



# Analisi e Visualizzazione delle Reti Complesse

**NS16 - Cascading behaviors in networks**

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# Agenda

- Diffusion in Networks
- Modeling Diffusion through a Network
- Cascades and Clusters
- Diffusion, Thresholds, and the Role of Weak Ties
- Extensions of the Basic Cascade Model
- Knowledge, Thresholds, and Collective Action
- The Cascade Capacity

# The diffusion of innovations

- Let's consider how new behaviors, practices, opinions, conventions, and technologies spread from person to person through a social network, as people influence their friends to adopt new ideas
- diffusion of innovation: long list of classic studies in the middle of the 20th century
  - focus on informational and network effects
- **direct benefit effects** to understand how technologies spread: incentives to adopt telephone, fax, emails based on friends that already adopted those technologies
- we need to take into account **network effects on a local network level**
- Objective of this lecture: **formulating a model for the spread of an innovation through a social network**

## A diffusion of a new behavior

- Assumption: individuals make decisions based on the choices of their neighbors
  - focus on links
- In this lecture, let's focus on **direct-benefit effects** instead of informational effects
- Natural model introduced by Stephen Morris in 2000
- Reading material:
  - [Stephen Morris. Contagion. Review of Economic Studies, 67:57–78, 2000.](#)



## Examples



VHS

vs



Betamax



vs



# A networked coordination game

- It is natural to use a **coordination game**
  - each node has a choice between two possible behaviors, A and B
  - players have an incentive to adopt the same behavior

		<i>w</i>
<i>v</i>	<i>A</i>	<i>A</i>
	<i>B</i>	<i>B</i>

		<i>w</i>
<i>v</i>	<i>A</i>	<i>A</i>
	<i>B</i>	<i>B</i>

		<i>w</i>
<i>v</i>	<i>A</i>	<i>a, a</i>
	<i>B</i>	<i>0, 0</i>

		<i>w</i>
<i>v</i>	<i>A</i>	<i>0, 0</i>
	<i>B</i>	<i>b, b</i>

## A networked coordination game

$p$  fraction of neighbors adopting  $A$

$1 - p$  fraction of neighbors adopting  $B$

$d$  is the number of neighbors

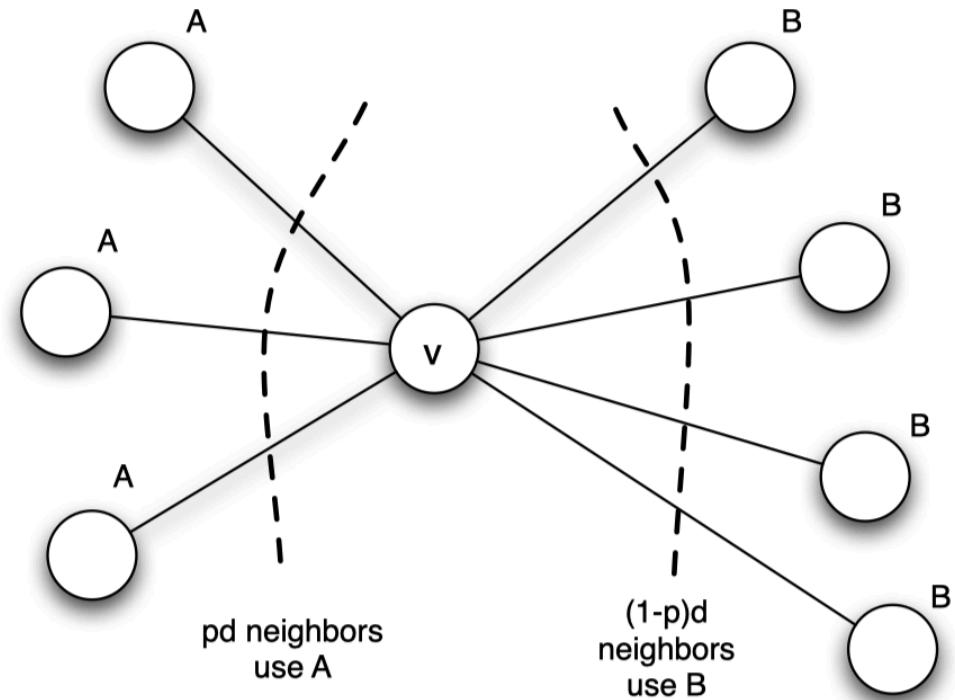
Node  $v$  chooses  $A$  if

$$pda \geq (1 - p)db$$

$$pa \geq b - pb$$

$$(a + b)p \geq b$$

$$p \geq \frac{b}{a + b} = q$$



## Threshold rule

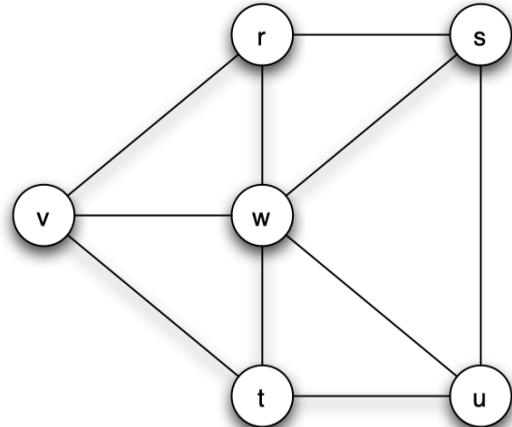
- In order to adopt  $A$  or  $B$ , we just need to check if  $p \geq q$ .
- Very simple - and myopic - model of individual decision making
- It is a research question to think about richer models that incorporate more long-range considerations

# Cascading behavior

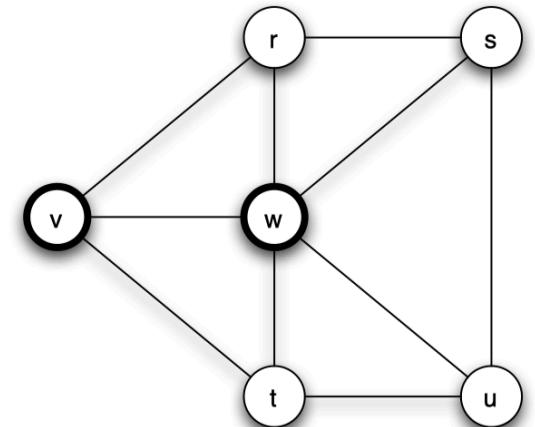
- Two obvious equilibria in the network:
  - one in which everyone adopts  $A$
  - another in which everyone adopts  $B$
- We want to understand how easy is to get one of these equilibria
- Also, we want to understand if other intermediate equilibria exist and how they look like
- **Assumptions:**
  - everyone is using  $B$  at the beginning
  - $S$ : small set of **initial adopters** of  $A$
- Will the spread of  $A$  make everyone to switch to the new technology, or will the spread stop?
- **Answer:** it depends on the network structure, the choice of nodes in  $S$ , and the value of  $q$ .

