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**THERMAL EVOLUTION OF URANUS AND NEPTUNE WITH  
CONDENSATION-INHIBITED CONVECTION**

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requirements for the degree of

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in

ASTROPHYSICS

by

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## 1

# Introduction

Giant planets radiate away their latent heat of formation through the top of their atmosphere. If they cool to a point where the amount they radiate away is equal to the amount of radiation they receive from their parent star, they enter thermal equilibrium with their surroundings. At the present time, the giant planets in our solar system: Saturn, Jupiter, Uranus, and Neptune, all have effective temperatures greater than their equilibrium temperature, except for Uranus. Observations of Uranus show a planet that appears to be in thermal equilibrium with its parent star, a 'dead' planet with no intrinsic temperature, cooler than its more distant neighbor, Neptune. Thermal evolution models for Uranus have not matched observation, instead predicting a warmer effective temperature at 4.6 Gyr, the current age of the solar system. (Fortney et al., 2011), (M. Podolak, 1991), (W.B. Hubbard, 1995), (L. Scheibe, 2019) [There are other papers by Nettelmann 2013, Linder 2019 that I haven't looked at yet].

There have been various attempts to explain the cool Uranus. [The remainder of this paragraph is poorly worded] One of the first explanations posited a stratified interior,



## 2

# Model

## 2.1 Background: Interior Structure with Dry Convection

Understanding the physics of solar system gas and ice giant planets, and attempts to model their thermal evolution date back to the mid to late twentieth century (Hubbard, 1977b), (Hubbard, 1977a), and (M. Podolak, 1991). The description of the planetary interior begins with the following equations. Conservation of mass:

$$\frac{dm}{dr} = 4\pi r^2 \rho \quad (2.1)$$

where  $dm$  is the mass contained within a sphere of radius  $r$ .  $\rho(r)$  is the density at radius  $r$ .

Hydrostatic equilibrium is assumed and described by:

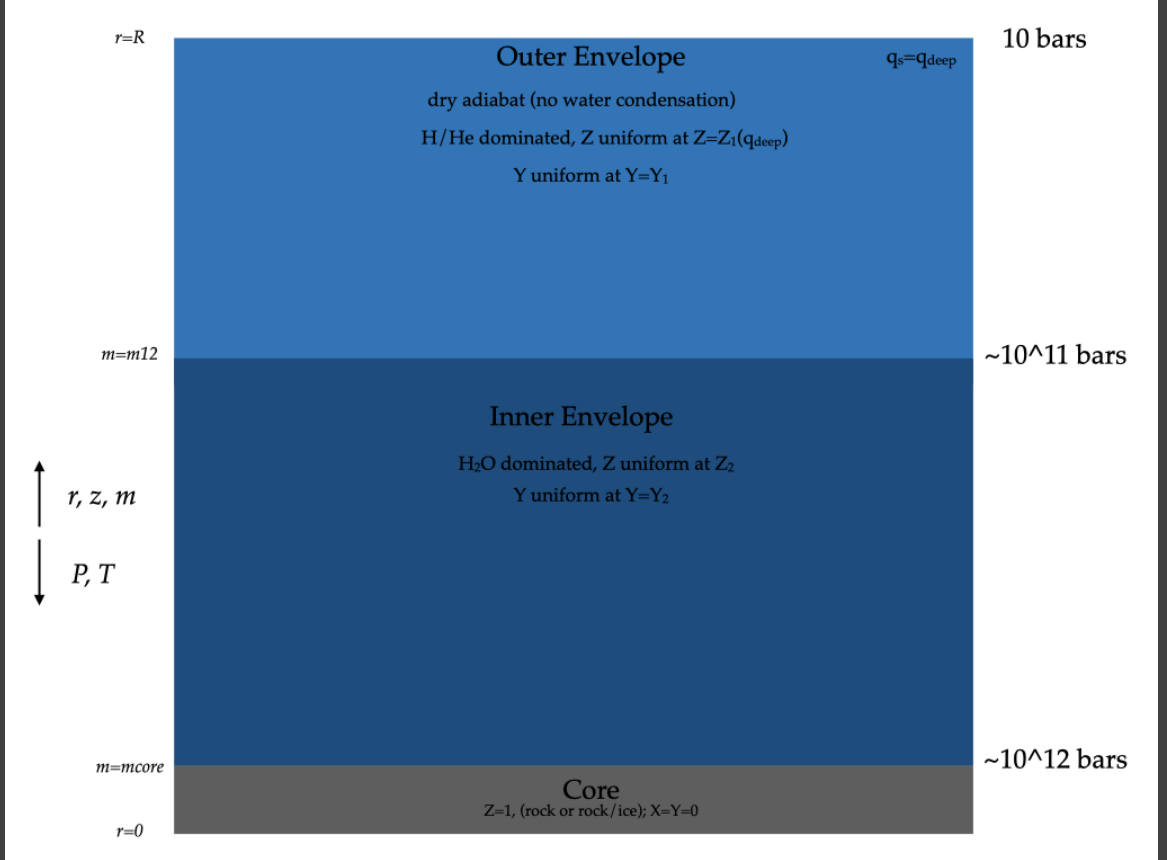
$$\frac{dP}{dr} = -\frac{Gm\rho}{r^2} \quad (2.2)$$

