Time-dependence of volatility in energy markets: comparitive analysis and forecasting of essential energy ETFs using GARCH*

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Abstract

In this paper we consider a basic introduction to the GARCH class of statistical models with the intent of modeling and forecasting fundamental energy data. In particular, we show that time-independent volatility is insufficent when dealing with real-world data in the form of returns for the ETFs USO and UGA. We introduce a basic version of the GARCH model as a potential solution to this problem. We first study a GARCH model alone without any underlying ARMA-related process. We then allow R to select the optimal parameters for USO and UGA and forecast the data out 90 (trading) days and comment on the results. We then compare to the well-known ARIMA model and discuss the related information criteria, showing that the GARCH model has better explanatory power for energy data. Lastly, we consider some limitations and other methods in the literature that can motivate future directions of research.

Introduction & Background

Statistical modeling and forecasting can be a powerful tool to help understand the dynamics of open systems, especially when the driving factors of a given dynamical system are not known. One of the most important ways to characterize the dynamics of a system is by considering its own autocorrelation; this is because systems with autocorrelation exhibit a sort of statistical causality–that is, we can use the value of some function of a random variable now to predict its value at a later time. In particular, this gives us a way to forecast the values of the function of the random variable. This way of thinking has many applications, and in particular is useful in the domain of econometrics and market analysis.

To clarify this idea further, let's consider a fundamental statistical process. The simplest non-trivial case of autocorrelation involves only a single lag, so that we can consider the form

$$x_t = \beta x_{t-1} + \varepsilon_t$$

where we take the intercept to be zero for simplicity and ε_t is a white noise process with zero mean and $V(\varepsilon_t) = \sigma^2$. If we assume x_{t-1} to be known, then

$$E(x_t) = \beta x_{t-1} \tag{1}$$

$$V(x_t) = V(\varepsilon_t) = \sigma^2 \tag{2}$$

*I'd like to thank Dr. Stewart for the helpful conversations surrounding time series analysis in general and the idea for this project specifically. I've learned a lot!

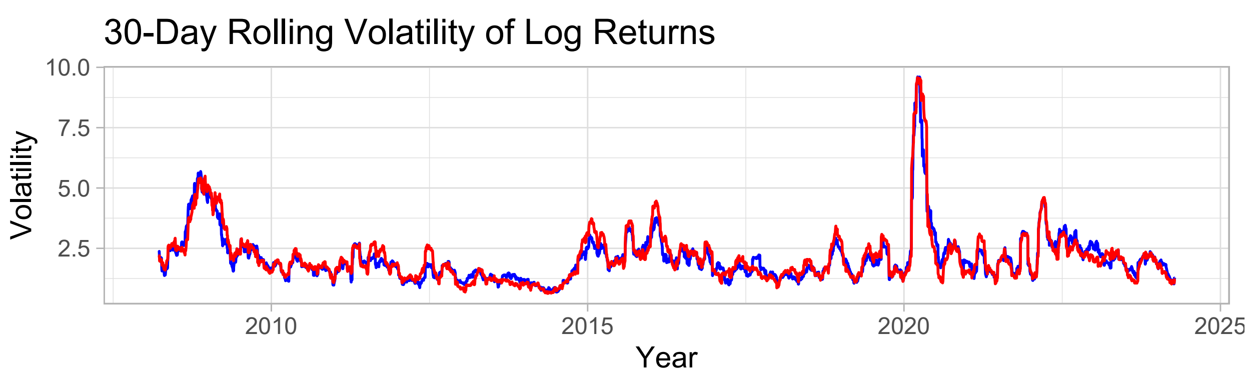
so that the conditional variance in the random variable is constant[1]. In particular, this is because we assume that the fundamental statistical process that drives the value of the random variable here is the white noise process. This leads us to the question of whether this is an accurate assumption in general. If we look at basic economic data, we can test to see whether this assumption is plausible.

For the sake of this paper, we will consider energy ETF data, and in particular we will restrict ourselves to considering the ETFs USO and UGA. First inspecting the price graphs

There is nothing in either of them that would suggest that the driving process is white noise is insufficient; we need to extract the rolling volatility to determine whether we need to consider a model that is more general. To do this, we want to consider the log returns of the ETFs, and then we can extract the rolling volatility from there. To do this, we transform the price data

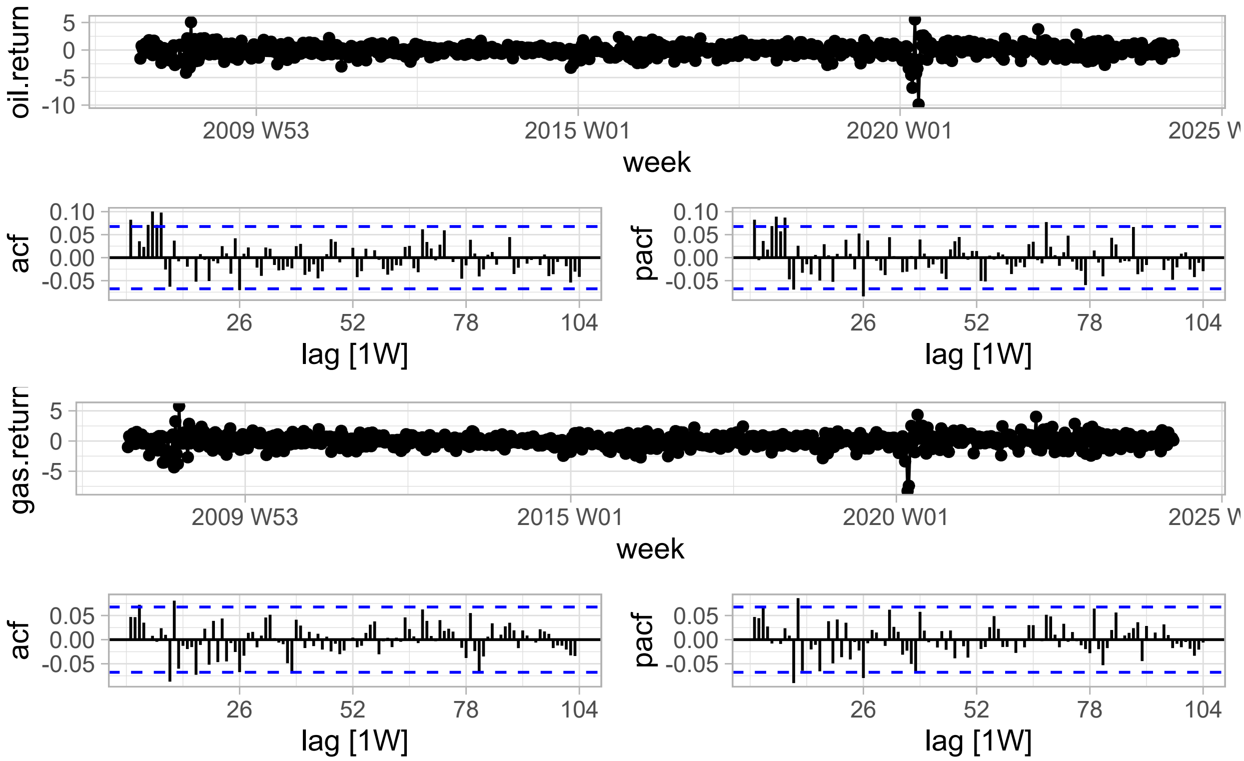
$$p_t \to \log(p_t) - \log(p_{t-1}) = \log(p_t/p_{t-1})$$

