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**THERMAL EVOLUTION OF URANUS AND NEPTUNE WITH
CONDENSATION-INHIBITED CONVECTION**

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by

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Abstract

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This will be the last section written, once we have finished our results and conclusion.

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To Who,

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I'd like to thank....

1

Introduction

During the mid twentieth century, Frank Low observed that Jupiter was radiating away more energy than it received from the Sun (Hubbard, 1967), (Low, 1966). Shortly after this observation, theoretical work began on investigating the physics of the interior structure of the solar system gas and ice giants (Hubbard, 1977b), (Hubbard, 1977a), (M. Podolak, 1991).

At the present time, most of the giant planets in our solar system: Saturn, Jupiter, and Neptune, all have effective temperatures greater than their equilibrium temperature. Uranus is the exception. Observations of Uranus show a planet that appears to be in thermal equilibrium with its parent star, a planet with no intrinsic temperature, cooler than its more distant neighbor, Neptune, a planet of similar mass and composition. Thermal evolution models for Uranus have not matched observation, instead predicting a warmer effective temperature at 4.6 Gyr, the current age of the solar system. (Fortney et al., 2011), (M. Podolak, 1991), (W.B. Hubbard, 1995), (L. Scheibe, 2019) [There are other papers by Nettelmann 2013, Linder 2019 that I haven't looked at yet].

There have been various attempts to explain Uranus' cool temperature. Early investigations posited that a stratified interior(s), stable against convection, would allow heat to be trapped deep within the the interior (M. Podolak, 1991). Later work built on this idea, investigating the formation of stable condensation zones(Friedson & Gonzales, 2017), (Leconte et al., 2017), and (Guillot, 1995), and thermal boundary layers(N. Nettelmann, 2016), that would inhibit convection. It was speculated that the presence of these thermal boundary layers, or condensation zones, could trap heat deep within the interior, allowing the envelope above to cool more rapidly, thereby lowering the planet's effective temperature.

1.1 Condensation in Hydrogen Dominated Atmospheres

On Earth, moist air is lighter than dry air. For example, water vapor, the primary condensate in Earth's atmosphere is lighter (not by much) than the background air which is composed primarily of N_2 . Thus, when H_2O condenses out of the atmosphere, there is small vertical gradient in mean molecular weight. This small gradient does not impose a significant barrier to convection. In hydrogen dominated atmospheres such as Neptune and Uarnus, the background gas is now much lighter than the condensate. When H_2O condenses out of the atmosphere a stong vertical gradient in mean molecular weaith can be established, resulting in a negative bouyancy for the parcel of gas, and hence creatiaing a situation where the zone is stable against convection(Guillot, 1995), (Friedson & Gonzales, 2017), (Leconte et al., 2017).

In this paper, we investigate where and when, assuming a variety deep water concentrations, whether such stable water condensation zones could form within in the outer envelope, and present their potential impact on the thermal evolution of the these

planets. We decribe our model in Chapter 2. We present our results in Chapter 3. And, we discuss our results in Chapter 4.

2

Model

2.1 Three-layer Model with Dry Adiabat

We begin with a description of the physics of our baseline structure model, starting with the conservation of mass:

$$\frac{dm}{dr} = 4\pi r^2 \rho \quad (2.1)$$

where dm is the mass contained within a sphere of radius r , and $\rho(r)$ is the density at radius r . Hydrostatic equilibrium is also assumed and described by:

$$\frac{dP}{dr} = -\frac{Gm\rho}{r^2} \quad (2.2)$$

where P is the pressure and G is the gravitational constant.

We employ a three-layer interior structure, seen schematically in Figure 2.1. At the center of the planet is a core made of rock and ice. Moving outward, the inner envelope is H_2O dominated, with uniform concentrations of H , He , and H_2O . We use the MAZEVET

equation of state (EOS) (S. Mazevet & Potekhin, 2019) to define the structure of the inner envelope. The outer envelope, below 10 bars, contains trace amounts of H_2O , but is mostly H and He , dominated, and utilizes the MH13SCVH EOS (Y. Miguel & Fayon, 2018).

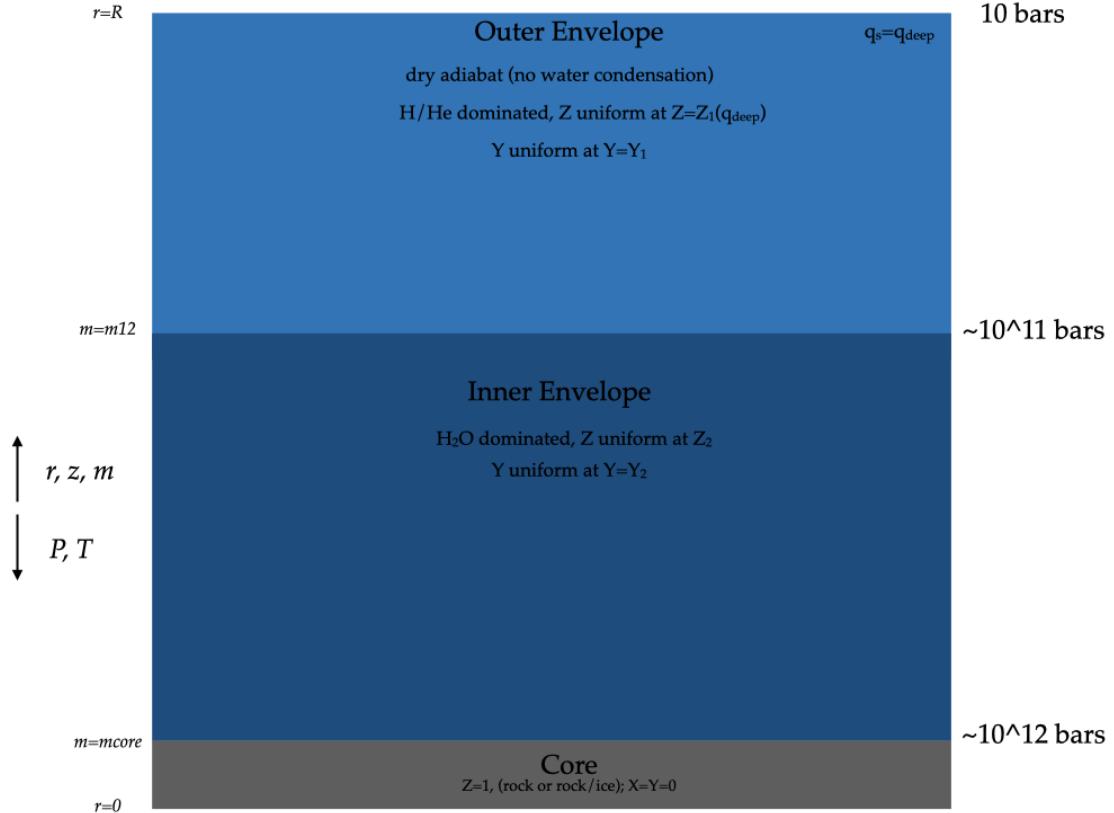


Figure 2.1: The structure for a fully convective, dry adiabatic interior. In this model, the inner and outer envelopes are assumed to be well mixed, fully convective, and following a dry adiabat. The core is composed of rock and ice. The inner envelope is water dominated, with uniform concentrations of hydrogen, helium, and water; whereas, the outer envelope is hydrogen and helium dominated, with trace amounts of water. The 'atmosphere' exists above 10 bars.

