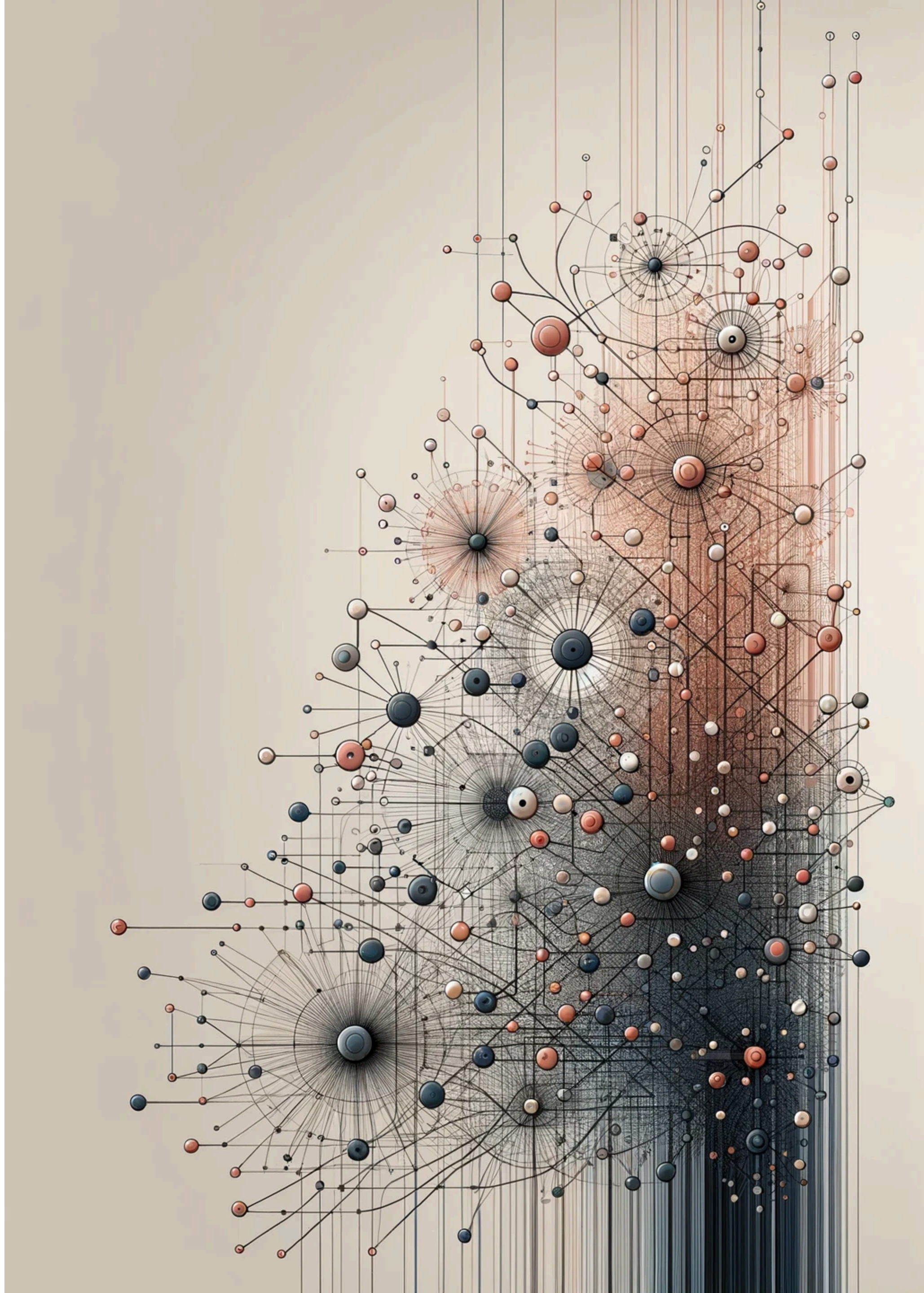
UNIVERSITÀ DI TORINO

Analisi e Visualizzazione delle Reti Complesse

NS16 - Cascading behaviors in networks

Prof. Rossano Schifanella



Agenda

- Diffusion in Networks
- Modeling Diffusion through a Network
- Cascades and Clusters
- Diffusion, Thresholds, and the Role of Weak Ties
- Extensions of the Basic Cascade Model

The diffusion of innovations

- Let's consider how new behaviors, practices, opinions, conventions, and technologies spread from person to person through a social network, as people influence their friends to adopt new ideas
- diffusion of innovation: long list of classic studies in the middle of the 20th century
    - focus on informational and network effects
- **direct benefit effects** to understand how technologies spread: incentives to adopt telephone, fax, emails based on friends that already adopted those technologies
- we need to take into account **network effects** on a **local network level**
- Objective of this lecture: **formulating a model for the spread of an innovation through a social network**

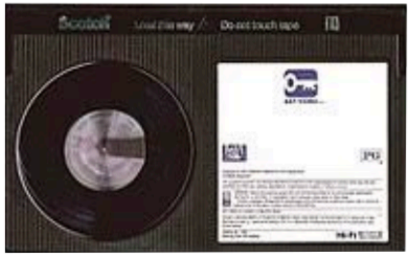
A diffusion of a new behavior

- Assumption: individuals make decisions based on the choices of their neighbors
  - focus on links
- In this lecture, let's focus on **direct-benefit effects** instead of informational effects
- Natural model introduced by Stephen Morris in 2000
- Reading material:
  - Stephen Morris. Contagion. Review of Economic Studies, 67:57–78, 2000.

