UNIVERSITY of CALIFORNIA
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# THERMAL EVOLUTION OF URANUS AND NEPTUNE WITH CONDENSATION-INHIBITED CONVECTION

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in

ASTROPHYSICS

by

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**Abstract**

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This will be the last section written, once we have finished our results and conclusion.

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To Who,

M ention to who, if anyone, here

## Acknowledgements

I'd like to thank....

# 1

# Introduction

Observations of Uranus show a planet that appears to be in thermal equilibrium with the Sun. Observations have also shown that Uranus is cooler than its more distant neighbor, Neptune. Thermal evolution models for Uranus have not matched observation, instead predicting a warmer effective temperature during the current epoch(Fortney et al., 2011), (M. Podolak, 1991), (W.B. Hubbard, 1995), (L. Scheibe, 2019) [There are other papers by Nettelmann 2013, Linder 2019 that I haven't looked at yet].

There have been various attempts to explain the underluminous Uranus. The formation of stable layers, trapping internal energy in the the interior of Uranus and Neptune was proposed by (M. Podolak, 1991). There has also been much work done investigating the formation of stable condensation zones that inhibit convection (Friedson & Gonzales, 2017), (Leconte et al., 2017), and (Guillot, 1995).

# 2

# Theory

## 2.1 Background: Thermal Evolution with Dry Convection

Understanding the physics of solar system gas and ice giant planets, and attempts to model their thermal evolution date back to the mid to late twentieth century (Hubbard, 1977b), (Hubbard, 1977a), and (M. Podolak, 1991). Their interior structure is determined by the following equations. Conservation of mass:

$$\frac{dm}{dr} = 4\pi r^2 \rho \tag{2.1}$$

where $dm$ is the mass contained within a sphere of radius $r$. $\rho(r)$ is the density at radius $r$. Hydrostatic equilibrium is assumed and described by:

$$\frac{dP}{dr} = -\frac{Gm\rho}{r^2} \tag{2.2}$$

where $P$ is the pressure and $G$ is the gravitational constant. NEED STATEMENT ON CONSERVATION OF ENERGY AND THERMAL EVOLUTION of the planet, we relate the intrinsic luminosity profile $L(m)$ to the rate of change of specific entropy $s$ in the planet as follows:

$$\frac{dL}{dm} = -T\frac{\delta s}{\delta t}. \tag{2.3}$$

Integrating over the mass of the planet and solving for the timestep, $\delta t$, yields

$$\delta t = -\frac{1}{L_{\text{int}}}\int_0^M T\,\delta s\,dm \tag{2.4}$$

where $L_{\text{int}}$ is the intrinsic luminosity, given by:

$$L_{\text{int}} = 4\pi r^2 \sigma_{\text{SB}} T_{\text{int}}^4 \tag{2.5}$$

A common practice is to assume a three-layer interior structure (CITE), which we assume for our model as well, as seen in Figure 2.1. At the center of the planet is a core made of ??. The inner envelope is $H_2O$ dominated, with uniform concentrations of $H$, $He$, and $H_2O$, using the MAZEVET EOS (S. Mazevet & Potekhin, 2019). The outer envelope, below 10 bars, contains trace amounts of $H_2O$, but is mostly $H$ and $He$, dominated, and utilizes the MH13SCVH EOS (Y. Miguel & Fayon, 2018). Both inner and outer envelopes are fully convective, with the pressure-temperature profile following a dry adiabat, $\nabla_{\text{ad}}$, given by :

$$T(P > P_{\text{base}}) = T_{\text{base}} + \int_{P_{\text{base}}}^{P} \left(\frac{dT}{dP}\right)_{\text{ad}} dP \tag{2.6}$$

Finally, beyond the outer envelope is the atmosphere. When modeling the thermal evolution of gas and ice giants, it has long been recognized that model atmospheres

