Cloud formation of condensible species in exoplanet atmospheres using the Clausius-Clapeyron equation

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Abstract

Clouds can have a major impact on a planet's energy balance. They can increase a planet's albedo, reflecting the parent star's radiation. They can also be strongly absorbing [1]. As such, they impact the transport of radiation, the chemistry of the atmosphere, and the planet's surface temperature [2]. In this paper, we will investigate whether particular molecular species observed in an exoplanet's spectra can condense to form clouds. Using the Clausius-Clapeyron framework outlined in Sanchez-Lavega et al.[4], we will use equations 7,12,16,20, and data from tables I, II, and III to validate our implementation of their framework using solar system planet atmospheres and their reference exoplanet, HD209458b. Our implementation attempts to determine the possibility of cloud formation in any exoplanet's atmosphere. If clouds are present, we attempt to determine the location of the cloud base and the cloud's vertical extent. At present, our results are mixed. We find agreement with the solar system terrestrial atmospheres, but our results differ by several orders of magnitude when considering the high temperature condensation of iron and enstatite. Furthermore, we find a paucity of data on exoplanet atmospheres. These findings and issues are outlined in section 1 below.

Chapter 1

Equations and Data

The equations and data used in the model described in this paper are taken from Sanchez-Lavega et al.[4] We start with Clausius-Clapeyron:

$$P = C_L e^{\frac{-L_s}{R_{gas} T}}$$

For calculating the P-T profile of a planet's atmosphere, we use eqn.(7)[4].

$$P(T) = P_o \left(\frac{T}{T_o} \right)^{\frac{g}{\Gamma_a R^*}}$$

For calculating the saturation vapor pressure of a condensible species, we use eqn.(16)[4].

$$ln(P_V) = ln(C) + \frac{1}{R_V} \left[-\frac{L_o}{T} + \Delta\alpha T + \frac{\Delta\beta}{2} \right]$$

The location of the cloud base is located where:

$$P_{Cl}(T_{Cl}) = \frac{P_V(T_{Cl})}{X_C}$$

The model relies on the tables on pages 3 and 4 for properties in the equations described above.

Table I. Basic properties of planetary atmospheres. The quantities are defined in Sec. II.

Planet or satellite	Main components (%)	μ (g mol^{-1})	R* (J g^{-1} K^{-1})	C_p (J g^{-1} K^{-1})	g (m s^{-2})	Γ_a (K Km^{-1})	T_0 (K)	P_0 (bar)
Venus	CO_2 (0.96) N_2 (0.035)	44.01	0.19	0.85	8.89	10.50	731	92
Earth	N_2 (0.78) O_2 (0.21)	28.97	0.29	1.00	9.80	9.80	288	1.013
Mars	CO_2 (0.953) N_2 (0.027)	44.01	0.19	0.83	3.74	4.50	214	0.07
Jupiter	H_2 (0.864) He (0.136)	2.22	3.75	12.36	24.25	2.00	165	1.00
Saturn	H_2 (0.85) He (0.14)	2.14	3.89	14.01	10.00	0.70	134	1.00
Titan	N_2 (0.65–0.98) Ar (0.25–0)	28.67	0.29	1.04	1.35	1.30	94	1.50
Uranus	H_2 (0.85) He (0.15)	2.30	3.61	13.01	8.80	0.70	76	1.00
Neptune	H_2 (0.79) He (0.21)	2.30	3.61	13.01	11.10	0.85	76	1.00
HD 209458	H_2 (1.0)	2.00	4.16	14.00	8.00	0.60	1300	1.00

Figure 1.1: Reproduced from Sanchez-Lavega [4]

Table II. Saturation vapor pressure and latent heat. The quantities are defined in Sec. III.