where  $P$  is the pressure and  $G$  is the gravitational constant.

[include thermal evolution and conservation of energy discussion here]



$$\nabla_{\text{ad}} = \left( \frac{\partial \ln T}{\partial \ln P} \right)_s \quad (2.6)$$

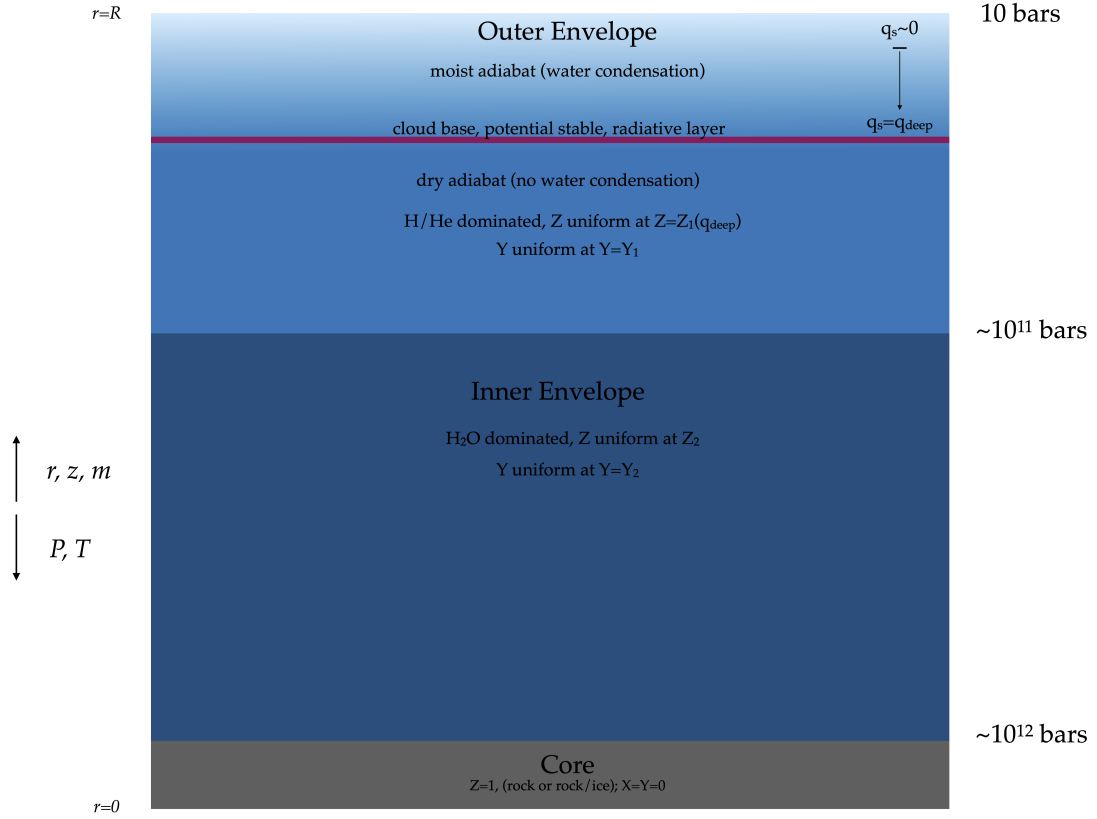


**Figure 2.1:** The structure for a fully convective, dry adiabatic interior. In this model, the inner and outer envelopes are assumed to be well mixed, fully convective, and following a dry adiabat. The core is composed of rock and ice. The inner envelope is water dominated, with uniform concentrations of hydrogen, helium, and water; whereas, the outer envelope is hydrogen and helium dominated, with trace amounts of water. The 'atmosphere' exists above 10 bars.

