ICARUS 30, 305–310 (1977)

The Jovian Surface Condition and Cooling Rate

W. B. HUBBARD

Department of Planetary Sciences, Lunar and Planetary Laboratory, University of Arizona, Tucson, Arizona 85721

Received April 6, 1976; revised June 2, 1976

A theory which is almost fully analytic is used to investigate Jupiter's cooling rate. We find that a simple model of contraction with adiabatic interior structure gives a total cooling time to the present which is in good agreement with the age of the solar system. The interplay between the surface condition and the cooling rate is exhibited and discussed. The current rate of change of the effective temperature is calculated to be $-1°K/0.145 \times 10^9$ yr. Discrepancies with fully numerical investigations of the Jovian age and cooling rate are noted.

I. INTRODUCTION

Hubbard (1968) showed that the intrinsic thermal radiation from Jupiter could be attributed, in order of magnitude, to Kelvin contraction–cooling of a degenerate hydrogen fluid body. Since that paper, a number of investigations have attempted to study the thermal history of Jupiter quantitatively. The most elaborate calculations available to date are those of Graboske *et al.* (1975; henceforth, GPGO). GPGO found that a solar-composition object of Jovian mass would evolve from an initially gaseous and luminous state to a state resembling the present Jupiter in a time period of 2.6×10^9 yr. The discrepancy between this cooling time and the expected age of Jupiter, roughly 4.5×10^9 yr, was attributed to possible inaccuracies in constituent physics, including thermodynamic equations and departures from an adiabatic temperature distribution. The thermodynamics of liquid metallic hydrogen has been intensively studied recently (DeWitt and Hubbard, 1976; henceforth, MCIV), and the thermal properties are now known well enough to preclude time-scale uncertainties exceeding about 10%. The present paper reports the result of independent calculations of Jovian cooling using the thermodynamics of MCIV together with an analytic representation of the same surface condition used by GPGO. Because the calculations are almost entirely analytic rather than numerical, any interested worker can reproduce the results and observe the impact of varying different parameters on the cooling time scale. These calculations are published here because the shortfall in the cooling time of Jupiter found by GPGO is not confirmed. The discrepancies between the time scales of the present investigation and those of GPGO are produced almost entirely during the late, degenerate phase of cooling; we have not carried out independent calculations for the early, nondegenerate phase. Our results indicate that the simplest possible model of Jovian cooling, namely, fully adiabatic structure determined by the atmospheric surface condition with no energy sources such as unmixing due to immiscibility, is able to give a cooling time compatible with the expected age of Jupiter.

ISSN 0019–1035

II. APPROXIMATIONS AND CONSTITUENT PHYSICS

The initial assumptions are (a) cooling at constant radius, and (b) fully adiabatic structure determined by the surface condition. Assumption (a) will be examined further below. We also assume (c) that the relevant thermodynamic parameters are those for the liquid metallic hydrogen core, which comprises 80% of the mass of Jupiter at present. Finally, (d) it is assumed that the composition of the metallic core is approximately solar, with a mass fraction 0.77 of hydrogen.

From Hubbard (1973), the interior temperature distribution can be represented to within 5% by

$$T = 120T_1(\rho/5)^{\gamma}, \tag{1}$$

where T is the temperature in degrees Kelvin, ρ is the total mass density, and T_1 is the temperature at 1 bar of pressure. The calculations of MCIV indicate that the Grüneisen parameter, γ, can be taken equal to 0.64 with satisfactory accuracy over the range of conditions of interest here. Calculations for a molecular hydrogen fluid (Slattery and Hubbard, 1976) indicate that (1) is also accurate to within a few per cent for $\rho \gtrsim 0.2$ g/cm^3.

The interior adiabat (1) is then coupled to the planet's luminosity by means of a grid of model atmospheres. For this purpose, we use an analytic representation of the surface condition used by GPGO. For a surface gravity $g \approx 10^3$ cm/sec^2 and an effective temperature $T_e \approx 10^2$ °K, GPGO's atmospheric adiabats can be represented by

$$P = 0.263g^{0.5}T_e^{-3.73}T^3, \tag{2}$$

where P is the pressure in bars. Equation (2) is valid for $P \approx 1$ bar. Combining (1) and (2), the interior temperature is coupled to the atmospheric parameters:

$$T = 66.8g^{-1/6}T_e^{1.243}\rho^{\gamma}, \tag{3}$$

valid for $\rho \gtrsim 0.2$.

We now write an expression which equates the Jovian luminosity (neglecting solar input for the moment) to the loss of interior heat:

$$L = 4\pi R^2\sigma T_e^4$$
$$= -\int dm\left(\frac{dE}{dt} + \frac{P}{\rho^2}\frac{d\rho}{dt}\right), \tag{4}$$

where L is the luminosity, R is the planet's radius, σ is the Stefan–Boltzmann constant, and E is the internal energy per gram, and the integration is carried out over the mass of the planet.