Component	ln(C) (C in bars)	L_0 (J g^{-1})	$\Delta\alpha$ (J g^{-1} K^{-1})	$\Delta\beta/2$ (J g^{-1} K^{-2})	Reference
SO_4H_2	16.256	865.8	···	···	15
H_2O	25.096	3148.2	···	-8.7×10^{-3}	8
CO_2	26.100	639.6	···	-1.7×10^{-3}	16
NH_3	27.863	2016	-0.888	···	10
NH_4SH	75.678	2915.7	-1.760	7.8×10^{-4}	10
CH_4	1.627	553.1	1.002	-4.1×10^{-3}	17
SH_2	17.064	747	···	-2.9×10^{-3}	17
C_2H_6	10.136	521.4	···	···	17
Fe	1.894	7097	···	···	9
$MgSiO_3$	11.554	4877.5	···	···	9

Figure 1.2: Reproduced from Sanchez-Lavega [4]

Table III. Planetary atmospheres condensates (l=liquid, s=solid) and cloud characteristics. The quantities are defined in Sec. IV.

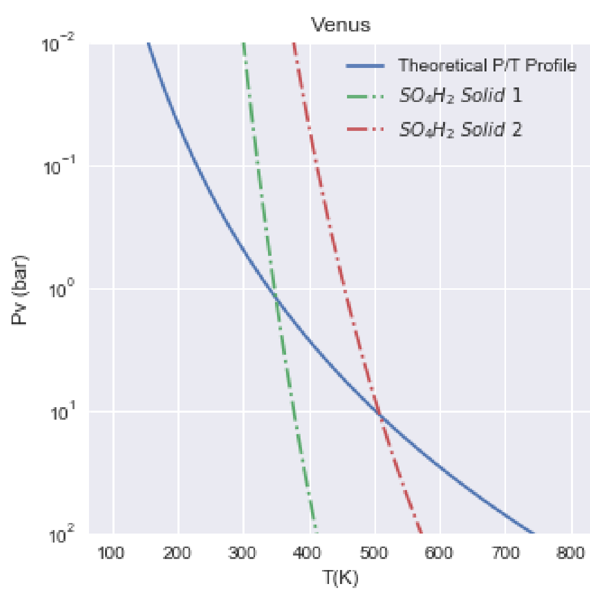
Cloud and planet	X_c	μ_c	P (bar)	T (K)	H (km)	H_c (km)	H_c/H	Density (g cm^{-3})	Γ_s (K km^{-1})	Γ_s/Γ_a
Venus										
SO_4H_2 (l)	2.0×10^{-6}	98.08	1.0	348	7.4	1.1	0.15	4.5×10^{-8}	10.5	1.000
SO_4H_2 (l)	2.0×10^{-3}	98.08	11.3	510	11.0	2.4	0.22	2.4×10^{-4}	9.3	0.880
Earth										
H_2O (s)	2.5×10^{-4}	18.02	0.30	229	6.8	0.9	0.13	5.3×10^{-7}	9.4	0.957
H_2O (l)	0.015	18.02	0.96	285	8.4	1.5	0.18	5.9×10^{-5}	5.0	0.508
Mars										
CO_2 (s)	0.95	44.01	2.0×10^{-4}	127	6.4	1.1	0.17	4.6×10^{-6}	0.77	0.17
H_2O (s)	3.0×10^{-4}	18.02	1.0×10^{-3}	190	9.6	1.3	0.14	2.5×10^{-9}	3.90	0.87
Jupiter										
NH_3 (s)	2.0×10^{-4}	17.00	0.75	150	23	3.0	0.13	1.6×10^{-6}	1.93	0.967
NH_4SH (s)	3.6×10^{-5}	50.00	2.20	210	32	1.2	0.04	6.1×10^{-6}	1.90	0.948
H_2O (Galileo)(s)	5.0×10^{-5}	18.02	3.20	228	35	4.3	0.12	1.2×10^{-6}	1.98	0.990
H_2O (Solar) (l)	1.7×10^{-3}	18.02	5.7	280	43	7.4	0.17	4.4×10^{-5}	1.73	0.867
Saturn										
NH_3 (s)	2.0×10^{-4}	17.00	1.2	150	58	8.4	0.14	2.3×10^{-6}	0.68	0.970
NH_4SH (s)	3.6×10^{-5}	50.00	4.0	215	84	3.4	0.04	1.0×10^{-5}	0.67	0.954
H_2O (Solar) (s)	1.7×10^{-3}	18.02	9.3	285	111	21.0	0.19	6.3×10^{-5}	0.62	0.883
Titan										
CH_4 (s)	0.05	16.04	0.90	84	18	4.6	0.26	4.0×10^{-4}	0.59	0.456
C_2H_6 (s)	10^{-5}	30.00	1.04	87	19	3.1	0.16	2.6×10^{-7}	1.3	0.995
Uranus										
CH_4 (s)	0.02	16.04	0.7	68	28	5.9	0.21	1.9×10^{-4}	0.36	0.511
SH_2 (s)	3.7×10^{-5}	34.06	4.0	117	48	7.0	0.15	3.6×10^{-6}	0.70	0.994
NH_3 (s)	2.0×10^{-4}	17.00	15.0	169	69	11.1	0.16	2.3×10^{-5}	0.68	0.977
NH_4SH (s)	3.6×10^{-5}	50.00	50.0	240	99	4.5	0.05	9.9×10^{-5}	0.67	0.963
H_2O (s)	1.7×10^{-3}	18.02	200.0	350	144	36.0	0.25	8.4×10^{-4}	0.65	0.934
Neptune										
CH_4 (s)	0.02	16.04	0.9	69	22	4.8	0.22	2.4×10^{-4}	0.440	0.520
SH_2 (s)	3.7×10^{-5}	34.06	3.8	117	38	5.5	0.15	3.4×10^{-6}	0.845	0.994
NH_3 (s)	2.0×10^{-4}	17.00	14.0	168	55	8.7	0.16	2.1×10^{-5}	0.830	0.977
NH_4SH (s)	3.6×10^{-5}	50.00	50.0	240	78	3.6	0.05	9.8×10^{-5}	0.820	0.963
H_2O (s)	1.7×10^{-3}	18.02	200.0	350	114	29.0	0.25	8.3×10^{-4}	0.790	0.933
HD209458b										
$MgSiO_3$ (s)	7.52×10^{-5}	100.4	2.4	1620	840	78	0.09	1.5×10^{-6}	0.58	0.974
Fe (s)	6.77×10^{-5}	55.84	3.2	1750	909	112	0.12	6.8×10^{-7}	0.59	0.987

