E & M Review

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1 Electrostatics

1.1 E Fields

1.1.1 Definitions

- Quantization of charge:

$$Q = Ne$$

- Coulomb's law:

$$F_E = \frac{kq_1q_2}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}$$

- Electric field: region of space in which charged particle experiences electric force

$$E = \frac{F_E}{q}$$

1.1.2 E Field Derivations

- Finite line of charge:

$$E = \frac{kQ}{x_0(x_0 - L)}$$

- Above infinite line:

$$E = \frac{2k\lambda}{r}$$

- Above ring of charge:

$$E = \frac{kQZ}{(R^2 + Z^2)^{3/2}}$$

- At center of arc of charge:

$$E = \frac{2k\lambda\sin\theta}{R}$$

- Above disk of charge:

$$E = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{Z}{(R^2 + Z^2)^{3/2}}\right]$$

- Above infinite sheet:

$$E = \frac{\sigma}{2\epsilon_0}$$

1.1.3 Electric Flux

- Gauss' Law:

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$

1.1.4 Gauss' Law E Derivations

- Point charge:

$$E = \frac{kQ}{r^2}$$

- Conducting sphere: same for hollow/solid;

$$E = \frac{kQ}{r^2}$$

- Insulated sphere:
  - Hollow:

$$E = \begin{cases} 0 & r < R \\ \frac{kQ}{r^2} & r > R \end{cases}$$

  - Solid, uniform ρ:

$$E = \begin{cases} \frac{kQr}{R^3} & r < R \\ \frac{kQ}{r^2} & r > R \end{cases}$$

- Infinite line:

$$E = \frac{2k\lambda}{r}$$

- Hollow infinite cylinder: same for conducting/insulating;

$$E = \begin{cases} 0 & r < R \\ \frac{2k\lambda}{r} & r > R \end{cases}$$

- Solid infinite cylinder:
  - Conducting:

$$E = \begin{cases} 0 & r < R \\ \frac{2k\lambda}{r} & r > R \end{cases}$$

  - Insulating:

$$E = \begin{cases} \frac{2k\lambda r}{R^2} & r < R \\ \frac{2k\lambda}{r} & r > R \end{cases}$$

- Infinite sheet:

$$E = \frac{\sigma}{2\epsilon_0}$$

- Parallel sheets:

$$E = \frac{\sigma}{\epsilon_0}$$

1.2 Electric Potential

1.2.1 Definitions

- Electric potential:

$$\Delta V = -\int_{\infty}^{r} E \cdot dr$$

- Work:

$$W = -q\,\Delta V$$

- Point charge/generalization:

$$V = \frac{kQ}{r} = k\int \frac{dq}{r}$$

- Parallel plates:

$$V = -E \cdot s = Ed$$

1.2.2 V Derivations

- Above line of charge:

$$V = k\lambda \ln\left[\frac{L + (L^2 + d^2)^{1/2}}{d}\right]$$

- Ring of charge:

$$V = \frac{kQ}{(R^2 + x^2)^{1/2}}$$

- Disk of charge:

$$V = 2k\sigma\pi\left[(R^2 + x^2)^{1/2} - x\right]$$

1.2.3 V Derivations Using E

- Charged hollow sphere:

$$V = \begin{cases} \frac{kQ}{R} & r < R \\ \frac{kQ}{r} & r > R \end{cases}$$

- Charged solid insulating sphere, uniform ρ:

$$V = \begin{cases} \frac{3kQ}{2R} - \frac{kQr^2}{2R^3} & r < R \\ \frac{kQ}{r} & r > R \end{cases}$$

2 Circuits

2.1 Capacitance

2.1.1 Definitions

- Capacitance:

$$C = \frac{\epsilon_0 A}{d} = \frac{Q}{V} = \kappa\, C_0$$

- Energy: $E = \frac{1}{2}CV^2 = \frac{Q^2}{2C} = \frac{1}{2}QV$

2.1.2 CQVE Circuits

- Series:
  - $C_{tot} <$ any C

$$\frac{1}{C_{tot}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ...$$

  - Q same
  - V adds up
- Parallel:
  - $C_{tot} >$ any C

$$C_{tot} = C_1 + C_2 + C_3 + ...$$

  - V same
  - Q adds up

2.1.3 C Derivations

- Parallel plate:

$$C = \frac{\epsilon_0 A}{d}$$

- Cylindrical:

$$C = \frac{2\pi\epsilon_0 L}{\ln(\frac{r_o}{r_i})}$$

- Spherical shells:

$$C = 4\pi\epsilon_0 \frac{r_o\, r_i}{r_o - r_i}$$

- Isolated sphere:

$$C = 4\pi\epsilon_0 r_i$$

2.1.4 Dielectrics

$$E = \frac{E_0}{\kappa} \iff \int E \cdot dA = \frac{q_{enc}}{\kappa\epsilon_0}$$