Examples

VHS

vs



Betamax

Blu-ray Disc™

vs

HD DVD™

A networked coordination game

- It is natural to use a **coordination game**
  - each node has a choice between two possible behaviors, A and B
  - players have an incentive to adopt the same behavior

$$
\begin{array}{cc|c|c|}
 & & \multicolumn{2}{c}{w} \\
 & & A & B \\
\cline{3-4}
\multirow{2}{*}{v} & A & a, a & 0, 0 \\
\cline{3-4}
 & B & 0, 0 & b, b \\
\cline{3-4}
\end{array}
$$

A networked coordination game

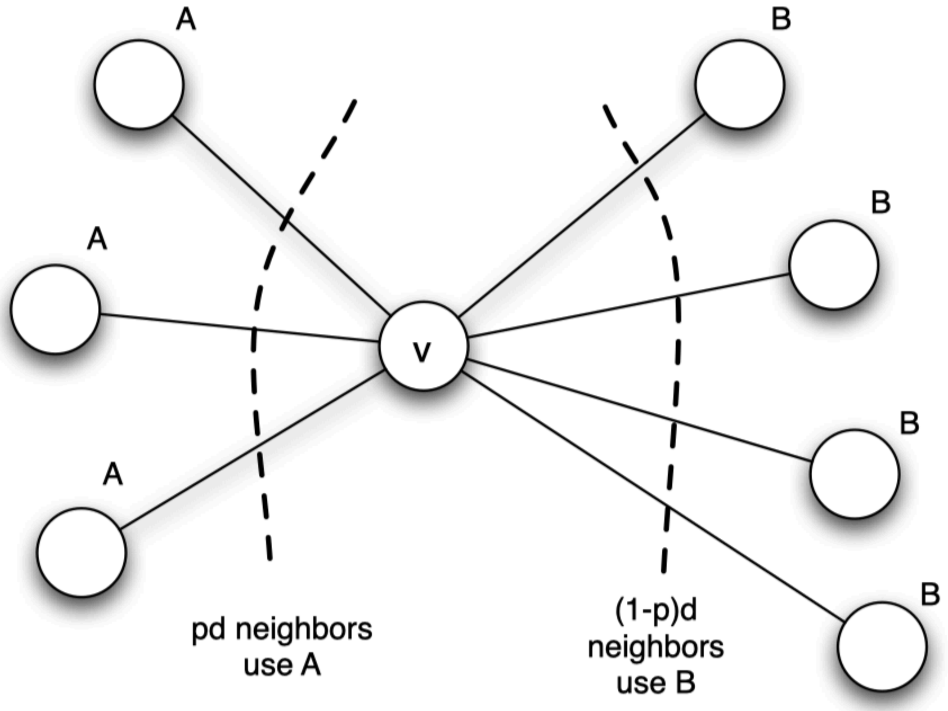
p `fraction of neighbors adopting` A

$1-p$ `fraction of neighbors adopting` B

d `is the number of neighbors`

Node v chooses A if

$$pda \geq (1-p)db$$
$$pa \geq b - pb$$
$$(a+b)p \geq b$$
$$p \geq \frac{b}{a+b} = q$$



Threshold rule

- In order to adopt A or B, we just need to check if $p \geq q$.
- Very simple - and myopic - model of individual decision making
- It is a research question to think about richer models that incorporate more long-range considerations

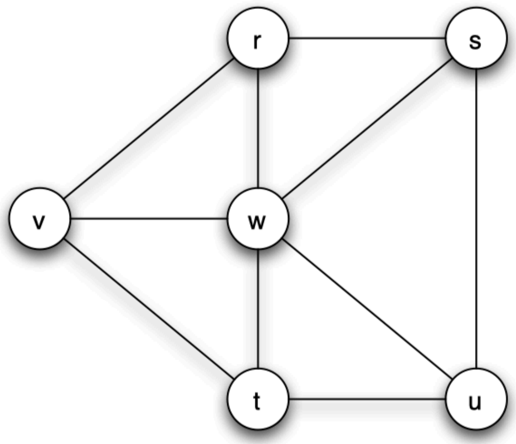
UNIVERSITÀ DI TORINO

Cascading behavior

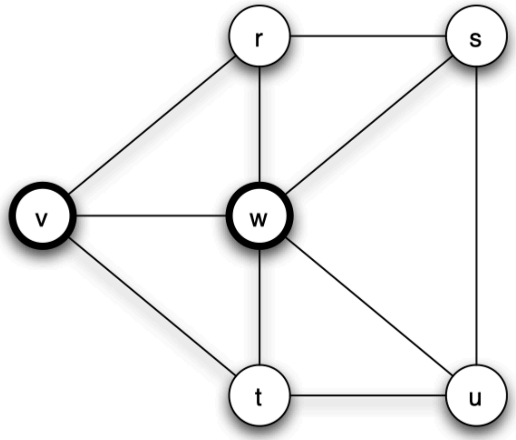
- Two obvious equilibria in the network:
  - one in which everyone adopts A
  - another in which everyone adopts B
- We want to understand how easy is to get one of these equilibria
- Also, we want to understand if other intermediate equilibria exist and how they look like
- **Assumptions**:
  - everyone is using B at the beginning
  - S: small set of **initial adopters** of A
- Will the spread of A make everyone to switch to the new technology, or will the spread stop?
- **Answer:** it depends on the network structure, the choice of nodes in S, and the value of q.