Figure 1.3: Reproduced from Sanchez-Lavega [4]

Chapter 2

Checking the Model

As we verified the implementation against various solar system planets, we noticed much agreement, but also some discrepancies which we describe below on a per planet basis.



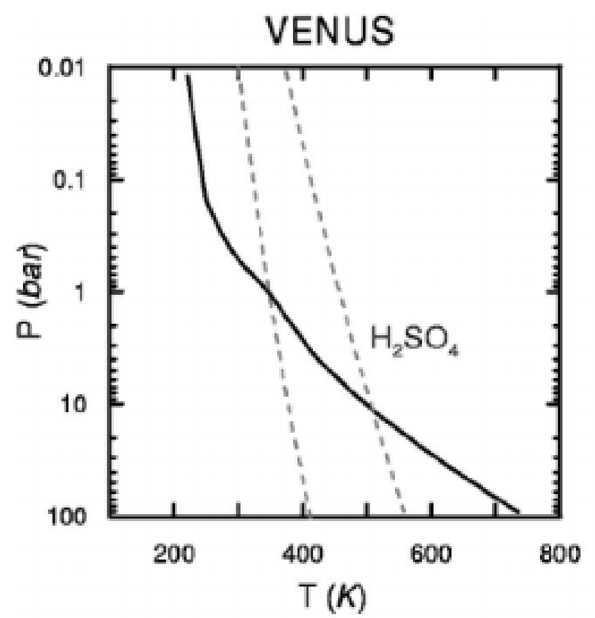


Figure 2.1: The figure on the left is our model's plot of Venus's P-T profile. The plot from Sanchez-Lavega is on the right. As in all of the Sanchez-Lavega plots, their P-T profiles seem to be based on actual data; whereas, our model uses a theoretical P-T profile