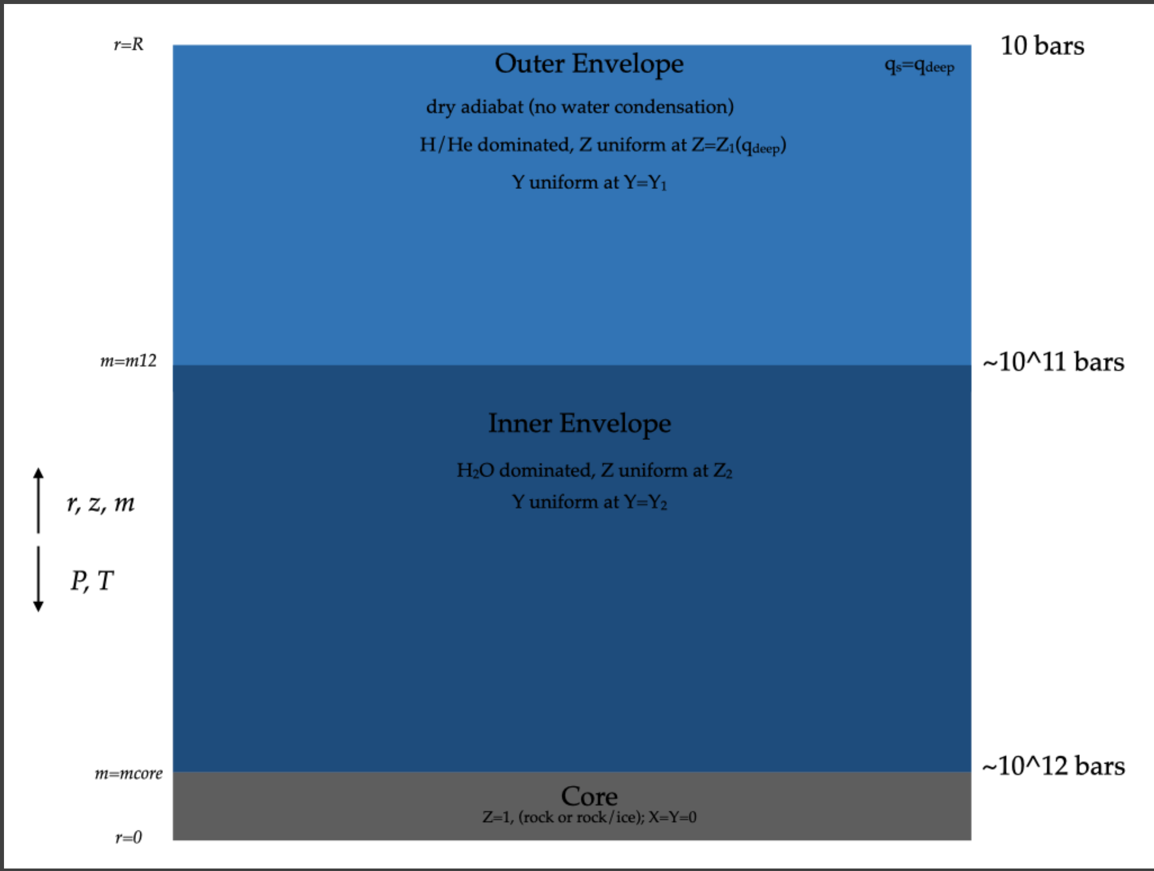


**Figure 2.1:** The structure for a fully convective, dry adiabatic interior.

constitute an outer boundary condition for interior structure models, providing key inputs that impact cooling times for interior structure models. Our work considers both (Harold C. Graboske & Olness, 1975) and (Fortney et al., 2011) model atmospheres. Unless otherwise stated, our results will utilize the Fortney 2011 model atmospheres.

## 2.2 Inclusion of Moist Convection Within Outer Envelope

Our interior structure model departs from the standard model described above by adding a moist adiabatic layer to the outer envelope. In our case, this allows for the condensation of water. We do not account for other condensates such as $CH_4$ and $NH_3$.

As seen in Figure (Y). Within the outer envelope, we implement moist adiabatic layer, such that when conditions are suitable for condensation, we allow for the formation of a stable water condensation zone. ADD PHYSICS FOR CONDENSATION (Include equations for $q_s$, latent heat, xi, saturation vapor pressure, ... other equations that define the moist adiabat.)

