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Analisi e Visualizzazione delle Reti Complesse

NS18 - Cascading behaviors in networks

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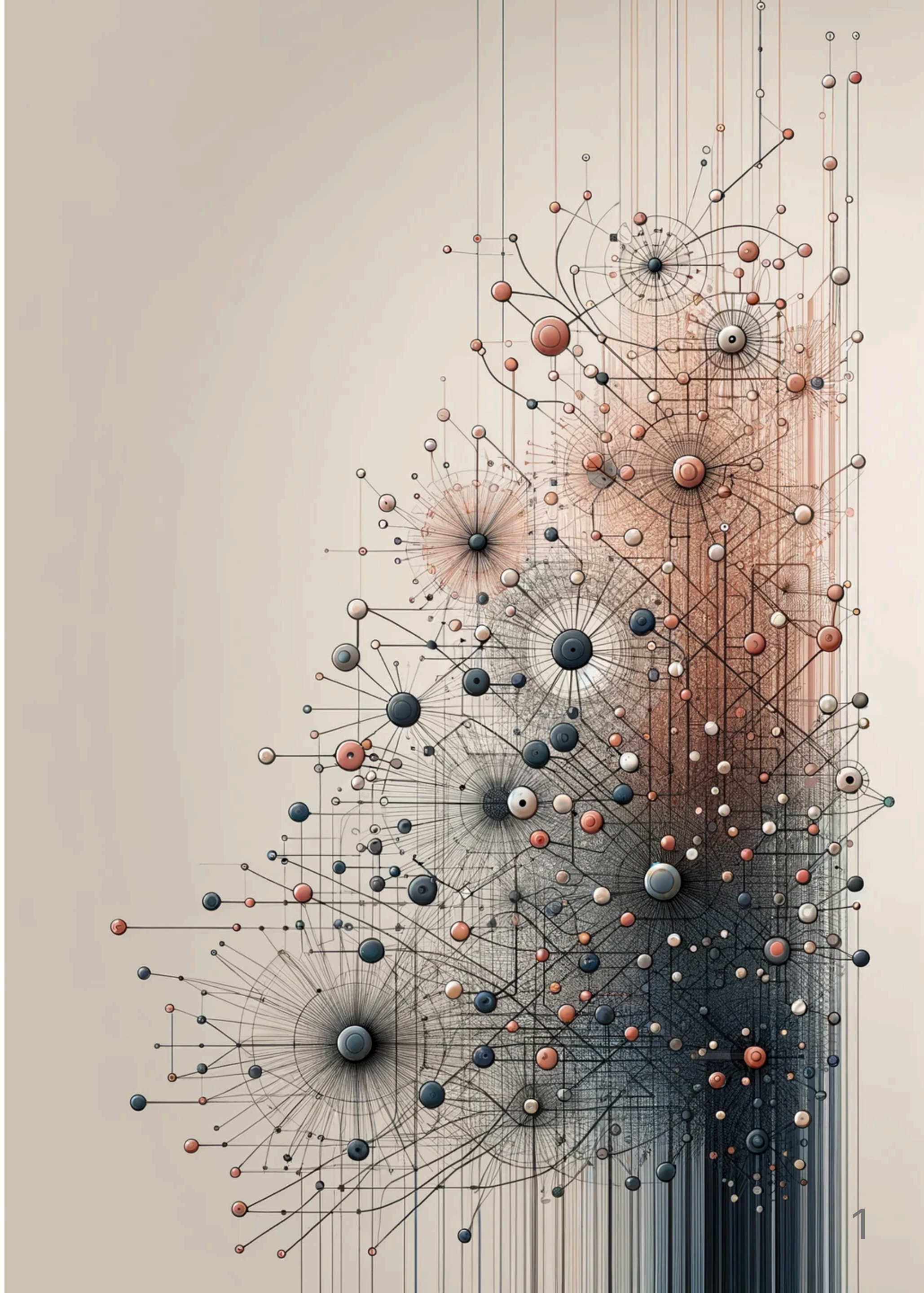

Agenda

- **Diffusion in Networks:** How innovations and behaviors spread socially
- **Modeling Diffusion through a Network:** Formal models of influence
- **Cascades and Clusters:** When spread continues or stops
- **Diffusion, Thresholds, and the Role of Weak Ties:** Simple vs. complex contagions
- **Extensions of the Basic Cascade Model:** Heterogeneous thresholds, collective action, cascade capacity, and compatibility

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The Diffusion of Innovations

- Many innovations—new behaviors, practices, opinions, conventions, technologies—spread person-to-person via social influence. Neighbors or friends adopting something new can influence others to follow.
- *Diffusion of innovation* research (mid-20th century) emphasized both **informational effects** (learning from others) and **network effects** (influence through social ties). Classic studies by Rogers (1962) and others identified adoption categories (innovators, early adopters, etc.).
- We focus on **direct-benefit effects**: situations where adopting a technology or behavior yields increasing benefits as more friends or contacts adopt it too. Examples:
    - Telephones or social media: each friend who joins increases its usefulness (a network externality).
    - Products like fax machines or email: more adopters among acquaintances provide direct benefits for you to adopt.

The Diffusion of Innovations

- **Network effects at the local level** are crucial: it's not just global popularity, but whether *your* friends have adopted.
- **Objective:** Formulate a simple model for the spread of an innovation through a social network, capturing how local network structure and incentives drive diffusion.

Diffusion of a New Behavior

- **Key assumption:** Individuals base their adoption decisions on the choices of their neighbors. This is a *local* decision rule focusing on peer influence along network links.
    - We ignore external marketing or media influence here and concentrate on peer-to-peer spread.
- We emphasize **direct-benefit (coordination) effects** rather than pure information or "hype." Each person's payoff for adopting increases as more neighbors adopt the same behavior.
- A natural modeling framework: a **networked coordination game** (first introduced by Morris, 2000).
    - Each node (individual) chooses between two behaviors (or technologies), say A (new behavior) and B (status quo).
    - People have an incentive to align with their neighbors' choices to gain coordination benefits (compatibility, communication, etc.).
- **Reference:** Stephen Morris (2000), *Contagion*, which provided one of the first formal analyses of such network diffusion processes.

Examples of Diffusion in Networks

VHS

vs

Betamax

Blu-ray Disc

vs

HD DVD

A Networked Coordination Game

- To formalize, consider a **coordination game on a network**:
  - Each node chooses either behavior A or behavior B.
  - **Payoffs:** If a node and a neighbor choose the same behavior, both gain a benefit. Let's say:
    - If both use A, each gets payoff a from that link.
    - If both use B, each gets payoff b from that link.
    - If they differ, payoff is 0 on that link (mismatch yields no benefit).
  - Typically assume a and b are positive, and one might be larger (A might be superior but new; B is inferior but established).
  - This captures direct-benefit effects: you benefit from aligning choices.

- Each node wants to coordinate with neighbors. Thus, the more neighbors choosing A, the more attractive A becomes.
- This leads to a simple decision rule based on the **fraction of neighbors** on A vs. B and the payoffs *a* and *b*.

$$
\begin{array}{cc|c|c|}
 & & \multicolumn{2}{c}{w} \\
 & & \multicolumn{1}{c}{A} & \multicolumn{1}{c}{B} \\
\cline{3-4}
\multirow{2}{*}{v} & A & a, a & 0, 0 \\
\cline{3-4}
 & B & 0, 0 & b, b \\
\cline{3-4}
\end{array}
$$

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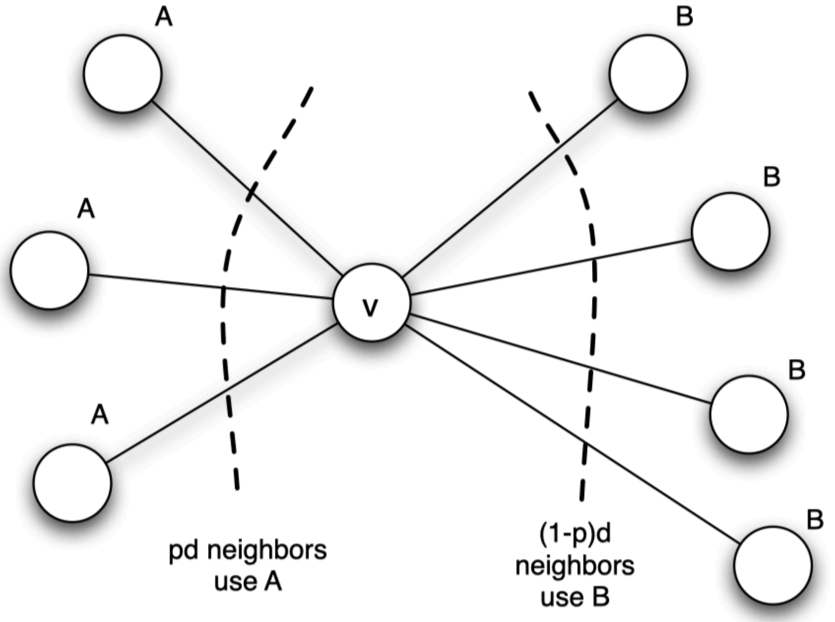
A Networked Coordination Game: Threshold Condition

Let:

- p = *fraction* of a node's neighbors who have adopted A.
- $(1-p)$ = *fraction* of a node's neighbors who have adopted B.
- d = total number of neighbors (node's degree).