Example

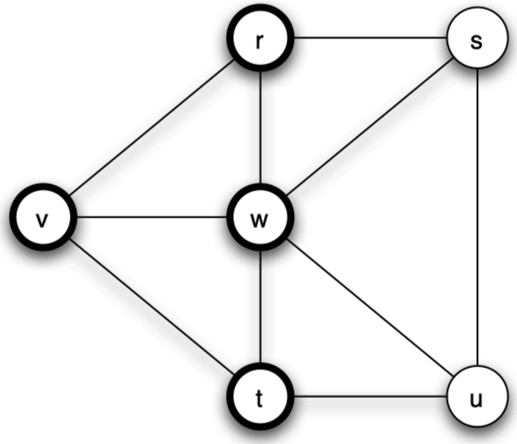
- $a = 3, b = 2 \Rightarrow q = \frac{2}{5}$
- $S = \{u, v\}$
- **complete cascade**



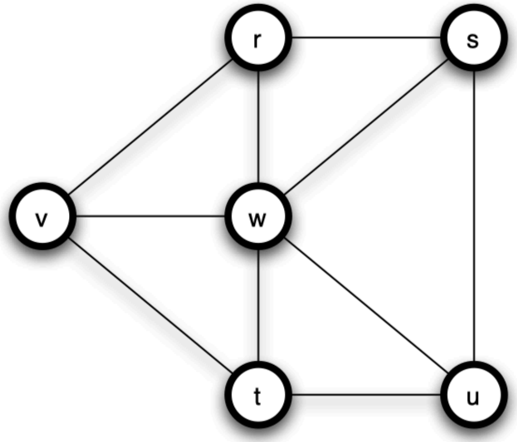
(a) *The underlying network*



(b) *Two nodes are the initial adopters*



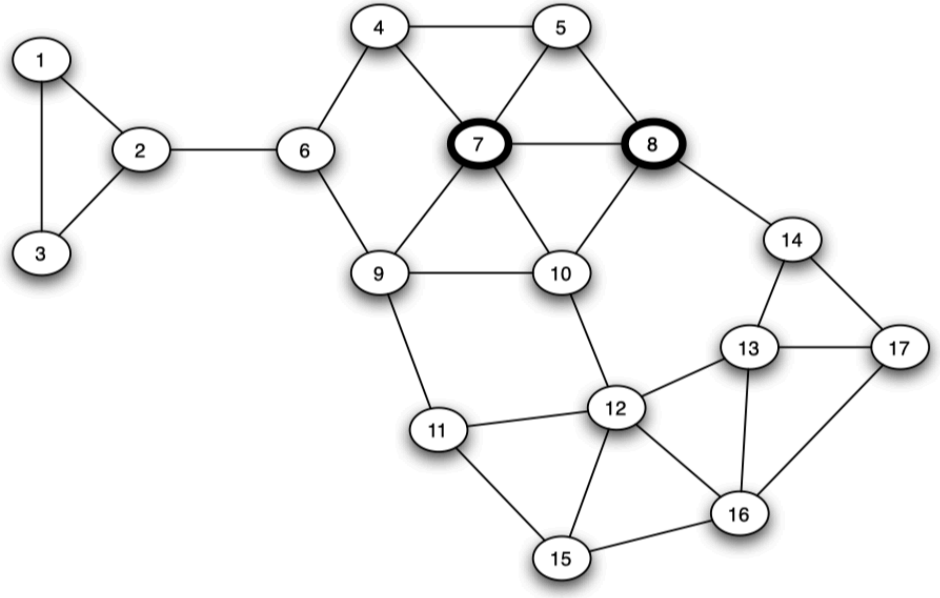
(c) *After one step, two more nodes have adopted*



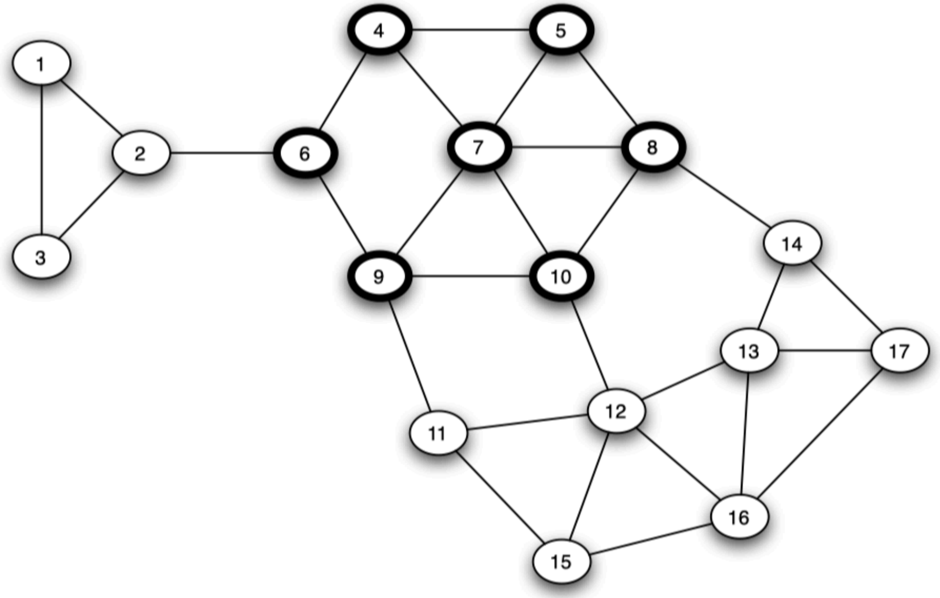
(d) *After a second step, everyone has adopted*

