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Comparative Thermal Evolution of Uranus and Neptune

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We extend a Jovian convective-cooling model to Uranus and Neptune. The model assumes that efficient interior convection prevails, so that escape of interior heat is governed by the radiative properties of the atmosphere. A comparison of the thermal evolution of Uranus and Neptune indicates that the larger amount of solar radiation absorbed in Uranus' atmosphere tends to differentially suppress the escape of interior heat. The model is shown to be consistent with recent infrared observations of the thermal balance of Uranus and Neptune, and with the presumed age of these planets.

I. INTRODUCTION

Far-infrared observations of Uranus by Fazio *et al.* (1976) and Loewenstein *et al.* (1977a), and of Neptune by Loewenstein *et al.* (1977b), indicate that Uranus has no evident internal source of heat emission, while Neptune radiates approximately twice as much power as it receives from the Sun. Various possibilities have been advanced to explain this discrepancy between the two planets, which are similar in mass and mean density. (a) Trafton (1974) proposed that Neptune derives its intrinsic heat source from the decay of Triton's orbit due to tidal friction. However, the characteristic decay time for Triton's orbit which is required to account for the emission rate is much less than the age of the solar system. (b) Uranus' atmosphere presumably undergoes great seasonal variation in solar heat input, due to the near coincidence of the planet's rotation axis with its orbital plane. In principle, this could have an effect on the global energy balance of the planet. However, as shown by Fazio *et al.* (1976), the atmospheric temperature distribution is unlikely to change greatly over half a Uranian year, even if advection plays no role in equalizing temperatures between the day and night hemispheres. (c) Uranus receives about 2.4 times as much solar flux as does Neptune, and the corresponding change in the global energy balance is perhaps enough to significantly affect the thermal evolution of one planet with respect to the other, even if both planets started their evolution with identical interior structure.

In the following, we will present simple arguments, which are reasonably insensitive to detailed interior models, to show that possibility (c) provides a satisfactory explanation of currently available data.

II. THE CONVECTIVE COOLING MODEL

A thermal model for Jupiter in which the planet derives its internal power from the radiation of primordial heat, delivered to the planetary photosphere by efficient convection, has been reasonably successful in accounting for infrared heat balance measurements (Hubbard, 1977). According to

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this model, interior convection is able to transport heat at such a rate that the atmospheric opacity at a pressure ~ 1 bar and lower in effect serves as the bottleneck for the escape of interior heat. This concept is incorporated in quantitative models by means of the following well-known procedure from stellar evolution calculations. We consider a model planetary atmosphere with effective temperature T_e, and a heat flux given by σT_e^4, where σ is the Stefan–Boltzmann constant. It is assumed that any heat input from the Sun is thermalized deep within the atmosphere, so that σT_e^4 includes flux of solar heat as well as flux of intrinsic heat. Calculations for a hydrogen–helium atmosphere under Jovian planet conditions (Trafton, 1967; Graboske *et al.*, 1975) indicate that the atmosphere is convectively unstable at pressures exceeding ~ 1 bar. If the planet is liquid and convective throughout, the interior adiabat thus becomes a function of T_e, which is in turn determined by the average heat flux through the atmosphere. For a hydrogen–helium planet such as Jupiter, the corresponding relationship is (Hubbard, 1977)

$$T = 66.8 g_0^{-1/6} T_e^{1.243} \rho^\gamma, \qquad (1)$$

where T is the interior temperature, g_0 is the surface gravity, ρ is the mass density, and γ is the Grüneisen parameter (adiabatic exponent). In (1), all quantities are in cgs units. For Jupiter, the interior temperature given by (1) is of order 20,000°K, a value sufficiently small that the object can be regarded as cooling at constant radius during the current epoch. For such a model, the heat balance equation reads

$$4\pi R^2 \sigma (T_e^4 - T_\odot^4) = -\int dm C_v \frac{dT}{dt}, \qquad (2)$$

where R is the radius of the planet, $T_\odot$ is the effective temperature corresponding to equilibrium with sunlight in the absence of interior heat emission, C_v is the heat capacity of the interior per gram, t is the time, and the integral is taken over the mass of the planet. Substituting (1) in (2), we find

$$dt/\alpha = -T_e^{0.243} dT_e / (T_e^4 - T_\odot^4), \qquad (3)$$

where the convection-cooling time constant α is given by

$$\alpha = 83 g_0^{-1/6} \int dm C_v \rho^\gamma \Big/ 4\pi R^2 \sigma. \qquad (4)$$

For Jupiter, $\alpha = 2.8 \times 10^{23}$ cgs units. Integration of (3) from $t = 0$ ($T_e = \infty$) to $t = 4.6 \times 10^9$ yr gives $T_e \sim 130$°K, in accordance with observations.

