) = \pi\left(x_i | \{x_j\}_{j \neq i}\right) \ \mu\text{-almost surely} \tag{1}$$

for all $i \in V$ and $x_i \in S$ (Spitzer 1971). The conditional probabilities are said to be consistent if at least one probability measure μ exists that satisfies (1). Next, following Preston (1974), we note that if

$$\pi\left(x_i | \{x_j\}_{j \neq i}\right) = \pi\left(x_i | \{x_j\}_{j \in N(i)}\right),$$

then the local characterization satisfies the Markov property and the regional price distribution is a Markov random field. We shall make use of this fact to identify a number of properties of such price distributions.

Now assume that each seller determines his price from a combination of both intrasite influences (on-site decision making or the influence of external agents that affect each individual seller independently) and intersite influences (pairwise price comparisons with neighboring or reference-group sellers). In this case conditional probabilities satisfying the Markov property are constructed as follows (Averintzev 1970). Define a function (often called a potential), $J(\mathbf{x})$, where

$$J(\mathbf{x}) = \sum_{i \in V} R_I(x_i, \alpha) + \sum_{\substack{i \in V \\ j > i}} \sum_{j \in N(i)} R_J(x_i, x_j, \beta),$$

where $R_J(x_i, x_j, \beta) > 0$ if $i \neq j$ and $j \in N(i)$. Then if the conditional probability distribution is from the exponential family and if we set $\mathbf{x}_i = (x_1, \ldots, x_{i-1}, 0, x_{i+1}, \ldots, x_n)$,

$$\begin{aligned} \pi\left(x_i | \{x_j\}_{j \in N(i)}\right) &= \left[Z^*(\cdot)\right]^{-1} \exp\left[J(\mathbf{x}) - J(\mathbf{x}_i)\right] \\ &= \left[Z^*(\cdot)\right]^{-1} \exp\left[R_I(x_i, \alpha) + \sum_{\substack{j \in N(i) \\ j > i}} R_J(x_i, x_j, \beta)\right], \end{aligned} \tag{2}$$

where Z^* is a normalizing constant. The joint distribution is given by

$$\mu(\mathbf{x}) = \left[Z(\cdot)\right]^{-1} \exp\left[J(\mathbf{x})\right]. \tag{3}$$

Finally,

$$Z(\cdot) = \sum_{\mathbf{x} \in \Omega} \exp\left[J(\mathbf{x})\right]$$

and is called the partition function. The form (2) is necessary and sufficient for the existence of at least one regional price probability distribution μ (Averintzev 1970).

(The structure of these models is considered in much more detail in Haining (1982).)

Now the local characterization of the economy is said to admit a phase transition if there is more than one regional distribution μ that is compatible with Π. If V is finite, the collection of conditional probabilities (Π) uniquely determines the regional price distribution (Spitzer 1971). There is also no phase transition if either

(i) $\max|R(x_j, x_i, \beta)|$ is small for all i and j, or
(ii) the structure of interaction is one dimensional (such as a linear system, where sellers are located for example along a road, and only nearest neighbor prices are consulted, i.e., $N(i) = i \pm 1$) (Spitzer 1971).

Two points need emphasizing from these results. First, where intermarket interaction is sufficiently strong and the configuration of the reference groups sufficiently "complex," there may be several distinctive regional price distributions that may be generated by a given system of local intermarket relationships. Second, although the previous results hold only for infinite graphs, finite forms of phase transition graphs possess measures, μ, with several local maxima and with the regional price distribution displaying the polarized price distributions characteristic of phase transitions for fixed levels of intermarket interaction (Kindermann and Snell 1980, chap. 3).

4. TWO SPACE-TIME MODELS FOR SPATIAL PRICE COMPETITION

In this section we develop two space-time Markov processes. Let $G_i(y, \mathbf{x})\,dt$ denote the probability that site i, $i \in V$, charges price y at time $t + dt$ given that the price configuration at time t was $\mathbf{x}(x_i \neq y)$. We assume that prices cannot change by more than one unit in any time interval so that we can define the following transition rates (Bartlett 1975, p. 43):

$$G_i(y, \mathbf{x})\,dt = \begin{cases} (m - x_i)\gamma(\mathbf{x})\,dt; & y = x_i + 1 \\ x_i \delta(\mathbf{x})\,dt; & y = x_i - 1, \end{cases}$$