Example

- $a = 3, b = 2 \Rightarrow q = \frac{2}{5}$
- $S = \{u, v\}$
- The diffusion of A stops here
  - **partial cascade**



(a) *Two nodes are the initial adopters*



(b) *The process ends after three steps*

Recap

- Consider a set of initial adopters who start with a new behavior A, while every other node starts with behavior B.
- Nodes then repeatedly evaluate the decision to switch from B to A using a threshold of q.
- If the resulting cascade of adoptions of A eventually causes every node to switch from B to A, then we say that the set of initial adopters causes a **complete cascade at threshold** q

Definition

- We say that a **cluster of density p** is **a set of nodes** such that each node in the set has **at least a p fraction of its network neighbors in the set**.

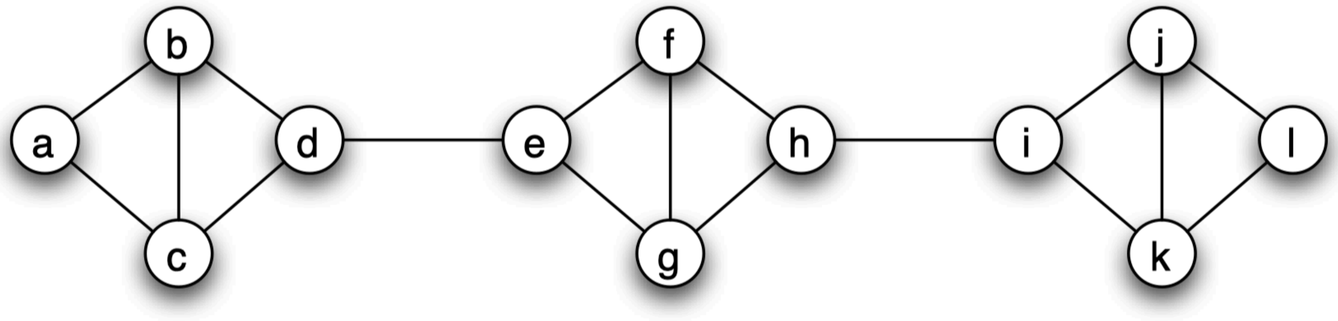
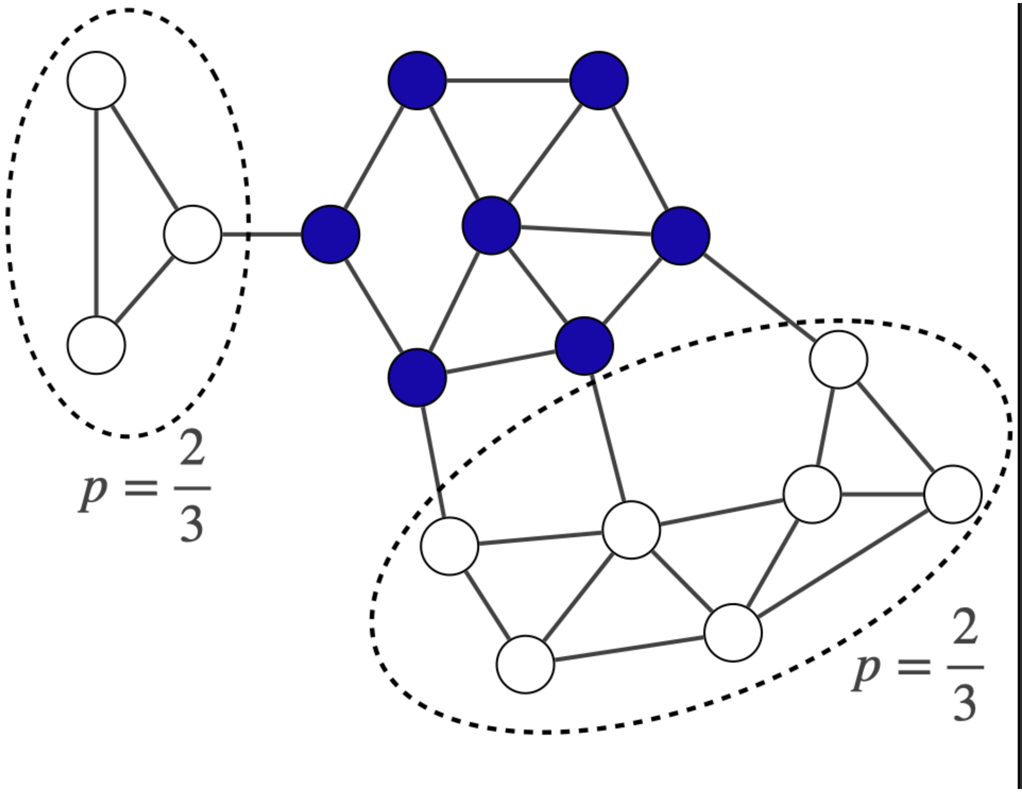


Figure 19.6: A collection of four-node clusters, each of density 2/3.