## Example

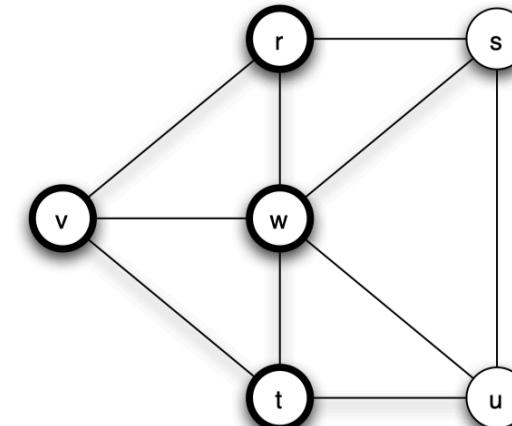
- $a = 3, b = 2 \Rightarrow q = \frac{2}{5}$
- $S = \{u, v\}$
- **complete cascade**



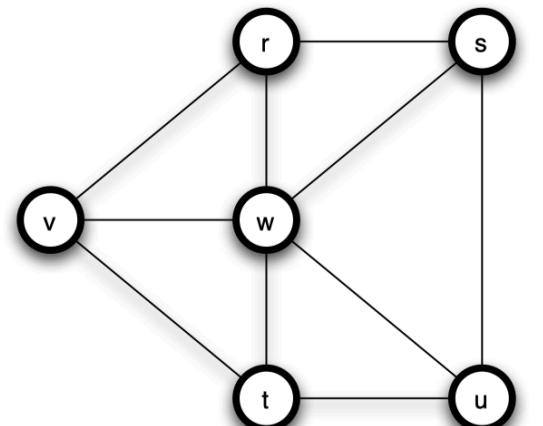
(a) *The underlying network*



(b) *Two nodes are the initial adopters*



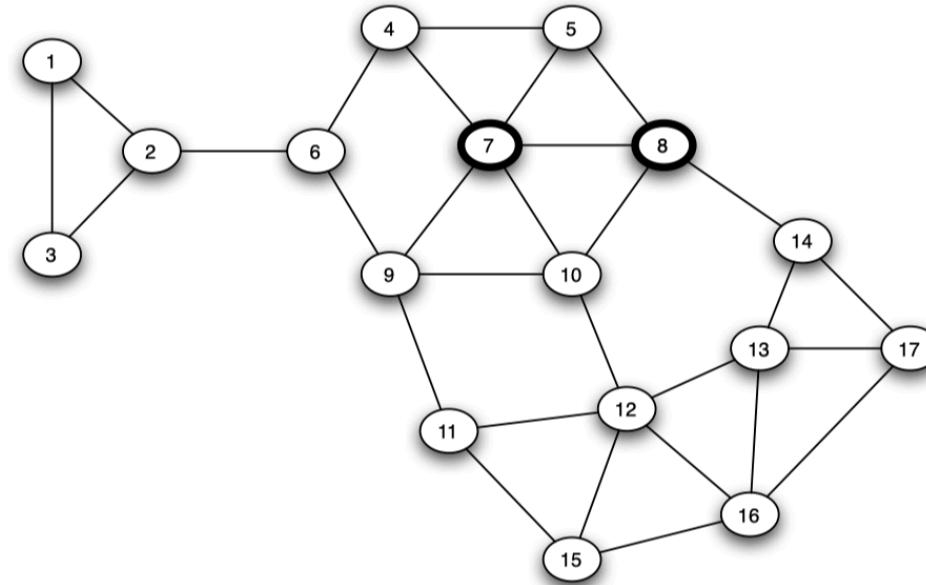
(c) *After one step, two more nodes have adopted*



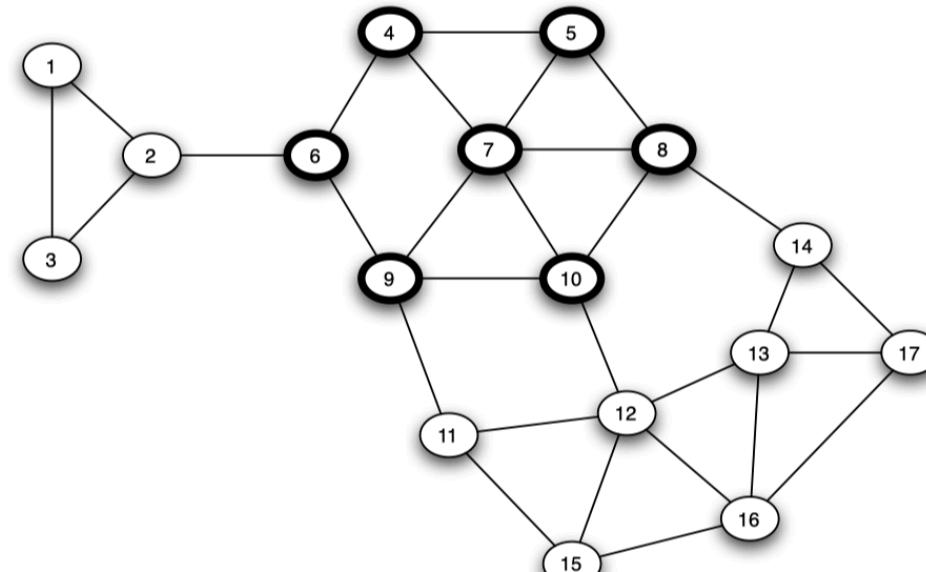
(d) *After a second step, everyone has adopted*

## Example

- $a = 3, b = 2 \Rightarrow q = \frac{2}{5}$
- $S = \{u, v\}$
- The diffusion of A stops here
  - partial cascade



(a) Two nodes are the initial adopters



(b) The process ends after three steps

## Recap

- Consider a set of initial adopters who start with a new behavior  $A$ , while every other node starts with behavior  $B$ .
- Nodes then repeatedly evaluate the decision to switch from  $B$  to  $A$  using a threshold of  $q$ .
- If the resulting cascade of adoptions of  $A$  eventually causes every node to switch from  $B$  to  $A$ , then we say that the set of initial adopters causes a **complete cascade at threshold  $q$**

## Definition

- We say that a **cluster of density  $p$**  is a **set of nodes** such that each node in the set has **at least a  $p$  fraction of its network neighbors in the set**.