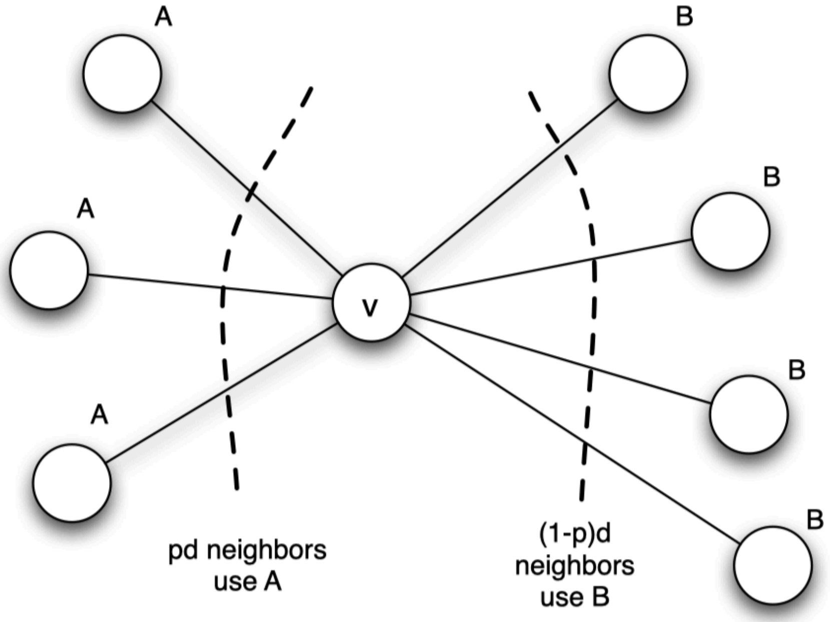
Node v will choose A if:

$$p \cdot d \cdot a \geq (1-p) \cdot d \cdot b$$
$$pa \geq (1-p)b \quad \text{(dividing both sides by } d > 0\text{)}$$
$$pa \geq b - pb$$
$$pa + pb \geq b$$



A Networked Coordination Game: Threshold Condition

- We define $q = \frac{b}{a+b}$ as the **threshold fraction** of neighbors adopting A that makes the node indifferent. If at least a fraction q of your neighbors choose A, you prefer A.
- Intuition: If $a > b$, then $q < 0.5$, meaning even a minority of neighbors on A might tip the balance (since A yields higher payoff per neighbor). If $a < b$ (A is lower quality), $q > 0.5$, meaning you need a majority of neighbors on A to make it worthwhile.



Threshold Rule

- The adoption decision rule for each node:
  - **Adopt A if** $p \geq q$, i.e., if at least a fraction q of your neighbors have adopted A.
  - Otherwise, stick with B.
- This is a very simple, **myopic threshold model** of decision-making:
  - “Myopic” because each node just looks at the current state of neighbors (one step look-ahead, no strategic long-term planning).
  - All nodes use the same threshold q (for now) derived from payoffs. Later we will consider heterogeneity in q.
- Such threshold models are widely used to study **social contagion**.

- **Research note:** In reality, people might consider more complex strategies, expectations, or long-range outcomes. Here we assume synchronous rounds where each node updates based on neighbors, which is a baseline model. Richer models can incorporate things like **inertia**, **decay**, or **foresight**.
  - **Inertia**: it refers to a **resistance to change**—how **"sticky"** a current behavior or decision is—even when others switch. Adds friction. For instance, even if $p \geq q$, a node might still require a slightly higher threshold (e.g., $p \geq q + \delta$) to overcome inertia and switch.
  - **Decay**: it introduces a **temporal weakening of influence or commitment**, since they fades over time without reinforcement.
  - **Foresight** Decision based on **anticipated** future states, not just the current one. Instead of being myopic, nodes might estimate whether adopting A now will benefit them more in the long run.

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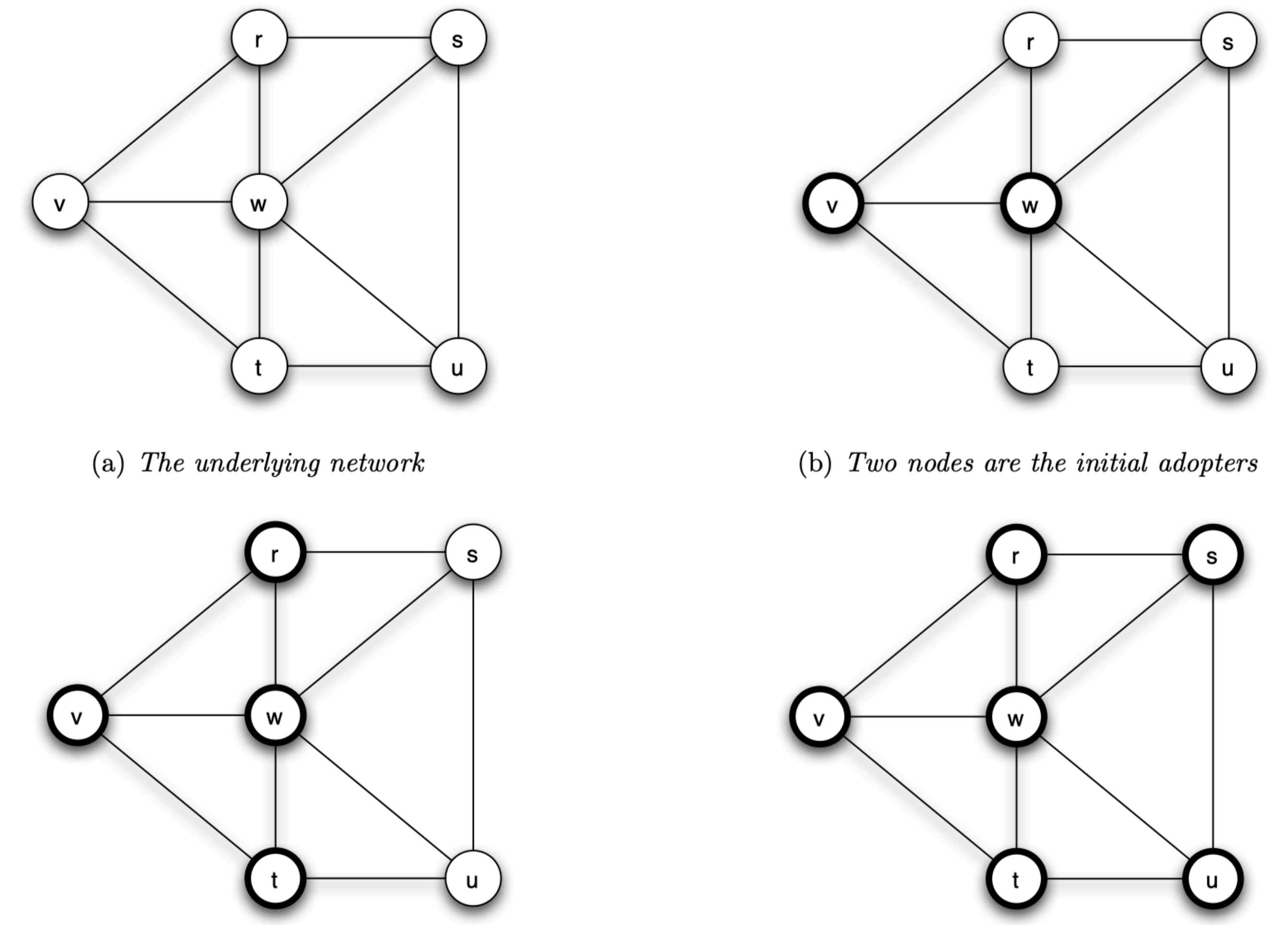
Cascading Behavior

- Given this threshold rule, the network can exhibit multiple **equilibria** (stable outcomes):
    - Everyone adopts A: all nodes switched to the new behavior **(global cascade)**.
    - Everyone sticks with B: no one adopts the innovation.
    - Partial adoption **(mixed equilibrium)**
- Key questions:
    - Under what conditions will the all-A equilibrium emerge starting from a few initial adopters? How easy is it to get a full cascade to A?
    - Under what structures do *partial* adoption equilibria (some mix of A and B stable) occur?

- **Assumptions for cascade analysis:**
    - Initially, everyone is using B (the old behavior/technology).
    - A small set S of initial adopters is "seeded" with A.
- The process: We then allow the network to evolve in **discrete steps**. In each round, all nodes with at least a q fraction of neighbors on A will switch to A (B → A). B-adopters with fewer than q fraction A-neighbors remain B.
- **Question:** Does this process result in A spreading to every node (a **complete cascade**) or does it stop at some point (a **partial cascade**)?
- **Answer:** It depends on three factors:
    i. The network structure (who is connected to whom),
    ii. The choice of initial adopters S (which nodes start with A),
    iii. The threshold q (based on relative payoff of A vs. B).

Example: Complete Cascade

- Payoffs $a = 3, b = 2$. Then the threshold $q = \frac{b}{a+b} = \frac{2}{5} = 0.4.$
- Each node needs at least 40% of its neighbors on A to be willing to switch.
- Let the initial adopters be $S = u, v$ (two nodes that start with A).
- In the network shown (figure on the right), starting from S we observe that eventually **every node adopts A**. The innovation percolates through the entire network
- This is a **complete cascade** at threshold q.



(a) *The underlying network*

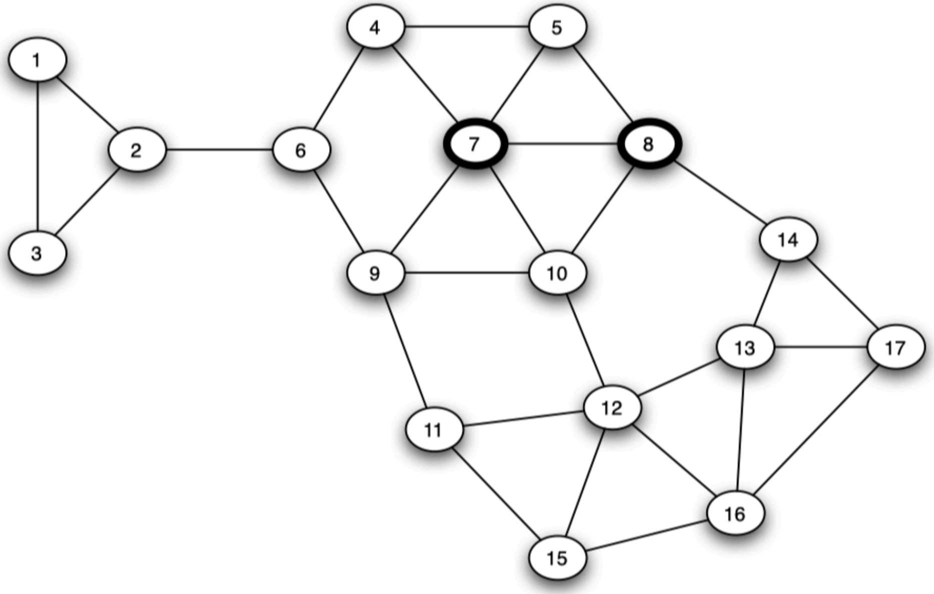
(b) *Two nodes are the initial adopters*