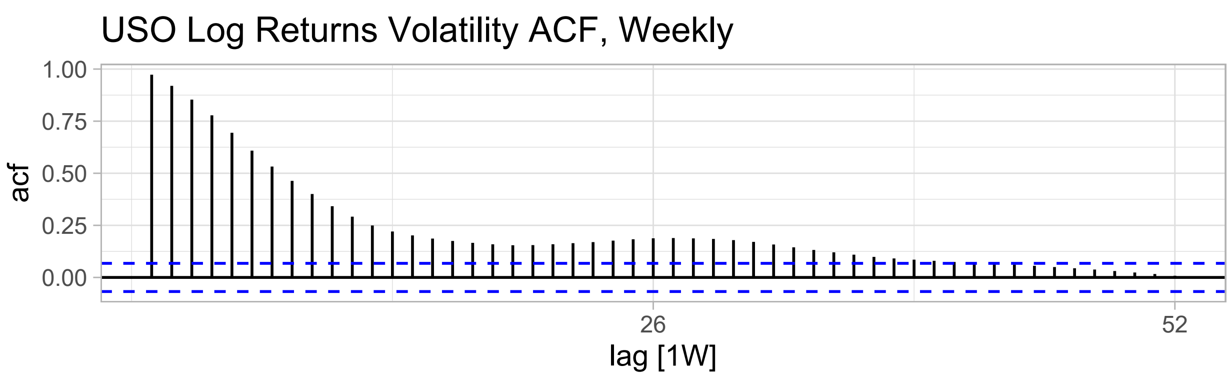
and then plot the 30-day rolling volatility

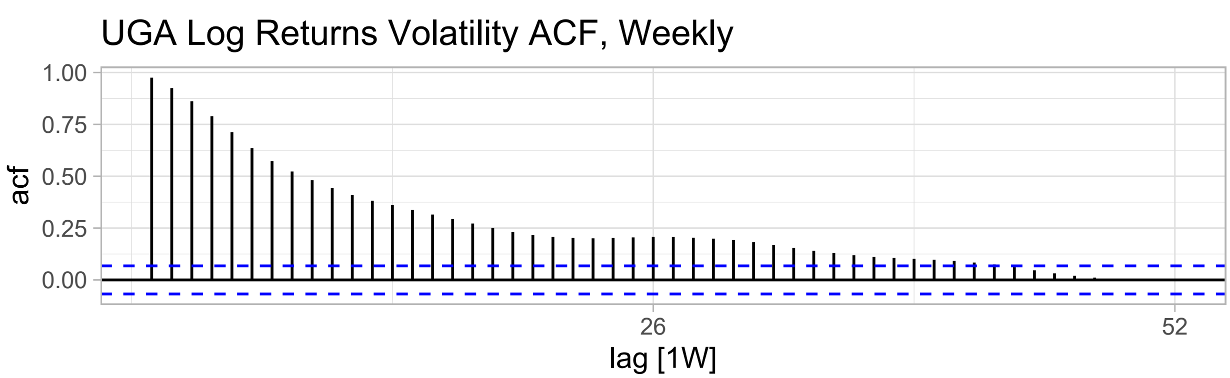


Here the blue curve corresponds to UGA's 30-day rolling volatility of the log returns, and the red curve corresponds to USO's 30-day rolling volatility. These curves are certainly *not* constant, which means that we can postulate that these too follow an underlying statistical process. To further clarify this, we can look at the ACF and PACF of the returns and volatility

[1] In fact, one can show that this is true even if one assumes the form of $x_t = \sum_k \beta_k x_{t-k} + \sum_{k'} \alpha_{k'} \varepsilon_{t-k'}$, where one allows for lags in the white noise process as well.







Here we consolidate the returns into weekly average values in order to fill the gaps in the data. Both the ACF and PACF of the log returns break the significance bands within the first five to ten lags, and the ACF functions for the volatility of both commodities exhibit decaying autocorrelation, suggesting cycle-trend behavior. Consequently, we view this as sufficient evidence to modify the hypothesis of the fundamental form of the driving statistical process and instead turn our attention to consider a time-dependent volatility model.

Methodology

Model Specification

One of the most prominent candidate models for time-varying volatility is the ARCH model (autoregressive conditional heteroskedasticity), and its generaliation, the GARCH model (generalized autoregressive conditional heteroskedasticity) (Bollerslev, 1986; Engle, 1982). We will consider the GARCH model directly, and specifically we will consider the GARCH(1,1) model. Let us outline briefly how GARCH works.

The key modification now consists in the time-dependence of the volatility. The GARCH model in general can be described by the process

$$y_t = x_t + \varepsilon_t \tag{3}$$

$$\varepsilon_t \sim \mathcal{N}(0, \sigma_t^2) \tag{4}$$

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{5}$$

so that we now have a fundamental time-dependence in the volatility. Note in particular that we can take the exogenous variable $x_t = \mu$ so that the statistical process completely drives the endogenous variable. If instead we allow x_t to be time-dependent, then we can introduce other exogenous factors which may explain the variation in y_t. For our purposes, we will not be considering other exogenous factors in our model primarily for the sake of simplicity and to exhibit the key properties of the GARCH model. The model we consider explicitly below (and the one we detailed just above) is the GARCH(1,1) model. Let us now turn to an analysis of this model.

We proceed in the following way: the GARCH model allows for the specification of an ARMA model along side its parameters characterizing the MA and AR processes in the volatility. We first look at the case where we only forecast based on the GARCH parameters. Since this is to be construed as a 'proof-of-principle', we only look at how it models and forecasts for USO. We then allow R to optimize the parameter values for the GARCH and related ARMA processes. We compare this to the GARCH(1,1) that we directly enforce initially. After this, we look at a comparison to an ARIMA process for USO and UGA. Finally, we discuss these results and potential other methods for instilling a time-dependence in the volatility, such as a stochastic volatility model. We point to the literature to suggest other areas of exploration for future investigations.