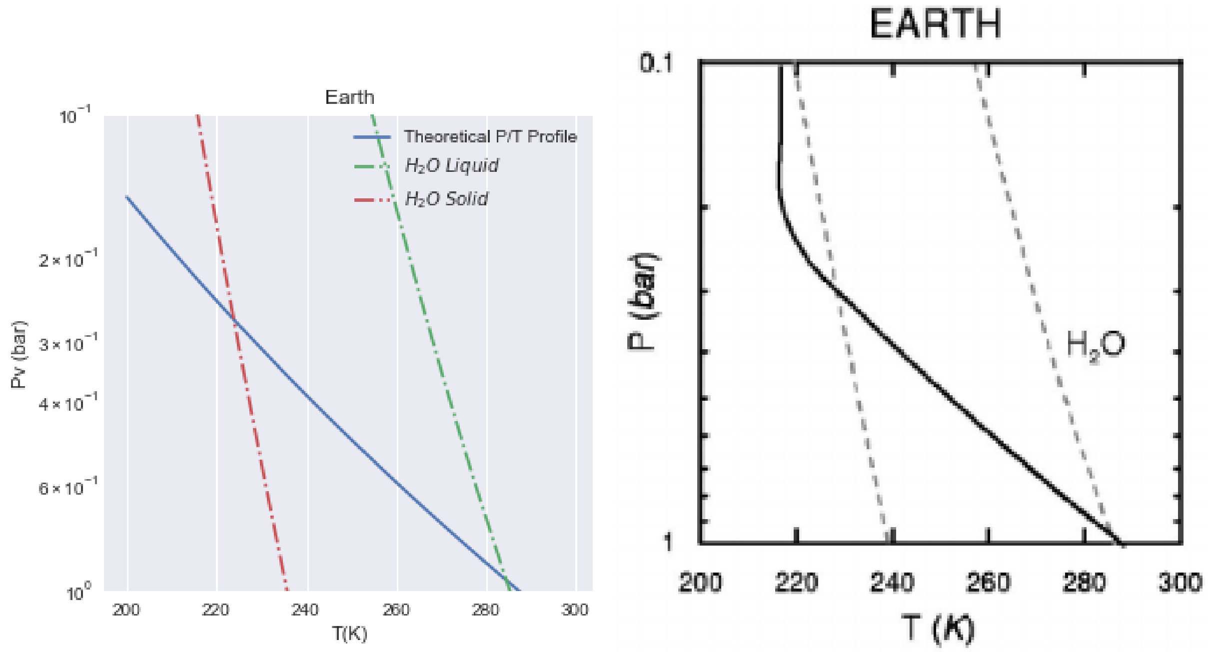


Figure 2.2: On the left, our model. On the right, Sanchez-Lavega[4]. Again, the primary difference is the shape of the P-T profiles. The saturation vapor pressure c[t]urves for water seem to be in fairly good agreement.