(c) *After one step, two more nodes have adopted*

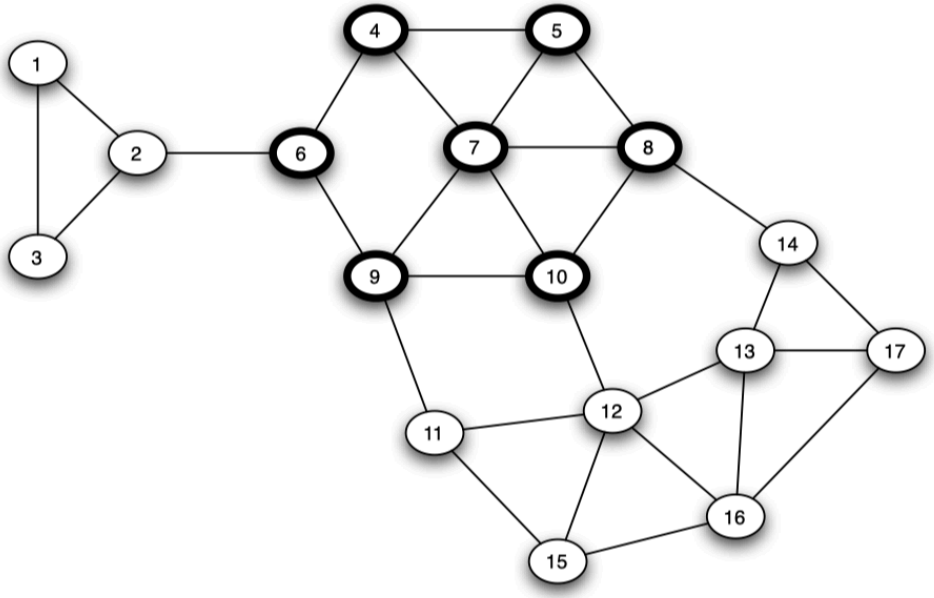
(d) *After a second step, everyone has adopted*

Example: Partial Cascade

- Same payoffs $a = 3, b = 2$ ($q = 2/5$) and initial seed $S = u, v$, but a slightly different network structure.
- In this case, A spreads to some additional nodes but **stops before reaching everyone**. The diffusion of A stops after a few rounds; some nodes remain using B indefinitely.
- This is a **partial cascade** – A gains some foothold but cannot overcome the resistant part of the network.



(a) *Two nodes are the initial adopters*



(b) *The process ends after three steps*

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Recap: Cascade Dynamics

- We have a process where:
  - Start with a set S of initial adopters of A (all others initially B).
  - Iteratively, any node with at least a q fraction of neighbors on A will switch from B to A.
  - Nodes that switch to A stay with A thereafter (we assume no switching back to B in this model).
- **Complete cascade (at threshold q):** If eventually every node ends up adopting A (B → A everywhere). Then we say the initial set S *causes* a complete cascade at threshold q.
- **Partial cascade:** If the process converges to a mix of A and B (some nodes remain B), meaning A's spread reaches a limit and stops.
- Key insight: Whether a complete cascade happens depends on network structure relative to q. Highly interconnected clusters of B can resist if the cluster density is high (we formalize this next).
- This model captures phenomena like adoption reaching a "tipping point" and then taking over, versus stalling due to insufficient peer support.

Defining Clusters (as Obstacles)

- To analyze where cascades stop, we introduce the notion of a **cluster** in this context:
- A **cluster of density** p is a set of nodes in the network such that **every node in the set has at least a** p **fraction of its neighbors also in the set**.
  - In other words, each member's neighborhood overlaps significantly with the cluster itself (at least p of their neighbors are also in the cluster).
  - Example: A tightly knit community could form a cluster of high density (close to 1 if everyone is connected).
- Intuition: A cluster of high density means strong internal cohesion—members are connected mostly to each other, with relatively few outside neighbors.

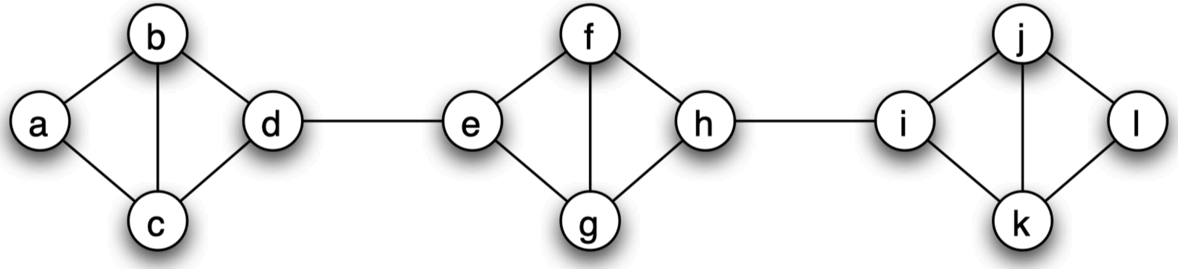
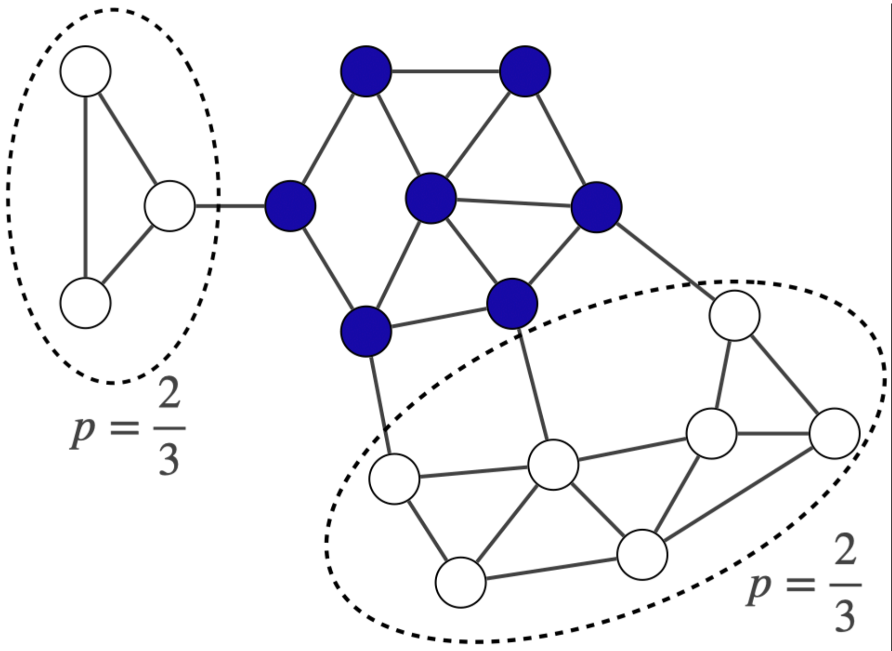


Figure 19.6: A collection of four-node clusters, each of density 2/3.

Stopping Cascades: Intuition

- **What prevents a cascade from spreading to the whole network?**
  - A cascade will fail to propagate if it encounters a subnetwork where newcomers (A-adopters) can't get a foothold.
  - **Homophily and community structure** can be barriers: an innovation introduced outside a tightly-knit community may have trouble penetrating that community.
- A tightly connected cluster of B-users can resist A if its internal links dominate external links.
  - Each member sees so many neighbors sticking with B that $p_A < q$ (fraction of neighbors on A stays below threshold).



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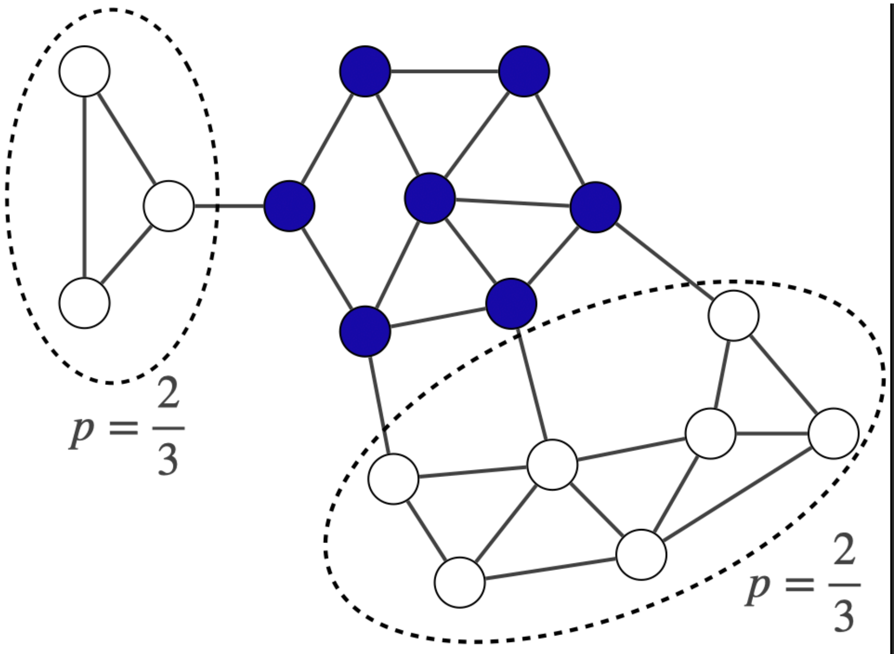
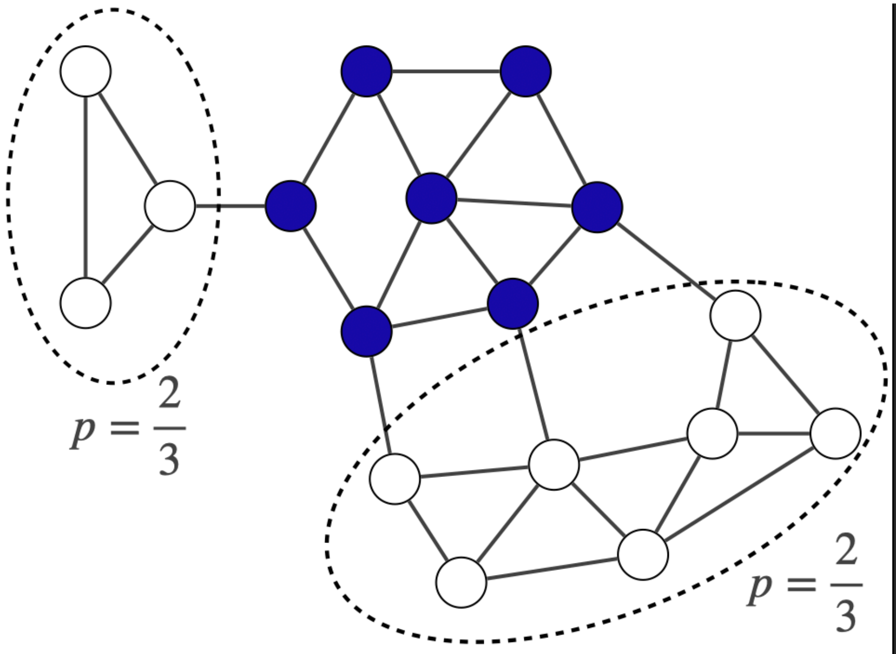
Stopping Cascades: Intuition (2)

- We quantify this with cluster density:
  - A **cluster of density** p (recall definition) provides "internal cohesion" for B: everyone in it sees at least p of neighbors inside the cluster (thus at most $(1-p)$ outside).
  - If $(1-p)$ (the max fraction of outside neighbors) is below the threshold q, then no one in the cluster will ever switch to A. Each member is always short of enough A-influence.