2.2 VIRP Circuits

$$I = \frac{dQ}{dt} = n\,q\,v_d\,A$$

2.2.1 Kirchhoff's Rules

1. Loop Rule: $\Sigma V_{gains} = \Sigma V_{drops}$
   - Battery voltage = sum of voltages in loop connected to battery
2. Junction Rule: $\Sigma I_{in} = \Sigma I_{out}$
   - Current that enters junction must split between paths

2.2.2 Resistors

- Ohm's Law: $V = \varepsilon = IR$; $R = \frac{\rho l}{A}$
- Power: $P = IV = I^2 R = \frac{V^2}{R}$
- Series:
  - $R_{tot} >$ any R

    $$R_{tot} = R_1 + R_2 + R_3 + ...$$
  - I same
  - V adds up
- Parallel:
  - $R_{tot} <$ any R

    $$\frac{1}{R_{tot}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ...$$
  - V same
  - I adds up

2.2.3 RC Circuits

$$\tau = RC$$

1. Charging:

$$q = \varepsilon\,C\left(1 - e^{-t/\tau}\right)$$

$$I = I_{MAX}\,e^{-t/RC}$$

$$V_R = I_{MAX}\,R\,e^{-t/RC}$$

$$V_C = \varepsilon\left(1 - e^{-t/RC}\right)$$

2. Discharging:

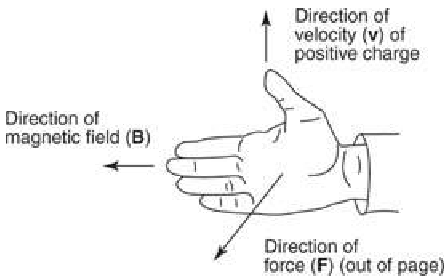
$$q = Q\,e^{-t/RC}$$

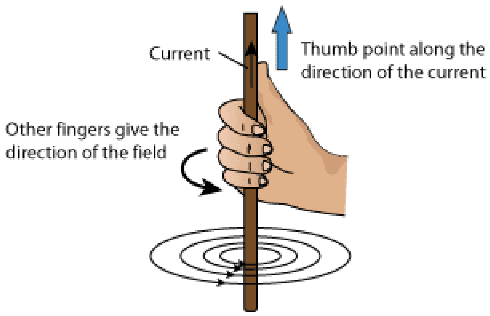
$$I_R = -I_{MAX}\,e^{-t/RC}$$

$$I_C = \frac{Q}{C}\,e^{-t/RC}$$

3 Magnetism

3.1 Right Hand Rule





Right Hand Grip Rule

3.2 Magnetic Force

- Charged particle: $\vec{F}_B = q\,(\vec{v} \times \vec{B})$
- Current-carrying wire: $\vec{F}_B = L\,(\vec{I} \times \vec{B})$
- Use right hand rule to determine direction; F_B, v, and B all mutually perpendicular

3.3 Biot-Savart

- Currents:

$$dB = \frac{\mu_0}{4\pi} \frac{I\,d\ell\,\sin\theta}{r^2}$$

- Point charges:

$$dB = \frac{\mu_0}{4\pi} \frac{q\,v\,\sin\theta}{r^2}$$

3.3.1 Biot-Savart Derivations

- Above finite wire:

$$B = \frac{\mu_0 I}{4\pi}\left(\sin\theta_R - \sin\theta_L\right)$$

- Above infinite wire:

$$B = \frac{\mu_0 I}{2\pi r}$$

- Above loop of wire:

$$B = \frac{\mu_0 I R^2}{2(R^2 + Z^2)^{3/2}}$$

3.4 Ampere's Law

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0\, I_{enc}$$

3.4.1 Ampere's Law Derivations

- Infinite cylinder:

$$B = \begin{cases} \frac{\mu_0 I r}{2\pi R^2} & r < R \\ \frac{\mu_0 I}{2\pi r} & r > R \end{cases}$$