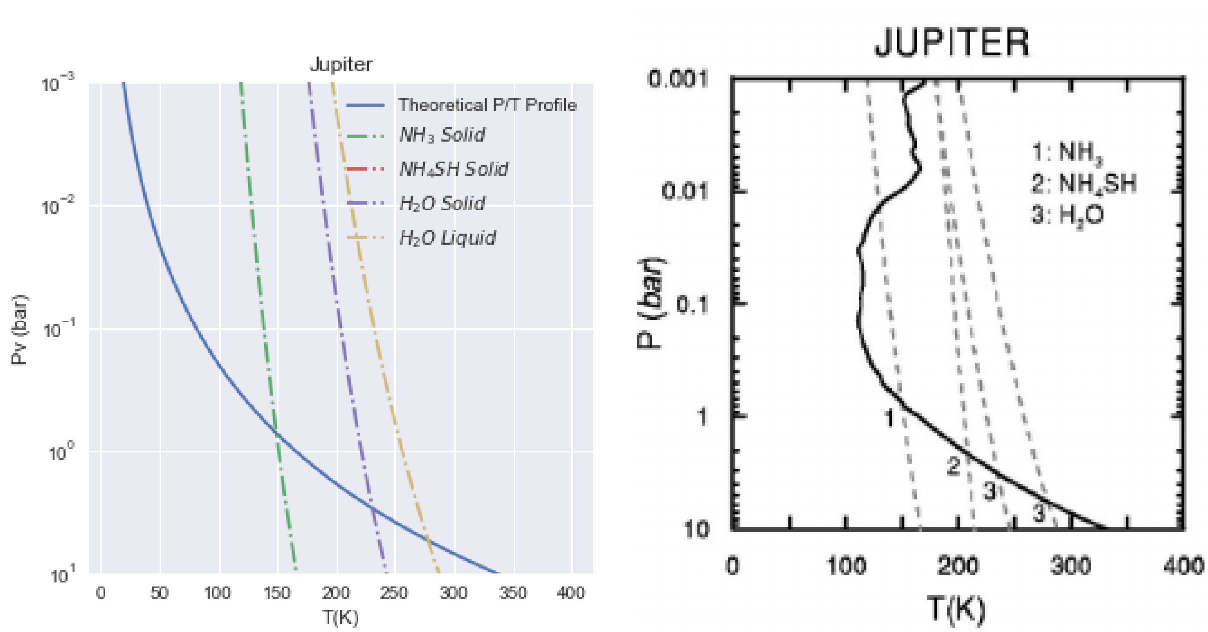


Figure 2.3: On the left, our model. On the right, Sanchez-Lavega[4]. Again, the primary difference is the shape of the P-T profiles. The saturation vapor pressure curves for water seem to be in fairly good agreement. However, in our model, NH_4SH does not render. The reason for this is not clear. Tabular data has been checked for accuracy. This same problem exists for this molecule on Saturn, Uranus, and Neptune as well.

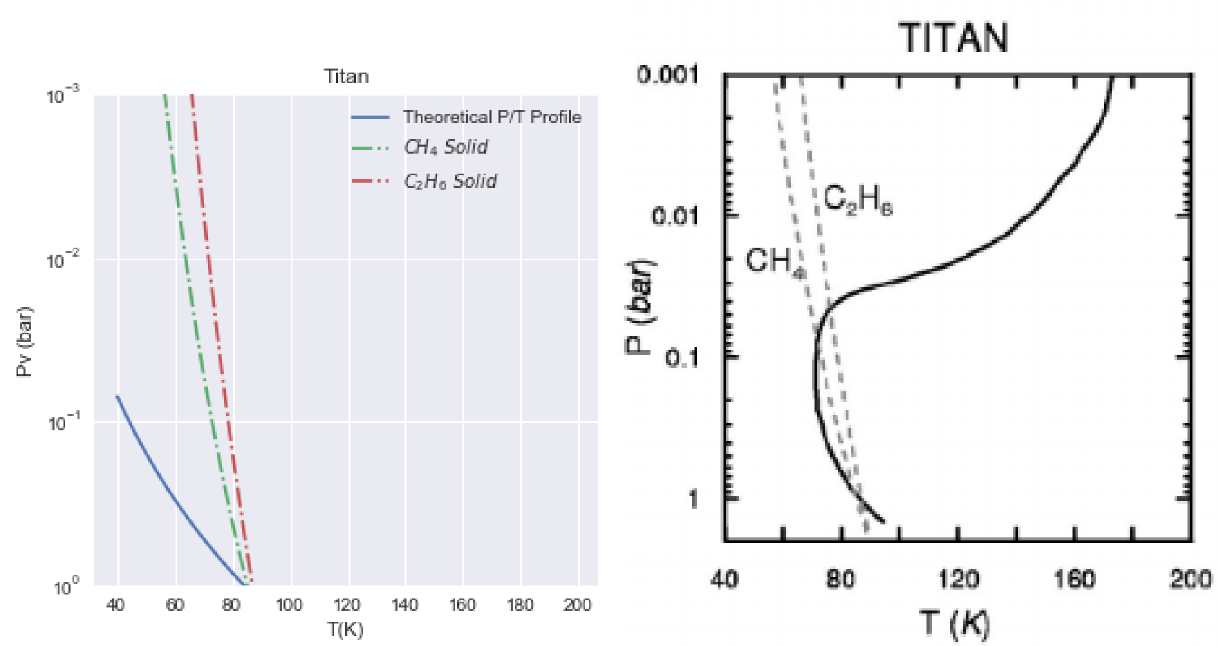


Figure 2.4: On the left, our model. On the right, Sanchez-Lavega[4]. Again, the primary difference is the shape of the P-T profiles, resulting in a miss on cloud formation between pressures of 0.01 and 0.1 bar. The saturation vapor pressure curves for CH_4 and C_2H_6 seem to be in fairly good agreement. Actual P-T data for solar system planets exists at atmos.nmsu.edu. If time permits, I will overlay the data on these graphs.