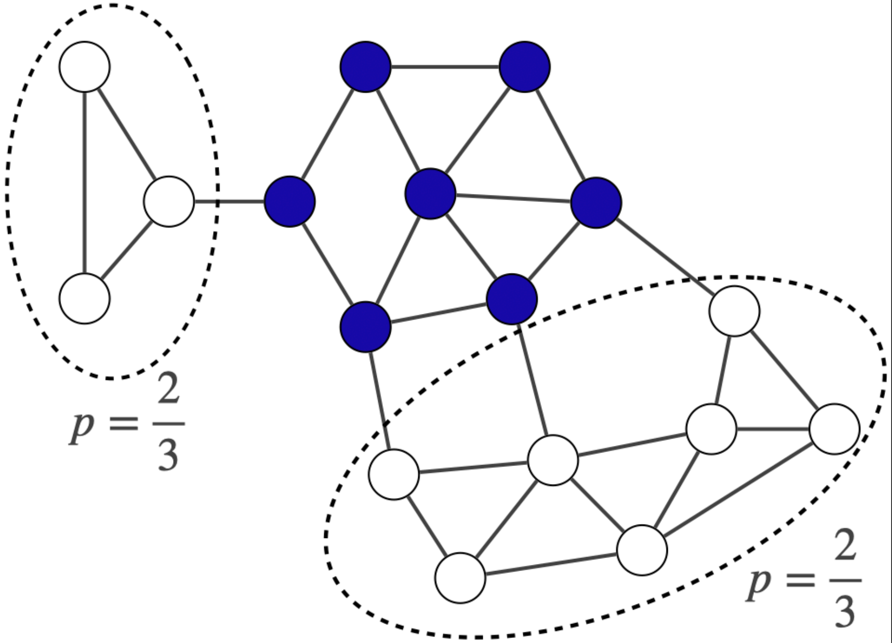
Internal Cohesion and Diffusion

- **Internal cohesion:** Nodes in a cluster support each other's current behavior. Each node has a significant fraction of friends within the same cluster (hence likely sharing the same behavior initially).
  - Note: Nodes in a cluster need not be identical or share an ideology; it's about network structure (who's connected to whom).
  - Extreme cases:
    - The entire network is a cluster of density $p = 1$ (trivial: everyone is interconnected).
    - The union of two clusters of density p is also a cluster of at least that density p. Clusters can exist at multiple scales overlapping.



Internal Cohesion and Diffusion (2)

- For diffusion: A cluster with high density can serve as an "immune system" against external contagion:
    - So long as each node in the cluster sees enough peers in B (the cluster's behavior) to stay below threshold q for A, they will all resist switching.
    - More formally, if a cluster's density p is greater than $(1-q)$, then each member has fewer than q fraction neighbors outside. So even if all outside neighbors adopt A, it's still not enough pressure.



Clusters and Cascades: Formal Relationship

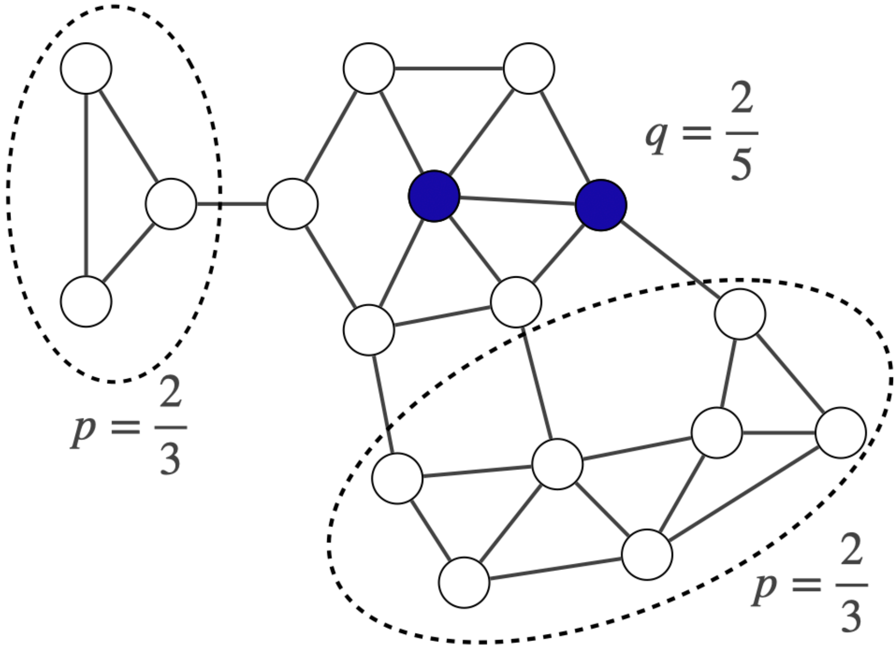
- **Key claim connecting clusters to cascades**:

  i. If the remaining network (after removing initial adopters S) contains a cluster of density $p > (1 - q)$, then S **cannot trigger a complete cascade**.

  - Such a cluster has too strong internal support for B, halting the spread of A.

  ii. Conversely, if S fails to cause a complete cascade, then there must exist a cluster in the network (among those who remained B) with density $p > (1 - q)$.

  - In other words, whenever a cascade stops short, it's precisely because it ran into a cluster dense enough to block it.



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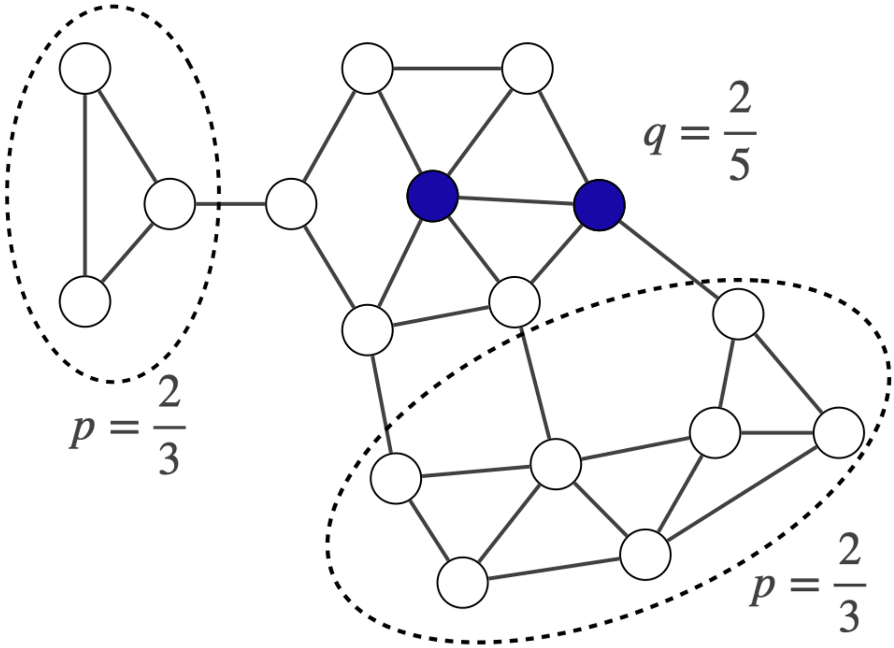
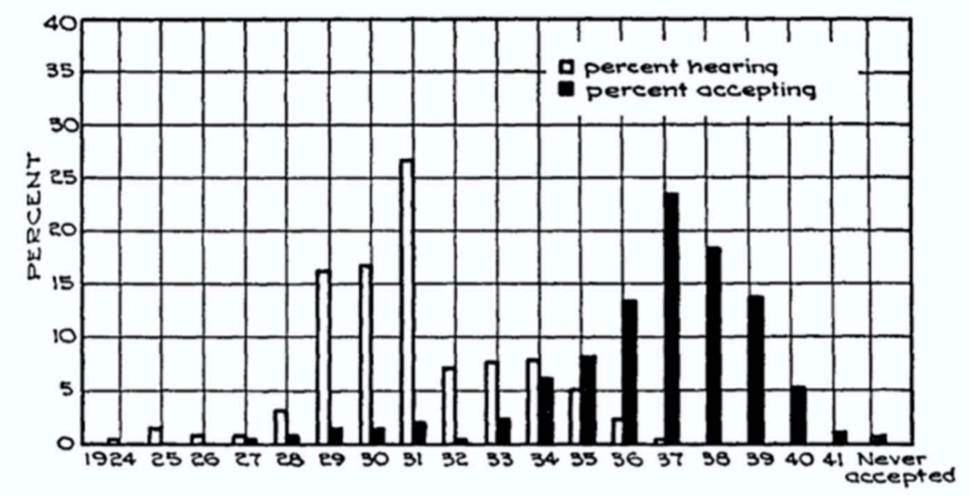
Clusters and Cascades: Formal Relationship (2)

- Therefore, **clusters are the natural obstacles to cascades**:
  - A cluster of density greater than $(1-q)$ is sometimes called a **blocking cluster** (it blocks A's spread).
  - This gives a structural criterion to evaluate if a given initial seed set S will fully take over or not.