Finally, beyond the outer envelope is the atmosphere. When modeling the thermal evolution of gas and ice giants, it has long been recognized that model atmospheres constitute an outer boundary condition for interior structure models, providing key inputs



dominated. In this case, a strong vertical gradient in mean molecular weight arises, creating a negative buoyancy. This may result in the formation of a layer that is stable against convection (citations).



**Figure 2.2:** The structure for moist adiabatic interior, allowing for condensation-inhibited convection.

If condensation zone forms, it may be stable against convection. Convection is inhibited due to the formation of a stable condensation zone when  $\alpha < 1$ , where  $\alpha$  (Friedson & Gonzales, 2017) is given by:

$$\alpha = 1 + \xi(q_s L / R_W T_0) \quad (2.8)$$

If condensation is found to be inhibited,

At pressure where  $\alpha < 1$ , the cloud base of the water condensation zone forms.

This thin, stable radiative layer has a temperature profile that is governed by:

$$T(P) = T_{\text{top}} + \int_{P_{\text{top}}}^P \left( \frac{dT}{dP} \right)_{\text{rad}} dP \quad (2.9)$$

$$\left( \frac{dT}{dP} \right)_{\text{rad}} = \frac{T}{P} \nabla_{\text{rad}} = \frac{T}{P} \times \frac{3}{16} \frac{\kappa_R P}{g} \frac{T_{\text{int}}^4}{T^4} \quad (2.10)$$

$$T_{\text{base}} \equiv T(P + \Delta P) = T_{\text{top}} + \left( \frac{dT}{dP} \right)_{\text{rad}} \Delta P. \quad (2.11)$$

$$x_{\text{vap}}(P, T) = x_{\text{vap}}^{\text{sat}}(P, T) = \frac{e_s(T)}{P}, \quad P < P_{\text{base}}. \quad (2.12)$$

$$x_{\text{vap}}^{\text{sat}}(P_{\text{base}}, T_{\text{base}}) = \frac{e_s(T_{\text{base}})}{P_{\text{base}}} = x_{\text{vap}}^{\text{deep}} \implies \Delta P \equiv P_{\text{base}} - P_{\text{top}} = \frac{e_s(T_{\text{base}})}{x_{\text{vap}}^{\text{deep}}} - P_{\text{top}} \quad (2.13)$$

$$T_{\text{base}} = T_{\text{top}} + \left( \frac{dT}{dP} \right)_{\text{rad}} \left( \frac{e_s(T_{\text{base}})}{x_{\text{vap}}^{\text{deep}}} - P_{\text{top}} \right) \quad (2.14)$$

Below the base of the radiative layer, the temperature-pressure profile again follows a dry adiabat, given by:

$$T(P > P_{\text{base}}) = T_{\text{base}} + \int_{P_{\text{base}}}^P \left( \frac{dT}{dP} \right)_{\text{ad}} dP \quad (2.15)$$