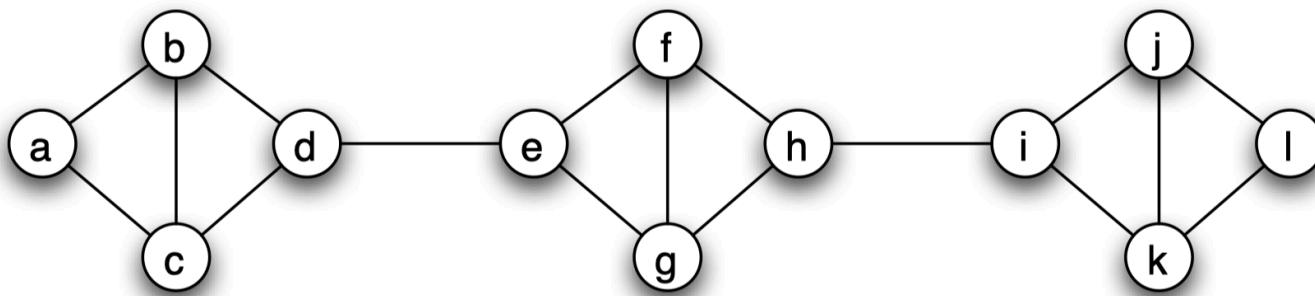
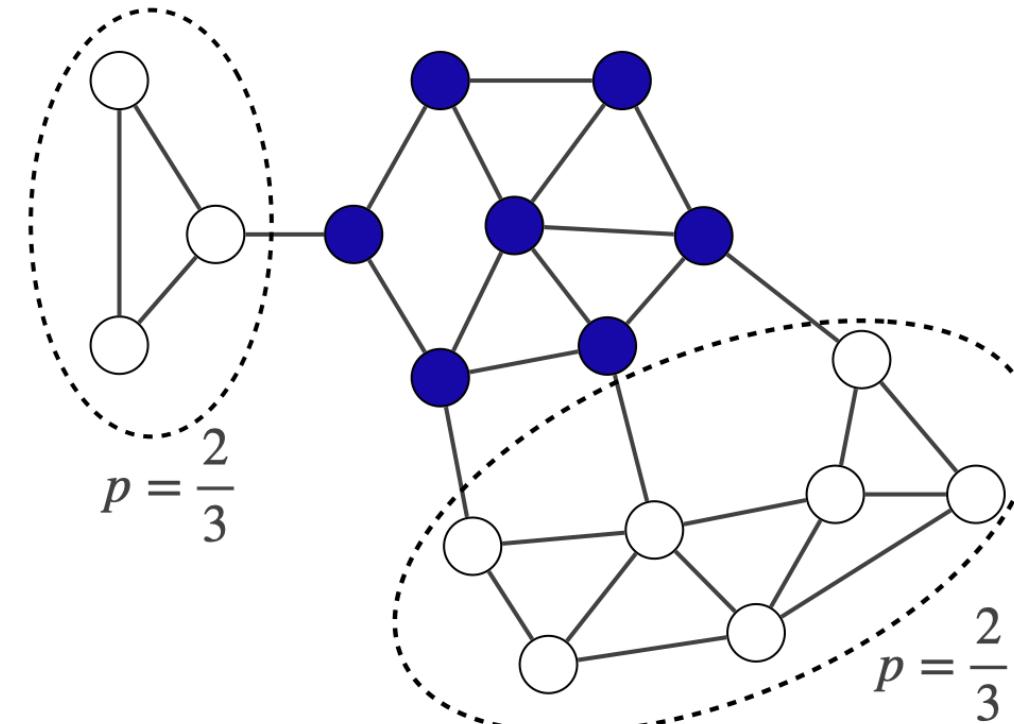


Figure 19.6: A collection of four-node clusters, each of density  $2/3$ .

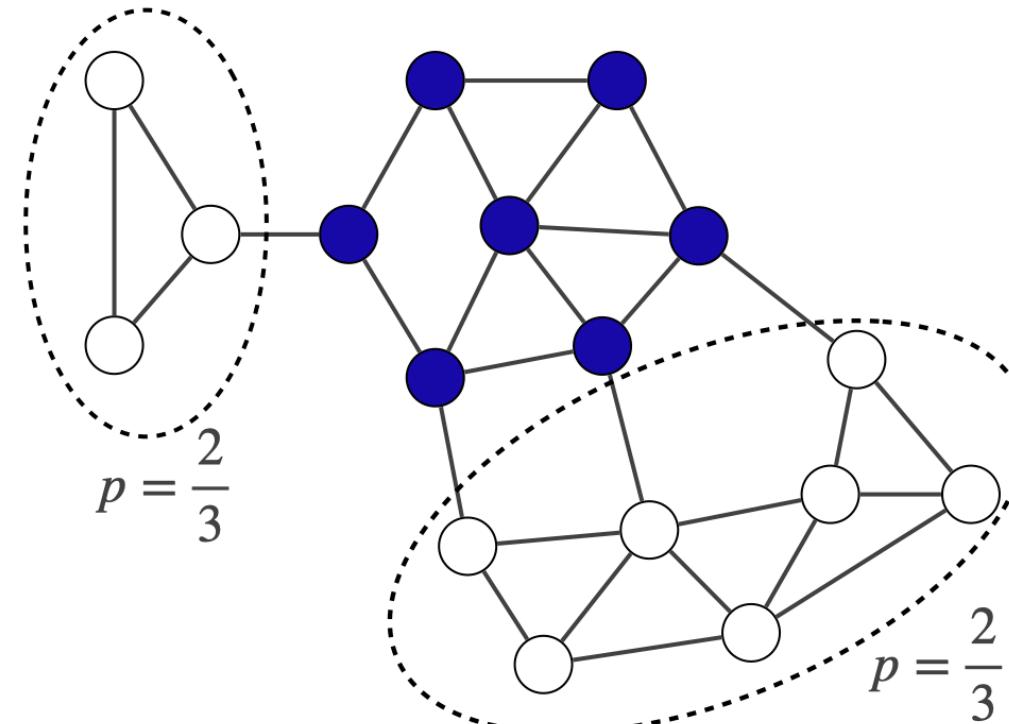
## Stopping cascades

- What prevents cascades from spreading?
  - **Homophily** can serve as a barrier to diffusion: it is hard for innovation to arrive from outside densely connected communities
- Let's try to quantify this intuition:
  - **Cluster of density  $p$**