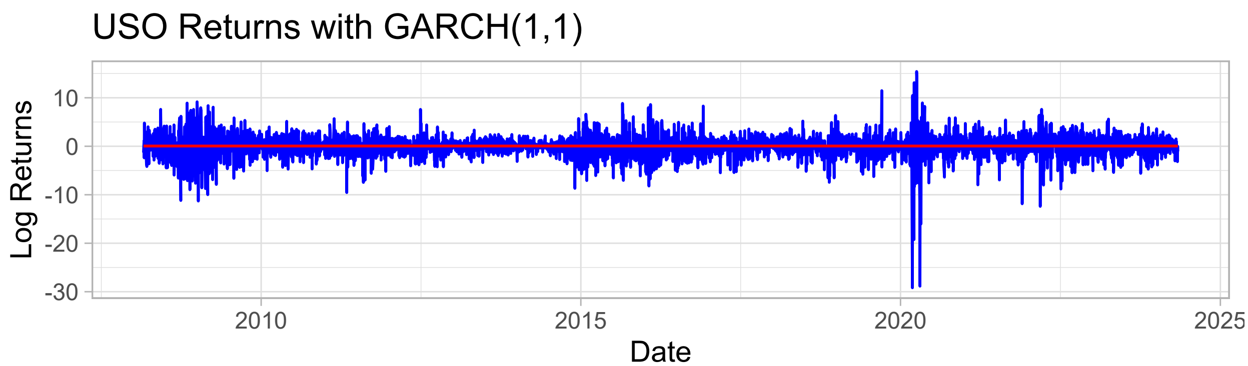
Empirical Results

GARCH without ARMA - Proof of Principle

In order to gain a sense of the statistical ability of the GARCH process, we first run a GARCH(1,1) model with ARMA(0,0). Here are the coefficients characterizing the GARCH process against the log returns of oil. We include the full statistical breakdown in the appendix at the end of the paper.

```
##            Estimate   Std. Error   t value   Pr(>|t|)
## mu     4.766352e-04 2.689760e-04  1.772036 0.07638853
## omega  7.983332e-06 3.253168e-06  2.454018 0.01412700
## alpha1 1.039059e-01 9.434122e-03 11.013838 0.00000000
## beta1  8.843599e-01 9.469454e-03 93.390800 0.00000000
```

We note in particular from the p-values that all coefficients are significant at the 10% level, with ω, α_1, and β_1 significant below the 5% level. If we plot the fit over the log returns to gain a sense of the goodness of fit, we see



We observe that because we do not include a model of the moving average, that this value is effectively constant, so that this model functions similarly to a `mean` model. We conduct a Ljung-Box test on the standardized residuals at a lag of 1 year

```
##
##  Box-Ljung test
##
## data:  .
## X-squared = 234.75, df = 252, p-value = 0.7754
```

The p-value is such that we lack sufficient evidence to reject the null of autocorrelation up to one year out. From here, we can forecast[2]

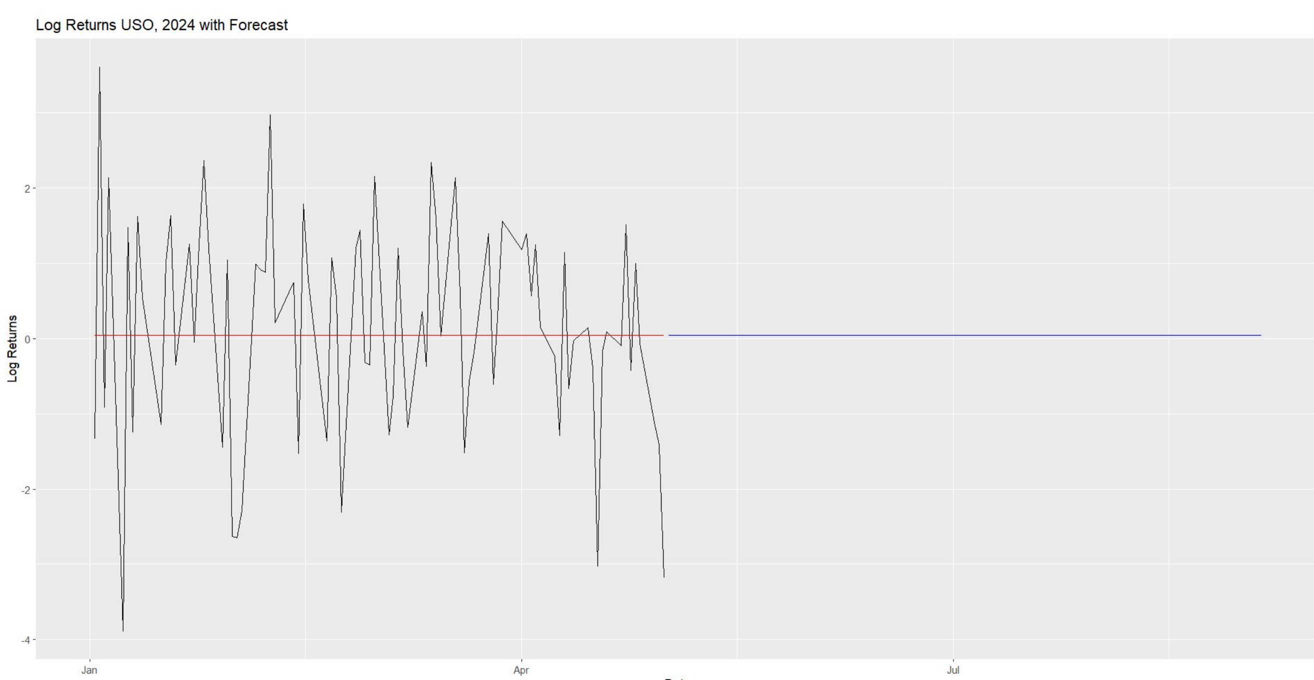


Figure 1: Log Returns with Forecast

[2]See the RMD file (line 244) for a short note regarding the GARCH forecasted figures.

where we've chosen to forecast out 90 (trading) days.