Using the thermodynamic identity

$$dE = C_V dT - (P/\rho^2)d\rho - C_V(\partial T/\partial\rho)_s d\rho,$$

(4) takes the form

$$4\pi R^2\sigma T_e^4$$
$$= -\int dm C_V\left(\frac{dT}{dt} - \gamma\frac{T}{\rho}\frac{d\rho}{dt}\right), \tag{5}$$

where C_V is the heat capacity per gram. From (3),

$$\frac{dT}{dt} = T\left(-\frac{1}{6}\frac{d\ln g}{dt} + 1.243\frac{d\ln T_e}{dt} + \gamma\frac{d\ln\rho}{dt}\right). \tag{6}$$

The g dependence of the interior temperature distribution is so weak that it can be safely neglected. Dropping the term $d\ln g/dt$, then, (5) becomes

$$dt = -\alpha T_e^{-3.757}dT_e, \tag{7}$$

where

$$\alpha = (4\pi R^2\sigma)^{-1}\int dm C_V(22.4\rho^{0.64})$$
$$= \frac{22.4}{R^2\sigma}\int r^2 dr C_V\rho^{1.64}, \tag{8}$$

and r is the radial distance from the center of the planet in centimeters. The surface gravity g is assumed constant and equal to 2690 cm/sec^2, corresponding to a nonrotating model of the present Jupiter. Similarly, we take $R = 6.86 \times 10^9$ cm.

It is clear that the constant α is the crucial parameter for the cooling rate during the late stage of Jovian evolution. To evaluate α, we need the heat capacity per gram, C_V, and an interior model, $\rho(r)$. From the results of MCIV, for a hydrogen mass fraction 0.77, and for the temperature range $T \approx 10\,000$ to $40\,000°$K, the heat capacity is, to within 5%, about $2.0k_B$ per heavy particle; thus

$$C_V = 1.66k_B/m_H, \qquad (9)$$

where k_B is Boltzmann's constant and m_H is the mass of a hydrogen atom. This result for the heat capacity is essentially independent of the hydrogen density for densities of concern here. In the molecular hydrogen phase, the heat capacity per gram (Slattery and Hubbard, 1976) is lower than the value given by (9) by about 50%. However, this region of the planet contributes only about 5% of the integral for α, Eq. (8), so that this approximation is quite acceptable. A further approximation is the assumption that Jupiter is of solar composition. Detailed static models constrained by the gravity coefficients and high-precision pressure–density relations indicate that the actual Jovian interior composition may comprise of the order of 10% of elements other than hydrogen and helium by mass (Hubbard and Slattery, 1976). Since the heat capacity per gram is roughly inversely proportional to the mean molecular weight, expression (9) may need to be reduced by as much as 10%, depending upon the precise composition of the interior model. The time scale for Jovian cooling will clearly be reduced by the same amount.

We evaluated integral (8) for two plausible Jovian mass distributions which satisfy the constraints imposed by gravity field measurements (Hubbard and Slattery, 1976), for a nonrotating model with the same pressure–density relation as the rotating model. The value of α increases by about 5% as the heavy material in the envelope is transferred to the core. In this calculation, we took the lower value corresponding to a chemically homogeneous model, with the result

$$\alpha = 2.79 \times 10^{23} \text{ cgs units.} \qquad (10)$$

The time required for the planet to cool from $T_e = \infty$ to the currently observed value is then obtained by integrating (7). For $T_e = 134°$K at present, this time is 4.4×10^9 yr. For $T_e = 127°$K, the time is 5.1×10^9 yr.

III. VARIOUS CORRECTIONS TO THE TIME SCALE

The calculations presented above give the basic time scale for Jovian evolution. Here we consider a number of different corrections which can affect the time scale, but by no more than 20 to 30%.

(a) *Insolation Correction*

The observed heat flux from Jupiter is produced partly from the loss of heat from the interior and partly from the conversion of sunlight to infrared in the Jovian atmosphere. We assume that this conversion occurs well within the convective portion of the atmosphere (a further discussion of this point is given in Section IV). Thus the surface condition for the interior adiabats is given, as before, in terms of T_e, corresponding to the total flux through the Jovian photosphere. The analysis then proceeds as before, except that (4) is now replaced by

$$4\pi R^2\sigma(T_e^4 - T_\odot^4) = -\int dm\left(\frac{dE}{dt} + \frac{P}{\rho^2}\frac{d\rho}{dt}\right), \qquad (11)$$

where $T_\odot$ is the same parameter used by GPGO to represent the equivalent blackbody temperature of the converted solar radiation. Differential (7) is then replaced by

$$dt = -\alpha T_e^{-3.757} \times [1 - (T_\odot/T_e)^4]^{-1} dT_e. \quad (12)$$

This equation can be integrated by expanding the quantity in brackets in powers of $T_\odot/T_e$, with the result

$$t = (\alpha/2.757) T_e^{-2.757}[1 + 0.41(T_\odot/T_e)^4 + 0.26(T_\odot/T_e)^8 + \cdots]. \quad (13)$$

Thus the effect of insolation is to increase the cooling time. This is not a major effect. For $T_\odot = 105°K$ and $T_e = 134°K$ (Aumann *et al.*, 1969), the quantity in brackets is 1.20. For $T_\odot = 108°K$ and $T_e = 127°K$, it is 1.31.