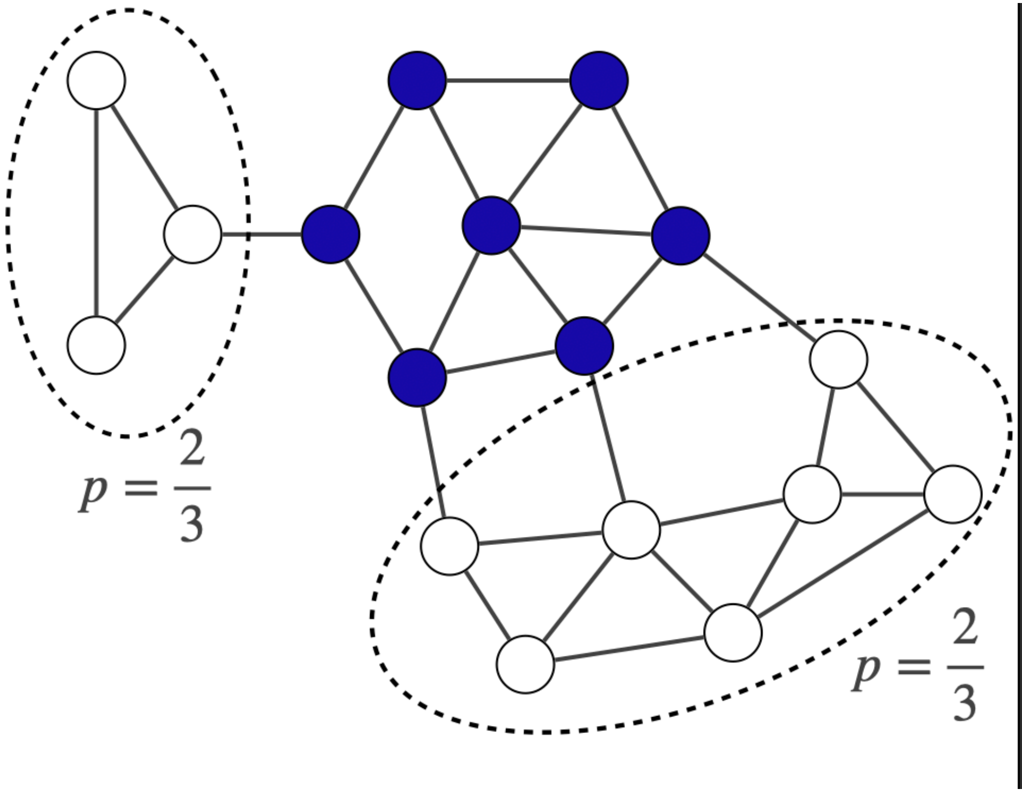
Stopping cascades

- What prevents cascades from spreading?
  - **Homophily** can serve as a barrier to diffusion: it is hard for innovation to arrive from outside densely connected communities
- Let's try to quantify this intuition:
  - **Cluster of density p**



Internal cohesion

- Each node in a cluster has a prescribed fraction of its friends residing in the cluster: **cohesion**
- Nodes in the same clusters do not necessarily have much in common
    - Any network is a cluster of density $p = 1$.
    - The union of two clusters of density p is still a cluster of density p.
- In fact, clusters in networks can exist simultaneously at different scales.