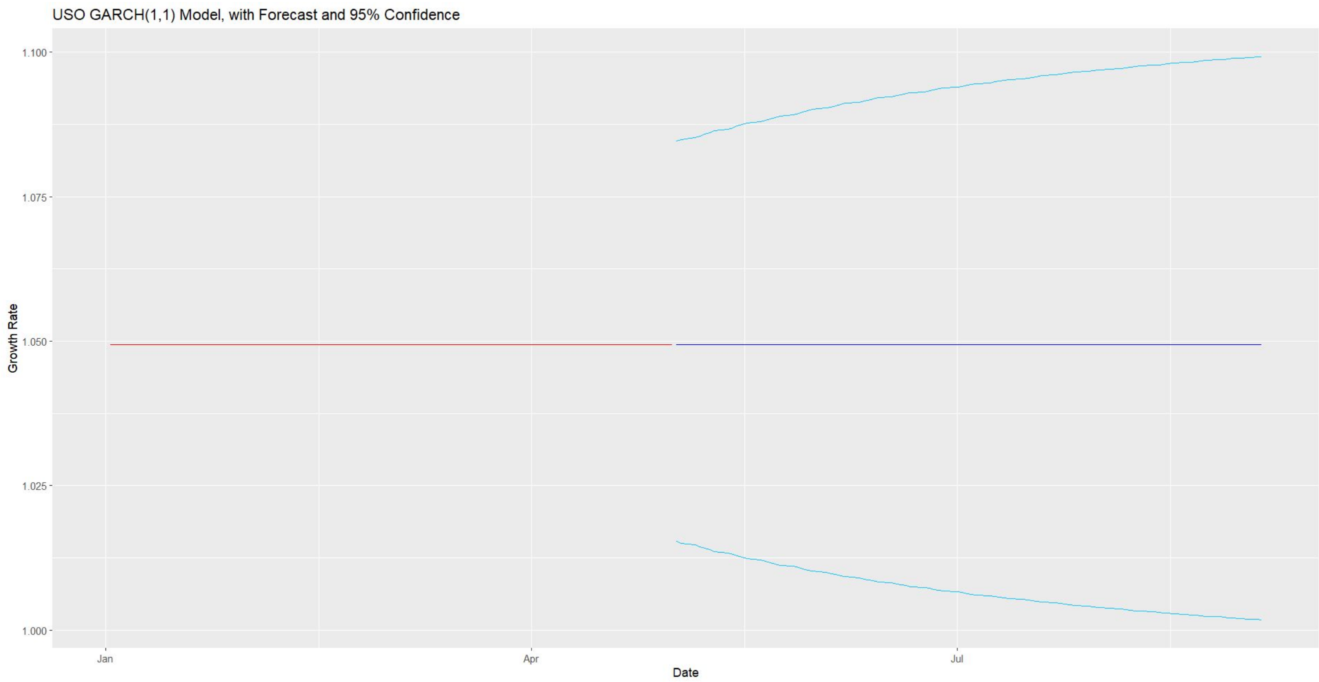


Figure 2: Modeled and Forecasted values with 95% confidence intervals

Here we display the real growth rate together with the 95% confidence intervals. We observe that the GARCH(1,1) predicts small but positive daily growth in the returns, about 5% daily. Let's see how this compares to the inclusion of an ARMA term.

GARCH with ARMA

We now allow R to select the optimal parameters for fitting the data. We display oil data

```
##              Estimate   Std. Error   t value     Pr(>|t|)
## mu       4.717807e-04 2.668035e-04  1.768270 7.701571e-02
## ar1     -8.006590e-01 1.398937e-01 -5.723338 1.044510e-08
## ma1      7.840415e-01 1.449112e-01  5.410498 6.284979e-08
## omega    7.941526e-06 3.211261e-06  2.473024 1.339751e-02
## alpha1   1.034087e-01 9.335582e-03 11.076834 0.000000e+00
## beta1    8.848966e-01 9.419001e-03 93.948036 0.000000e+00
```

```
##
##  Box-Ljung test
##
## data:  .
## X-squared = 232.48, df = 252, p-value = 0.8059
```

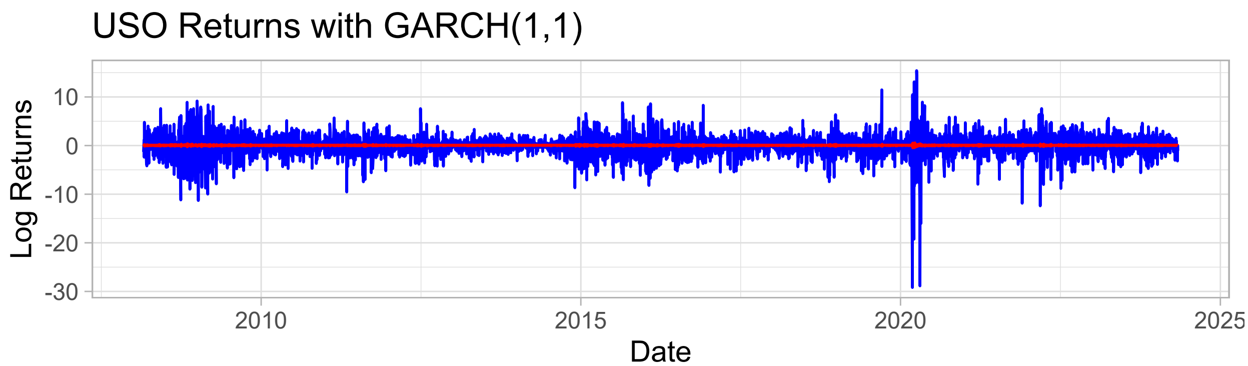
then gas data

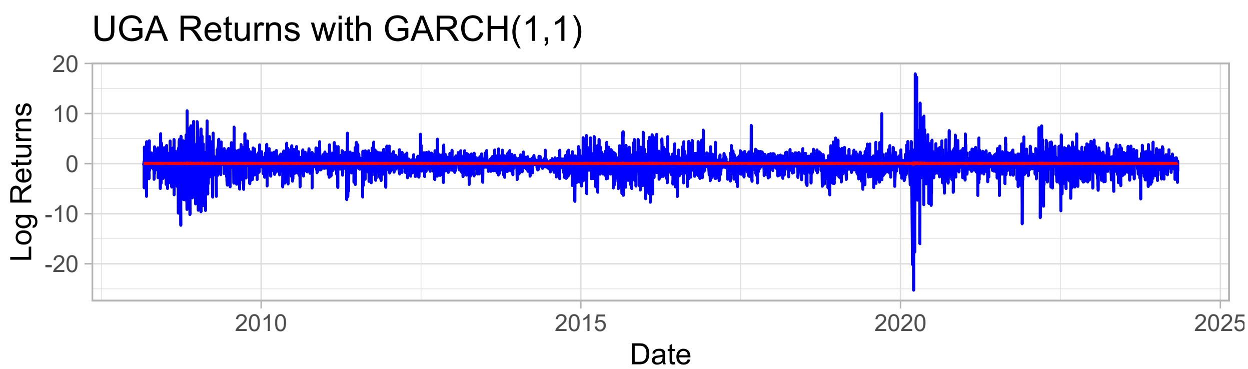
```
##              Estimate   Std. Error    t value   Pr(>|t|)
## mu       4.527536e-04 2.815360e-04  1.6081557 0.10780109
## ar1     -4.386707e-01 1.105630e+00 -0.3967610 0.69154370
## ma1      4.443278e-01 1.102444e+00  0.4030391 0.68691949
## omega    5.737201e-06 2.326829e-06  2.4656739 0.01367558
```

```
## alpha1  8.084827e-02 8.121999e-03  9.9542326 0.00000000
## beta1   9.092179e-01 9.325744e-03 97.4954768 0.00000000
```

```
##
##  Box-Ljung test
##
## data:  .
## X-squared = 266.11, df = 252, p-value = 0.2589
```

where both now have an ARMA(1,1) process supplementing the GARCH process. We also see that the Ljung-Box tests suggest that we've accounted for autocorrelation appropriately.





and we can again forecast over a 90-day interval.