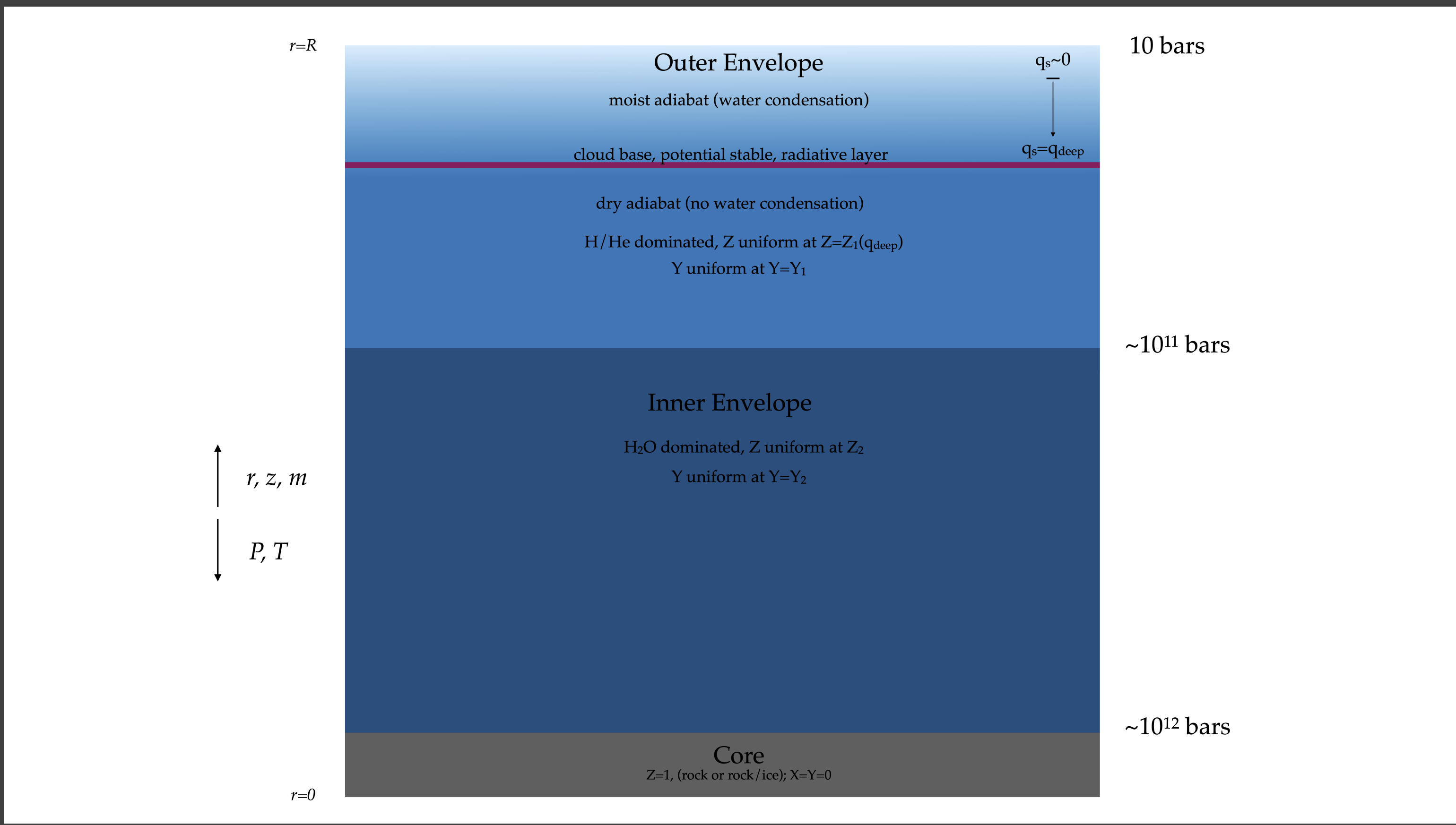


**Figure 2.2:** The structure for moist adiabatic interior, allowing for condensation-inhibited convection.

If condensation zone forms, it may be stable against convection. Convection is inhibited due to the formation of a stable condensation zone when $\alpha < 1$, where is $\alpha$ (Friedson & Gonzales, 2017) is given by:

$$\alpha = 1 + \xi(q_s L / R_W T_0) \tag{2.7}$$

If condensation is found to be inhibited,

At pressure where $\alpha < 1$, the cloud base of the water condensation zone forms. This thin, stable radiative layer has a temperature profile that is governed by:

$$T(P) = T_{\rm top} + \int_{P_{\rm top}}^{P} \left(\frac{dT}{dP}\right)_{\rm rad} dP \tag{2.8}$$

$$\left(\frac{dT}{dP}\right)_{\rm rad} = \frac{T}{P}\nabla_{\rm rad} = \frac{T}{P} \times \frac{3}{16}\frac{\kappa_R P}{g}\frac{T_{\rm int}^4}{T^4} \tag{2.9}$$

$$T_{\rm base} \equiv T(P + \Delta P) = T_{\rm top} + \left(\frac{dT}{dP}\right)_{\rm rad} \Delta P. \tag{2.10}$$

$$x_{\rm vap}(P,T) = x_{\rm vap}^{\rm sat}(P,T) = \frac{e_s(T)}{P}, \qquad P < P_{\rm base}. \tag{2.11}$$

$$x_{\rm vap}^{\rm sat}(P_{\rm base}, T_{\rm base}) = \frac{e_s(T_{\rm base})}{P_{\rm base}} = x_{\rm vap}^{\rm deep} \Longrightarrow \Delta P \equiv P_{\rm base} - P_{\rm top} = \frac{e_s(T_{\rm base})}{x_{\rm vap}^{\rm deep}} - P_{\rm top} \tag{2.12}$$

$$T_{\rm base} = T_{\rm top} + \left(\frac{dT}{dP}\right)_{\rm rad} \left(\frac{e_s(T_{\rm base})}{x_{\rm vap}^{\rm deep}} - P_{\rm top}\right) \tag{2.13}$$