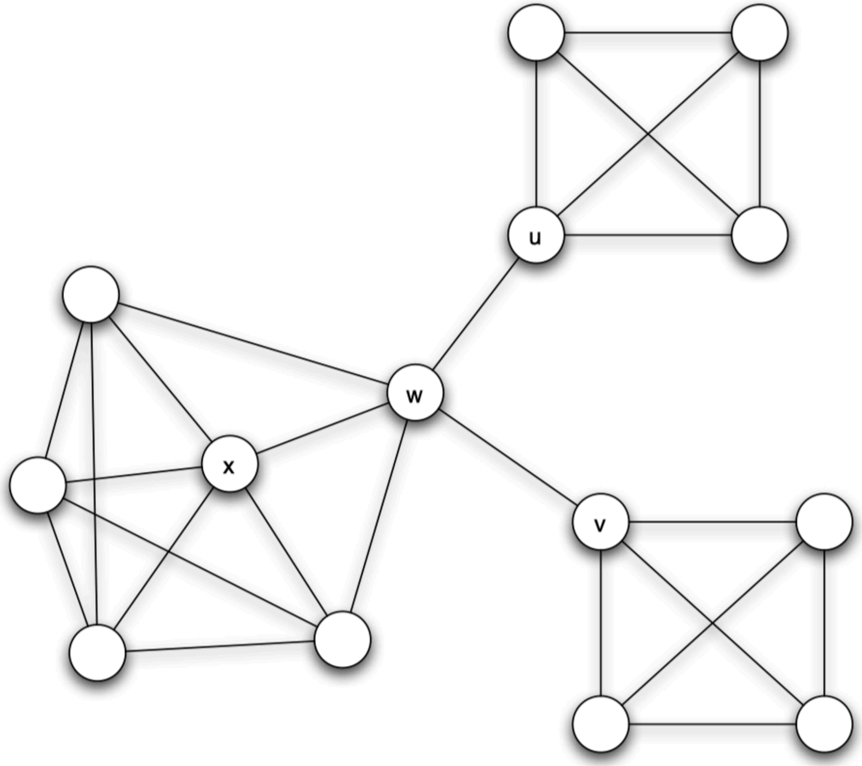


- **Historical example – Ryan & Gross (1943):** Studied adoption of hybrid seed corn in Iowa. Farmers learned about the innovation (many heard about hybrid seeds), yet adoption lagged until they saw enough peers try it successfully.
    - Demonstrates the gap between awareness and adoption – similar to individuals on the edge of a cluster who hear about A but don't switch because not enough of their close peers have switched.
- This is analogous to nodes on the boundary of a cluster: they get exposed to the new idea (from outside neighbors) but *still* don't adopt because their internal ties dominate and reinforce the old behavior.
    - In the seed corn study, even after almost all farmers knew about hybrid corn, some waited to see a larger fraction of their neighbors adopt before they did (a threshold behavior).
- **Takeaway:** Viral marketing must often overcome this reinforcement problem: simply getting the word out (information) isn't enough if people are waiting for social proof from their network.

Reading: Ryan, B. & Gross, N. (1943). "The diffusion of hybrid seed corn in two Iowa

The Role of Weak Ties

- Threshold models shed light on Granovetter's **"strength of weak ties"** theory:
  - **Weak ties** (acquaintanceships bridging different communities) are great for spreading simple information widely (news, gossip) because they connect distant parts of the network.
  - But for behaviors that require reinforcement (our threshold q models that need multiple confirmations), weak ties alone might not be enough to trigger adoption.
- Nodes connected by weak ties often get the *first news* of innovations from outside their circle, but one weak tie might not provide enough peer pressure to adopt if q is high. For example, if $q = 1/2$, a single friend (especially an outsider) isn't sufficient.



- **Strong ties** (close friends/family) often cluster together, providing redundant signals. Thus, strong ties can be more influential in causing actual behavior change, even if the idea originally came via a weak tie.
- Reference: Centola, D., & Macy, M. (2007). Complex contagions and the weakness of long ties. *American Journal of Sociology, 113*(3), 702–734. Their work showed that high clustering can facilitate diffusion of behaviors requiring social reinforcement.

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Simple vs. Complex Contagion

- **Simple contagion:** One contact might be enough to transmit the “infection.” Analogy: diseases or viral memes. In network models:
    - *Independent Cascade model:* each infected node gets a chance to infect each neighbor (one-to-one chance-based influence). This does not require multiple confirmations – just one successful transmission event.
    - *Simple threshold case:* effectively q is low enough (or probabilistic contagion) that a single activation can trigger you.
    - Weak ties are very powerful here: one connection can spread it onward.
- **Complex contagion:** Adoption requires multiple reinforcing signals. Our threshold model is a prime example:
    - If $q > 0.5$, you need more than one neighbor to convince you (a “critical mass” in your neighborhood).
    - Each additional friend adopting increases the likelihood you do – there’s synergy or reinforcement.
- In complex contagion, *each new adopting neighbor has greater influence than the previous ones*. The first friend might not sway you, but the third or fourth might tip you over.
- **Key distinction:** Simple contagions thrive on network reach (many long-range weak ties = good). Complex contagions thrive on clustered reinforcement (multiple friends in the same circle adopting).

Health Behavior and Artificial Communities (Centola 2010 Experiment)

- Sociologist **Damon Centola (2010)** ran a landmark experiment on health behavior diffusion in online communities:
  - Recruited 1,528 participants interested in health, and created an online "health community."
  - Participants were randomly placed into one of two network conditions:
    a. A **clustered lattice** network (highly clustered, lots of local ties, longer path lengths).
    b. A **random network** (low clustering, many long-range ties, shorter path lengths akin to "small-world").
  - Each participant had a few "health buddies" (neighbors) in their network condition and could see their activities.
- Participants were asked to consider adopting a health behavior (registering for a health forum site).
  - When a participant's buddy adopted (registered), they'd get a notification (signal).
  - If multiple buddies adopted, they received multiple signals (one per adopting neighbor).

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Health Behavior and Artificial Communities (Centola 2010 Experiment) (2)

- Thus, Centola’s setup directly tests simple vs. complex contagion:
    - In the random network, individuals are likely to have *single* contacts from various parts of the network (weak ties).
    - In the clustered network, individuals might receive *redundant* signals from a tight-knit cluster of buddies.

Reading: Centola, D. (2010). “The Spread of Behavior in an Online Social Network Experiment.” Science 329(5996):1194-1197.

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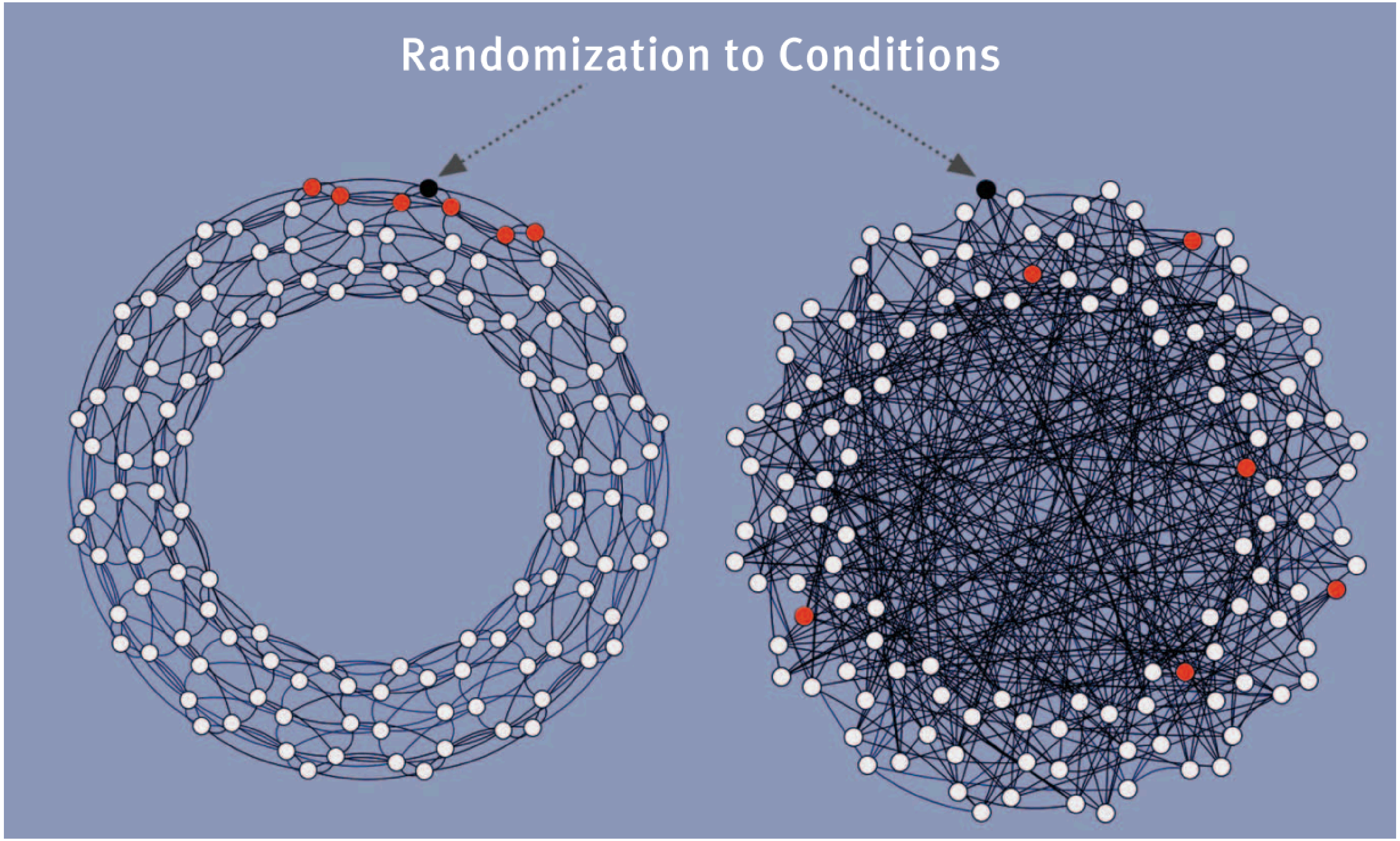


Results of Centola's Experiment

Findings: Network structure had a significant effect on behavior diffusion.