where $m + 1$ denotes the order of the set S and is the number of possible prices that can be charged at any site. Since probabilities must sum to one, the remainder is the probability of no change in any time interval. Assuming the process is reversible and irreducible, then (Preston 1974, prop. 2.2)

$$\frac{\mu(\mathbf{x})}{\mu(\mathbf{y}_i)} = \frac{G_i(x, \mathbf{y}_i)}{G_i(y, \mathbf{x})} = \frac{(m - x_i + 1)}{x_i} \frac{\gamma(\mathbf{y}_i)}{\delta(\mathbf{x})},$$

where $\mathbf{y}_i = (x_1, \ldots, x_{i-1}, x_i - 1, x_{i+1}, \ldots, x_n)$, and x and y denote values in position i. Now assume

$$\gamma(\mathbf{x}) = \exp\left[\alpha' + \sum_{j \in N(i)} \beta' x_j\right]$$

$$\delta(\mathbf{x}) = \exp\left[\alpha'' + \sum_{j \in N(i)} \beta'' x_j\right],$$

then

$$\frac{\mu(\mathbf{x})}{\mu(\mathbf{y}_i)} = \frac{m - x_i + 1}{x_i}\exp\left[\alpha + \sum_{j\in N(i)} \beta x_j\right],$$

where $\alpha = (\alpha' - \alpha'')$ and $\beta = (\beta' - \beta'')$. It follows that the conditional distribution at site i is binomial with parameters m and $\beta(\mathbf{x})/(1 + \beta(\mathbf{x}))$, where $\beta(\mathbf{x}) = \gamma(\mathbf{x})/\delta(\mathbf{x})$, and $\mu(\mathbf{x})$ is given by (3) with

$$R_j(x_i, \alpha) = x_i(\alpha' - \alpha'') \tag{4}$$

$$R_j(x_i, x_j, \beta) = x_i x_j(\beta' - \beta''). \tag{5}$$

An important special case is given when the random variables $\{X_i\}$ are binary valued. Thus the price of the good is either "high" or "low," or, alternatively, the price range is partitioned so that $x_i \in [S_1, S_2]$ or $x_i \notin [S_1, S_2]$, where $S_1, S_2 \in S$. The conditional distribution is now logistic with parameters as given above but with $m = 1$. This special case has been referred to as the pure competition prices model (Haining 1984).

The parameter β describes the sensitivity of any seller to different types of price shift in the set of reference markets. On the other hand α measures individual market sensitivity to site factors or the actions of external agents. Much of what follows in the next section is concerned with identifying the properties of μ for variations in α, β and $\{N(i)\}_i$, but we conclude this section with general results on the nature of the distributions and phase transitions in the pure competition model.

The distribution μ is homogeneous if it is translation invariant. Let Φ denote the set of homogeneous distributions that are consistent with the local characterization Π. The extreme points of Φ are called "pure phases" (cf. Follmer 1974). In the pure competition model there are at most two pure phases and we shall call these μ_+ and μ_- corresponding to the situations where sites on the boundary of the graph are all fixed in one state or all fixed in the other state (Kindermann and Snell 1980, chap. 3).[1] If $\mu_+(\mathbf{x}) \neq \mu_-(\mathbf{x})$ a phase transition is said to occur, which means that different limiting probabilities are assigned to identical price configurations on G. If $\alpha \neq 0$ or if $\alpha = 0$ and $\beta \leqslant \beta_{crit}$ then $\mu_+(\mathbf{x}) = \mu_-(\mathbf{x})$, whereas if $\alpha = 0$ and $> \beta_{crit}$ in two dimensions and on certain graphs a phase transition occurs, that is, $\mu_+(\mathbf{x}) \neq \mu_-(\mathbf{x})$ (Spitzer 1971). All homogeneous distributions with given local conditional probabilities are convex combinations of the two pure phases such that

$$\mu(\mathbf{x}) = t\mu_+(\mathbf{x}) + (1 - t)\mu_-(\mathbf{x}); \qquad 0 \leqslant t \leqslant 1$$

and in the case where boundary site values are not fixed (free boundaries) $t = 0.5$.