Historically, interior structure models have assumed that the interiors are composed of compressible gasses that are statically unstable and fully convective. In a dry-convective model such as this, a parcel of gas rises as its temperature increases while its

pressure remains constant. This process happens without the addition or loss of heat from the parcel, a process referred to as adiabatic. Furthermore, while there may be a critical concentration for a condensable species, this dry model does not allow for condensation. The temperature-pressure profile follows a dry adiabat gradient (R. Kippenhahn, 2012), given by:

$$\nabla_{\text{ad}} = \left(\frac{\partial \ln T}{\partial \ln P} \right)_s \quad (2.3)$$

Finally, beyond the outer envelope is the atmosphere. When modeling the thermal evolution of gas and ice giants, it has long been recognized that model atmospheres constitute an outer boundary condition for interior structure models, providing key inputs that impact cooling times for interior structure models. Our work considers both (Harold C. Grboske & Olness, 1975) and (Fortney et al., 2011) model atmospheres. Unless otherwise stated, our results will utilize the Fortney 2011 model atmospheres.

2.2 Inclusion of Moist Adiabat Within Outer Envelope

Our interior structure model modifies the baseline structure described above by adding a moist adiabatic layer to the outer envelope that, under favorable conditions, allows for the condensation of H_2O . We define the moist adiabat as follows:

$$\nabla_{\text{moist}} = \left(1 + \frac{\frac{x_{\text{vap}} L}{R_{\text{gas}} T}}{\nabla_{\text{ad}} + \frac{L^2}{R_{\text{gas}}^2 T^2}} \right) \quad (2.4)$$

where,

$$\frac{dT}{dP} = \frac{T}{P} \nabla_{\text{moist}} \quad (2.5)$$

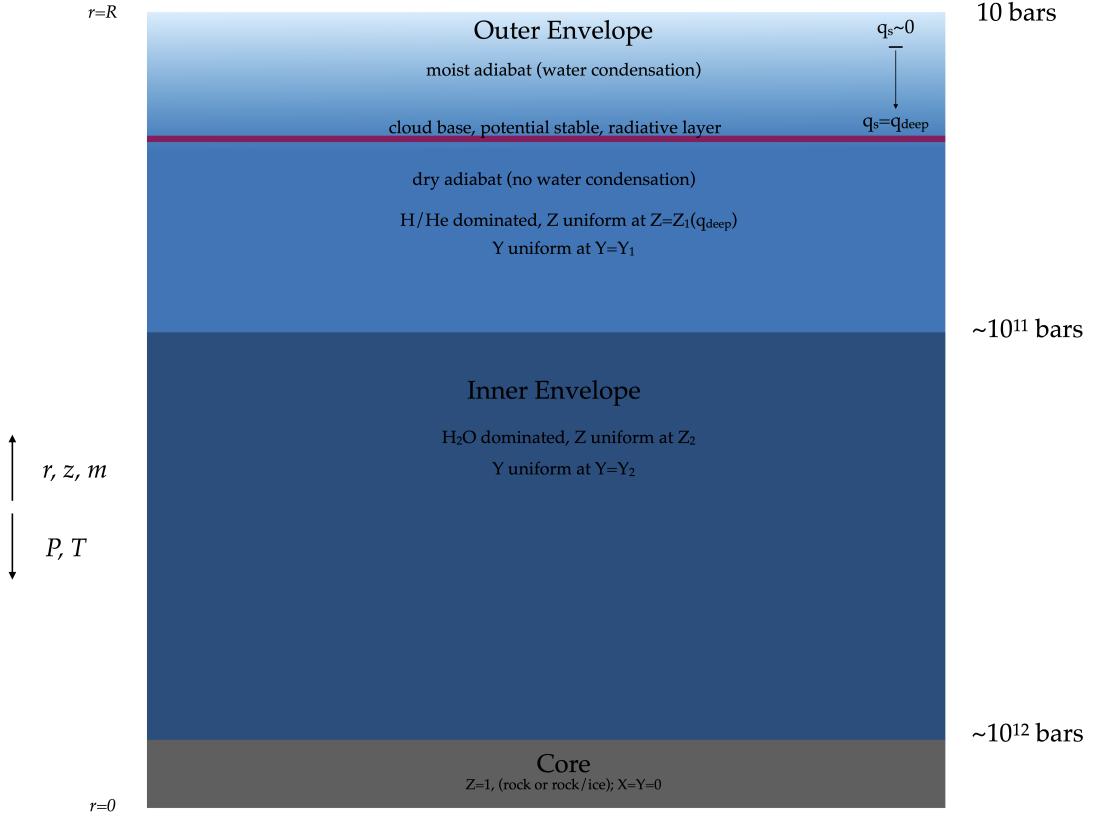


Figure 2.2: The structure for moist adiabatic interior, allowing for condensation-inhibited convection. In this model, a stable water condensation zone may form. The pressure and temperature at the base of the condensation zone is set by the condition that x_{vap} has reached the deep value $x_{\text{vap}}^{\text{deep}}$. Below the condensation zone, the temperature and pressure follow a dry adiabat.

and the gradient of the water vapor mole fraction is given by,

$$\frac{dx_{\text{vap}}}{dP} = \frac{x_{\text{vap}} L}{R_{\text{gas}} T^2} \frac{dT}{dP} - \frac{x_{\text{vap}}}{P} \quad (2.6)$$

Gases condense at sufficiently low temperatures or high pressures. Condensation of a gas is characterized by its saturation vapor pressure (Lavega, 2011), P_{sat} , given by:

$$P_{\text{sat}}(T) = P_{\text{sat}}(T_0) e^{-\frac{L+C_p T_0}{R_{\text{gas}}} (\frac{1}{T} - \frac{1}{T_0}) - \frac{C_p}{R_{\text{gas}}} \ln \frac{T}{T_0}} \quad (2.7)$$

where $T_0 = 273.16K$.

where R_{gas} is the gas constant for the condensable species. When the partial pressure of a gas, P_{gas} , is less than P_{sat} , the parcel of gas is 'unsaturated'. When $P_{\text{gas}} = P_{\text{sat}}$, the gas is 'saturated'. And, when $P_{\text{gas}} > P_{\text{sat}}$, the parcel is 'supersaturated'. Each condensable species has its own saturation vapor pressure.