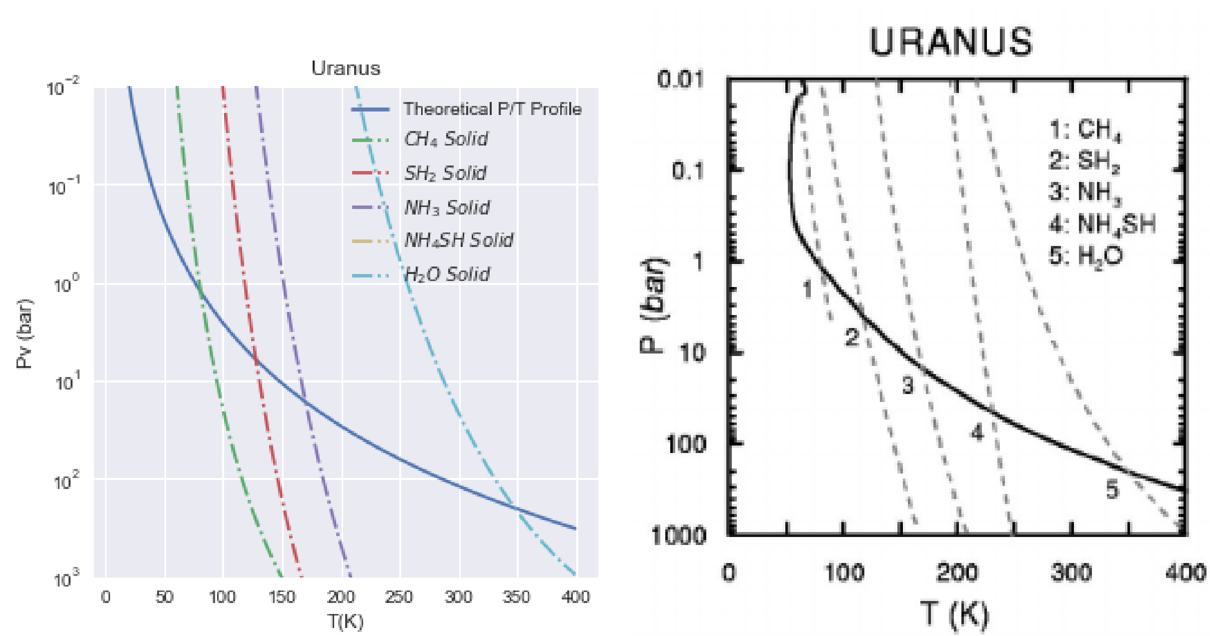


Figure 2.5: On the left, our model. On the right, Sanchez-Lavega[4]. Again, the primary difference is the shape of the P-T profiles. However, as with Jupiter, in our model, NH_4SH does not render.