If $\beta = 0$ the states at each site are independent and classical limit laws apply to $M_n = \Sigma_{i=1}^n x_i$ ($x_i = \pm 1$) (Kindermann and Snell 1980). The law of large numbers holds for M_n (M_n approaches 0 as $n \to \infty$ with probability 1) and the central limit theorem holds for $M_n/\sigma(M_n)$, where $\sigma(M_n)$ denotes the standard deviation of M_n. If $\beta \neq 0$ but there is no phase transition ($\beta \leqslant \beta_{crit}$), the distribution M_n/n satisfies

[1] The boundary of any graph can be thought of as a set of sites that enclose the graph G. So, for a rectangular lattice with sites of the form (i, j), $1 \leqslant i \leqslant n$ and $1 \leqslant j \leqslant n$ and with four nearest neighbor edges for each site, the boundary sites are the additional points $(0, j)$, $(n + 1, j)$ $1 \leqslant j \leqslant n$ and $(i, 0)$, $(i, n + 1)$ for $1 \leqslant i \leqslant n$. Boundary definitions are discussed in more detail in Kindermann and Snell (1980, chap. 3), and the appropriate specification for boundary sites in the prices problem is considered again in section 5.

a law of large numbers and a central limit theorem but $\sigma(M_n)$ increases as β increases so that the normalization becomes increasingly severe. Where a phase transition does occur for $\beta > \beta_{crit}$,

$$\lim_{n\to\infty} \frac{M_n}{n} = W,$$

where W is a random variable and (in the case of free boundaries)

$$W = \begin{cases} \left(1 - \left((\sinh\beta)^4\right)^{-1}\right)^{\frac{1}{8}} & \text{with prob } 0.5 \\ -\left(1 - \left((\sinh\beta)^4\right)^{-1}\right)^{\frac{1}{8}} & \text{with prob } 0.5. \end{cases}$$

The distribution M_n/n approaches a mixture of the pure phases. As β increases and passes through β_{crit} the distribution changes from a normal distribution with increasing variance ($\beta < \beta_{crit}$) as noted above to a bimodal distribution with decreasing variance as values are all either $+1$ or -1 ($\beta > \beta_{crit}$). Each of the pure phases individually satisfies the central limit theorem and the law of large numbers, and these results are closely approximated even for relatively small lattice configurations (Kindermann and Snell 1980).

These results indicate how subregional price shifts may produce extensive price adjustment as interaction effects are diffused through the region. More importantly they indicate how a stable price system may undergo short-term shifts if sellers become more sensitive to neighborhood price levels, or if they change the nature of their reference set $N(i)$ (see sec. 5), or if the activities of external price agents diminishes.

5. SPATIAL STRUCTURE AND PROPERTIES OF THE PURE COMPETITION MODEL

We consider phase transitions in the pure competition model defined in section 4 in more detail and in particular the role of the graph structure as specified by $\{N(i)\}$.

Let Φ denote the set of all phases compatible with π, then $|\Phi| = 1$ (there are no phase transitions) if

$$R_j(x_i, \alpha) + \sum_{\substack{j\in N(i)\\ j>i}} |R_j(x_i, x_j, \beta)| < \log(0.5)$$

for all i and j (Preston 1974, theorem 6.2). This shows that phase transitions will not occur if interactions are weak (either due to a small value of β or a lack of interacting neighbors) or if $R_j(x_i, \alpha)$ is large and negative, corresponding either to an element of autonomous pricing or to strong external influences on individual sites—perhaps due in the present instance to the pricing policies of external agents.

We now establish that the phase transition properties of a graph depend critically on the number of ways in which influence can be transmitted through the graph and hence to the structure of competition between sellers. In the following, the term *phase transition graph* is used to denote a graph for which a phase transition occurs for $R_j(x_i, x_j, \beta) > 0$ and all $\beta > \beta_{crit}$, where β_{crit} is positive, finite valued and its value depends on the particular edge structure of the graph.

Let η be a finite subset of E and let $\langle z_i, z_{i+1}\rangle$ denote the edge linking sites i and $i+1$. Define $I(v,\eta) \subset V$:

$$I(v,\eta) = \{v\} U \{w \in V\text{: there exists a path } v = z_1, \ldots, w = z_g \text{ in } V \text{ such that } \langle z_i, z_{i+1}\rangle \in \eta, i = 1, \ldots, g-1\}.$$

If $\wedge$ is a finite subset of V we shall say that η *blocks* v in $\wedge$ if $I(v,\eta) \subset \wedge$. If η blocks v in $\wedge$ but no proper subset of η does, then η is called a *border* round v in $\wedge$ and $|\eta|$ is called the *length* of the border. Let $r_k(v)$ denote the *number* of borders around v of length k (Preston 1974).