(*b*) *Corrections for Contraction*

Equation (12) cannot be validly integrated back to $T_e = \infty$ because the assumption of cooling at constant radius will break down at some point. Accordingly, we integrate (12) only over a range of T_e for which the assumptions and representations of constituent relations are expected to be accurate. For $T_e = 200°K$, internal temperatures are increased by a factor of about 1.85 over the present epoch. From the calculations of MCIV, this causes an increase of about 20% in internal pressures for a given density. Assuming that Jupiter can be represented approximately as a homologously contracting polytrope of index unity (Hubbard, 1975), we find that the corresponding radius change is about 10% above the present epoch, in good agreement with GPGO. Densities are reduced by about 30%. Although these changes are appreciable and act to reduce the time scale, their cumulative effect on the cooling time is not large, primarily because cooling-time increments increase rapidly for each degree Kelvin decrease in T_e.

Over the range $130°K \lesssim T_e \lesssim 200°K$, the combined effect of varying density and radius can be represented in the form

$$\alpha \simeq \alpha_0[1 - 0.8(T_e - 134)/134], \quad (14)$$

where α_0 is the value of α at present, given by (10). When this expression is substituted in (12) and then integrated, we find a total correction to the time required to evolve from $T_e = 200°K$ to $T_e = 134°K$ of -0.4×10^9 yr, while (13) gives 3.8×10^9 yr. Thus we find a net evolution time for this phase of 3.4×10^9 yr, compared with GPGO's value of 2.0×10^9 yr.

For $T_e > 200°K$, most of our assumptions are vitiated. The formal result from (13) for the time required to cool from $T_e = \infty$ to $T_e = 200°K$ is 1.4×10^9 yr, while GPGO finds the corresponding time to be 0.6×10^9 yr from a full stellar evolution calculation. We will simply accept this latter result, which leads to a further correction of -0.8×10^9 yr.

(*c*) *Summary of Corrections*

The simple theory of Section II gave a total time to cool to $T_e = 134°K$ of 4.4×10^9 yr. To this we add the corrections $+0.8 \times 10^9$ yr for insolation, -0.4×10^9 yr for contraction after $T_e = 200°K$, -0.8×10^9 yr for contraction before $T_e = 200°K$, for a total age of 4.0×10^9 yr.

If we continue the evolution to $T_e = 127°K$, the simple theory gives an age of 5.1×10^9 yr. The insolation correction increases to $+1.5 \times 10^9$ yr, while the other two corrections remain the same, leading to a total age of 5.4×10^9 yr.

IV. EFFECT OF ROTATION: LUCY'S LAW

The foregoing discussion should make clear the overriding importance of the surface condition in determining the cooling time of a Jovian-type planet. We have incorporated the effect of insolation on the surface condition in the same way as GPGO. The Jovian photosphere lies at levels of about 0.3 bar, and adiabatic conditions prevail at depths greater than about 0.5 bar (Trafton, 1967). It is assumed that solar energy is deposited at considerably deeper levels, where it is effectively redistributed by convection without directly modifying the surface condition for the adiabats. It is clear that a consequence of this assumption is that the effective temperature of Jupiter should be uniform from equator to pole.[1] If one takes into account the variation in the surface gravity of Jupiter due to its rotation, one is led to an interesting prediction. If convection is sufficiently vigorous to maintain the interior very close to an adiabatic temperature distribution, then the same surface condition must apply at equator and pole in order to maintain the same adiabat $T(P)$ in both regions of the planet. From (2), it is seen that this requires

$$T_e g^{-0.134} = \text{const}, \qquad (15)$$

which is equivalent to a relation proposed by Lucy (1967) for the effect of gravity on convective stellar envelopes. Equation (15) predicts that the polar effective temperature of Jupiter should be about 2°K *higher* than the equatorial effective temperature.

At a subsequent stage, it will be desirable to incorporate the effects of rotation fully into a Jovian evolution calculation. For the moment, however, these corrections can be ignored since they are at the level of current imprecision in the equations of state and surface conditions. Hubbard (1970) showed that the increase in rotational kinetic energy of Jupiter due to present contraction is less than 5% of the rate of loss of heat from the interior. The biggest correction due to rotation may be due to the variation of T_e over the surface, which could change the cooling time by as much as 0.2×10^9 yr.