We observe that the inclusion of the ARMA structure yields oscillatory behavior in the mean, which is to be expected. We note that the oscillations dampen out

We again show the real growth rate as the ratio of today's value to yesterday's value. Plotting the model and forecast allows us to see that the inclusion of the ARMA term more accurately models the dynamics of the log returns curves, and this propagates into forecast as well. We also can compare to the case of USO without the inclusion of an ARMA term–there we see the mean and forecast predict a slightly higher growth rate than in the case where we include the ARMA terms.

If we display the information criteria for the two cases of USO, we have

```
##
## Akaike       -4.977612
## Bayes        -4.971762
## Shibata      -4.977613
## Hannan-Quinn -4.975547
```

```
##
```

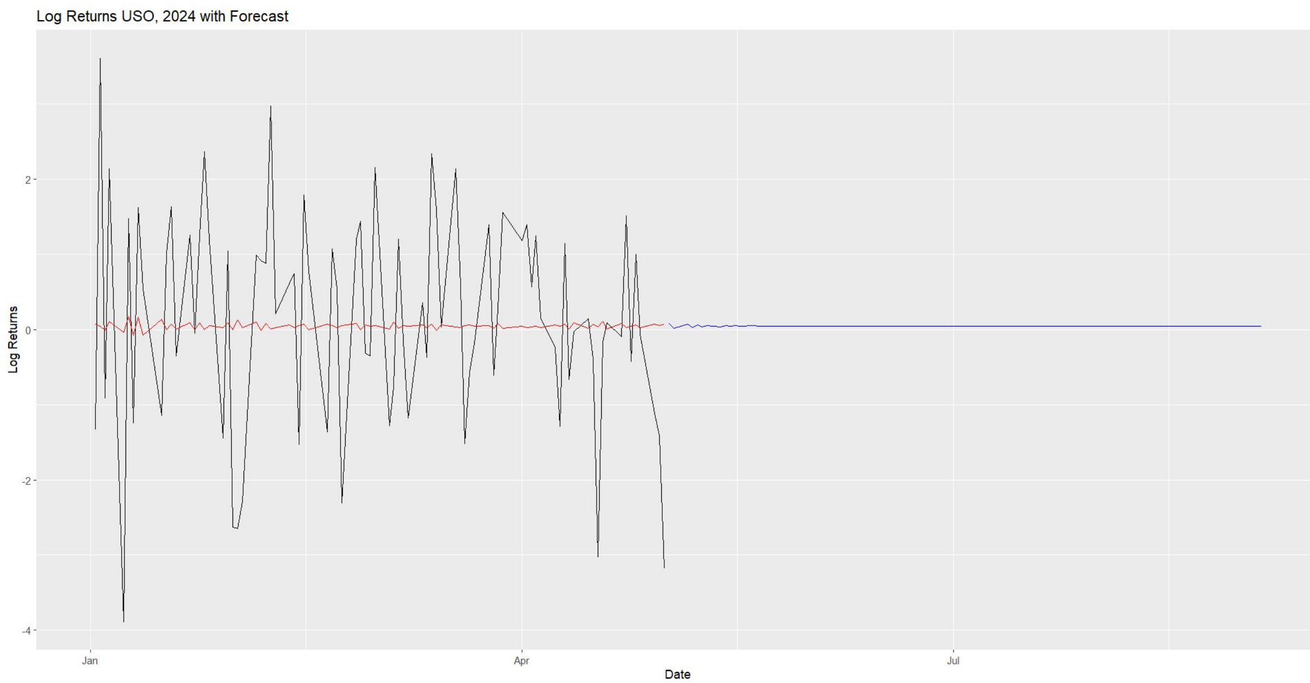


Figure 3: USO Log Returns with Forecast