THEOREM 1 (Preston 1974, prop. 10.3). *Let $G = \{V, E\}$ and suppose there exists $v \in V$ and $\overline{W} > 0$ such that $r_k(v) \leqslant \exp(\overline{W}k)$ for all $k \geqslant 1$. Then G is a phase transition graph.*

This result can be used to show that infinite rectangular, triangular, and hexagonal lattices, for example, with nearest neighbor edge systems are phase transition graphs. The proof for the rectangular lattice is given by Preston (1974, prop. 10.5) and depends on showing that the number of borders around any $v \in V$ of length k is $r_k(v) \leqslant k^2 3^k$. Similar proofs can be constructed for the other two regular lattice structures in two dimensions. The hexagonal lattice and its dual, the triangular lattice, are of particular interest since they arise in the theory of central place systems (Losch 1967). The value of β_{crit} for these regular, two-dimensional lattices has in fact been frequently identified (see Biggs 1977; Kindermann and Snell 1980, p. 57).

The tree graph may be of particular interest in the study of hierarchial spatial systems and those pricing systems where "leader/follower" relationships exist between sellers. We say that a graph G is a *tree* if it is connected and contains no circuits so that any two vertices on the graph are joined by a unique path. For $h \geqslant 2$ let T^h denote the tree in which every $v \in V$ has exactly h neighbors. A linear system with $N(i) = i \pm 1$ is a T^2 graph and is the simplest such system.

THEOREM 2 (Preston 1974, prop. 10.9). *A T^h graph, G, is a phase transition graph iff the equation*

$$\left(\frac{1+x_1}{b+x_1}\right)^{h-1} = ax_1$$

has more than one nonnegative solution in x_1, where

$$a = \exp(R_j(x_i, \alpha))$$

$$b = \exp(R_j(x_i, x_j, \beta)).$$

Since the equation has only one nonnegative solution for $h = 2$, linear (one-dimensional) nearest neighbor systems are not phase transition graphs. For $h = 3$, T^h is a phase transition graph providing $b > (h/(h-2))^3$. Thus ordered central place systems that preserve dominance relationships on the lattice ($h = 3$, 4, or 7) are phase transition graphs (Christaller 1966). Note that the dominance structure of central place lattices is not nearest neighbor but rather determined by size characteristics associated with the elements of V, independent of S.

Theorem 2 seems to imply that graphs with "too few" edges will not be phase transition graphs while theorem 1 seems to imply that graphs with "too many" will not be either. Biggs (1977, p. 15), using different methods that depend on identifying singularities in the partition function Z of the joint distribution μ, has shown that the "complete graph," where $N(i) = \{v \in V: v \neq i\}$, is not a phase transition graph.

Caution is needed in relating these results to finite graphs as Kindermann and Snell note (with reference to binary valued nearest neighbor interaction models on lattices with n sites). "For any finite lattice in either one or two dimensions we would expect that as β tends to infinity, the distribution would have two maxima, one at each of the values corresponding to a configuration with all (values) the same. The fundamental difference between one and two dimensions is that for any fixed β, in one dimension the distribution will have a single maximum for large enough n but in two dimensions the double maximum will persist for arbitrarily large boxes (groups of sites) when $\beta > \beta_{crit}$" (1980, p. 42).