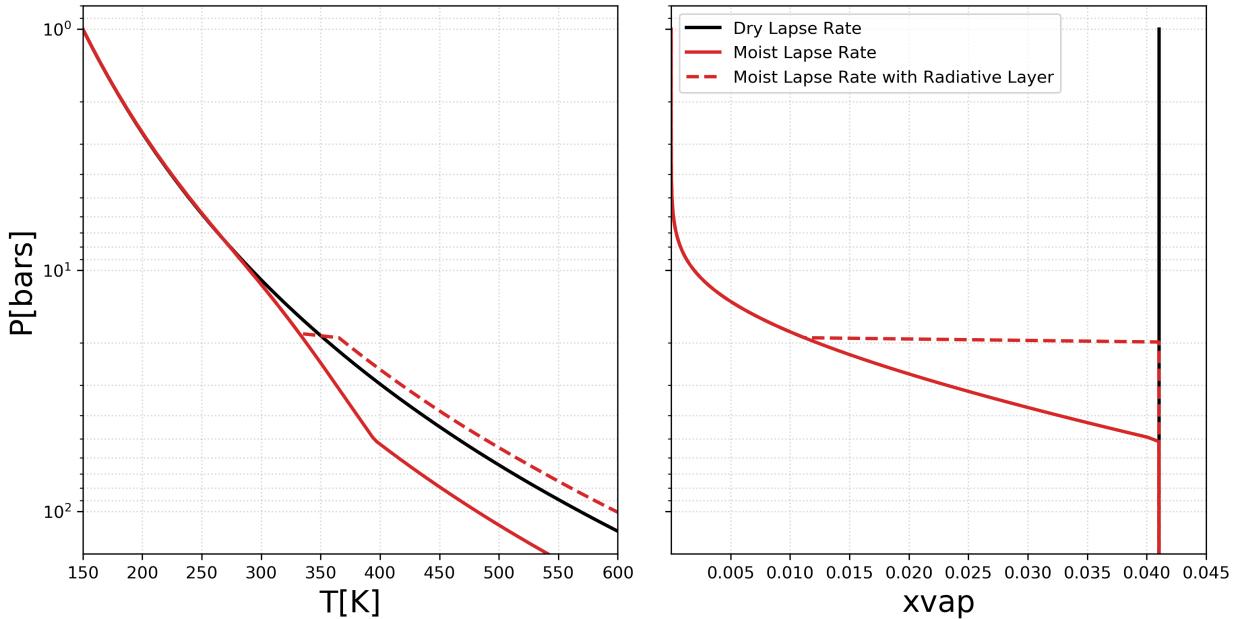


Figure 2.3: The solid red line is the moist adiabatic lapse rate. The solid black line is the dry lapse rate. The dashed red line is a moist lapse rate with the addition of a stable radiative layer (water condensation zone). The simple moist lapse rate is cooler at depth than either of the other two lapse rates. The presence of a stable radiative layer results in a warmer interior. These lapse rates assume $q_{\text{deep}} = 0.25$ and $T_1 = 150K$, which is during the period when the onset of condensation-inhibited convection occurs.

If condensation occurs, we assume that it may be stable against convection if a fast rainout occurs such that the vertical gradient in mean molecular weight is large enough to counteract the positive buoyancy of the parcel of gas (Leconte et al., 2017) (Friedson

& Gonzales, 2017). In this scenario, condensation-inhibited convection occurs when α is negative, where α (Friedson & Gonzales, 2017) is given by:

$$\alpha = 1 + \xi(q_s L / R_W T_0) \quad (2.8)$$

where R_{vap} is the gas constant for the vapor (water), T_0 is the local temperature, L is the latent heat of vaporization for water, q_s is the saturation specific humidity, and ξ is given by $\xi = \frac{1}{\epsilon} - 1$, where ϵ is the ratio of the molecular weight of vapor to the mean molecular weight of dry atmosphere. When α is negative, the vertical gradient in molecular weight results in a stabilizing effect, overwhelming the effects due to latent heat release.

2.3 Temperature Jump Across the Water Condensation Zone

Our model treats the radiative layer as a discontinuous increase in temperature. This, stable, radiative layer has a temperature profile that is governed by:

$$T(P) = T_{\text{top}} + \int_{P_{\text{top}}}^{P_{\text{base}}} \left(\frac{dT}{dP} \right)_{\text{rad}} dP \quad (2.9)$$

with P_{top} and T_{top} denote the pressure and temperature at the top of the stable water condensation(radiative) zone, and P_{base} represents the bottom of the zone. The integrand is the radiative temprature gradient (Leconte et al., 2017), defined as:

$$\left(\frac{dT}{dP} \right)_{\text{rad}} = \frac{T}{P} \nabla_{\text{rad}} = \frac{T}{P} \times \frac{3}{16} \frac{\kappa_R P}{g} \frac{T_{\text{int}}^4}{T^4} \quad (2.10)$$

The radiative temperature gradient across the layer is nearly constant, so that the integral above simplifies to:

$$T_{\text{base}} \equiv T(P + \Delta P) = T_{\text{top}} + \left(\frac{dT}{dP} \right)_{\text{rad}} \Delta P \quad (2.11)$$

where ΔP is the extent of the pressure-space of the radiative layer, given by:

$$\Delta P \equiv P_{\text{base}} - P_{\text{top}} = \frac{P_{\text{sat}}(T_{\text{base}})}{x_{\text{vap}}^{\text{deep}}} - P_{\text{top}}. \quad (2.12)$$

$$x_{\text{vap}}^{\text{sat}}(P_{\text{base}}, T_{\text{base}}) = \frac{P_{\text{sat}}(T_{\text{base}})}{P_{\text{base}}} = x_{\text{vap}}^{\text{deep}} \quad (2.13)$$

Within the condensation zone, the vapor mole fraction, x_{vap} is equal to the saturated vapor mole fraction:

$$x_{\text{vap}}(P, T) = x_{\text{vap}}^{\text{sat}}(P, T) = \frac{P_{\text{sat}}(T)}{P}, \quad P < P_{\text{base}}. \quad (2.14)$$

The pressure and temperature at the base of the condensation zone is set by the condition that x_{vap} has reached the deep value $x_{\text{vap}}^{\text{deep}}$.

Below the condensation zone, the region is subsaturated and hence no condensation occurs. Deeper temperatures are then obtained by integrating the dry adiabat ∇_{ad} :

$$T(P > P_{\text{base}}) = T_{\text{base}} + \int_{P_{\text{base}}}^P \left(\frac{dT}{dP} \right)_{\text{ad}} dP \quad (2.15)$$

2.4 Energy Conservation and Thermal Evolution of Model

We borrow from conservation of energy and stellar structure and evolution to determine the timestep needed to evolve our model (R. Kippenhahn, 2012). We begin with the planets' intrinsic luminosity, defined as:

$$L_{\text{int}} = 4\pi R^2 \sigma_{\text{SB}} T_{\text{int}}^4 \quad (2.16)$$

And, the first law of thermodynamics states:

$$dQ = du + Pdv \quad (2.17)$$

where dQ is the heat added per unit mass, u is the internal energy per unit mass, and $v = \frac{1}{\rho}$ per unit mass. So, for the heat per unit mass shell, we may write:

$$dQ = (\epsilon - \frac{dL}{dm})dt \quad (2.18)$$

Substituting our equation for the first law of thermodynamics, we arrive at:

$$\frac{\partial L}{\partial m} = \epsilon - \frac{\partial u}{\partial t} + \frac{P}{\rho^2} \frac{\partial \rho}{\partial t} \quad (2.19)$$

Here, we are not dealing with any nuclear reactions, so $\epsilon = 0$, giving us:

$$\frac{\partial L}{\partial m} = \frac{\partial u}{\partial t} + \frac{P}{\rho^2} \frac{\partial \rho}{\partial t} \quad (2.20)$$