# 3

## Results

### 3.1 Condensation-inhibited Convection

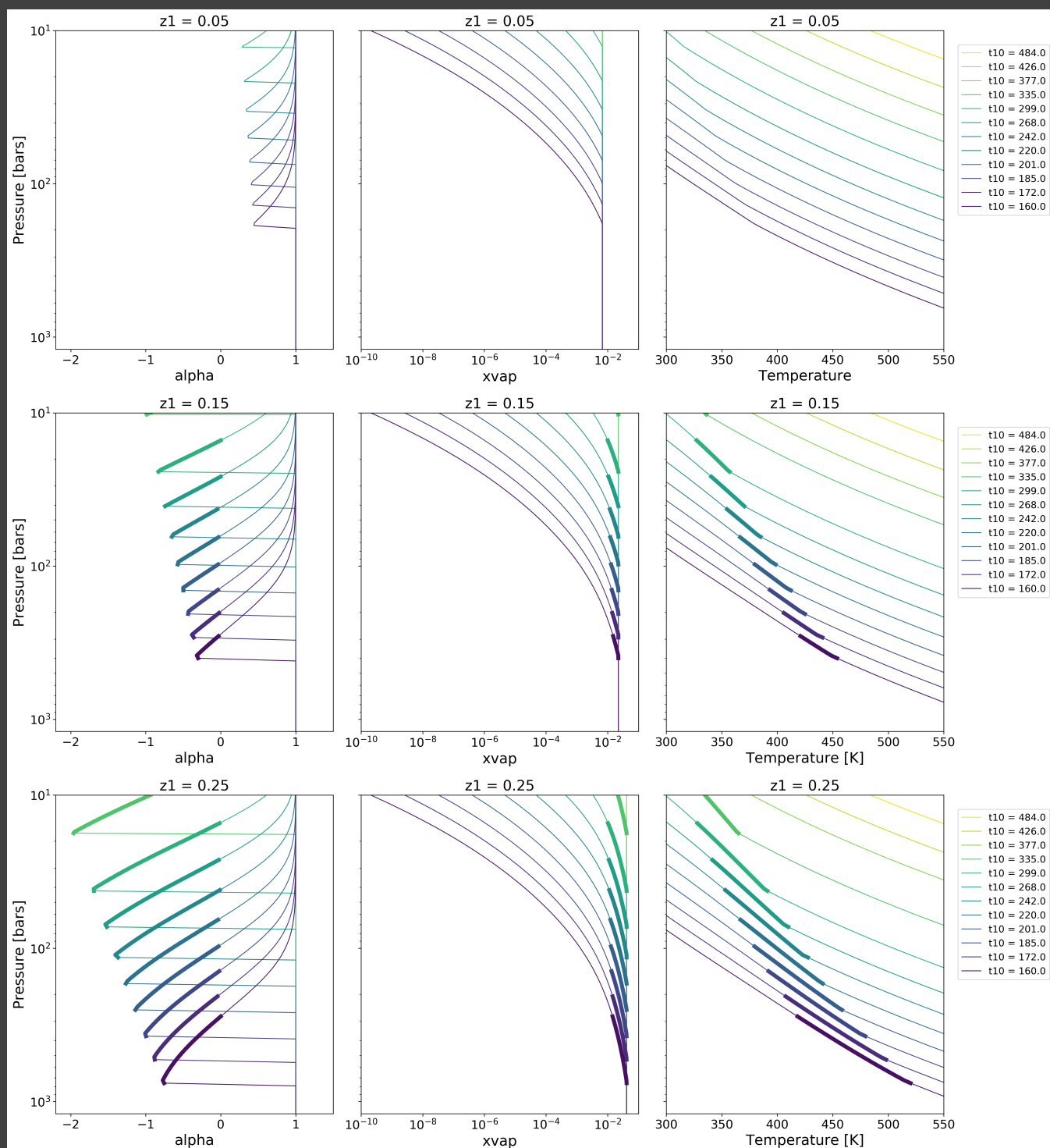
Talk about Figure 3.1.

### 3.2 Formation of Radiative Layer

Talk about Figure 3.2.

### 3.3 Thermal Evolution

Talk about Figure 3.3.



**Figure 3.1:** add description re: moist adiabat, when/where convection is inhibited and where the radiative zone base is in these plots. explain differences