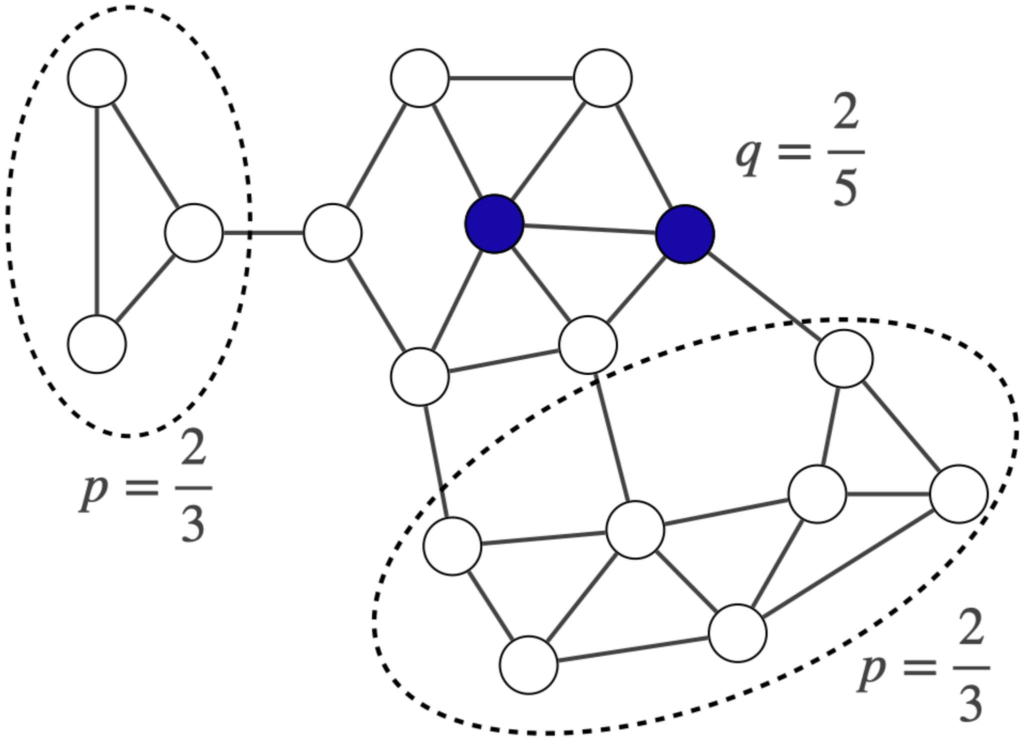


Relationship between clusters and cascades

- A cascade stops **iff** it runs into a dense cluster
  - i.e., **clusters are natural obstacles to cascades**
- **Claim**: Let S be the set of **initial adopters** of A, with a threshold of q

  -(i) if the remaining network contains a cluster of density $p > (1-q)$ then **S cannot cause a complete cascade**

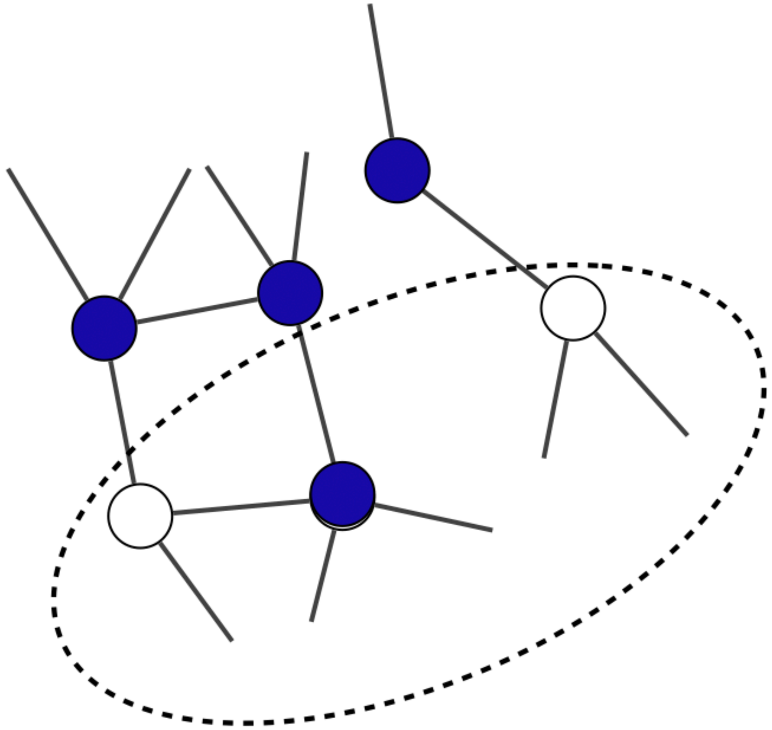
  (ii) if S fails to cause a cascade, then **there is a cluster of density** $p > (1-q)$ **in the remaining network**



Clusters implies no cascade

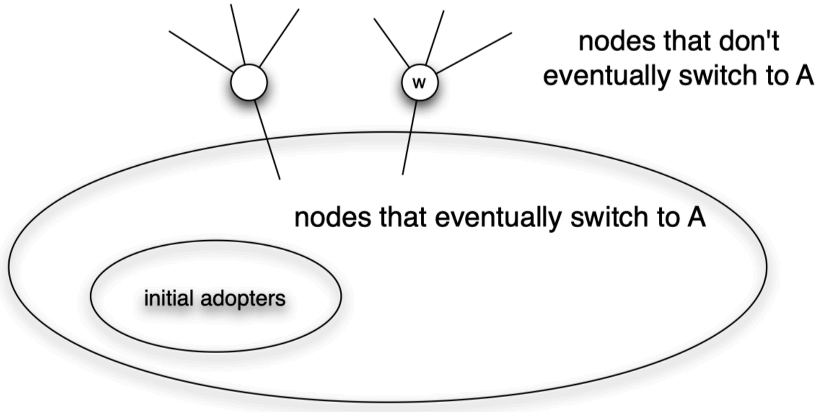
Proof by contradiction:

- some node inside the cluster C of density $p > (1 - q)$ will adopt A at time t (the first to adopt in its cluster);
- this means that the decision was made at time $t - 1$, when no other node in the cluster C adopted A;
- since the cluster C has density $p > (1 - q)$, less than a q fraction of nodes are outside the cluster;
- these nodes are the only that could have adopted A and they were outside the cluster at time $(t - 1)$
- so at time $t - 1$ it is impossible that the threshold rule fired for v since $p < q$: **contradiction**