3

Results

3.1 Condensation-inhibited Convection

We began by looking at if and when condensation-inhibited convection would occur. We ran static models for a variety of T_{10} 's, or points in Uranus' past, moving from warmer (past) temperatures to cooler (recent) temperatures. The output of these static simulations are plotted in Figure 3.1. We ran our model with three different values of q_{deep} . The goal was to determine for each deep water concentration, at what pressures and temperatures, would condensation-inhibited convection occur. We found that for $q_{\text{deep}} = 0.05$, no condensation-inhibited convection occurred. In other words, α (Eqn. 2.8) was never negative. However, for larger values of q_{deep} , we found that α did take on negative values in the planet's past (see rows 2 and 3 in Figure 3.1). The shaded regions of the plots indicate the pressure-space over which α is negative. We show α with respect to pressure, vapor mole fraction, and temperature. Note: the top of the shaded region in each plot would indicate the base of the condensation zone. [not sure if i should go into the issue of the

remainder of the shaded region having no physical meaning]. Looking at these plots, for $q_{\text{deep}} = 0.15$ and $q_{\text{deep}} = 0.25$, we can see that condensation inhibited convection sets in at approximately $T_{10} = 335K$. Note: The T_{10} at which the onset of condensation occurs is of course dependent on the resolution of the simulation. As such, these plots are meant to show the likelihood of condensation-inhibited convection rather than the exact moment or specific deep water concentration for which it occurs.

3.2 Formation of Radiative Zone

The plots in Figure 3.3 show the lapse rate (first column), and the change in the vapor mole fraction for H_2O (second column), for three different values of q_{deep} , 0.05, 0.15, and 0.25, for rows 1 through 3, respectively. In the first row, we can see that for early T_{10} 's, there is no onset of condensation, and the lapse rate follows a dry adiabat. For later T_{10} 's, there is a visible kink in the lapse rate which indicates the onset of condensation, at which point the lapse rate has a shallower slope. For the larger values of q_{deep} , where α takes on negative values, we see the onset of condensation-inhibited convection and the establishment of a radiative zone. In the plots, these condensation (stable, radiative) zones are represented by the horizontal discontinuities moving from left to right. As the planet cools, these radiative zones descend deeper into the planet's interior. When the radiative zones are established, the temperature jump across the radiative zone creates a warmer interior. Looking at Figure 3.2, we have overlayed the lapse rate for $q_{\text{deep}} = 0.25$ (containing radiative zones) over the lapse rate for $q_{\text{deep}} = 0.05$ (no radiative zones). From this plot, one can see that the presence of a radiative zone creates a temperature jump such that a given T_{10} appears to look like an earlier T_{10} . In other words, in the presence of radiative

zones, the interior appears as warm as an earlier interior.

Looking at the adjacent x_{vap} plots, we can see that x_{vap} follows its saturated value. At the bottom of the radiative zone, the vapor mole fraction equals its deep water value, which sets the conditions for the creation of the base of the condensation zone.

3.3 Thermal Evolution of Uranus and Neptune

We simulated the thermal evolution of Uranus and Neptune with a variety of parameters. In Figure 3.4, we consider separately the evolution of a dry adiabat, a moist adiabat with condensation but no stable radiative zone, and finally a moist adiabat with condensation containing stable radiative zones. For all of these evolutionary tracks, we assumed $q_{\text{deep}} = 0.25$. Looking at these evolutionary tracks, the coolest scenario at present time, is a moist adiabat that is never stable against convection. The moist adiabat that is stable against convection has the warmest outcome. In Figure 3.5 and Figure 3.6, we consider the impact of different deep water concentrations on the thermal evolution of Uranus and Neptune, respectively. As the planets cool, their radiative zones descend deeper into the interior, as we saw in Figure 3.3. This feature is also noticeable in the thermal evolution plots. Looking at T_{eff} at just before 10^8 Gyr, the onset of condensation-inhibited convection occurs, resulting in a discontinuous temperature drop. The same behavior is seen in the T_{10} plot, however, by this time the radiative zone has descended deeper, later in time at around 10^9 Gyr. Larger q_{deep} 's result in warmer Uranus and Neptune at present time. We also look at the impact of q_{deep} on the evolution of planetary radius and find that larger values of q_{deep} tend to converge more closely toward the presently observed radius for both Uranus and Neptune in these simulations.