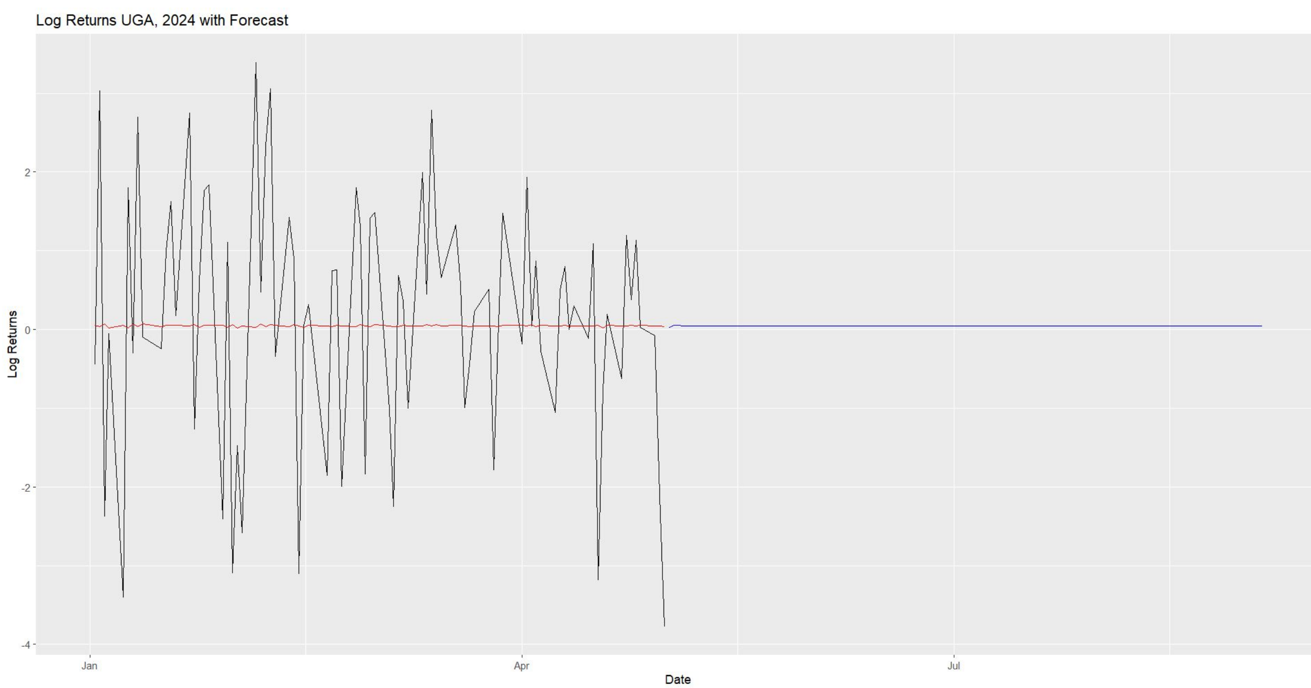


Figure 4: UGA Log Returns with Forecast

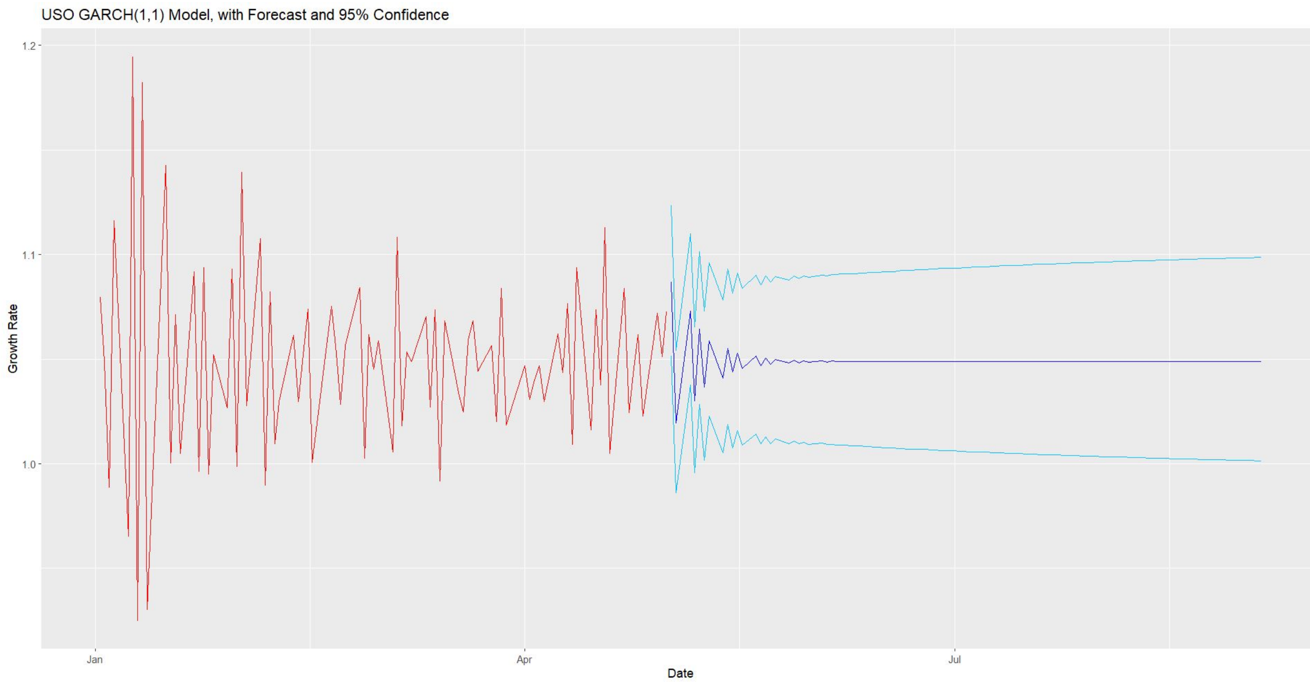


Figure 5: Modeled and Forecasted USO values with 95% confidence intervals

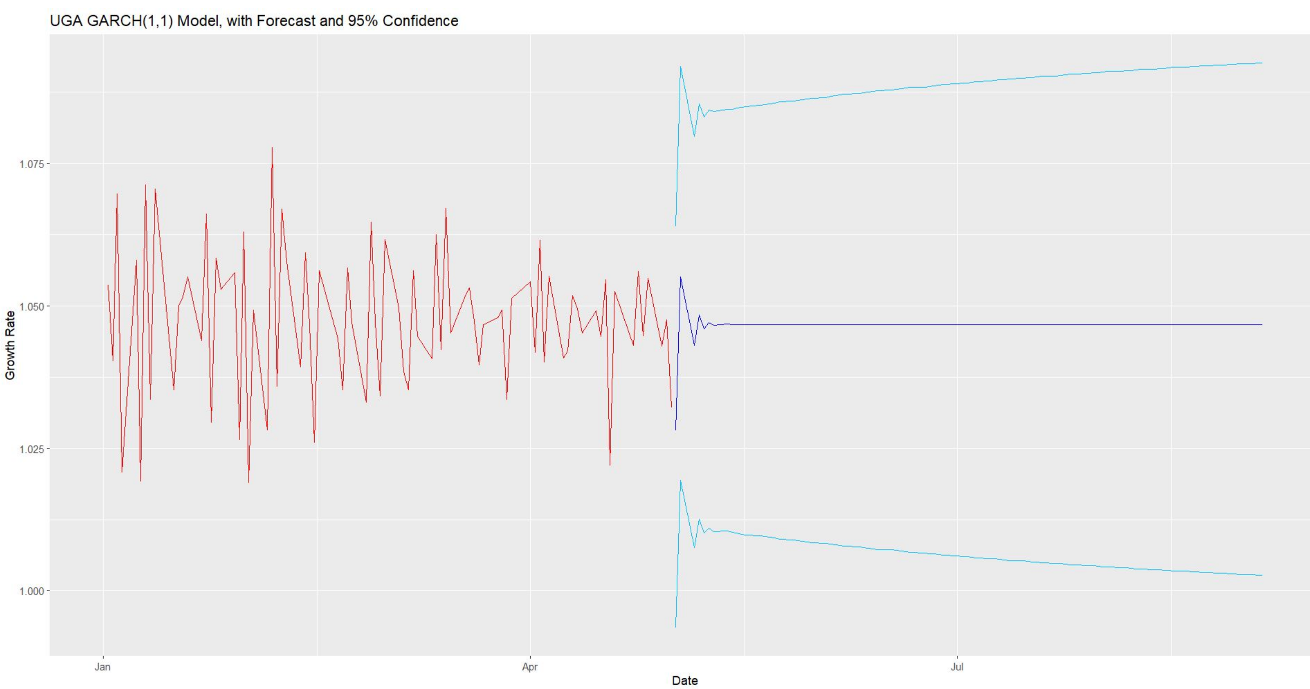


Figure 6: Modeled and Forecasted UGA values with 95% confidence intervals

```
## Akaike       -4.977371
## Bayes        -4.968596
## Shibata      -4.977375
## Hannan-Quinn -4.974274
```

Here the second set corresponds to the inclusion of the ARMA(1,1) process. We see that the inclusion of the ARMA term minimizes the various information criteria, especially the AIC and BIC, suggesting that the forecast the second method predicts is more accurate. This means we can expect a real growth rate of the value in USO of about 4.5% on average daily according to the GARCH(1,1) with an inclusion of an ARMA term. This also means we expect a similar growth rate in UGA of about 4.5%.