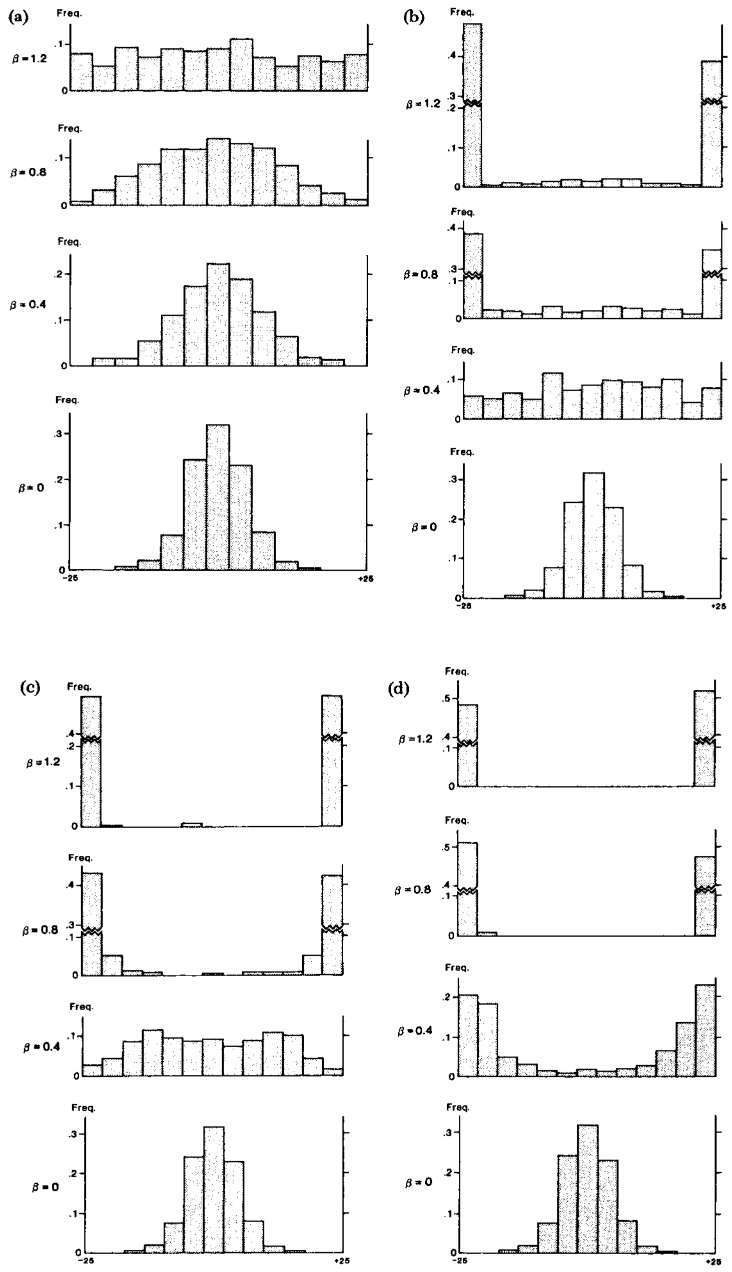
In the pure competition model ($X_i = \pm 1$) this means that the quantity M_n will have a bimodal frequency distribution on certain lattices (for $\alpha = 0$). Figure 1 shows the frequency distributions of M_n (the proportion of 200 simulations with particular M_n values) after 2,000 iterations for different, $n = 25$, lattice arrangements and $\beta \leqslant 1.2$. (The range of β values chosen reflects the range of empirical estimates of β (Haining 1983a). The simulation procedure is that defined in Kindermann and Snell (1980, pp. 53–54), starting with an initial random configuration of values, ± 1.) In the case of the one-dimensional systems (Figs. 1a, 1b) bimodality is evident for large β values in the simulation set for the second-order system ($N(i) = i \pm 1$, $i \pm 2$) but has not yet appeared for the first-order system. Phase transitions cannot occur in one dimension when interaction is only allowed between neighbors a finite distance away but can occur when interaction between all neighbors is allowed. Bimodality would disappear then for larger n in the case of the second-order system, for these values of β.

First-order rectangular and hexagonal lattices (Figs. 1c and 1d) have bimodal distributions for sufficiently large values of β as anticipated. The bimodality will persist as n is increased. The final lattice (Fig. 1e) uses a subset of the linkage map that was used in the analysis of gasoline prices and reported as Figure 1 in Haining (1983b). The subset is sites 30–48, 57–60, and 85 thus taking in one of the major areas that showed marked price uniformity. There is some evidence of a higher than average proportion of simulations with either all $+1$ or all -1 site values for large β values in the simulation set.

Finally the question of boundary values may need further consideration and from the results of section 4 may have important implications for the "direction" of the process. In the present context boundary sites presumably do not need to be sites on the perimeter of the graph. Two different types of boundary sites can be envisaged: first a small number of "leader" markets that start the process by dropping their prices. These values are held fixed and will presumably have the effect of forcing the other markets to follow suit. Alternatively there could be a small number of "key" markets that may not start the competition process but once they have dropped their prices keep them fixed.

6. CONCLUSIONS

Spatial price competition has been defined in terms of local conditional probabilities and these used to generate regional price distributions. Such an approach would seem appropriate to a situation in which groups of sellers compete for customers from common, overlapping, local market areas.