- Solenoid:

$$B = \mu_0\, N I$$

- Toroid:

$$B = \frac{\mu_0 N I}{2\pi r}$$

3.5 No Magnetic Monopoles

$$\oint \vec{B} \cdot d\vec{A} = 0$$

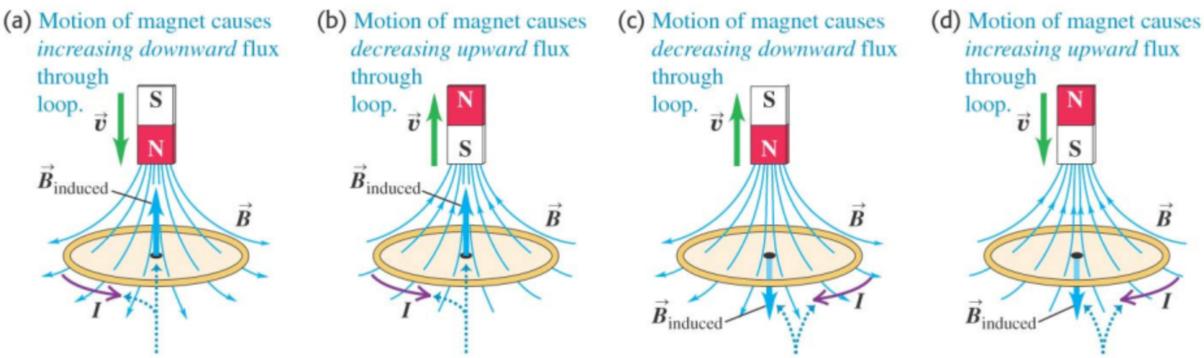
3.6 Faraday's Law

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\int B \cdot dA = -N\frac{d\Phi_B}{dt} \text{ if N turns}$$

$$\varepsilon = \oint \vec{E} \cdot d\vec{\ell}$$

3.7 Magnetic Flux

- Lenz's Law: induced B and I act to oppose change
- Equation: $\Phi = \int \vec{B} \cdot d\vec{A}$



3.8 Transformers

$$\frac{I_P}{I_S} = \frac{V_S}{V_P} = \frac{N_S}{N_P}$$

3.9 Motional EMF: Moving Rod

$$\varepsilon = B\ell v$$

3.10 Inductance

3.10.1 Definitions

- Inductance:

$$L = \frac{N\Phi_B}{I}$$

- Solenoid:

$$L = \mu_0 N^2 A\ell$$

- Toroid: H = toroid height, a, b are inner and outer radii

$$L = \frac{\mu_0 N^2 H \ln(b/a)}{2\pi}$$

3.10.2 Circuits

- Self-Inductance/Back EMF:

$$\varepsilon_B = -L\,\frac{dI}{dt}$$

- Energy: $U = \frac{1}{2}LI^2$

3.10.3 LR Circuits

1. Rise of current: $\tau = L/R$

$$I = \frac{\varepsilon}{R}\left(1 - e^{-\frac{R}{L}t}\right) = I_{MAX}\left(1 - e^{-t/\tau}\right)$$

2. Decay of current:

$$I = I_{MAX}\,e^{-t/\tau}$$

3.10.4 LC Circuits

$$\frac{dI}{dt} + \frac{Q}{LC} = 0 \implies \frac{d^2Q}{dt^2} + \frac{Q}{LC} = 0$$

$$\omega^2 = \frac{1}{LC} \implies \text{ general solution with } \sin\omega t \text{ or } \cos\omega t$$

4 Maxwell's Equations, Integral Form

Gauss' Law for Electricity: $\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0} = \frac{1}{\epsilon_0}\int \rho\,dV$

Gauss' Law for Magnetism: $\oint \vec{B}\cdot d\vec{A} = 0$

Faraday's Law: $\oint \vec{E}\cdot d\vec{\ell} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{A}$

Ampere's Law*: $\oint \vec{B}\cdot d\vec{\ell} = \mu_0\,I_{enc}$