GARCH vs ARIMA

Since the inclusion of the ARMA term improved the GARCH model, there may be an intuition that perhaps a simple ARIMA model would do a sufficient job at predicting the values. We consider this briefly by regressing the same data against an ARIMA model and then comparing the information criteria. Since the main focus is on the GARCH model, we do not proceed with a full analysis of the ARIMA models but instead just look at the information criteria.

If we model USO's log returns, we obtain the following

```
## Series: oil.return
## Model: ARIMA(1,0,1)
##
## Coefficients:
##           ar1     ma1
##       -0.1321  0.2166
## s.e.   0.7023  0.6933
##
## sigma^2 estimated as 1.226:  log likelihood=-1274.82
## AIC=2555.64   AICc=2555.67   BIC=2569.84
```

```
## # A tibble: 1 x 3
##   .model    lb_stat lb_pvalue
##   <chr>       <dbl>     <dbl>
## 1 oil.arima    89.7     0.802
```

The Ljung-Box test suggests that we've successfully accounted for autocorrelation, and the `gg_tsdisplay` plots we displayed above suggest that the data are stationary. While the model appears to have explanatory power, the AIC and BIC are much larger in the ARIMA case than they were in the GARCH case. By direct comparison, we have that the AIC in the GARCH case is -4.9768 and BIC is -4.9680. These are compared to an AIC of 2555.64 and BIC of 2569.84 in the ARIMA case. This persists across the commodities. Turning our attention to UGA

```
## Series: gas.return
## Model: ARIMA(1,0,1)(1,0,1)[52]
##
## Coefficients:
##          ar1      ma1     sar1    sma1
##       0.2724  -0.2185  -0.1453  0.1907
## s.e.  0.0032   0.0085   0.0002  0.0337
##
## sigma^2 estimated as 1.173:  log likelihood=-1254.44
```

```
## AIC=2518.88   AICc=2518.95   BIC=2542.54

## # A tibble: 1 x 3
##   .model    lb_stat lb_pvalue
##   <chr>       <dbl>     <dbl>
## 1 gas.arima    94.2     0.644
```

If we return to the plots for UGA, we can infer that it is stationary (a simple KPSS test confirms this) and from the Ljung-Box test that we've successfully accounted for autocorrelation. However, again the information criteria exclude the ARIMA as the effective candidate model for forecasting. For UGA, the AIC is -4.9828 and the BIC is -4.9735 for the GARCH model as compared to an AIC of 2518.88 and BIC of 2542.54 for the ARIMA model. Again this suggests that the GARCH model (and therefore the time-dependent assumption in the volatility) more accurately captures the dynamics of the log returns of the ETFs.

Other Possible Models

Although the main focus of this paper has been on the use of the GARCH model to study econometric data, this model is well-studied generally. There have been other proposals for generalization in the literature, especially for energy data (Chan and Grant, 2016; Kang et al., 2009). One of the clear limitations in our description is that it is not robust to shocks. We recall that the price of oil dropped to below zero during COVID (see the price graph at the start of the paper) which translates into a shock in the log returns. For example, the GARCH-J model can accommodate jumps in the data such as we see with COVID. The model also lacks the ability to encode long-term impacts of shocks; for things like this, one can look at using a FIGARCH model. In the case of the MSE and MAE, the FIGARCH outperforms the GARCH when modeling WTI data (Kang et al., 2009) and the GARCH-J model also outperforms GARCH for similar data sets (Chan and Grant, 2016).

Chan and Grant (2016) also suggests that for energy data, other models are relevant and have explanatory power. In particular, stochastic volatility models tend to outperform GARCH models across the board. This is in the context of crude oil prices for WTI and Brent crude, but since USO is a financial derivative of WTI spot prices, and UGA is highly correlated to USO in terms of price, it reasonable to conjecture that the explanatory interpretations will persist into the derivatives markets for these commodities. As one possible avenue for extending this study, we could also look at analyzing the ETFs using stochastic volatility models having been motivated by these papers.

Conclusion

In this paper, we've considered a basic generalization of volatility to a time-varying function of a random variable in the form of the GARCH model. We saw that this enhances our explanatory power of the data as measured by the AIC and BIC relative to other robust models such as the ARIMA model. In order to study its explanatory power, we considered its ability to model and forecast for the energy ETFs USO and UGA, and in both cases it predicts an average real growth rate of about 4.5% daily. We also discussed some other models and the drawbacks of working with the GARCH(1,1) directly. We would find it interesting to study classes of stochastic volatility models in the context of energy ETFs as inspired by Kang et al. (2009).

There also appears to be a natural and interesting question about at what point there is stabilization in the chain of time-dependence. For example, we might call a first-order model one which assumes that the endogenous variable experiences autocorrelation but whose conditional volatility is constant at each time step. Similarly, a second-order model might be considered to be one in which the volatility of the volatility is considered constant. Theoretically, one could continue this chain of rendering the volatility of the volatility of… (and so on) time-dependent. It would be interesting to study whether such a chain ultimately converges to a process with constant volatility. Does this depend on the assumed distribution of the error term? What would the effects be on the various models and their related predictive capacity? Can we motivate truncating at a specific order? These are interesting questions that we believe merit further investigation.