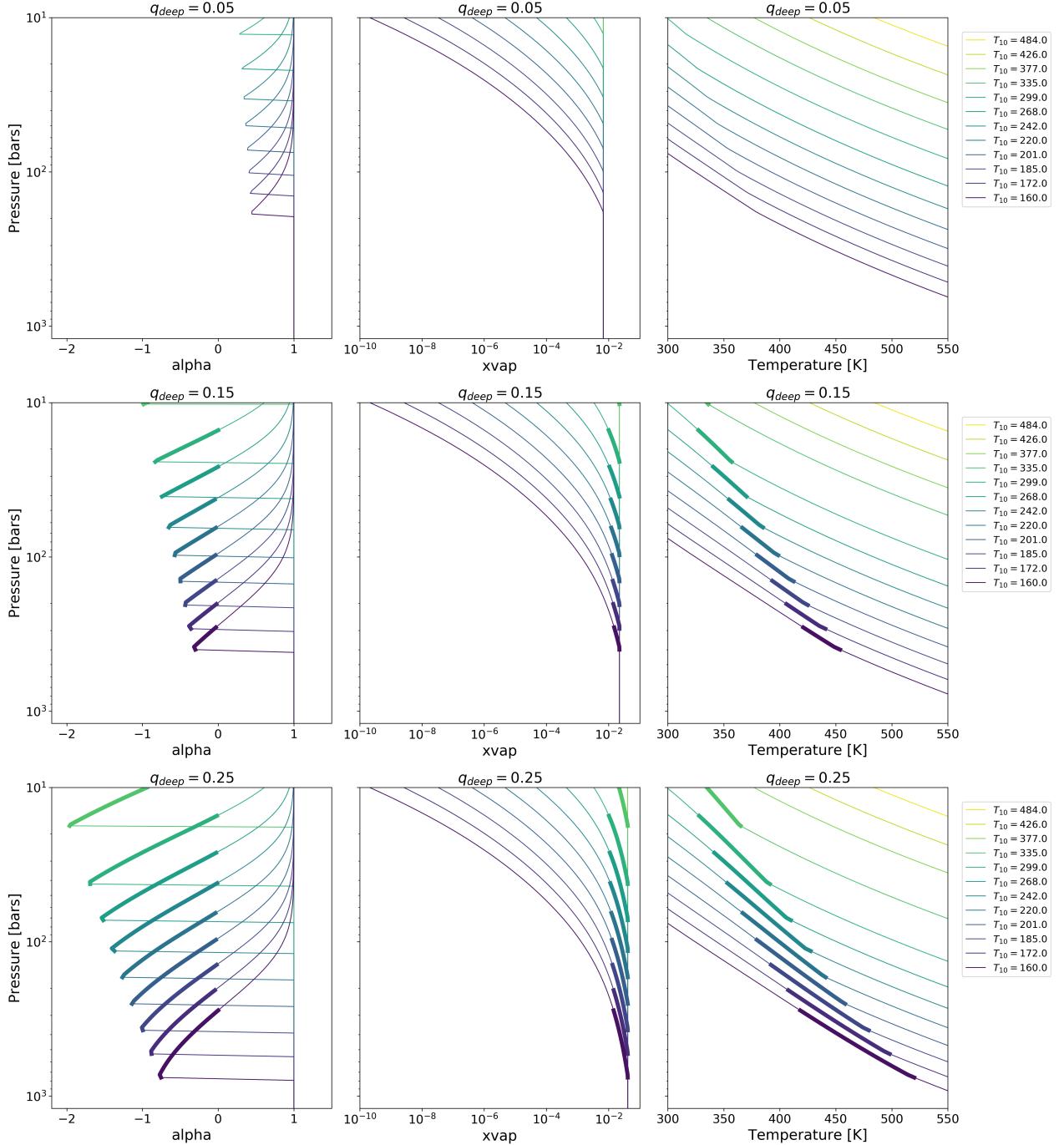


Figure 3.1: Each row represents a different value for q_{deep} . For the top row, $q_{\text{deep}} = 0.05$, for which no stable condensation zone forms. For the middle row, $q_{\text{deep}} = 0.15$, and for the third row $q_{\text{deep}} = 0.25$. For these concentrations, stable condensation zones can form. The shaded regions of the plots indicate zones where α is negative, or where stable zones can form. Note: Physically, the top of the shaded region is where the stable zone would form. Below the top of the shaded region has no physical meaning, as this model did not create a water condensation zone, but just checked for its existence.

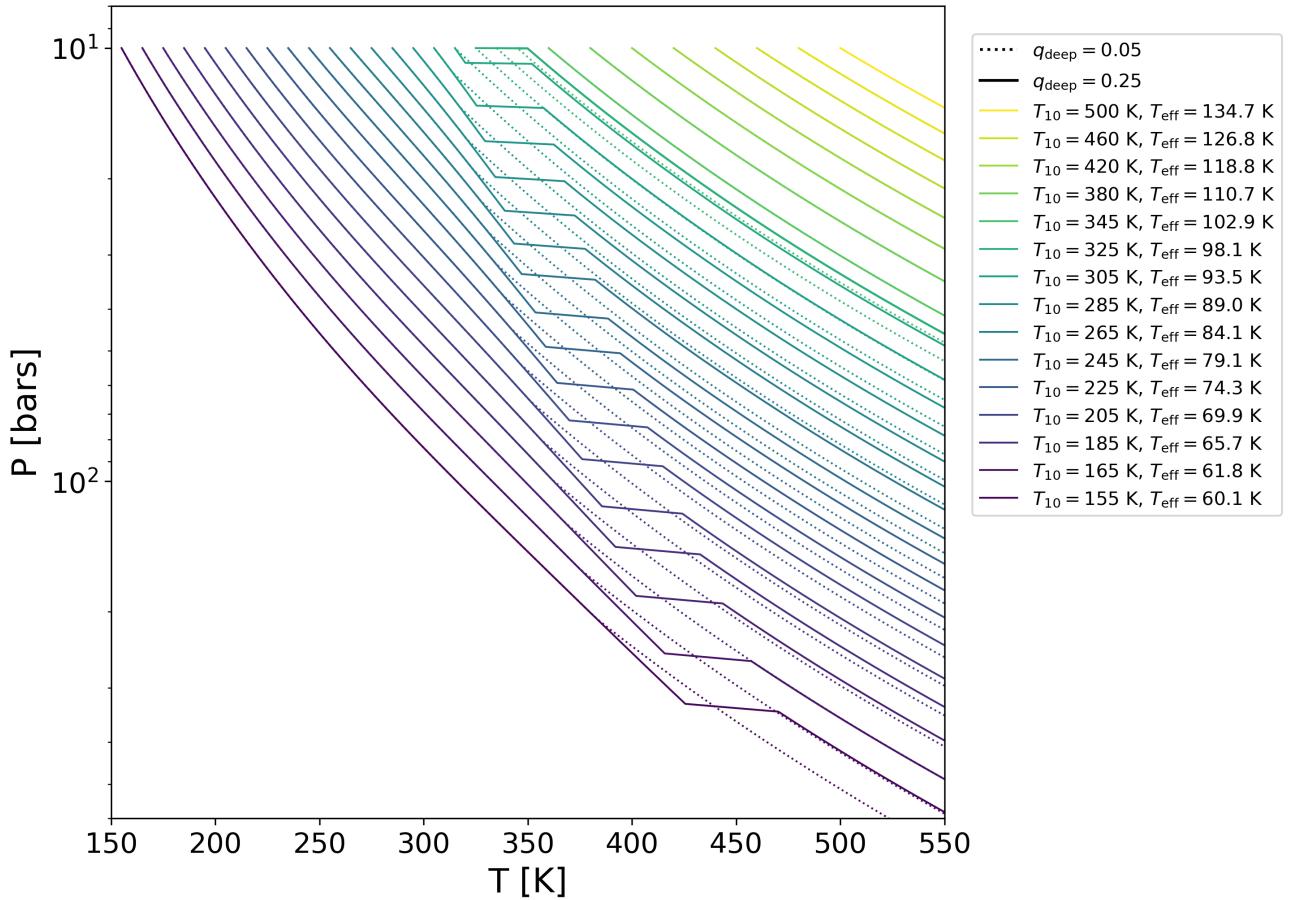


Figure 3.2: The solid lines represent the lapse rate for $q_{\text{deep}} = 0.25$, and the dashed lines for $q_{\text{deep}} = 0.05$. Looking at recent T_{10} 's, the lapse interior temperature for $q_{\text{deep}} = 0.25$ jumps to an earlier t_{10} . The radiative layer traps heat, such that the interior looks like the interior from an earlier T_{10} .