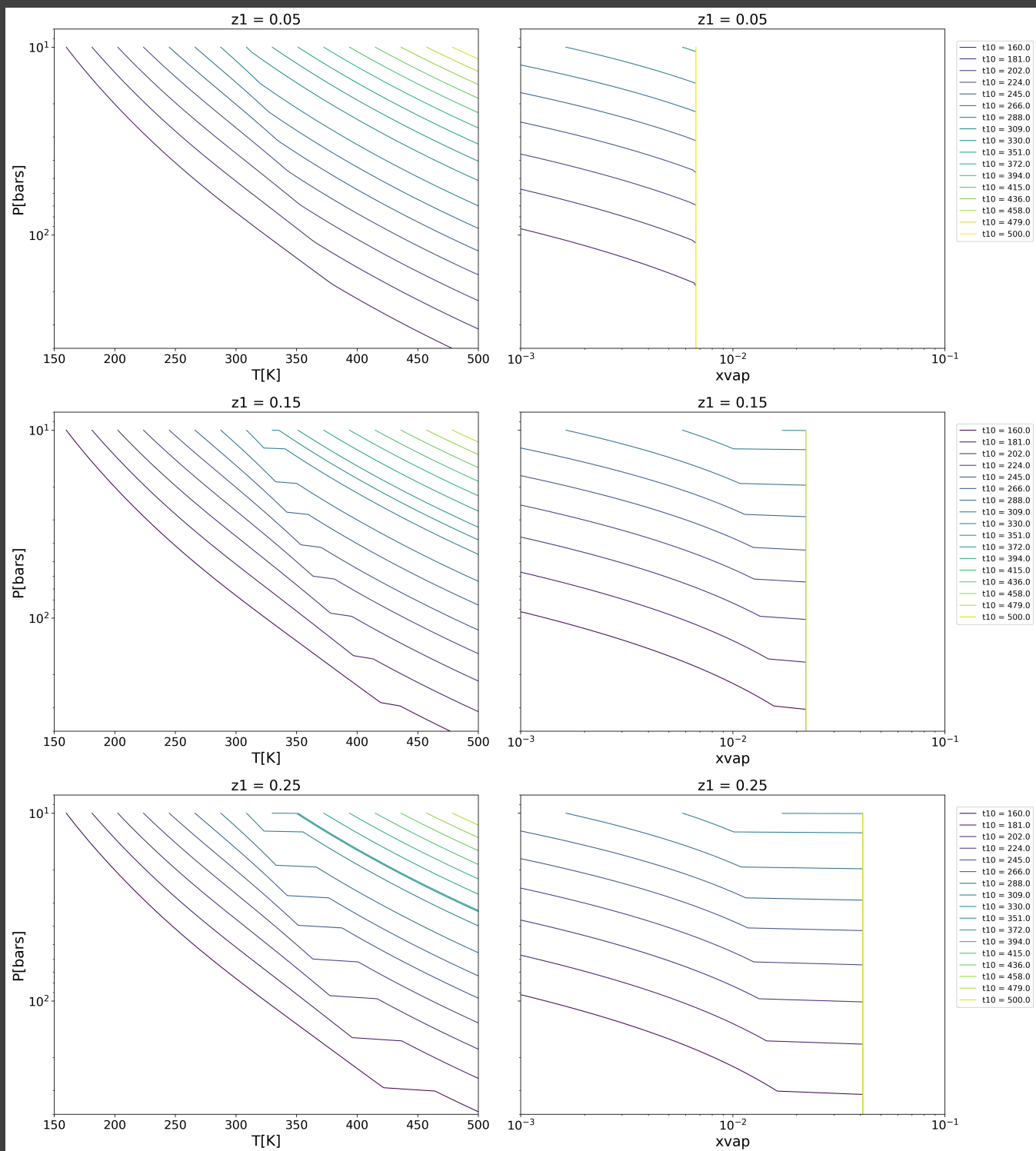


Figure 3.2: add description these plots. explain differences

## 4

## Discussion and Conclusions

## Appendix A

### Some Ancillary Stuff

# Bibliography

Fortney, J. J., Ikoma, M., Nettelmann, N., Guillot, T., & Marley, M. S. (2011). Self-consistent model atmospheres and the cooling of the solar system’s giant planets. *The Astrophysical Journal*, 729, 32.

Friedson, A. J. & Gonzales, E. J. (2017). Inhibition of ordinary and diffusive convection in the water condensation zone of the ice giants and implications for their thermal evolution. *Icarus*, 297, 160–178.

Guillot, T. (1995). Condensation of methane, ammonia, and water and the inhibition of convection in giant planets. *Science*, (pp. 1697–1699).

Guillot, T. (2019). Uranus and neptune are key to understand planets with hydrogen atmospheres. *arXiv.org*.

Harold C. Graboske, Jr., J. B. P. A. S. G. & Olness, R. J. (1975). The structure and evolution of jupiter: The fluid contraction. *The Astrophysical Journal*, 199, 265–281.

Hubbard, W. (1977a). Comparative thermal evolution of uranus and neptune. *Icarus*, , 35, 177–181.

Hubbard, W. (1977b). The jovian surface condition and cooling rate. *Icarus*, , 30, 305–310.