- Adoption reached a significantly **higher fraction of people in clustered networks** than in random networks. Example: in one condition ~54% adoption in clustered vs ~38% in random.
- **Speed:** The behavior diffused faster in clustered networks as well. On average, diffusion in clustered nets was more than four times faster than in random nets.
- **This is opposite to what we'd expect for a simple contagion.** It supports the complex contagion idea: redundant signals spur adoption.
- At the individual level, **redundant signals greatly increased adoption probability**. If you had two or three friends adopting, you were much more likely to follow than if you had just one.

- clustered-lattice (solid black circles) and random (open triangles)
- Z is the number of health buddies each person had
- N population size

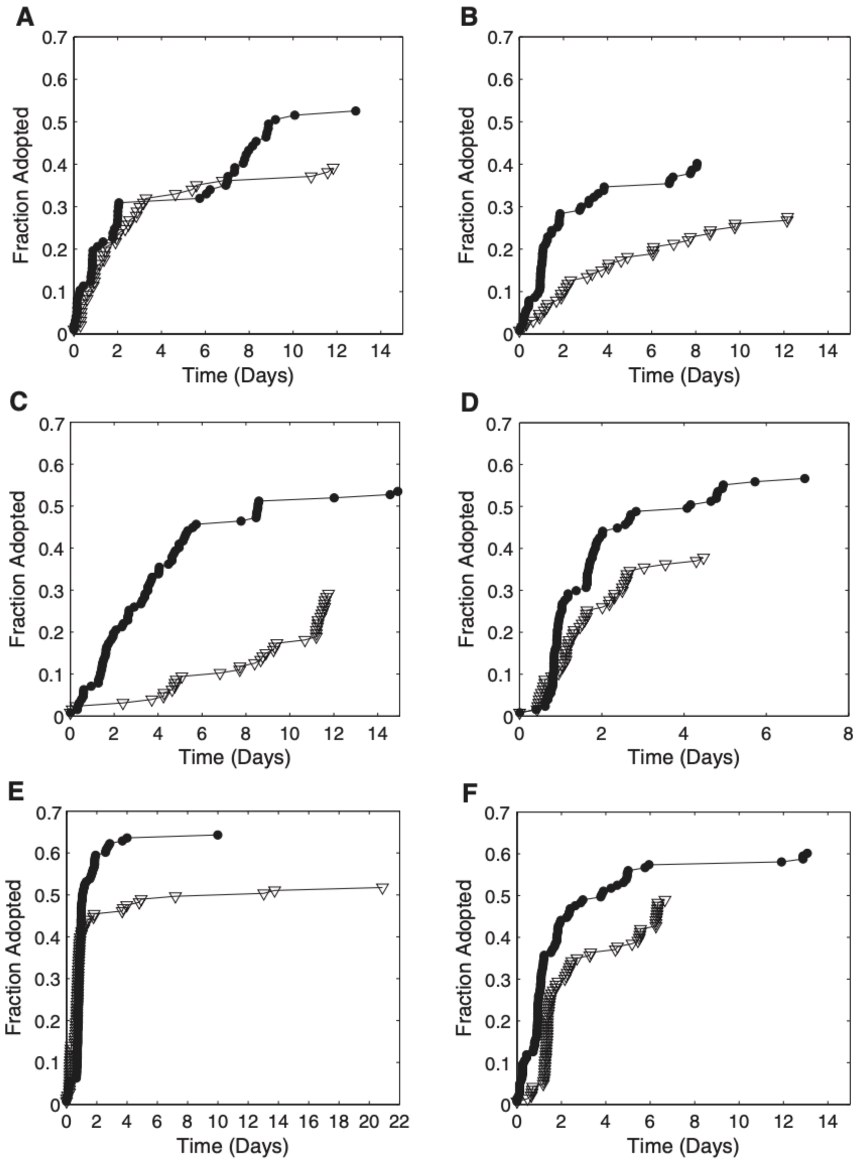
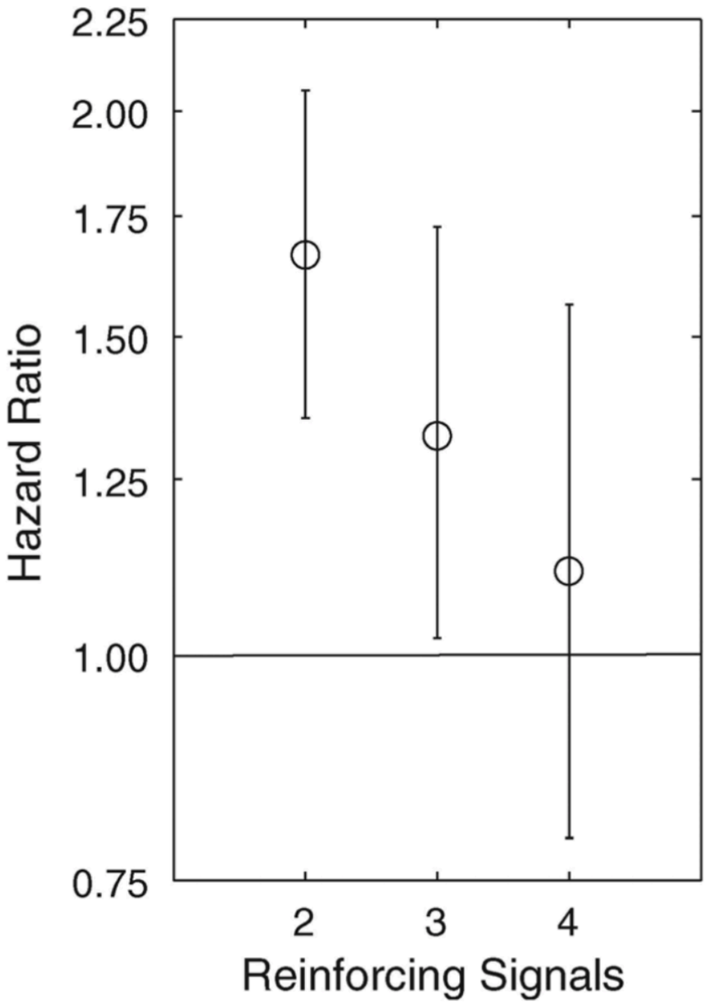
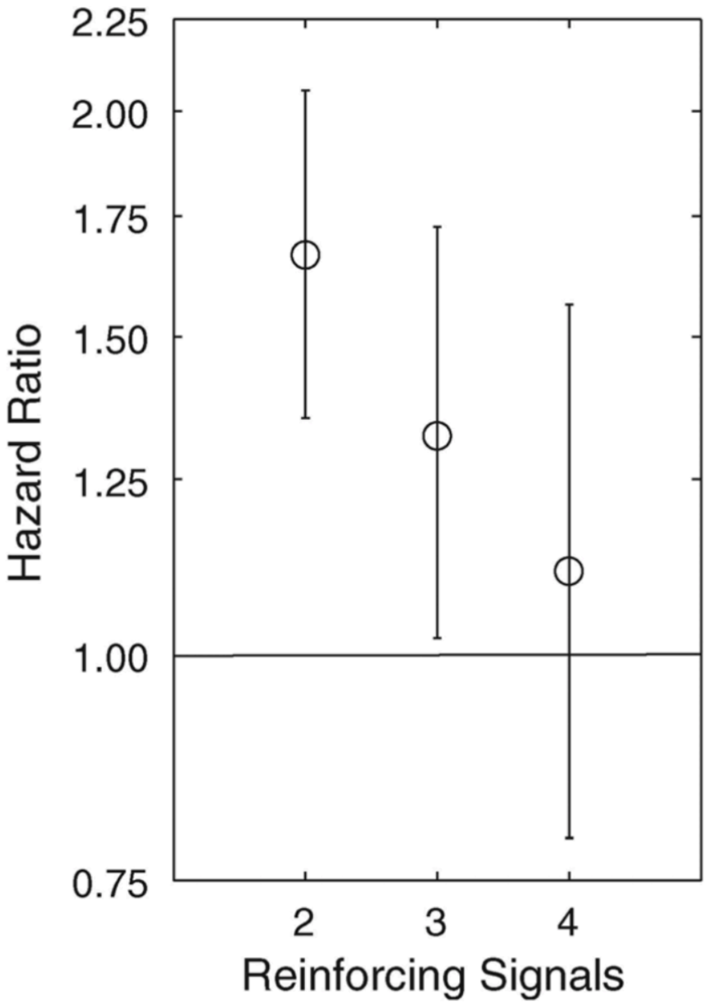


Fig. 2. Time series showing the adoption of a health behavior spreading through clustered-lattice (solid black circles) and random (open triangles) social networks. Six independent trials of the study are shown, including (**A**) $N = 98$, $Z = 6$, (**B** to **D**) $N = 128$, $Z = 6$, and (**E** and **F**) $N = 144$, $Z = 8$. The success of diffusion was measured by the fraction of the total network that adopted the behavior. The speed of the diffusion process was evaluated by comparing the time required for the behavior to spread to the greatest fraction reached by both conditions in each trial.

- The **hazard ratio** g quantifies how additional social signals affect adoption:
    - g > 1 indicates each extra neighbor adopting multiplies your chance of adopting. For example, if $g = 3$, two neighbors adopting makes you 3 times as likely as with one neighbor.
    - In the experiment, g was significantly > 1, confirming **social reinforcement**: each extra adopting friend sharply raised the probability of adoption.
- **Conclusions**:
    - Individual adoption is improved by having reinforcing signals from multiple social ties.
    - System-level: clustered networks (which provide those reinforcing ties) achieved larger and faster diffusion compared to random networks.
    - For simple contagions (like info or disease), redundant ties are unnecessary. But for complex contagions (behavior change, social movements), redundancy can make diffusion *more efficient*.



- **Implication for interventions:** To promote a new health behavior, it might be more effective to seed in a clustered community (ensuring people see multiple neighbors adopt) than to rely on a few influencers spreading the word broadly.
    - E.g., target tightly-knit residential communities for interventions like exercise programs or healthy eating, rather than hoping a mass media campaign alone will do it.