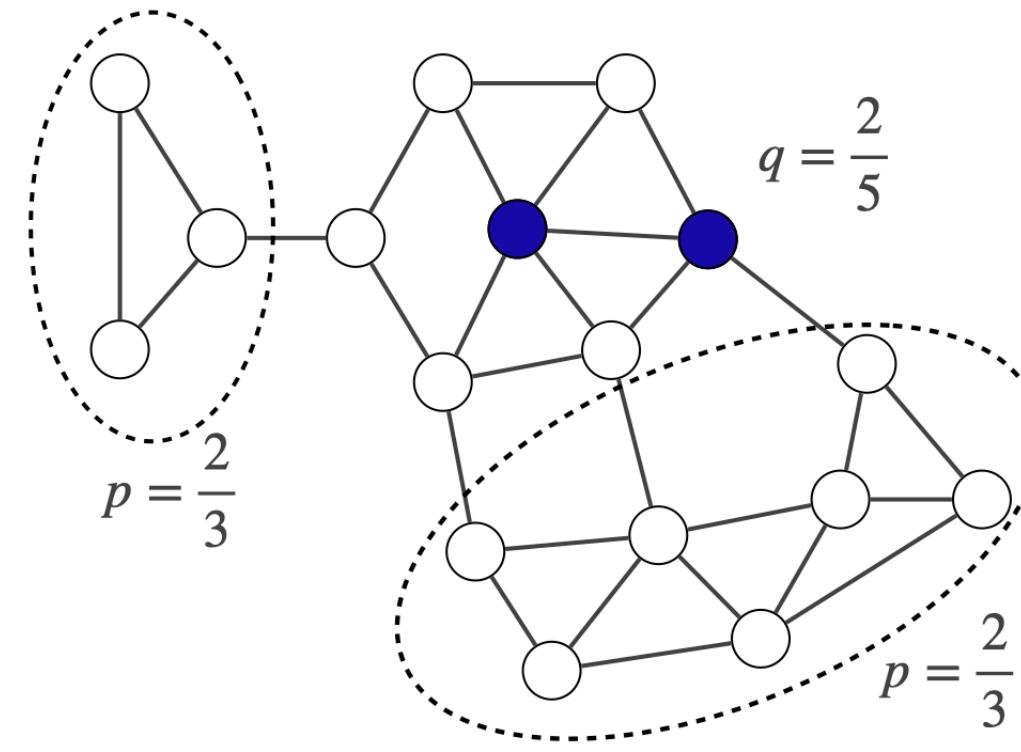
## Internal cohesion

- Each node in a cluster has a prescribed fraction of its friends residing in the cluster: **cohesion**
- Nodes in the same clusters do not necessarily have much in common
  - Any network is a cluster of density  $p = 1$ .
  - The union of two clusters of density  $p$  is still a cluster of density  $p$ .
- In fact, clusters in networks can exist simultaneously at different scales.



## Relationship between clusters and cascades

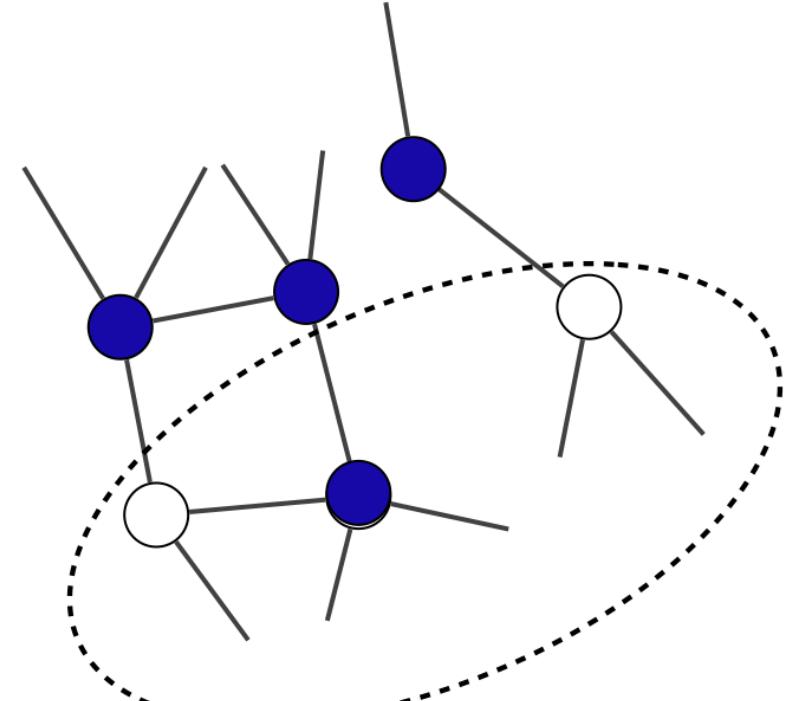
- A cascade stops iff it runs into a dense cluster
  - i.e., clusters are natural obstacles to cascades
- **Claim:** Let  $S$  be the set of **initial adopters** of  $A$ , with a threshold of  $q$ 
  - (i) if the remaining network contains a cluster of density  $p > (1 - q)$  then **S cannot cause a complete cascade**
  - (ii) if  $S$  fails to cause a cascade, then **there is a cluster of density  $p > (1 - q)$  in the remaining network**



## Clusters implies no cascade

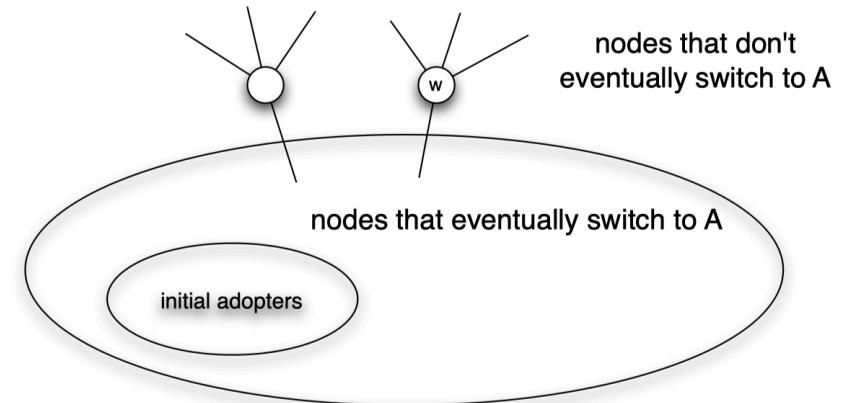
Proof by contradiction:

- some node inside the cluster  $C$  of density  $p > (1 - q)$  will adopt  $A$  at time  $t$  (the first to adopt in its cluster);
- this means that the decision was made at time  $t - 1$ , when no other node in the cluster  $C$  adopted  $A$ ;
- since the cluster  $C$  has density  $p > (1 - q)$ , less than a  $q$  fraction of nodes are outside the cluster;
- these nodes are the only that could have adopted  $A$  and they were outside the cluster at time  $(t - 1)$
- so at time  $t - 1$  it is impossible that the threshold rule fired for  $v$  since  $p < q$ : **contradiction**