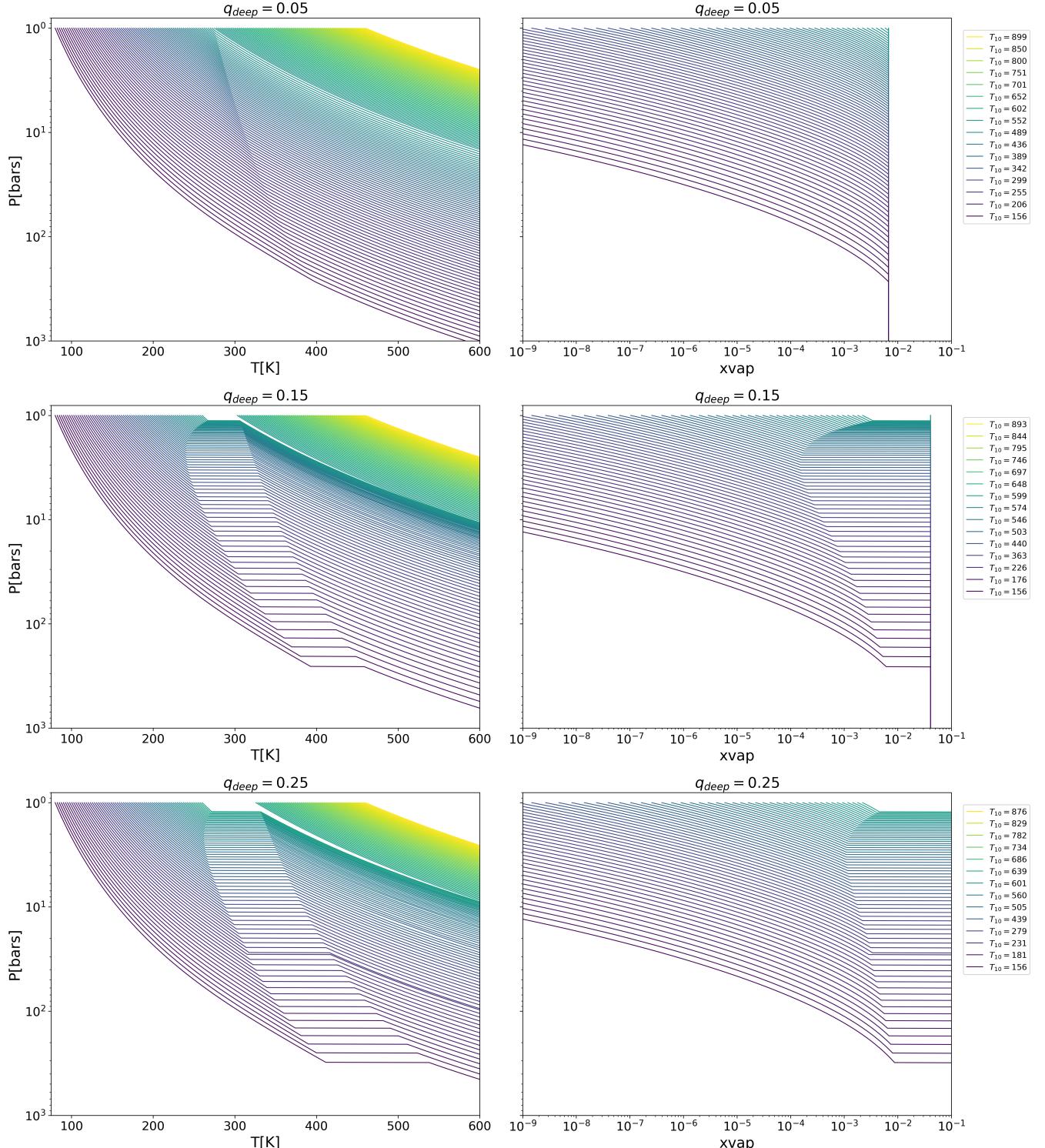


Figure 3.3: These plots were generated using our model Uranus. Again, from top to bottom row, we move from $q_{\text{deep}} = 0.05$, 0.15, and 0.25, respectively. T_{10} 's range from hotter (yellow) to cooler (purple), more recent temperatures. In the top row, no stable radiative zones are formed. The kink visible in the middle of the top left plot represents the transition from a moist to dry adiabat. Condensation occurs, but no stability is achieved. In rows two and three, stable radiative zones are

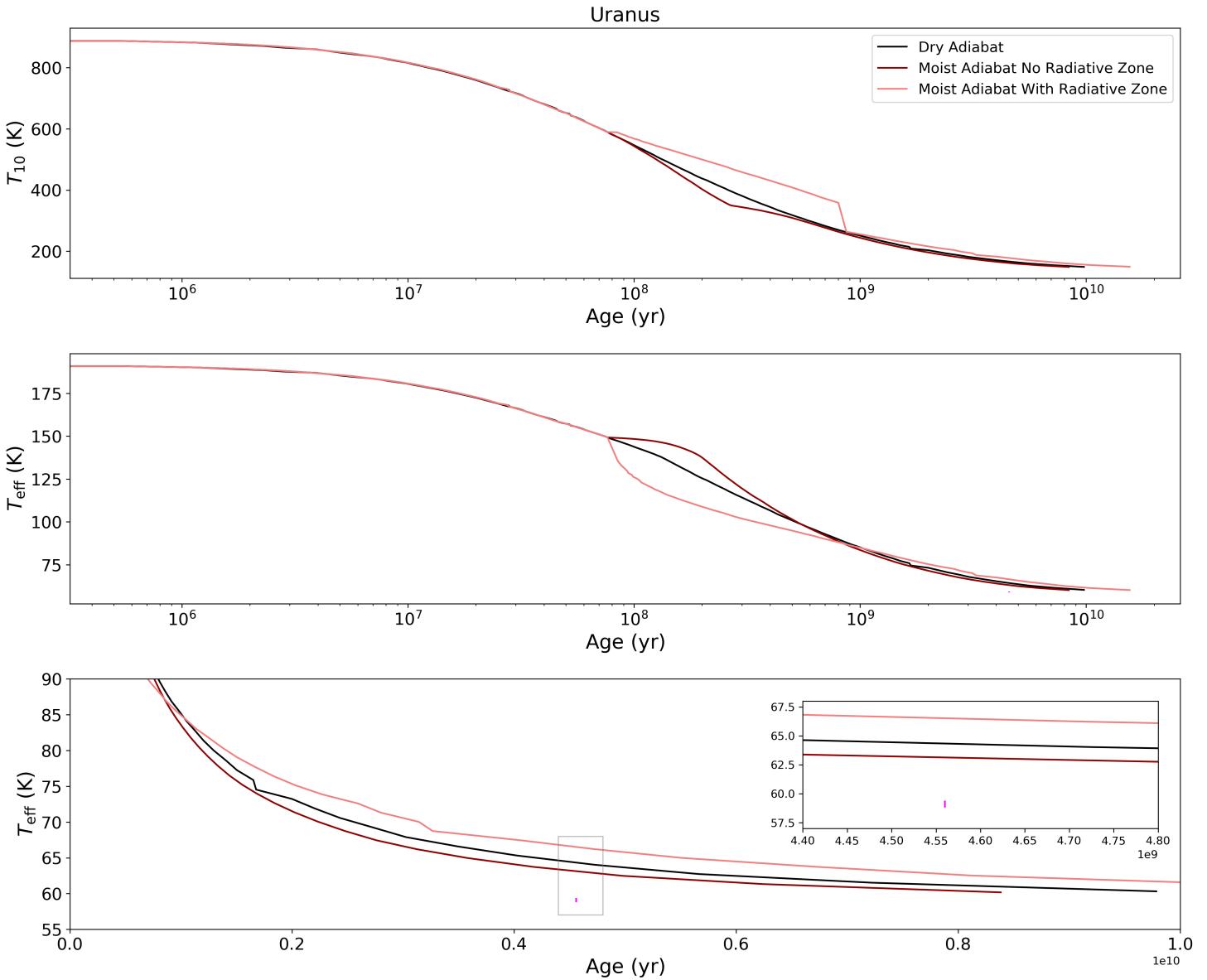


Figure 3.4: The black line represents the thermal evolution for a dry adiabat. The dark red line represents the thermal evolution for a moist adiabat that does not allow for the formation of a stable radiative layer. The light red line represents the thermal evolution of a moist adiabat that does allow for the formation of a stable radiative zone. The fuschia dot on the lower plot represent the currently observed effective temperature of Uranus [include temp], plus or minus [include temp error].