III. A SCALED "URANUS" COOLING MODEL

According to calculations by Zharkov and Trubitsyn (1972, 1977), roughly the outer 12% of the radius of Uranus, and about 6% of its mass, is composed primarily of molecular hydrogen, with other species present as impurities. The remainder of the planet is homogeneous and composed of H_2O ($\sim$40%, by mass), CH_4 ($\sim$22%, by mass), and heavy material or "rock" ($\sim$22%).

Because of the similar mass and mean density, a very similar interior model is valid for Neptune. Zharkov and Trubitsyn estimate the central temperature of an adiabatic Uranus or Neptune model to be $\sim 10^4$°K. For the temperature at the base of the hydrogen envelope, they estimate $\sim$3000°K.

Because of the stiffer equation of state for H_2O and CH_4, compared with hydrogen, the central density of Uranus and Neptune is calculated to be ~ 4 g/cm³, about the same as the central density of a homogeneous Jupiter model (Slattery, 1977), although the central pressure is ~ 6 times lower. One can obtain a first-approximation Uranus model by leaving the Jovian distribution unchanged, but by scaling all radii in the ratio of the planets' total radii, (25,000/70,000 km). This gives a Uranus model mass of $(2.5/7)^3 M_J = 8.7 \times 10^{28}$ g $= M_U$, in agreement with observation. In the following, we will use this scaling approach to estimate the appropriate value of α [see (4)] for Uranus and Neptune, and thus to calculate a convective-cooling model for these planets.

From (1), we deduce a central temperature for Uranus of 8000°K, using $T_e = 58$°K, $\rho = 4$ g/cm^3, and $\gamma = 0.6$. This value is in reasonable agreement with Zharkov and Trubitsyn's (1972) estimate, showing that (1) is qualitatively valid for the Uranus interior, although it is derived for metallic hydrogen. Similarly, at the base of the hydrogen envelope, where $\rho \sim 0.3$ g/cm^3, Eq. (1) gives $T \sim 1600$°K.

According to the above assumptions, we then estimate the value of α appropriate to Uranus:

$$\alpha_U = (g_{J0}/g_{U0})^{1/6}(R_U/R_J)(C_{vU}/C_{vJ})\alpha_J, \quad (5)$$

where the subscripts (U, J) refer to (Uranus, Jupiter). We have also assumed that the value of the heat capacity per g is roughly constant through the interior. For the metallic hydrogen Jovian interior, the heat capacity is about 2 k_B per heavy particle (k_B = Boltzmann's constant). No quantitative calculations of the heat capacity of H_2O–CH_4 mixtures at pressure ~1 Mbar and $T \sim 5000$°K are currently available. However, we may estimate, to within factors of order unity,

$$C_{vU}/C_{vJ} \cong \langle A \rangle_J / \langle A \rangle_U, \quad (6)$$

where $\langle A \rangle$ is the mean molecular weight in the planet's interior. We assume $\langle A \rangle_J \sim 1$, and if the H_2O and CH_4 are not pressure dissociated, $\langle A \rangle_U \sim 17$. Thus we obtain

$$\alpha_U \sim 5.9 \times 10^{21} \text{ cgs units.} \quad (7)$$

This estimate is obviously subject to great uncertainty, probably as much as a factor ~5. However, we are interested in explaining a difference between Uranus and Neptune. We shall assume that $\alpha_U \sim \alpha_N$, because of the similarity of interior structure, so that differences in the thermal evolution of the two planets will be primarily due to the difference in the value of $T_\odot$ in (3).