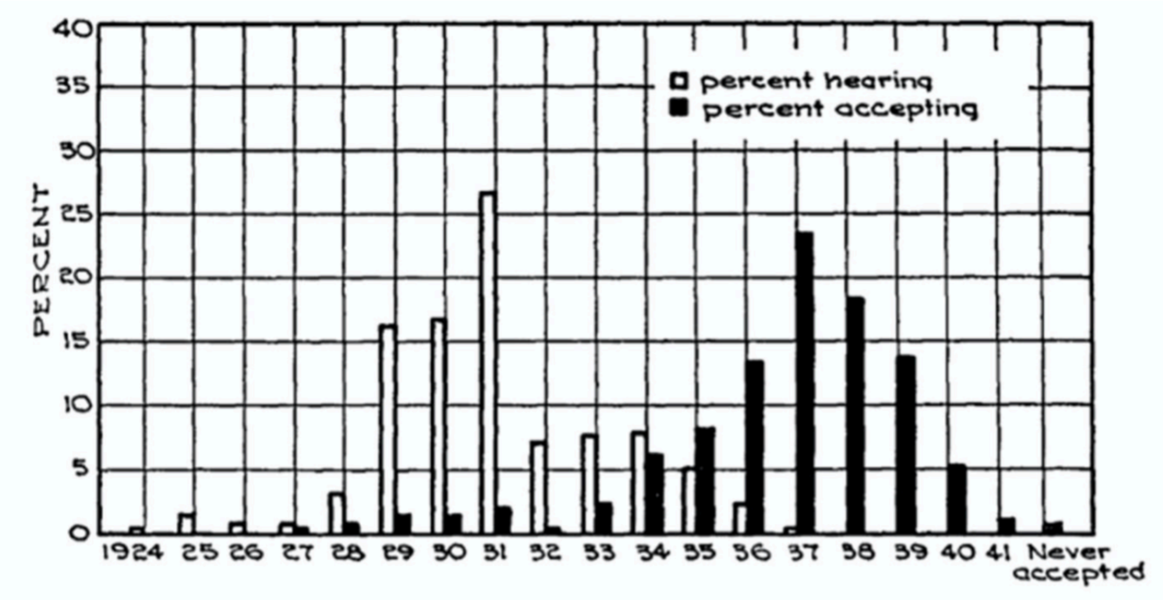
Clusters implies no cascade

- Let S be the set of initial adopters of A
- The spreading process stops: let S' be the maximum set of nodes that switched to A
- Let R be the set of nodes still using B at the end of the process
- Pick one node in R: it does not want to switch to A
  - the fraction of its friends using A is $< q$
  - the fraction of its friends belonging to R is $p > (1 - q)$
  - this holds for every node in R
  - R is a cluster of density $p > (1 - q)$



Viral marketing

- Lesson learned: tightly-knit communities in the network can work to stop the spread of an innovation
    - We have coexistence of both behaviors (or also competing products)
- In case of coexistence, if a firm can rise the quality of product A, than the spread of the new technology can produce a complete cascade
- Alternative strategy (when the firm cannot rise the quality of the product): trying to **convince a small number of key people** in the part of the network still using B
    - how to choose key people to trigger a viral marketing strategy?
    - their position in the underlying network can be very relevant - let's explore this now

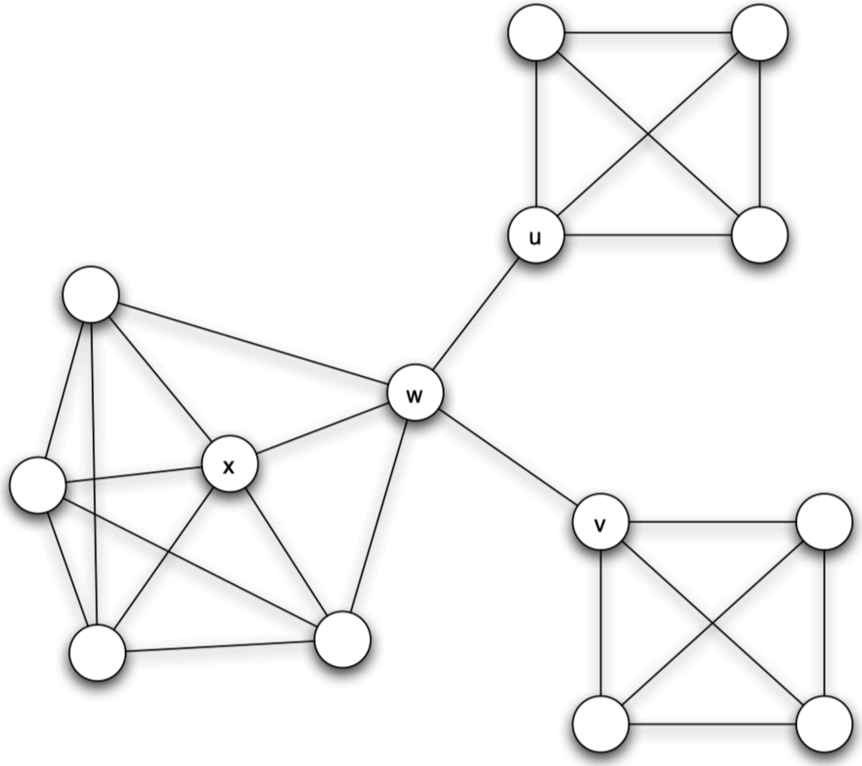


- In the Ryan-Gross study (1943) on the adoption of hybrid seed corn, it is possible to spot the difference between learning about a new idea and actually deciding to adopt it
- It is like nodes at the border in a cluster: they have been exposed to the idea, but still they decide not to adopt it.
- Reading material: Bryce Ryan and Neal C. Gross. The diffusion of hybrid seed corn in two Iowa commu- nities. Rural Sociology, 8:15–24, 1943.