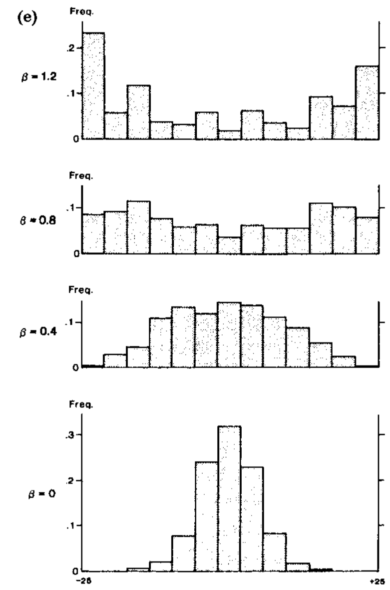


FIG. 1. Frequency Distributions for the Pure Competition Model ($X_i = \pm 1$) on a Lattice with $n = 25$, Increasing Levels of Intersite Sensitivity, and $\alpha = 0$. (Proportion of 200 simulations with particular values of M_n after 2,000 iterations of the process.) (a) First-order linear system. (b) Second-order linear system. (c) First-order rectangular system. (d) First-order hexagonal system. (e) Subset of linkage map from Haining (1983b).

Depending on the spatial structure of intermarket competition such a process has been shown to have inherent instabilities that might be triggered off by price shifts initiated in particular subregions, or behavioral changes in the market. (Instabilities that appear to share common, if uncharted, ground with catastrophe theory (Stewart 1981, p. 293).) The space-time dynamics of short-term price instability associated, for example, with regional price wars may then depend critically on the nature of the reference set adopted by each seller, the sensitivity of sellers to local patterns of pricing, and the actions of external agents. Empirical evidence on regional price distributions indicates some support for this conceptualization and suggests that attempts to model regional price variation should accommodate information on the spatial structure of intermarket competition.

Returning, finally, to the other main example discussed in Haining (1982), there seem to be at least superficial similarities between the process outlined here for the prices problem and the general stochastic epidemic in two dimensions considered by Bailey (1967), both in terms of the specification of the process and in terms of some important properties. In the epidemic model the probability of an individual becoming infected by an infectious neighbor, during an interval of time, is p. An individual is only infectious for one time period and is then removed. The interesting point of comparison however is that for sufficiently large values of p infection from one infectious individual ultimately spreads to infect all individuals on the lattice, whereas for small values of p no such spread occurs. Like the prices model, this spatial process also appears to show instability and the same tendency to generate global or long-range effects from local perturbations for suitable values of the interaction parameter. No doubt other examples exist.

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Research Notes and Comments

Humanism, Naturalism, and Geographical Thought
by J. Nicholas Entrikin

Geographers who have sought to characterize the distinctive qualities of a humanistic geography have relied heavily upon contrasts with the arguments of positivistic spatial scientists. This latter approach is described as being naturalistic (based upon the ideal of methodological unity in the natural and human sciences), and thus as seeking causal explanations to geographical questions. Humanistic geography is described in opposition to positivistic spatial science as being antinaturalistic and as seeking understanding rather than causal explanation (e.g., Buttimer 1976; Gregory 1979; Hay 1979; Ley 1977, 1981; Ley and Samuels 1978). This contemporary contrast has been applied to past traditions to create a picture of twentieth-century geographic thought as being divided between naturalist and antinaturalist perspectives.

The difficulty associated with applying this logical distinction to the history of geographical thought is best illustrated in recent interpretations of the idiographic study of landscape and region. For example, the Hettner-Hartshorne tradition has been interpreted as a forerunner of positivistic spatial science, while the Vidalian and Sauerian traditions have been judged as being part of the heritage of a contemporary humanism (Buttimer 1978; Capel 1981, pp. 446–47; Guelke 1977b, 1978; Ley 1977, 1981; Ley and Samuels 1978; Rose 1981). Associated with this alignment of the various schools of geographical thought has been a concomitant reinterpretation of the philosophical bases of their substantive concerns. For example, region and landscape have been described as concepts best studied phenomenologically or through other intuitional modes (Guelke 1977a; Ley 1981; Mugerauer 1981; Relph 1976). One of the consequences of this reinterpretation of the past in terms compatible with humanistic geography is the neglect of the influential role of natural science in these earlier conceptions of human geography. For geographers of the early twentieth century did not tend to identify positivism with naturalism as readily as do humanistic geographers.

Two individuals who have been described most often as the primary ancestors of geographical humanism, Carl Sauer and Paul Vidal de la Blache, were both concerned with the study of the interrelationship of people and their natural environment, and this interest in both physical and human phenomena precluded any rigidly antinaturalist position. Both appear to have been strongly antipositivist, but not antinaturalist in their conceptions of human geography. For example, although Sauer noted often that the human sciences could not discover laws similar

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