IV. EVOLUTIONARY COOLING SEQUENCES FOR URANUS AND NEPTUNE

Figure 1 shows the result of integrating (3) with $\alpha_U = 15.9 \times 10^{21}$ cgs units. The left-hand curve is calculated using $T_{\odot N} = 46$°K for Neptune (Loewenstein *et al.*, 1977b) and the right-hand curve uses $T_{\odot U} = 57$°K for Uranus (Fazio *et al.*, 1976). Here $\tau = t/4.6 \times 10^9$ yr. Observed values of T_e, with error bars, are also shown. First note that because $T_\odot$ is higher for Uranus than for Neptune, the effect of sunlight on the planet's thermal evolution is much more pronounced. Note also that it is possible to choose a common age ($\ln \tau \cong -0.1$, or $t \cong 4.0 \times 10^9$ yr) for both planetary models, which is consistent with the observational error bars, giving $T_{eN} \cong 53$°K, $T_{eU} \cong 60$°K. It should be recognized that α_N will not be precisely the same as α_U. Because Neptune is about 18% more massive than Uranus, while its radius is about 5% smaller, one expects the value of α_N to be at least ~18% larger than α_U. An increase in the value of α simply shifts a curve in Fig. 1 upward without changing its shape; the indicated relative shift for Neptune is shown with an arrow.

V. DISCUSSION

The observed heat flux from Neptune appears to be consistent with reasonable interior models. The value of the intrinsic flux from the interior, ~(280 ± 80) erg cm^{-2} s^{-1}, is much less than the value for Jupiter, ~(8500 ± 1590) erg cm^{-2} sec^{-1} (Hubbard, 1977). However, the reduced size of Neptune and its higher interior mean molecular weight, in comparison with Jupiter, provide a natural explanation for a smaller interior heat reservoir and thus a much lower emission rate at present.

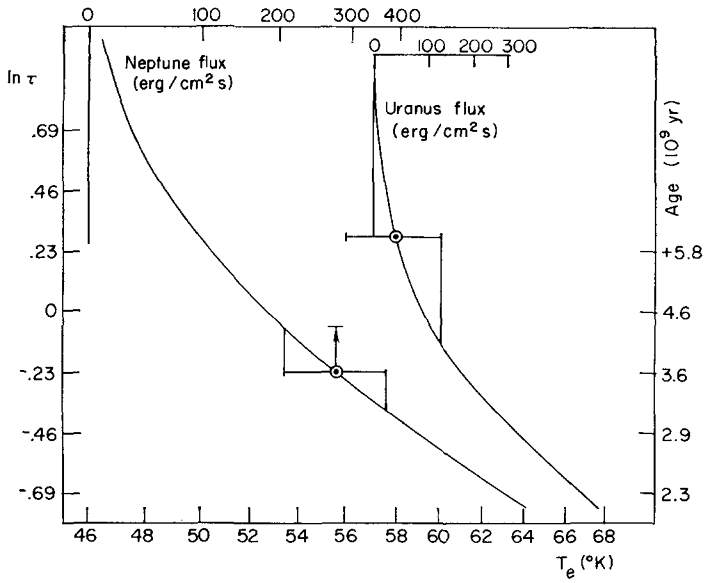

FIG. 1. Effective temperature T_e as a function of time, for Uranus (right) and Neptune (left). Here we have taken the solar equilibrium temperature $T_\odot$ to be 57°K for Uranus and 46°K for Neptune. Observed effective temperatures of 58°K for Uranus and 55.5°K for Neptune are plotted (circled dots), with error bars. The net flux, $\sigma(T_e^4 - T_\odot^4)$, is given at the top. The same value of α is used for both models.

A very similar model is applicable to Uranus. The principal difference with Neptune is that the increased solar thermal flux through Uranus' atmosphere has tended to suppress the rate of emission of interior flux. Provided that the sunlight is thermalized well below the planetary photosphere, efficient interior convection will equalize T_e at all points on the planet's surface, regardless of Uranian season or relative exposure to sunlight, in much the same way as occurs for Jupiter (Hubbard, 1977). Because $T_{\odot U} > T_{\odot N}$, evolution of identical models for Uranus and Neptune with the same starting time will eventually reach the point where solar radiation plays a dominant role in suppressing Uranus' intrinsic heat emission, but a relatively less important role in the case of Neptune (see Fig. 1). We claim that Uranus and Neptune have now reached this point in their thermal evolution. There should probably be some interior heat still escaping from Uranus, at a rate $\lesssim 100$ erg cm^{-1} sec^{-1} and possibly this can be observed after a better value of $T_{\odot U}$ is available. However, at some critical value of the interior heat flux, convection will cease to prevail throughout the planet, and the effective value of α will be reduced, so that the planet will approach $T_e = T_\odot$ even more rapidly than indicated by (3). Transport coefficients for the interior of Uranus and Neptune are not well understood, and so quantitative estimates of the critical flux are not available. It is conceivable that Uranus has evolved to the critical flux and below by the present epoch.

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