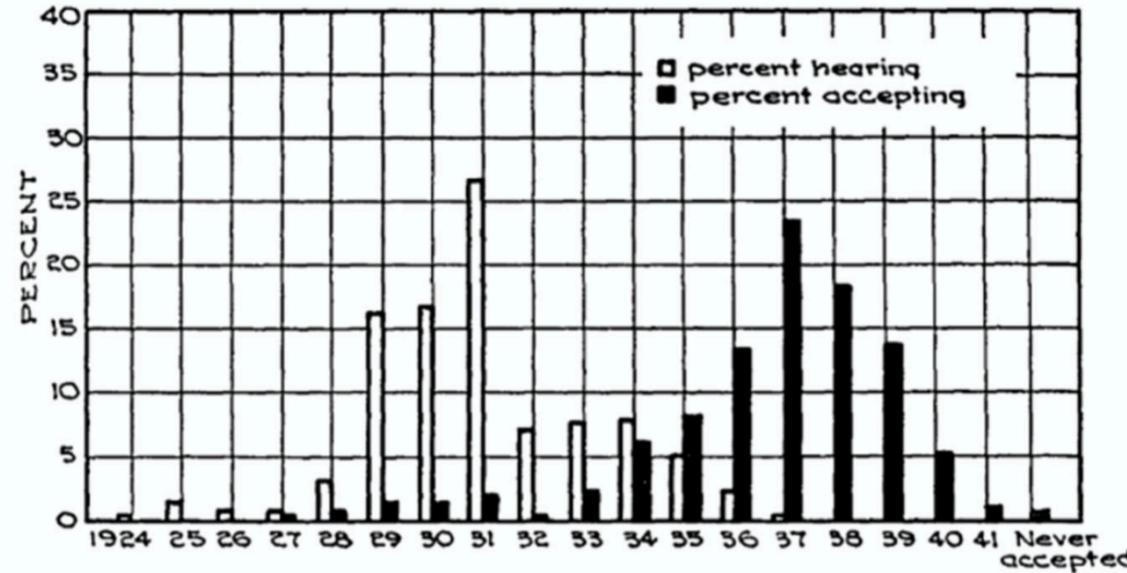
## Clusters implies no cascade

- Let  $S$  be the set of initial adopters of  $A$
- The spreading process stops: let  $S'$  be the maximum set of nodes that switched to  $A$
- Let  $R$  be the set of nodes still using  $B$  at the end of the process
- Pick one node in  $R$ : it does not want to switch to  $A$ 
  - the fraction of its friends using  $A$  is  $< q$
  - the fraction of its friends belonging to  $R$  is  $p > (1 - q)$
  - this holds for every node in  $R$
  - $R$  is a cluster of density  $p > (1 - q)$



# Viral marketing

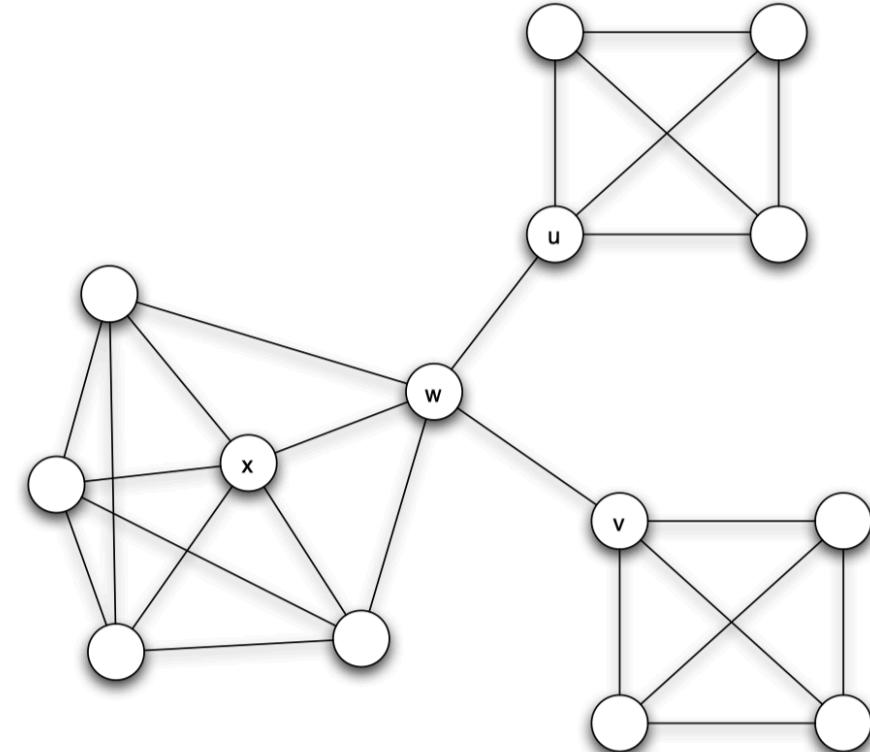
- Lesson learned: tightly-knit communities in the network can work to stop the spread of an innovation
  - We have coexistence of both behaviors (or also competing products)
- In case of coexistence, if a firm can rise the quality of product *A*, than the spread of the new technology can produce a complete cascade
- Alternative strategy (when the firm cannot rise the quality of the product): trying to **convince a small number of key people** in the part of the network still using *B*
  - how to choose key people to trigger a viral marketing strategy?
  - their position in the underlying network can be very relevant - let's explore this now



- In the Ryan-Gross study (1943) on the adoption of hybrid seed corn, it is possible to spot the difference between learning about a new idea and actually deciding to adopt it
- It is like nodes at the border in a cluster: they have been exposed to the idea, but still they decide not to adopt it.
- Reading material: [Bryce Ryan and Neal C. Gross. The diffusion of hybrid seed corn in two Iowa communities. Rural Sociology, 8:15–24, 1943.](#)

## The role of weak ties

- Threshold models highlight some important implications of the **strength of weak ties** theory
- They receive very fresh ideas from other communities; not enough for adoption and spread (try with  $q = \frac{1}{2}$ )
- Bridges and weak ties are great for spreading rumors or jokes across the network, but not for diffusion of innovation or social mobilization
- Strong ties can have more significant role for others in the community to take actions
- Reading material: [Damon Centola and Michael Macy. Complex contagions and the weakness of long ties. American Journal of Sociology, 113:702–734, 2007.](#)