[1] This effect was first noted and explained by Ingersoll (1976). The discussion presented here is essentially equivalent.

V. CONCLUSIONS

The results of this investigation indicate that a simple Jovian contraction model has a time scale which is in agreement with the age of the solar system. It does not appear to be necessary to invoke complications such as gravitational unmixing of hydrogen from helium, or a possible discontinuity in specific entropy across the metallic–molecular hydrogen interface (Stevenson and Salpeter, 1976). This may prove to be a useful, if indirect, clue to the nature of the metallic–molecular transition.

The time scale for Jovian cooling increases rapidly with decreasing T_e. The present calculations indicate that the value of T_e for Jupiter which would correspond to a total cooling age equal to the age of the solar system should lie somewhere between the Earth-based measurement of $T_e = 134$°K (Aumann *et al.*, 1969) and the spacecraft measurement of $T_e = 127$°K (Orton, 1975). At present there is no strong reason, from a theoretical point of view, for preferring either measurement. It should be noted that the uncertainty in the theoretical calculation presented here could easily be 10% because of imprecision in constituent relations and neglect of effects of rotation.

Our moderate discrepancy with GPGO remains unexplained. After all corrections are made, the time scale for our model to cool to an effective temperature of 134°K is 1.4×10^9 yr greater than the equivalent model of GPGO. At $T_e = 134$°K, the cooling rate, using (12) with $T_\odot = 105$°K, is

$$(dT_e/dt)_{T_e=134} = -1°\text{K}/0.145 \times 10^9 \text{ yr}. \qquad (16)$$

We conclude with experimental recommendations. First, it is clearly desirable to measure, by means of an entry probe, the conversion of sunlight to thermal radiation in the Jovian atmosphere, and to verify *in situ* the surface condition used for evolutionary calculations. Second, it is important to attempt to measure the variation in effective temperature from pole to equator with enough precision to verify Lucy's law (this experiment may prove more feasible for Saturn, where the variation would be larger). When data of this quality are available, the present Jovian interior structure should be tightly constrained by evolutionary considerations.

ACKNOWLEDGMENTS

This work was supported by Joint Research Interchange NCA2-OR040-602 with Ames Research Center, and by NASA Grant NSG-7045.

REFERENCES

AUMANN, H. H., GILLESPIE, C. M., JR., AND LOW, F. J. (1969). The internal powers and effective temperatures of Jupiter and Saturn. *Astrophys. J.* **157**, L69–L72.

DEWITT, H. E., AND HUBBARD, W. B. (1976). Statistical mechanics of light elements at high pressure. IV. A model free energy for the metallic phase. *Astrophys. J.* **205**, 295–301.

GRABOSKE, H. C., JR., POLLACK, J. B., GROSSMAN, A. S., AND OLNESS, R. J. (1975). The structure and evolution of Jupiter: The fluid contraction stage. *Astrophys. J.* **199**, 265–281.

HUBBARD, W. B. (1968). Thermal structure of Jupiter. *Astrophys. J.* **152**, 745–754.

HUBBARD, W. B. (1970). Structure of Jupiter: Chemical composition contraction, and rotation. *Astrophys. J.* **162**, 687–697.

HUBBARD, W. B. (1973). Observational constraint on the structure of hydrogen planets. *Astrophys. J.* **182**, L35–L38.

HUBBARD, W. B. (1975). Gravitational field of a rotating planet with a polytropic index of unity. *Soviet Astron.—AJ* **18**, 621–624.

HUBBARD, W. B., AND SLATTERY, W. L. (1976). Interior structure of Jupiter: Theory of gravity sounding. In *Jupiter* (A. M. J. Gehrels, Ed.). Univ. of Arizona Press, Tucson.

INGERSOLL, A. P. (1976). Pioneer 10 and 11 observations and the dynamics of Jupiter's atmosphere. *Icarus* **29**, 245–253.

LUCY, L. B. (1967). Gravity-darkening for stars with convective envelopes. *Z. f. Astrophys.* **65**, 89–92.

ORTON, G. S. (1975). The thermal structure of Jupiter. I. Implications of Pioneer 10 infrared radiometer data. *Icarus* **26**, 125–141.

SLATTERY, W. L., AND HUBBARD, W. B. (1976). Thermodynamics of a solar mixture of molecular hydrogen and helium at high pressure. *Icarus* **29**, 187–192.

STEVENSON, D. J., AND SALPETER, E. E. (1976). Interior models of Jupiter. In *Jupiter* (A. M. J. Gehrels, Ed.). Univ. of Arizona Press, Tucson.

TRAFTON, L. (1967). Model atmospheres of the major planets. *Astrophys. J.* **147**, 765–781.