Below the base of the radiative layer, the temperature-pressure profile again follows a dry adiabat, given by Equation (2.1).

# 3

# Results

## 3.1 Condensation-inhibited Convection

Talk about Figure 3.1.

## 3.2 Formation of Radiative Layer

Talk about Figure 3.2.

## 3.3 Thermal Evolution

Talk about Figure 3.3.

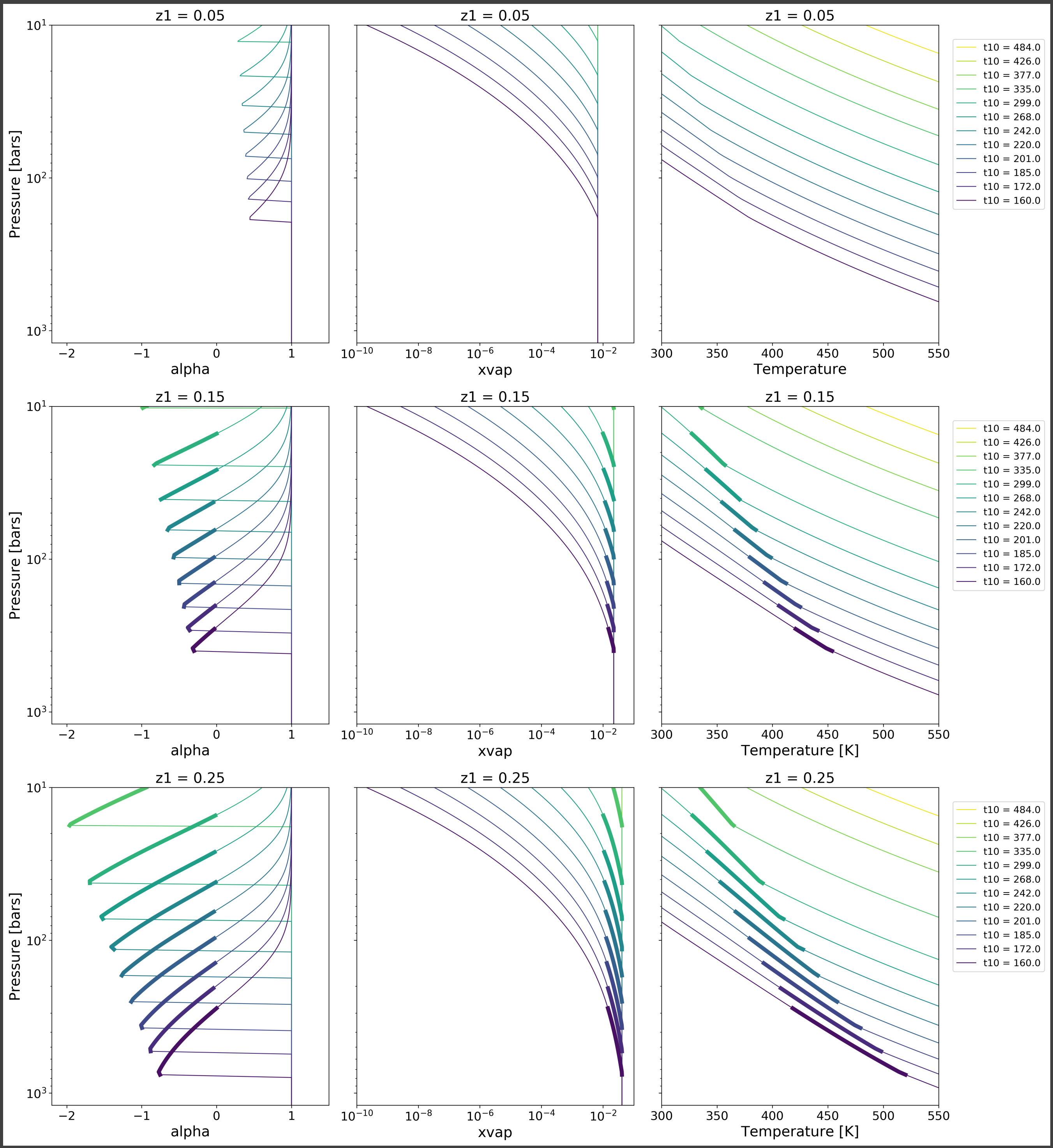


**Figure 3.1:** add description re: moist adiabat, when/where convection is inhibited and where the radiative zone base is in these plots. explain differences