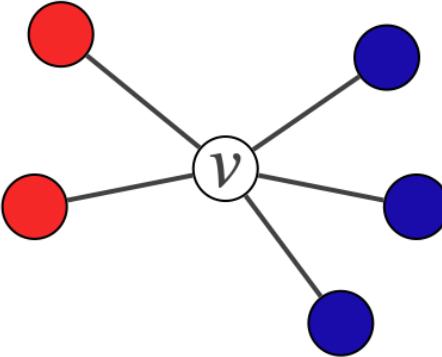
# Extensions of the Basic Cascade Model

## Heterogeneous thresholds

- Let's suppose each person gives values to  $A$  and  $B$  subjectively

		$w$	
		$A$	$B$
$v$		$A$	$a_v, a_w$
$v$		$B$	$0, 0$
			$b_v, b_w$

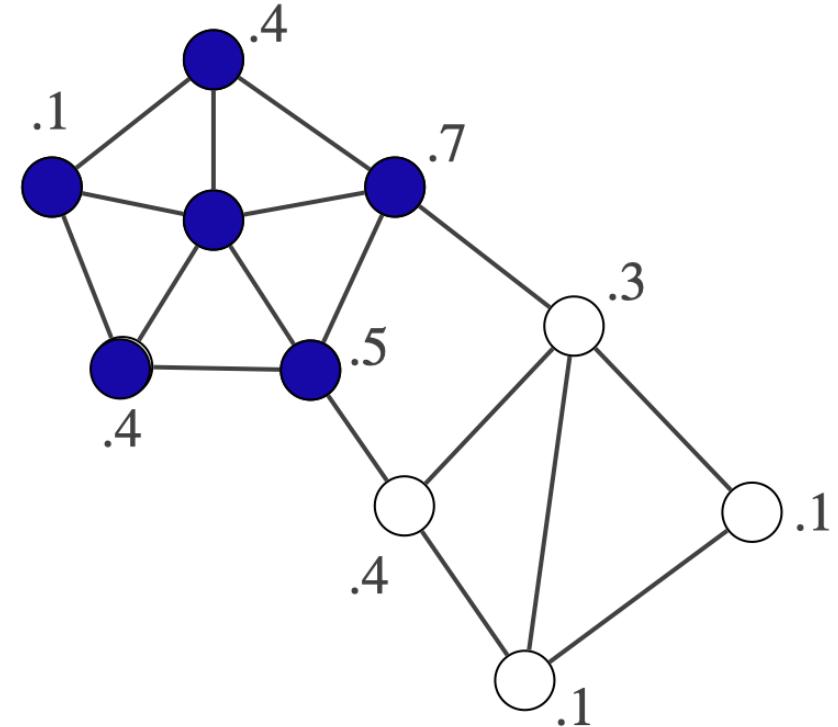
## Heterogeneous thresholds



- $p$  fraction of neighbors adopting  $A$
- $1 - p$  fraction of neighbors adopting  $B$
- $d$  is the number of neighbors
- The node chooses  $A$  if

$$pda_v \geq (1 - p)db_v \Rightarrow p \geq \frac{b_v}{a_v + b_v} = q_v$$

- Watts and Dodds: we need to take into account not just the power of influential nodes, but also the extent to which these influential nodes have access to easily **influenceable** people.
- Reformulating the notion of **blocking clusters**: set of nodes for which each node  $v$  has a fraction  $p > (1 - q_v)$  of its friends inside the set.
- The notion of density becomes **heterogeneous** as well: each node has a different requirement for the fraction of friends it needs to have in the cluster.
- Reading material: [Duncan J. Watts and Peter S. Dodds. Networks, influence, and public opinion formation. Journal of Consumer Research, 34\(4\):441–458, 2007.](#)



# Knowledge, Thresholds, and Collective Action

## Integrating network effects

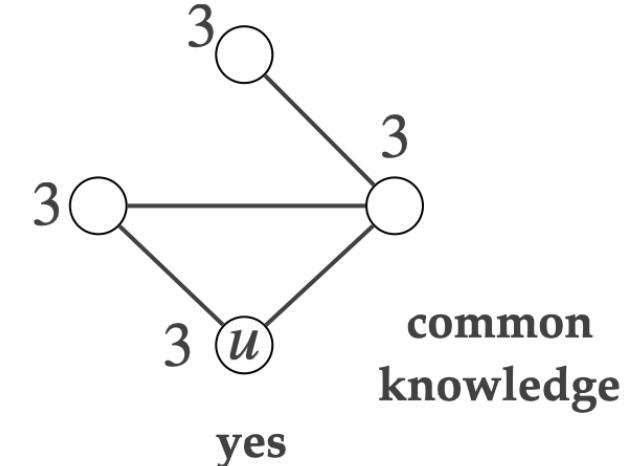
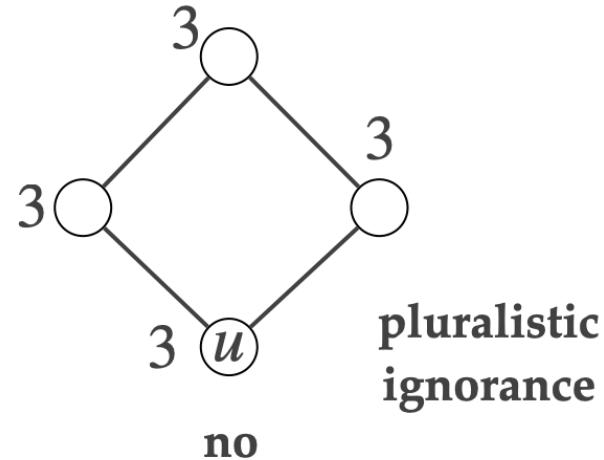
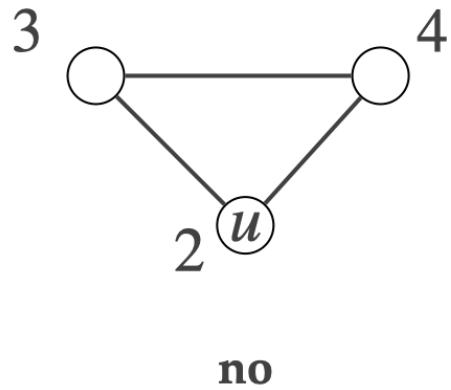
- In many domains you need to integrate network effects at both the **population level** and the **local network level**
  - collective actions and pluralistic ignorance
  - a model for the effect of knowledge on collective actions
  - common knoledge and social institutions

## Collective actions and pluralistic ignorance

- **Objective:** you want to organize a revolt under a repressive regime. Which is the probability to succeed?
- **Collective actions:** benefits only if enough people participate - but you can talk to a very limited number of persons you trust
- **Pluralistic ignorance:** when people have wildly erroneous estimates about the prevalence of certain opinions in the population at large

## A model for the effect of knowledge on collective actions

- Each individual has a personal threshold of  $k$ : "I will show up for the protest if I am sure that at least  $k$  people in total will show up"
- Assumption: every node knows the thresholds of all its neighbors in the network.
- Is it safe for  $u$  to join the protest in the following configurations?