Appendix

Statistical Details of Models

Oil - No ARMA

```
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : sGARCH(1,1)
## Mean Model   : ARFIMA(0,0,0)
## Distribution : norm
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000477    0.000269    1.772 0.076389
## omega   0.000008    0.000003    2.454 0.014127
## alpha1  0.103906    0.009434   11.014 0.000000
## beta1   0.884360    0.009469   93.391 0.000000
##
## Robust Standard Errors:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000477    0.000297  1.60587 0.108304
## omega   0.000008    0.000011  0.75156 0.452316
## alpha1  0.103906    0.026247  3.95871 0.000075
## beta1   0.884360    0.018266 48.41466 0.000000
##
## LogLikelihood : 10862.66
##
## Information Criteria
## ------------------------------------
##
## Akaike       -4.9776
## Bayes        -4.9718
## Shibata      -4.9776
## Hannan-Quinn -4.9755
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                     0.3070  0.5795
## Lag[2*(p+q)+(p+q)-1][2]    0.7368  0.5921
## Lag[4*(p+q)+(p+q)-1][5]    3.2886  0.3569
## d.o.f=0
## H0 : No serial correlation
##
```

```
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                          statistic p-value
## Lag[1]                       2.584  0.1080
## Lag[2*(p+q)+(p+q)-1][5]      3.961  0.2590
## Lag[4*(p+q)+(p+q)-1][9]      4.892  0.4454
## d.o.f=2
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[3]     1.076 0.500 2.000  0.2995
## ARCH Lag[5]     2.495 1.440 1.667  0.3719
## ARCH Lag[7]     2.757 2.315 1.543  0.5605
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  2.7262
## Individual Statistics:
## mu     0.2687
## omega  0.5009
## alpha1 0.1503
## beta1  0.1016
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          1.07 1.24 1.6
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                    t-value      prob sig
## Sign Bias           1.7715 0.0765550   *
## Negative Sign Bias  1.1004 0.2712122
## Positive Sign Bias  0.8255 0.4091124
## Joint Effect       16.3534 0.0009597 ***
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     72.52    3.487e-08
## 2    30     88.04    7.279e-08
## 3    40     97.83    5.846e-07
## 4    50    113.43    5.070e-07
##
##
## Elapsed time : 0.279763
```

Oil - Optimal

```
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : sGARCH(1,1)
## Mean Model   : ARFIMA(1,0,1)
## Distribution : norm
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000472    0.000267   1.7683 0.077016
## ar1    -0.800659    0.139894  -5.7233 0.000000
## ma1     0.784042    0.144911   5.4105 0.000000
## omega   0.000008    0.000003   2.4730 0.013398
## alpha1  0.103409    0.009336  11.0768 0.000000
## beta1   0.884897    0.009419  93.9480 0.000000
##
## Robust Standard Errors:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000472    0.000297  1.58932 0.111987
## ar1    -0.800659    0.092450 -8.66048 0.000000
## ma1     0.784042    0.094767  8.27332 0.000000
## omega   0.000008    0.000010  0.76565 0.443884
## alpha1  0.103409    0.025607  4.03824 0.000054
## beta1   0.884897    0.018055 49.01114 0.000000
##
## LogLikelihood : 10864.13
##
## Information Criteria
## ------------------------------------
##
## Akaike       -4.9774
## Bayes        -4.9686
## Shibata      -4.9774
## Hannan-Quinn -4.9743
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                      2.472  0.1159
## Lag[2*(p+q)+(p+q)-1][5]     3.614  0.1634
## Lag[4*(p+q)+(p+q)-1][9]     4.713  0.5204
## d.o.f=2
## H0 : No serial correlation
```

```
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                      2.804 0.09405
## Lag[2*(p+q)+(p+q)-1][5]     4.139 0.23718
## Lag[4*(p+q)+(p+q)-1][9]     5.046 0.42274
## d.o.f=2
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[3]     1.057 0.500 2.000  0.3039
## ARCH Lag[5]     2.447 1.440 1.667  0.3806
## ARCH Lag[7]     2.709 2.315 1.543  0.5701
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  3.6059
## Individual Statistics:
## mu     0.2744
## ar1    0.6126
## ma1    0.6310
## omega  0.4826
## alpha1 0.1548
## beta1  0.1034
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          1.49 1.68 2.12
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                    t-value     prob sig
## Sign Bias            1.436 0.150936
## Negative Sign Bias   1.287 0.198174
## Positive Sign Bias   1.043 0.297082
## Joint Effect        15.778 0.001259 ***
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     73.35    2.533e-08
## 2    30     90.20    3.397e-08
## 3    40    104.08    7.809e-08
## 4    50    106.18    4.205e-06
##
##
## Elapsed time : 0.58271
```

Gas - Optimal

```
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : sGARCH(1,1)
## Mean Model   : ARFIMA(1,0,1)
## Distribution : norm
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000453    0.000282  1.60816 0.107801
## ar1    -0.438671    1.105630 -0.39676 0.691544
## ma1     0.444328    1.102444  0.40304 0.686919
## omega   0.000006    0.000002  2.46567 0.013676
## alpha1  0.080848    0.008122  9.95423 0.000000
## beta1   0.909218    0.009326 97.49548 0.000000
##
## Robust Standard Errors:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000453    0.000300  1.51135 0.130698
## ar1    -0.438671    0.571427 -0.76768 0.442680
## ma1     0.444328    0.571086  0.77804 0.436546
## omega   0.000006    0.000006  1.02815 0.303879
## alpha1  0.080848    0.016775  4.81951 0.000001
## beta1   0.909218    0.019605 46.37618 0.000000
##
## LogLikelihood : 10154.33
##
## Information Criteria
## ------------------------------------
##
## Akaike       -4.9832
## Bayes        -4.9739
## Shibata      -4.9832
## Hannan-Quinn -4.9799
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                     0.3685  0.5438
## Lag[2*(p+q)+(p+q)-1][5]    1.2005  0.9999
## Lag[4*(p+q)+(p+q)-1][9]    2.4905  0.9525
## d.o.f=2
## H0 : No serial correlation
```

```
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                      1.391  0.2382
## Lag[2*(p+q)+(p+q)-1][5]     3.573  0.3122
## Lag[4*(p+q)+(p+q)-1][9]     5.818  0.3197
## d.o.f=2
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[3]  0.007274 0.500 2.000  0.9320
## ARCH Lag[5]  2.864341 1.440 1.667  0.3101
## ARCH Lag[7]  3.723568 2.315 1.543  0.3882
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  2.0057
## Individual Statistics:
## mu     0.17202
## ar1    0.25147
## ma1    0.25068
## omega  0.30415
## alpha1 0.16540
## beta1  0.09288
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          1.49 1.68 2.12
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                    t-value    prob sig
## Sign Bias           1.7355 0.08273   *
## Negative Sign Bias  0.7962 0.42598
## Positive Sign Bias  0.1591 0.87356
## Joint Effect       10.3336 0.01593  **
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     50.54    1.091e-04
## 2    30     75.60    5.012e-06
## 3    40     92.82    2.786e-06
## 4    50    103.27    9.579e-06
##
##
## Elapsed time : 0.395653
```

References

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