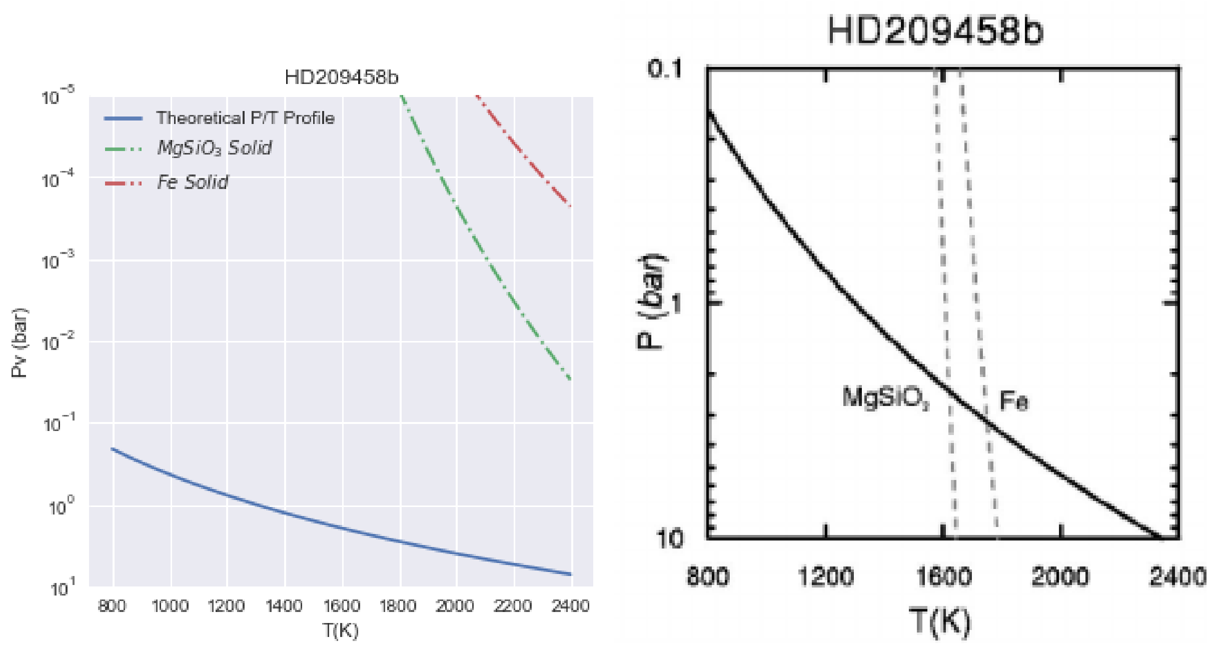


Figure 2.6: On the left, our model. On the right, Sanchez-Lavega[4]. Here the saturation vapor pressure curves are off by several orders of magnitude. Value have been checked for accuracy. It's unclear why these calculations are off. In the figure below, non-zero values for $\Delta\alpha$ are assumed to see if that is a possible explanation for the difference.

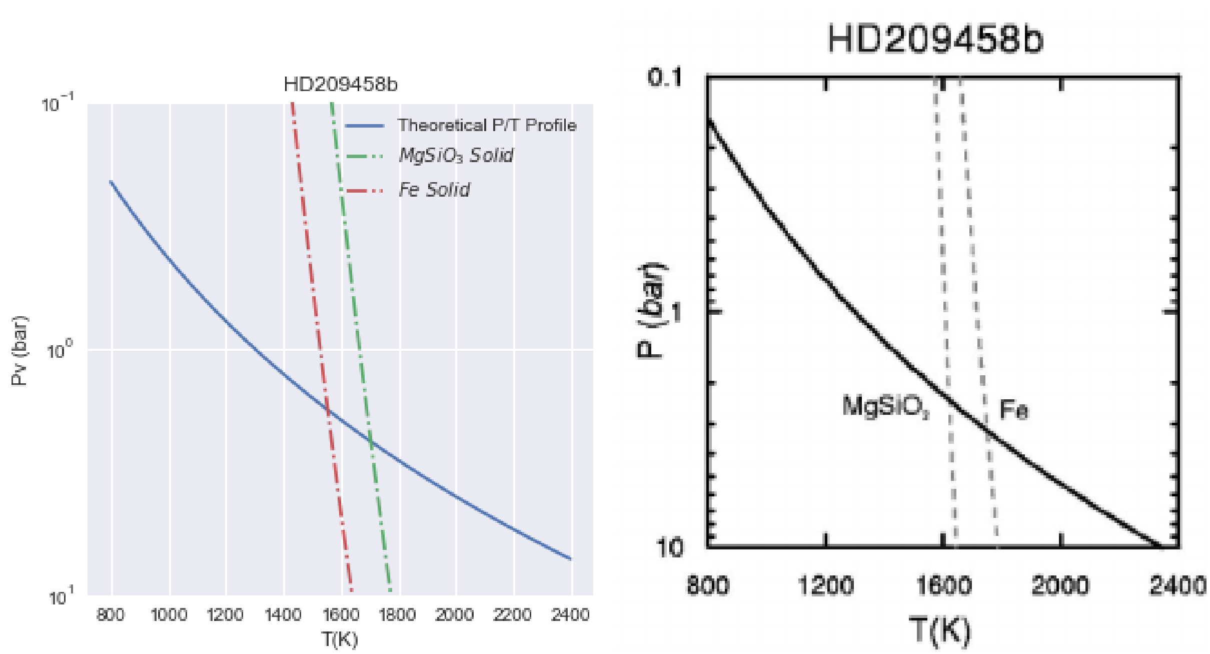


Figure 2.7: On the left, our model. On the right, Sanchez-Lavega[4]. Here, our model assumes a $\Delta\alpha = 0.4$ for iron, and $\Delta\alpha = 0.16$ for enstatite. This speculative exercise was doe to investigate what parameters might be missing.