## Common knowledge and social institutions

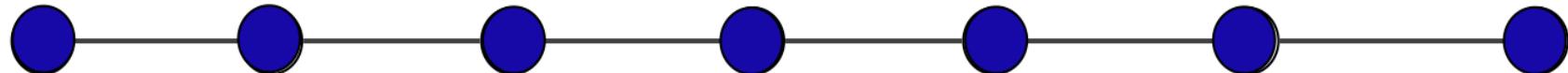
- Some social institutions have the purpose of helping people to achieve common knowledge
- Widely-publicized speech, article in a high circulation newspaper, and so on: people receive the message AND that a lot of other people are receiving the message
- Freedom of the press and freedom of assembly are important social institutions!
- Marketing: commercials during a very popular event (as the famous Apple Macintosh commercial during the 1984 Super Bowl)

# The Cascade Capacity

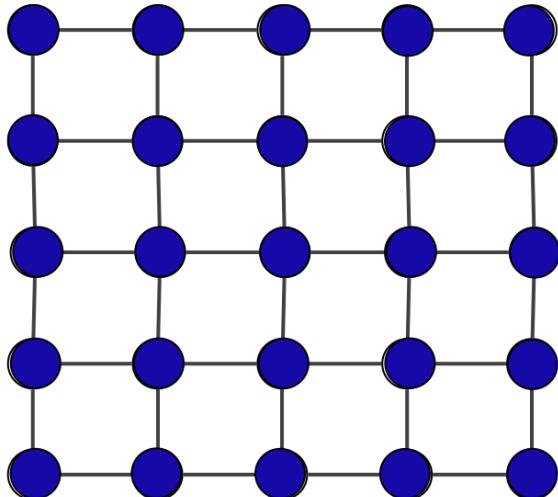
## Infinite Networks

- Understanding how different network structures are more or less prone to cascades
- **Cascade capacity:** the **maximum threshold** at which a small set of initial adopters can cause a complete cascade
- Let's consider **infinite graphs**
  - nodes have finite number of neighbors
- Let's say that a finite set  $S$  of initial adopters of  $A$  causes a cascade in  $G$  with threshold  $q$  if **eventually every node in  $G$  adopts  $A$**
- Example: Infinite path

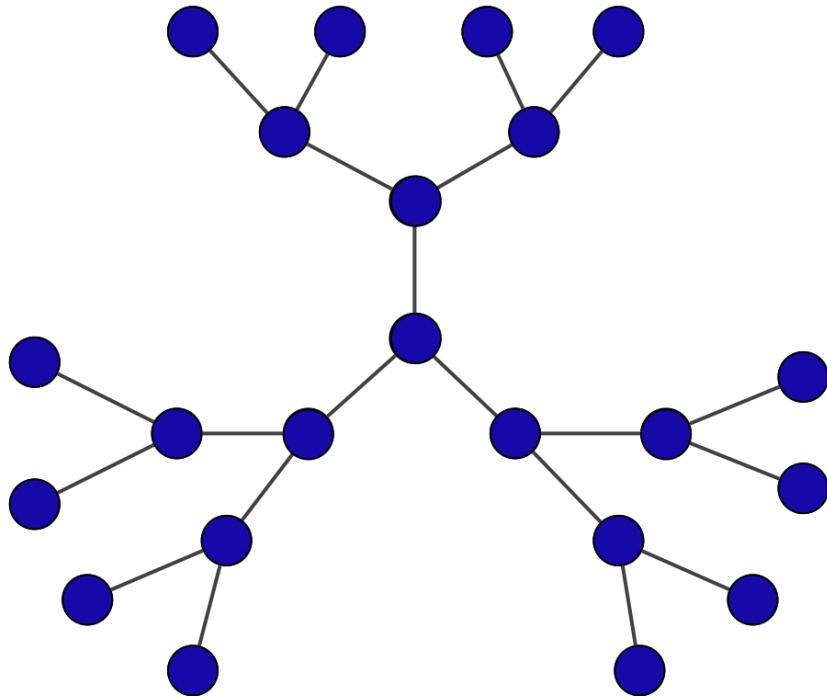
$$q \leq \frac{1}{2}$$



Infinite grid       $q \leq \frac{1}{4}$



Infinite tree       $q \leq \frac{1}{3}$



## How large can the cascade capacity be?

Claim: there is no network which the cascade capacity exceeds  $\frac{1}{2}$

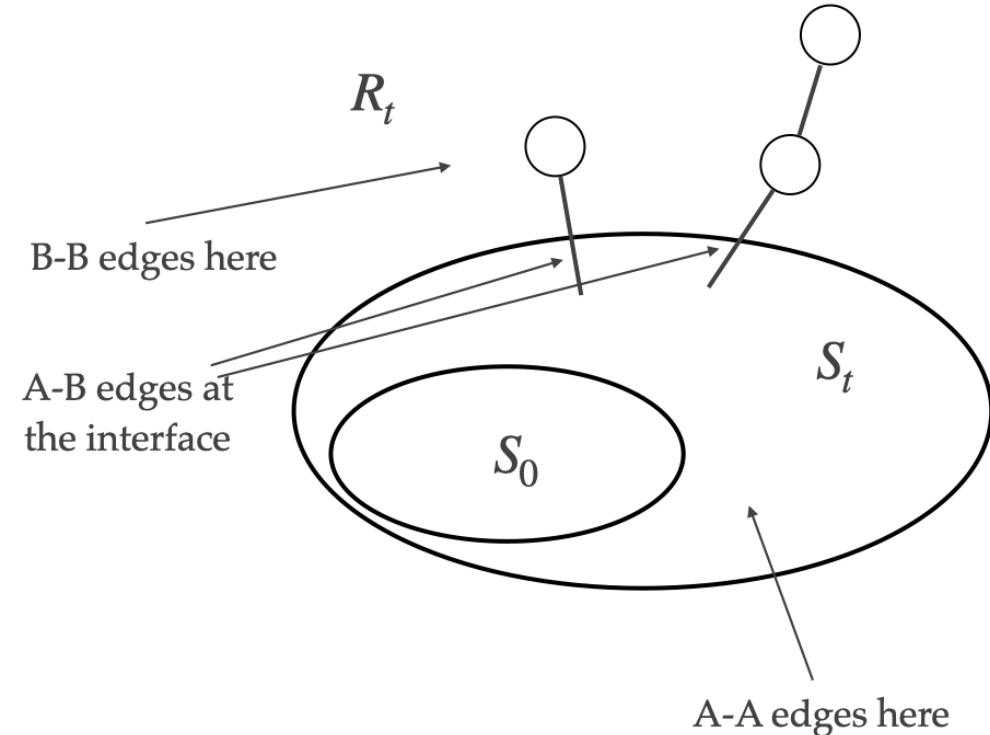
Proof by contradiction:

Let's suppose such  $G$  exists with  $q > \frac{1}{2}$

We want to find a contradiction, arguing that nodes stop switching from  $B \rightarrow A$  after a finite number of steps.

