We start with a model of co-repression taken from Buchler *et al*. In particular, we allow for a degree of repressor interaction captured by ω. There is no explicit term for leaky expression, since repression was found to completely shut off the strongest promoters. Furthermore, we assume that the induction is perfect, so the effect of a repressor can be removed completely. This function has two variables which range from 0 (full induced) to 1 (fully repressed) for each repressor. The three parameters account for the strength of repression by each, along with the interaction term ω, which is 1 when the repressors act independently, less than 1 when they interact competively, 0 when they are completely exclusive, and greater than one when they act competitively.

```
P[R1_, R2_, c1_, c2_, ω_] :=
                              A
   ─────────────────────────────────────────────
    1 + c1 * R1 + c2 * R2 + ω * c1 * c2 * R1 * R2
r[c1_, c2_, ω_] :=
 Log[10, P[0, 0, c1, c2, ω] / P[1, 1, c1, c2, ω]]
α[c1_, c2_, ω_] := Log[10, P[0, 0, c1, c2, ω] /
    P[1, 0, c1, c2, ω]] / r[c1, c2, ω]
β[c1_, c2_, ω_] := Log[10, P[0, 0, c1, c2, ω] /
    P[0, 1, c1, c2, ω]] / r[c1, c2, ω]
                    α[c1, c2, ω] + β[c1, c2, ω]
l[c1_, c2_, ω_] := ───────────────────────────
                                2
a[c1_, c2_, ω_] := α[c1, c2, ω] - β[c1, c2, ω]
```

Since there are three microscopic (c1, c2, ω) and three logical (r,a,l) parameters, we can solve for the microscopic parameters in terms of the logic parameters explicitly. First we solve for the repression coefficients c1 and c2. Note:

```
Simplify[a[c1, c2, ω] + 2 * l[c1, c2, ω]]
```

$$\frac{2\,\mathrm{Log}[1 + c1]}{\mathrm{Log}[1 + c1 + c2 + c1\,c2\,\omega]}$$

Where the denominator is just the regulatory range r. Therefore,

```
Solve[2 Log[10, 1 + c1]/r == a + 2 * l, c1]
```

$\{\{c1 \to -1 + 10^{\frac{a\,r}{2} + l\,r}\}\}$

```
c1[r_, a_, l_] := -1 + 10^(a r/2 + l r)
```

Similarly for c2:

```
Simplify[-a[c1, c2, ω] + 2 * l[c1, c2, ω]]
```

$\frac{2\,\mathrm{Log}[1 + c2]}{\mathrm{Log}[1 + c1 + c2 + c1\,c2\,\omega]}$

```
Simplify[Solve[2 Log[10, 1 + c2]/r == -a + 2 * l, c2]]
```

$\{\{c2 \to -1 + 10^{-\frac{a\,r}{2} + l\,r}\}\}$

```
c2[r_, a_, l_] := -1 + 10^(-a r/2 + l r)
```

To compute ω in terms of (r, a, l) we use the expression for r in terms of the microscopic parameters and the solution for the repression constants.

```
Solve[r[c1, c2, ω] == r, ω]
```

$\{\{\omega \to \frac{-1 + 10^{r} - c1 - c2}{c1\,c2}\}\}$

```
Solve[r[c1[r, a, l], c2[r, a, l], ω] == r, ω]
```

$$\left\{\left\{\omega \to \frac{-10^{\frac{a\,r}{2}} + 10^{l\,r} - 10^{r+\frac{a\,r}{2}} + 10^{a\,r+l\,r}}{-10^{\frac{a\,r}{2}} + 10^{l\,r} + 10^{a\,r+l\,r} - 10^{\frac{a\,r}{2}+2\,l\,r}}\right\}\right\}$$

$$\omega[r\_,\, a\_,\, l\_] := \frac{10^{\frac{a\,r}{2}} + 10^{\frac{1}{2}\,(2+a)\,r} - 10^{l\,r} - 10^{(a+l)\,r}}{10^{\frac{a\,r}{2}} - 10^{l\,r} - 10^{(a+l)\,r} + 10^{\frac{1}{2}\,(a+4\,l)\,r}}$$