Chapter 3

Codes

All code for the model can be found at: "https://github.com/rschroder/exoplanet$_{c}louds$"

The relevant calculations are found in the following functions and dicts.

```
def P(Po, T, To, g, gamma_ad, R, mu):
    pressure = Po * (T / To)**(g/(gamma_ad*R/mu)) # from sanchez-lavega 2003, (eqn. 7)
    return pressure
                                            # (eqn. 19)

def saturation_vapor_pressure(ln_c, T, Lo, beta, alpha, Rv, Xc):
    beta_term = (beta)/Rv
    alpha_term = (alpha)/Rv
    Lo_term = Lo/Rv
    svp = 1./Xc * np.e**(ln_c +(-Lo_term/T + \
        alpha_term*np.log(T) + T*beta_term))
    return svp
```

Data for the reference exoplanet can be found in the following dicts

---

```
FE_solid_1_HD209458b = {
    'name'    : "$Fe \ Solid$",
#    'ln_c'    : 15.71,         # From marley
    'ln_c'    : 1.894,          # From table II,sanchez-lavega 2003
    'Lo'      : 7097.,          # From table II,sanchez-lavega 2003
#    'alpha'   : 0.4,             # From CRC
    'alpha'   : 0.0,             # From table II,sanchez-lavega 2003
    'beta'    : 0.,              # From table II,sanchez-lavega 2003
    'Xc'      : 6.77*10**(-5),   # from table III, sanchez-lavega 2003
    'Rv'      : 8.3144598/55.84, # Remember to plug in mu_cl for cloud vapor
}

MGSIO3_solid_1_HD209458b = {
    'name'    : "$MgSiO_{3} \ Solid$",
#    'ln_c'    : 25.37,         # From marley
    'ln_c'    : 11.554,         # From table II,sanchez-lavega 2003
    'Lo'      : 4877.5,         # From table II,sanchez-lavega 2003
    'alpha'   : 0.0,             # From table II,sanchez-lavega 2003
#    'alpha'   : 0.16,             # made up
    'beta'    : 0,    # From table II,sanchez-lavega 2003
    'Xc'      : 7.52*10**(-5),   # from table III, sanchez-lavega 2003
    'Rv'      : 8.3144598/100.4, # Remember to plug in mu_cl for cloud vapor
}
```

```
hd209458b = {
    'name'     : 'HD209458b',
    'P_o'      : 1.0,          # bar, Table I, sanchez-lavega 2003
    'T_o'      : 1300.,          # K, Table I, sanchez-lavega 2003
    'g'        : 8.0,           # m/s^2, Table I, sanchez-lavega 2003
    'gamma_ad' : 0.60,     # K/m ----- 6.5 K/km -> taken from 'planetary sciences', de Pater
    'mu'       : 2.0,    # kg/mol, Table I, sanchez-lavega 2003
    'T_min'    : 800.,          # K, Figure 3, sanchez-lavega 2003
    'T_max'    : 2400.,          # K, Figure 3, sanchez-lavega 2003
    'T_step'   : 1.,            # K
    'P_min'    : 0.00001,           # bar, Figure 3, sanchez-lavega 2003
    'P_max'    : 10.0,           # bar, Figure 3, sanchez-lavega 2003
}
```

Bibliography

[1] Seager, S., *Exoplanet Atmospheres: Physical Processes*, Princeton University Press, 1st edition, 2010.

[2] Marley, M., Ackerman, S, Cuzzi, J., Kitzmann, D., *Clouds and Hazes in Exoplanet Atmospheres*, Comparative Climatology of Terrestrial Planets, University of Arizona Press, p.367-391, 2013.

[3] N. Madhusudhan, S. Seager, *A Temperature and Abundance Retrieval Method for Exoplanet Atmospheres*, The Astrophysical Journal, 707:2439, 2009 December 10.

[4] Sanchez,-Lavega, A., Perez-Hoyos, *Clouds in Planetary Atmospheres: A Useful Application of the Clausius-Clapeyron Equation*, American Association of Physics Teachers., DOI: 10.1119/1.1645279, 2003.