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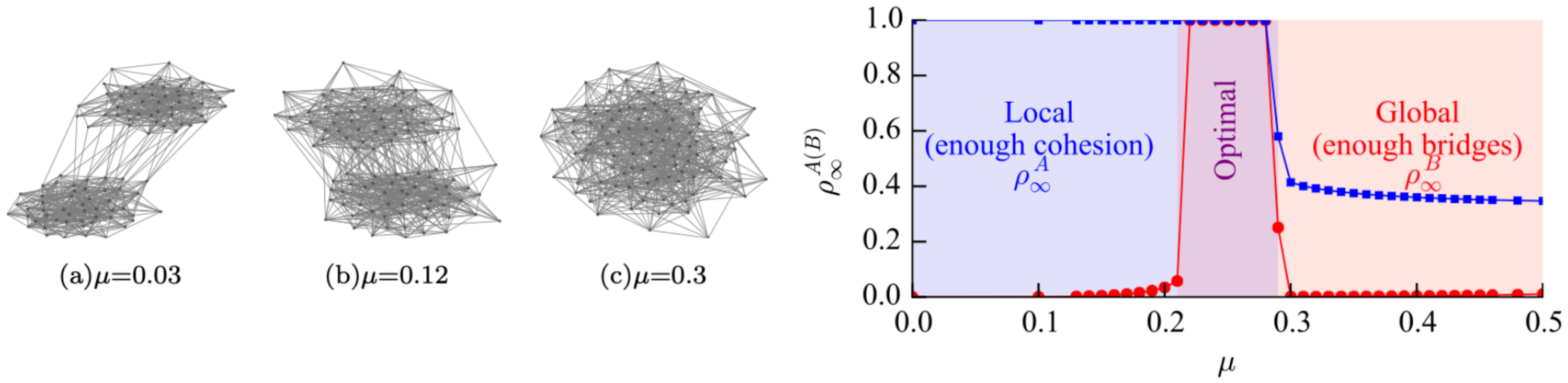
Finding the Optimal Clustering for Spreading



- There is a **trade-off** between intra-community spreading and inter-community spreading:
  - **High modularity / strong communities (low inter-connectivity, small μ):** Great for *within* community spread (local cascades easily among similar nodes) but poor for jumping to other communities.
  - **Low modularity / weak communities (lots of bridging ties, large μ):** Easier for something to jump between communities (global reach), but without local reinforcement it might not fully take off in each community.

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- Research by Nematzadeh et al. (2014) suggests an **optimal intermediate level of modularity** for diffusion:
    - At this optimal point, communities are clustered enough that social reinforcement works within them, yet enough inter-community links exist that once one community “ignites,” others can catch the flame.
    - Global diffusion requires minimal seeds at this sweet spot of modularity.
- In the figure:
    - μ represents fraction of edges that are between communities.
    - ρ_∞^A (and ρ_∞^B) represent final fractions adopting A (or B) in communities A and B.
    - Notice how intermediate μ yields both communities ending with high ρ_∞^A.
- **Takeaway:** Neither a completely fragmented network nor a completely random network is best for complex contagions. Some community structure helps, but too much creates blocking clusters. There’s an optimal balance.

Reading: Nematzadeh et al., “Optimal Network Modularity for Information Diffusion” (Phys. Rev. Lett. 113, 088701, 2014).

Extensions of the Basic Cascade Model

Heterogeneous Thresholds

- So far, we assumed everyone has the same payoff values a, b, hence the same threshold $q = b/(a+b)$. In reality, individuals value behaviors differently:
  - Some people might require fewer neighbors to adopt before they do (lower q – more *easily influenced* or find A very appealing).
  - Others need a lot of friends doing it before they switch (high q – *stubborn* or find A only marginally better than B).
- We can extend the model: each person v has their own payoff a_v for A and b_v for B, giving them a personal threshold $q_v = \frac{b_v}{a_v+b_v}$.

Heterogeneous Thresholds (2)

- Now the population is heterogeneous:
  - "Early adopters" might have low q_v (they switch with very little peer support or unilaterally if $a_v >> b_v$).
  - "Laggards" might have q_v near 1 (they need almost everyone around them to adopt first).
- This is closer to reality and connects to diffusion of innovations theory (innovators vs. early majority, etc., have different thresholds in effect).

$$
\begin{array}{cc|c|c|}
 & & \multicolumn{2}{c}{w} \\
 & & A & B \\
\hline
\multirow{2}{*}{v} & A & a_v, a_w & 0, 0 \\
\hline
 & B & 0, 0 & b_v, b_w \\
\hline
\end{array}
$$

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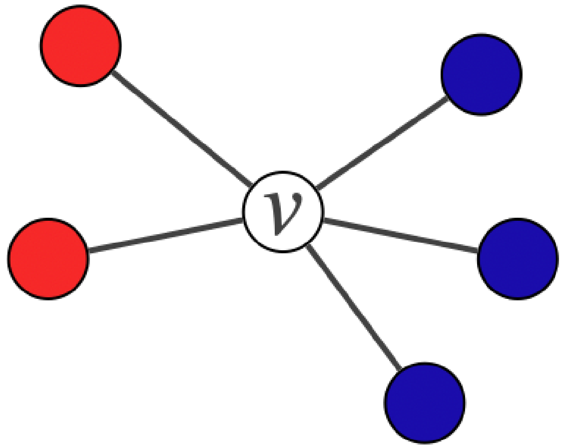
Heterogeneous Thresholds (Formalism)

- For a given node v:
    - Let p = fraction of neighbors adopting A (as before),
    - d = number of neighbors,
    - Payoffs: a_v, b_v (node's subjective benefit for coordinating on A vs B with a neighbor).
- v chooses A if
  $p \cdot d \cdot a_v \geq (1-p) \cdot d \cdot b_v$,
  $p \geq \frac{b_v}{a_v+b_v} = q_v$.
- Thus each node has its own threshold q_v. The diffusion dynamic now depends on the distribution of thresholds across the network.



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Blocking clusters with heterogeneous q

- A cluster generalization: A set C is a blocking cluster if every node v in C has more than $(1 - q_v)$ fraction of neighbors in C.
  - That is, each node is well supported given *its own* threshold.
- This is trickier since one very stubborn person (q_v near 1) can raise the needed density for the cluster.
- Takeaway: **Heterogeneity can either help or hinder cascades. A few low-threshold nodes (social "spark plugs") can get things going, but a few high-threshold individuals can hold up a region unless others circumvent them.**

Blocking clusters with heterogeneous q (2)

- Watts and Dodds (2007) argued we should consider not just **influentials** with low q, but also the **network of easily influenced people**.
- **Influentials hypothesis:** A small number of special individuals (influencers) drive trends and adoption
- **Their counter-argument:** The critical mass of easily influenced people may matter more

Two key groups in cascades:

Easily Influenced People:

- Individuals with **low thresholds** (q_v)
- Adopt with minimal social proof/peer pressure
- May have average or modest network connectivity
- Act as "social spark plugs" initiating adoption chains

Blocking clusters with heterogeneous q (3)

Super-Influencers:

- Individuals with many connections or high status
- Traditionally called "influencers" or "opinion leaders"
- Identified by network centrality measures
- Long believed to be essential for spreading innovations
- Large cascades often depend more on a critical mass of easily influenced individuals than a few super-influencers

Reading: Watts, D. & Dodds, P. (2007). "Influentials, Networks, and Public Opinion Formation."

J. of Consumer Research 34(4): 441–458.

Knowledge, Thresholds, and Collective Action

From Local to Global Coordination

- Not all diffusion processes are purely local. For collective action problems (protests, revolutions, etc.), people consider:
    - **Local network cues:** "How many of my neighbors will join?"
    - **Global expectations:** "How many people in total will participate?"
- The following aspects form an integrated framework for analyzing collective action in networks:
    - **Pluralistic ignorance:** When everyone privately supports action but believes others don't
    - **Knowledge thresholds:** People join when they believe enough others will join
    - **Common knowledge:** Information that everyone knows everyone knows
- **Key references:** Chwe (2000), "Communication and Coordination in Social Networks"; Granovetter (1978), "Threshold Models of Collective Behavior"

Pluralistic Ignorance and Collective Action Thresholds

- **Collective action problem:** An individual benefits from participating only if enough others also participate
  - Example: Protests require critical mass to succeed and avoid repression
  - **Reference:** Kuran (1991), "Now Out of Never: The Element of Surprise in the East European Revolution"
- **Individual decision rule:** "I'll join if I expect at least k_i total people will join" (a global threshold)
  - Unlike our previous model focusing on fraction of neighbors, here people care about absolute numbers
  - k_i varies across individuals (some require massive participation, others need just a few)
- **Pluralistic ignorance:** Everyone privately wants change but believes others support the status quo
  - Example: In authoritarian regimes, people hide their true preferences
  - **Reference:** Noelle-Neumann (1974), "The Spiral of Silence"

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Network Structure and Knowledge Constraints

- Individuals have limited information about others' thresholds:
  - You typically know only your immediate neighbors' thresholds
  - You must estimate global participation based on limited local information
- **Two key network variables affect collective action:**
  - **Network topology:** Who communicates with whom
  - **Threshold distribution:** What level of participation each person requires
- **When your neighborhood knowledge is insufficient:**
  - If you know 5 neighbors will join but your threshold is 10, you face uncertainty
  - **Strategies under uncertainty:**
    - **Conservative strategy:** Only join if your known participants exceed threshold
    - **Speculative strategy:** Join if your neighbors' commitments suggest threshold might be met
    - **Sequential revelation:** People gradually reveal intentions as others do
- **Reference:** Siegel (2009), "Social Networks and Collective Action", American Journal of Political Science

Thresholds and Information Cascades

- The neighborhood < threshold problem can be addressed through **cascading revelation** of intentions:
  - **First movers:** Individuals with very low thresholds (k_i = 1 or 2) act first
  - **Second wave:** Their action signals commitment to their neighbors
  - **Cascade:** If enough neighbors see participation, those with higher thresholds join
  - **Reference:** Lohmann (1994), "Dynamics of Informational Cascades"
- **Network position matters:**
  - **Bridges** between clusters can spread commitment information
  - **Clustered groups** provide reinforcing signals if early adopters are concentrated
  - The cascade can stall if:
    - No one has sufficiently low threshold to start (k_i is high for everyone)
    - Network disconnects prevent information flow
    - The distribution of thresholds creates gaps no one is willing to cross