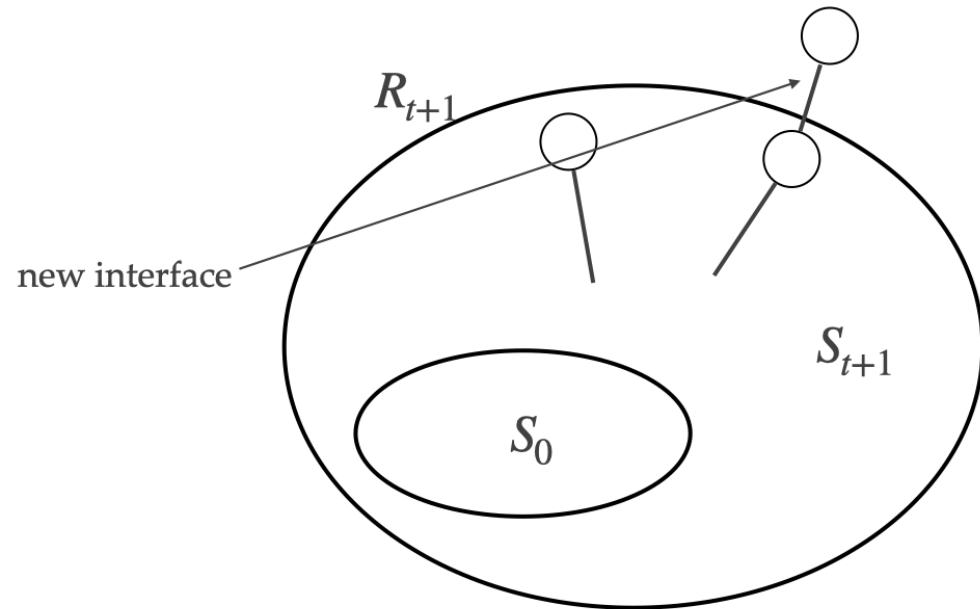
## Focus on the interface

- $S_0$ : finite set of initial adopters of  $A$
- $S_t$ : set of adopters of  $A$ , potentially larger than  $S_{t-1}$ 
  - size of the **interface** at time  $t$ :  $I_t$
- We show that  $I_t < I_{t+1}$
- Hence, if the size of the interface strictly decreases, the diffusion process will terminate after  $I_0$  steps



## Size of the interface

- at time  $t + 1$  some nodes in  $R$  switches from  $B \rightarrow A$
- Focus on a node that switches from  $B \rightarrow A$ 
  - it has edges to  $B$  nodes
  - and edges with  $A$  nodes
  - recall that  $q > \frac{1}{2}$ , so this node has more edges to nodes in  $S$  than to nodes in  $R$
  - more edges leaving the interface than edges joining it
  - $\Rightarrow I_{t+1} < I_t$



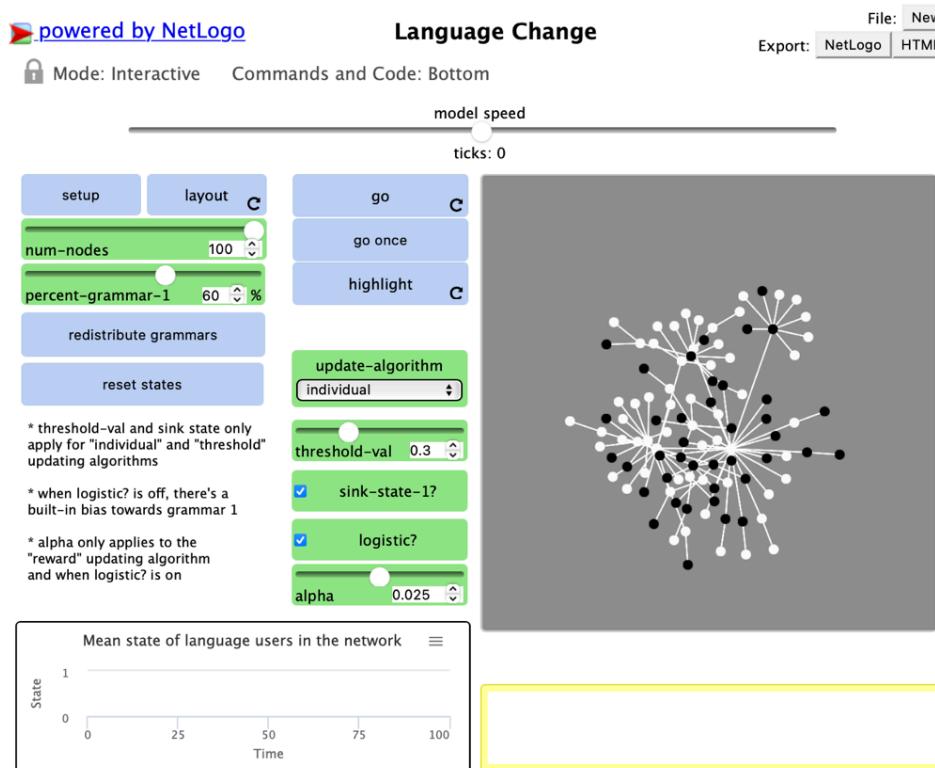
## Observations

- When  $q > \frac{1}{2}$  no finite set of nodes can cause a complete cascade in any network
- This threshold corresponds intuitively to a "lower quality" technology: a person will switch to it only if more than 50% of their friends already have.

## Another extension of the model

- Extending the model: allow people to **adopt both A and B** (compatibility)
- It is referred as the **bilingual option**
- Although the model is amazingly simple, we have very surprising and complex findings
  - compatibility can be a strategy of a firm to enter in a market and progressively cut out the competitor's product

# Language Change (netlogo model)



- [Play with the model]

## Reading material

[ns2] Chapter 19 (19.1-19.7) Cascading behavior in Networks

# Q & A