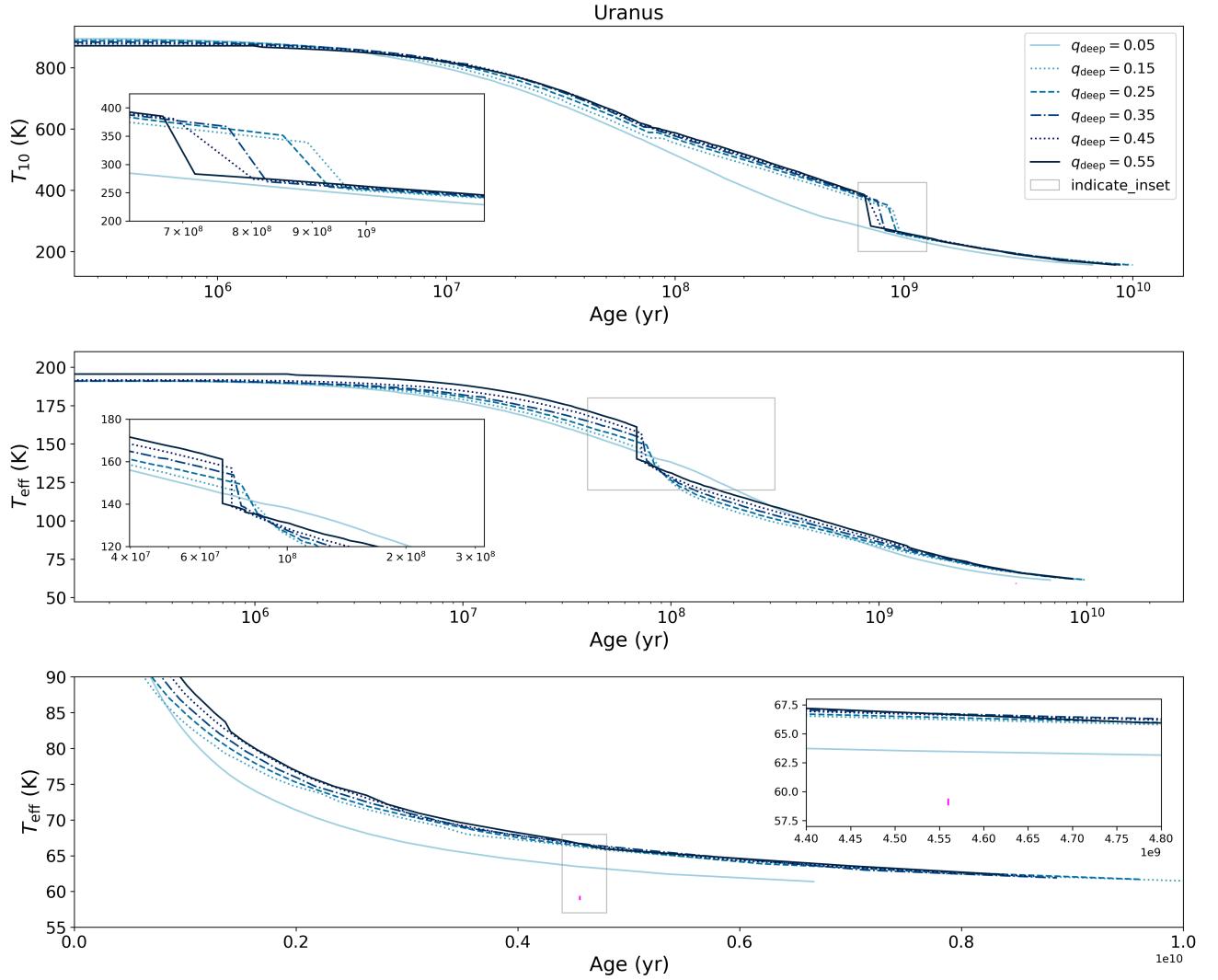


Figure 3.5: The curves in these plots represent thermal evolution tracks for different values of q_{deep} . Dark blue is the largest concentration of water vapor, at $q_{\text{deep}} = 0.55$ and the light blue line is the least concentration of water vapor at $q_{\text{deep}} = 0.05$. For $q_{\text{deep}} = 0.05$, there is no onset of condensation-inhibited convection and no rapid cooling episode. For larger values of q_{deep} there is a rapid cooling episode for T_{eff} at around 10^8 Gyr. Similarly, a rapid cooling episode is visible deeper down in the interior as seen in the T_{10} curves at around 10^9 Gyr.