UNIVERSITÀ DI TORINO

The role of weak ties

- Threshold models highlight some important implications of the **strength of weak ties** theory
- They receive very fresh ideas from other communities; not enough for adoption and spread (try with $q = \frac{1}{2}$)
- Bridges and weak ties are great for spreading rumors or jokes across the network, but not for diffusion of innovation or social mobilization
- Strong ties can have more significant role for others in the community to take actions
- Reading material: Damon Centola and Michael Macy. Complex contagions and the weakness of long ties. American Journal of Sociology, 113:702–734, 2007.



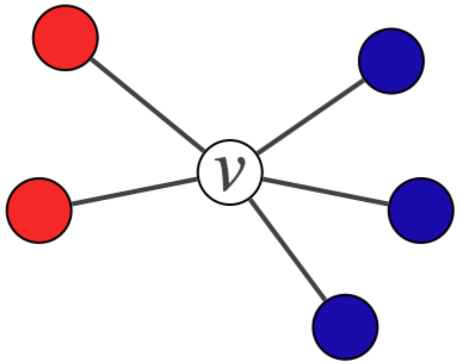
Extensions of the Basic Cascade Model

Heterogeneous thresholds

- Let's suppose each person gives values to A and B subjectively

$$
\begin{array}{cc|c|c|}
 & & \multicolumn{2}{c}{w} \\
 & & \multicolumn{1}{c}{A} & \multicolumn{1}{c}{B} \\
\cline{3-4}
\multirow{2}{*}{v} & A & a_v, a_w & 0, 0 \\
\cline{3-4}
 & B & 0, 0 & b_v, b_w \\
\cline{3-4}
\end{array}
$$

Heterogeneous thresholds

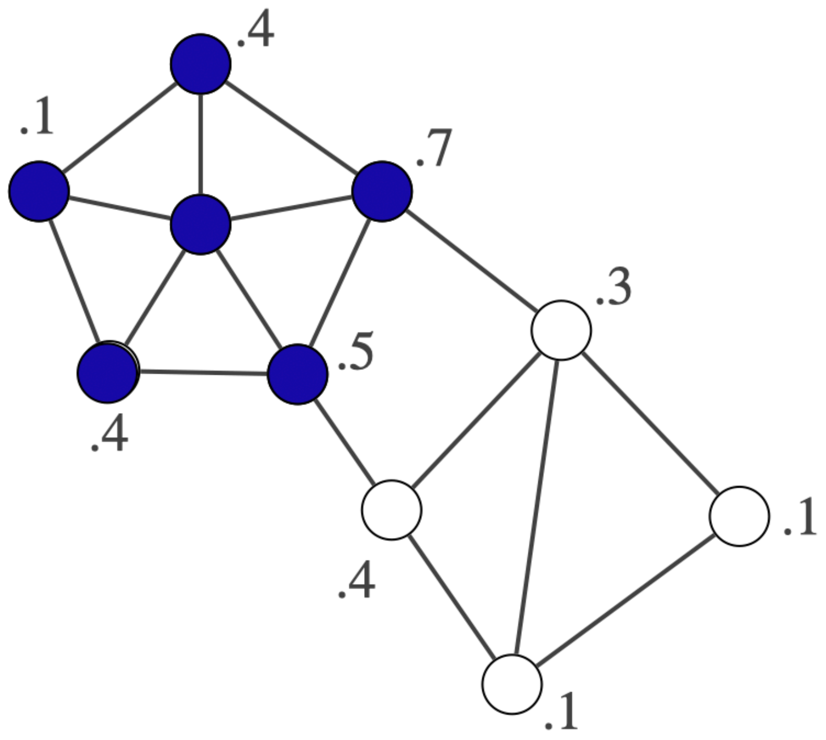


- p fraction of neighbors adopting A
- $1-p$ fraction of neighbors adopting B
- d is the number of neighbors
- The node chooses A if

$$pda_v \geq (1-p)db_v \Rightarrow p \geq \frac{b_v}{a_v + b_v} = q_v$$

UNIVERSITÀ DI TORINO

- Watts and Dodds: we need to take into account not just the power of influential nodes, but also the extent to which these influential nodes have access to easily **influenceable** people.
- Reformulating the notion of **blocking clusters**: set of nodes for which each node v has a fraction $p > (1 - q_v)$ of its friends inside the set.
- The notion of density becomes **heterogeneous** as well: each node has a different requirement for the fraction of friends it needs to have in the cluster.
- Reading material: Duncan J. Watts and Peter S. Dodds. Networks, influence, and public opinion formation. Journal of Consumer Research, 34(4):441–458, 2007.



Reading material

[ns2] **Chapter 19 (19.1-19.5)** Cascading behavior in Networks

Q&A