Common Knowledge and Coordination Problems

- **Common knowledge:** Information that is not just known by all, but everyone knows that everyone knows it (and so on)
  - More formal: A fact P is common knowledge if everyone knows P, everyone knows everyone knows P, etc.
  - **Reference:** Lewis (1969), "Convention"; Chwe (2001), "Rational Ritual"
- Common knowledge helps solve collective action problems by:
  - Eliminating uncertainty about others' knowledge
  - Creating focal points for coordination
  - Reducing the risk of being the "lone protester"
- **Institutional examples that create common knowledge:**
  - **Mass media:** Public broadcasts ensure everyone sees the same information simultaneously
  - **Public ceremonies:** Everyone witnesses the same event together
  - **Social media:** Can sometimes create common knowledge when content "goes viral"
  - **Reference:** Morris & Shin (2002), "Social Value of Public Information"

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Common Knowledge in Practice

- **Historical examples:**
  - **Arab Spring:** Social media created common knowledge about widespread discontent
  - **1989 East German protests:** Leipzig Monday demonstrations created common knowledge
  - **Reference:** Tufekci (2017), "Twitter and Tear Gas"
- **Creating common knowledge through institutions:**
  - **Freedom of assembly:** Allows people to visibly show numbers (signaling viability)
  - **Free press:** Broadcasts information uniformly to population
  - **Public squares:** Physical spaces for creating mutual awareness
  - **Digital technologies:** Can either facilitate or hinder common knowledge
- **Network implications:** Common knowledge effectively creates a complete graph where everyone "hears" everyone else
  - It can shortcut the need for cascade processes
  - It radically reduces the reliance on network structure

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The Cascade Capacity: Understanding Diffusion Limits

- So far we've analyzed cascades in finite networks. But what are the **fundamental limits** to diffusion in very large networks?
- **Cascade capacity** is a theoretical concept that helps us understand these limits:
  - Defined as the **maximum threshold** q such that a finite set of initial adopters can trigger a complete cascade throughout an infinite network
  - It measures how resistant a network is to complete behavioral takeover
  - Higher capacity means the network structure better facilitates diffusion
- **Key question:** Given a specific network structure, what's the largest threshold that still permits complete cascades from small seeds?
  - This helps us understand when global adoption is theoretically possible
  - It identifies the structural properties that enable or inhibit diffusion
- **Reference:** Morris, S. (2000). "Contagion." *The Review of Economic Studies*, 67(1), 57–78.

Cascade Capacity: Simple Examples

- **Infinite line (1D lattice):**
    - Each node has exactly 2 neighbors
    - If $q > 1/2$: No complete cascades possible from finite seeds
    - If $q \leq 1/2$: Complete cascades are possible
    - Therefore, **cascade capacity = 1/2**
- **Infinite 2D grid:**
    - Each node has 4 neighbors (north, east, south, west)
    - Cascade capacity is also exactly 1/2
    - More complex network structures follow similar patterns
- **Intuition:** If nodes need more than half their neighbors to adopt before they do, cascades will always stop at some point unless you start with infinitely many adopters

Predicting Cascade Capacity

- **For regular networks** (where each node has exactly d neighbors):
  - Cascade capacity = 1/2 regardless of degree d
  - Examples: line graphs (d=2), 2D lattices (d=4), etc.
- **For irregular networks** (varying degrees):
  - Capacity depends on clustering and the distribution of degrees
  - Higher clustering often reduces cascade capacity
  - Hubs (high-degree nodes) typically need greater neighbor adoption to switch
- **Challenge:** For most real-world network structures, calculating the exact cascade capacity is complex
  - Often requires simulation or numerical approaches
  - Depends on both local structure and global connectivity patterns

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Cascade Capacity in Practice

- **Practical implications for diffusion strategies:**
  - **For high-threshold behaviors** $(q > 0.5)$:
    - Viral campaigns alone won't achieve complete adoption
    - External interventions necessary to overcome the cascade capacity limit
    - Example: Technologies requiring significant investment or behavior changes
  - **For low-threshold behaviors** $(q < 0.5)$:
    - Complete cascades possible with the right initial seeds
    - Focus on identifying ideal early adopters
    - Example: Social media platforms where minimal commitment is required
  - **Strategic insight:** For behaviors with thresholds near or above 0.5, create additional incentives to effectively lower the threshold

Another Extension: Compatibility (Bilingual Option)

- Realistically, people can sometimes adopt a new behavior **without completely dropping the old one**. Think of technologies that can coexist:
  - E.g., one can use **both** WhatsApp and traditional SMS, or both a new streaming service and cable TV.
  - In language or convention terms, people can be **bilingual** (use A and B).
- Extension: nodes can choose A, B, **or both (AB)**. This is a form of *partial adoption* or compatibility.
  - E.g., a new product is backward-compatible with the old one, so you can interact with either.
- This is called the **bilingual option** in the literature.

Another Extension: Compatibility (Bilingual Option) (2)

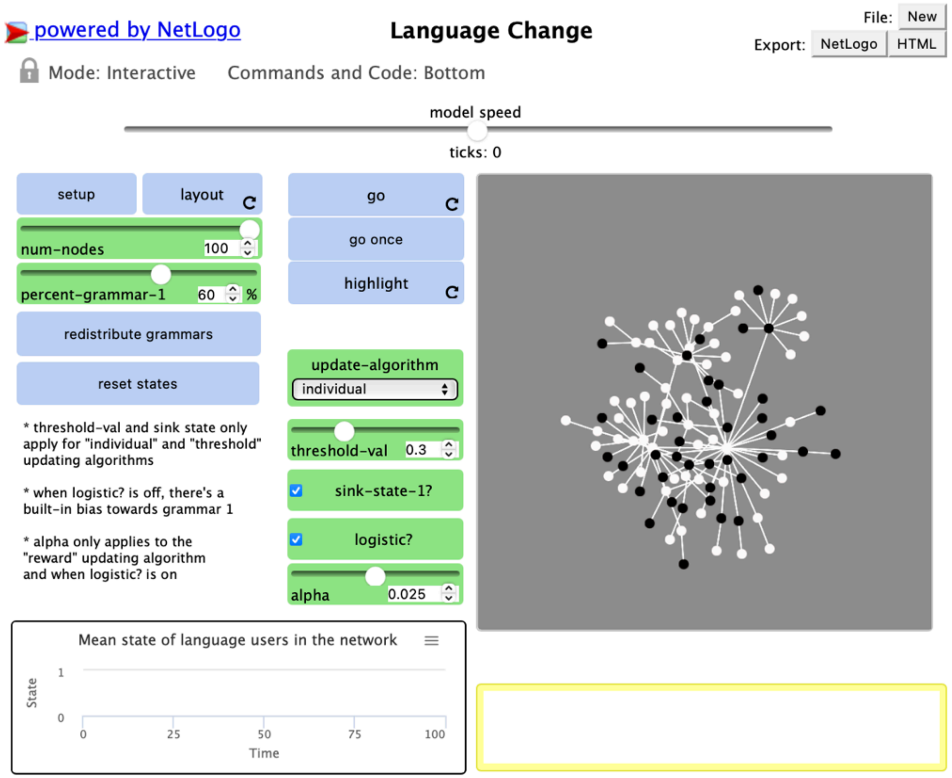
- Surprisingly, even this simple addition can lead to complex effects:
  - Sometimes offering a compatible option can accelerate the adoption of A (since people don't have to risk losing B, they can adopt A gradually).
  - Firms may use compatibility strategically to enter a market dominated by an incumbent, then gradually phase out the old tech.
- We won't go deep into equations, but conceptually:
  - AB users get payoff with both A and B neighbors (some reduced payoff possibly).
  - They can serve as "bridges" in the network, facilitating a later full shift to A if A proves better.

Language Change Model

- A classic example of compatibility is bilingualism in language adoption:
  - Imagine two languages A and B. People can speak either or both.
  - Bilingual individuals (AB) can communicate with both A-monolingual and B-monolingual individuals.
- Typically findings:
  - If there's even a small benefit to coordinating on a single language, bilinguals can act as a catalyst: they adopt the new language A while still interacting with B speakers, gradually tipping B speakers to add A as well, and eventually everyone might switch to A.
  - If A has higher payoff ($a > b$), even a few initial bilinguals can start an increase of A usage.
  - But if bilingualism has a cost or if q remains high, people might stick to B unless many go bilingual.
- **Strategic insight:** Compatibility (bilingual option) can lower the effective threshold for adoption because people don't have to abandon B immediately. It smooths the transition.

Language Change Model (NetLogo Simulation)

NetLogo's Language Change allows playing with parameters like payoff advantages and initial bilingual fraction.



Exploring the NetLogo Language Change Model

1. **Key Parameters to Experiment With:**
    - `a-payoff` and `b-payoff`: The utility of languages A and B (try setting A slightly higher)
    - `cost-of-bilingualism`: The penalty for maintaining two languages (0.0-1.0)
    - `initial-fraction-a` and `initial-fraction-bilingual`: Starting conditions
2. **Experiments to Try:**
    - **Experiment 1:** Set equal payoffs but vary initial bilingual population (0% vs. 20%)
    - **Experiment 2:** Give language A a slight advantage (e.g., a-payoff = 1.1, b-payoff = 1.0)
    - **Experiment 3:** Increase cost-of-bilingualism gradually and observe effects

Exploring the NetLogo Language Change Model (2)

3. **Observations to Record:**
   - How quickly does a language spread with/without bilinguals?
   - Under what conditions does the system stabilize with mixed usage?
   - What combination of parameters leads to complete takeover by language A?
   - How does network structure (try different values of "neighborhood") affect outcomes?
4. **Connection to Theory:** Observe how bilingualism serves as a "bridge" that eventually enables a complete cascade to language A, even when direct A adoption would be blocked by cluster effects

Reading Material

[NS2] Chapter 19 of "Networks, Crowds, and Markets" by D. Easley & J. Kleinberg – *Cascading Behavior in Networks*. Sections 19.1–19.5 cover diffusion games, cascades, clusters, and extensions, with examples and proofs.