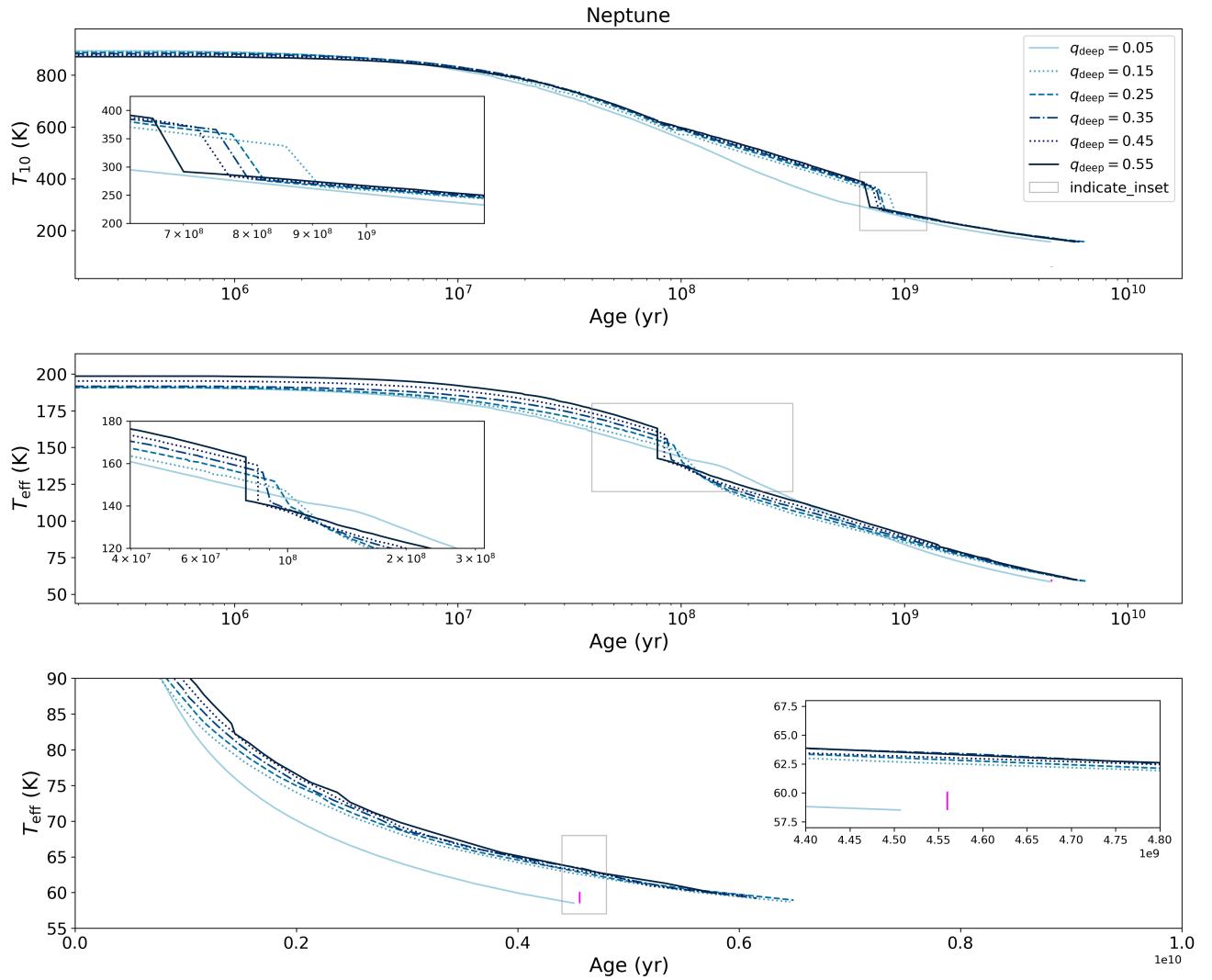


Figure 3.6: Similar to the Uranus plots, these curves represent cooling tracks for q_{deep} 's ranging from 0.05 to 0.55. Similar to Uranus, the rapid cooling episodes for T_{eff} and T_{10} occur at 10^8 Gyr and 10^9 Gyr, respectively. The vertical fucshia line in the bottom plot indicates the current observed effective temperature of Neptune [include temp], plus or minus [include temp error]

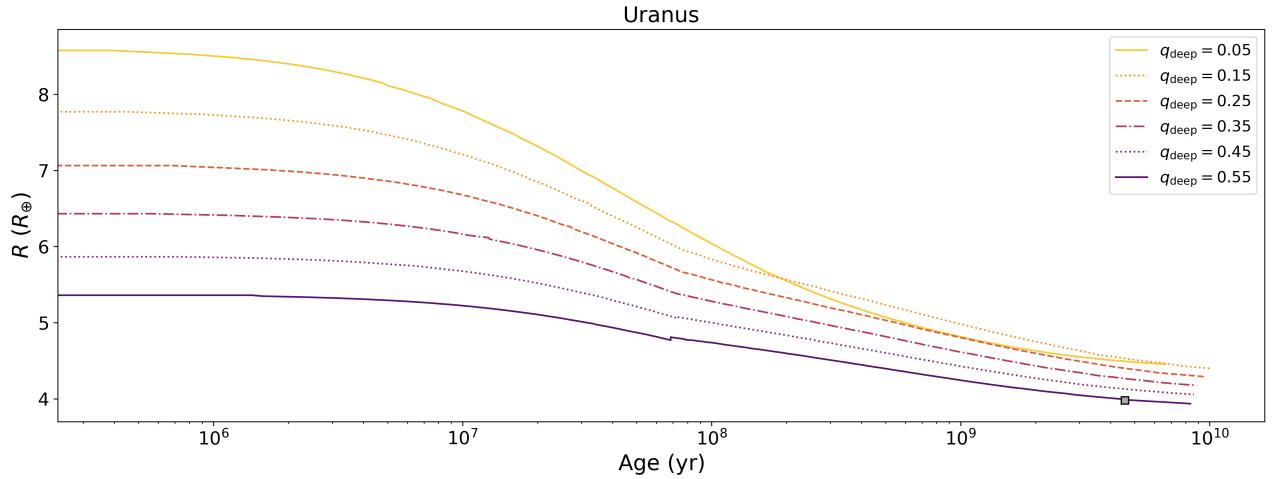


Figure 3.7: This thermal evolution plot shows the impact of different deep water concentration on the radius as the planet cools. The gray square represents the current observed radius.

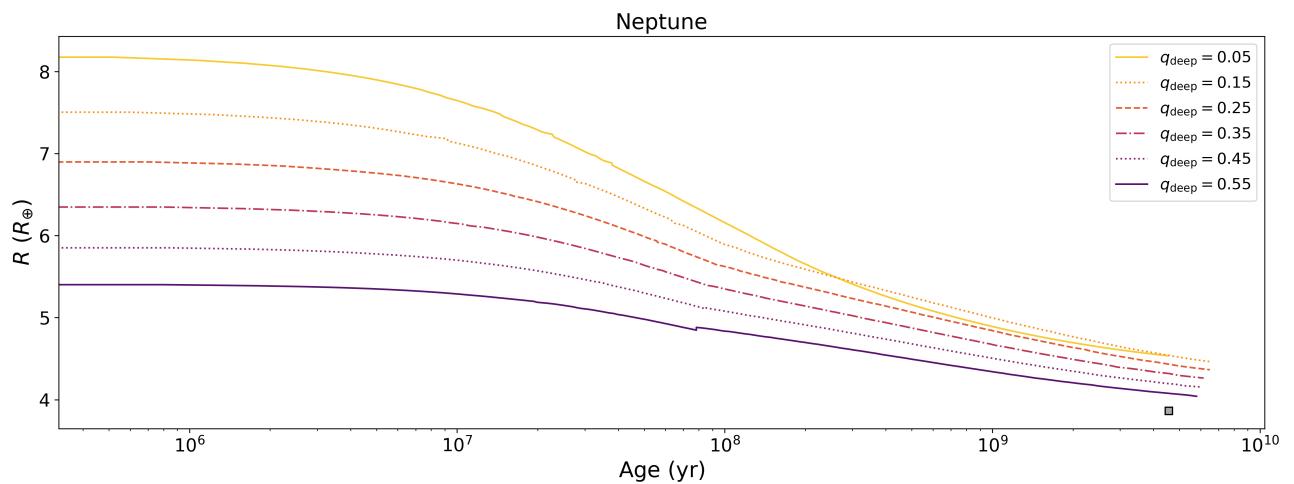


Figure 3.8: [Combine this plot with the above and have one caption.]

4

Discussion and Conclusions

We set out to investigate the impact of water condensation zones on the thermal evolution of our solar system ice giants. It has been speculated that such thermal boundary layers could act as an imperfect insulator, trapping heat below and allowing the envelope above the boundary layer to cool more rapidly (N. Nettelmann, 2016)(Friedson & Gonzales, 2017)(Leconte et al., 2017)(M. Podolak, 1991)(L. Scheibe, 2019). It seems plausible that interiors containing these thermal boundary layers could explain the problem with Uranus appearing to have no intrinsic temperature. Our findings are inconclusive. We do find that incorporating a moist adiabat into our interior structure model does result in a cooler planet, than would otherwise be seen with a purely dry model. However, when we add stable radiative zones to the interior, we do find in the planet's past a period of rapid cooling that results in a cooler effective temperature at around 10^8 Gyr, the planet eventually becomes warmer at present time than predicted by dry or simple moist adiabatic models. Reality is certainly more complex than the assumptions upon which our model is based. It is quite possible that reality resembles something in between the binary choice of a moist adiabat

with or without thermal boundary layers(Guillot, 2019). Our assumption of a stable shell of water condensation assumes that there are no other dynamics at play, such as upwellings or entrainment pressure (Friedson & Gonzales, 2017) eroding and punching holes in the stable radiative zone. Such scenarios could allow for more mixing of the warm gases below and above the condensation zone. We also considered only one condensate, H_2O . It would be worth considering NH_3 and CH_4 , and analyzing the impact of multiple stratified layers on the cooling of the planet over time.

Appendix A

Some Ancillary Stuff

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