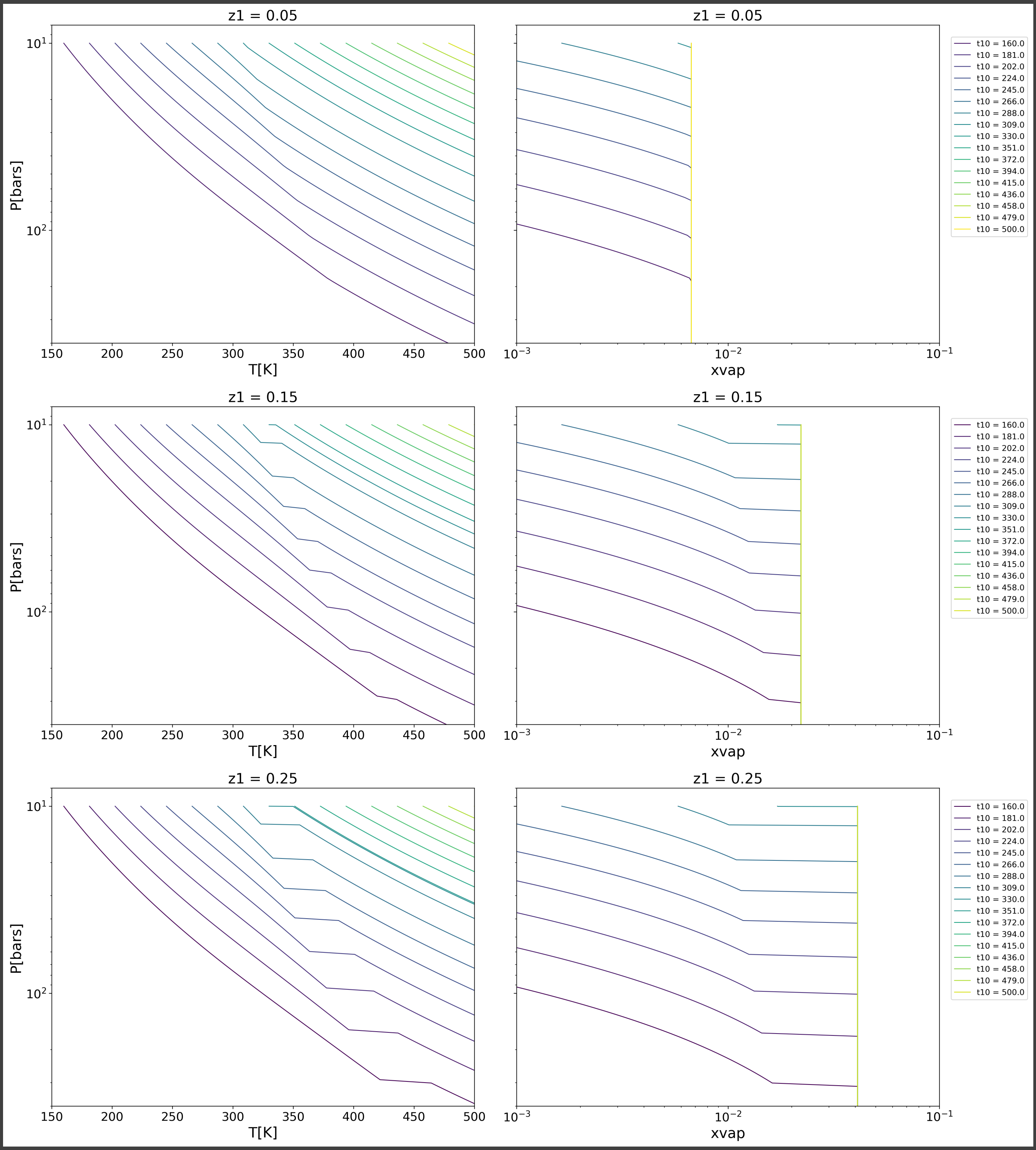


**Figure 3.2:** add description these plots. explain differences

# 4

# Discussion and Conclusions

# Appendix A

# Some Ancillary Stuff

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