# ETFs vs Options - Pricing
Draft

Ryan E. Scott

October 16, 2025

## 1 Introduction

Nick Restina made a great observation that the options derived from the ETFs we trade tend to have low volume and therefore high volatility. In the case of lower liquidity, convergence on a ‘real’ price is slowed so that the time to effective price discovery is longer than usual. On the other hand, these objects are ultimately derived from a basket of futures contracts that represent an underlying commodity, so it seems reasonable to wonder if we consider a basket of instruments that mimics the option on the ETF but which has higher liquidity if that allows for more rapid price discovery and therefore lower volatility. If this is the case, is seems reasonable to suppose that–as long as the option of the ETF tracks this portfolio–this new instrument may give a better sense of where the price of the ETF will actually go, together with the expected volatility of the price itself. The basic aim of this project is to ask whether a natural alternative, which we discuss below, is a better predictor in this sense of the basket of goods that is the portfolio of futures. If this is the case, it may lend plausibility to proposal and execution of trades on the options derived from the usual ETFs we trade. For simplicity, we will start with the CORN ETF (since it only holds futures contracts), and we will sketch a method of determining how to build the basket of options at the appropriate ratios (in this case 1/3, 1/3, 1/3). Let’s discuss the basic premise.

## 2 Sketch of the Problem

To build the new object, we need to introduce two assignments[1] that take in a futures contract (or collection of them) and give back a new financial object. The first we call the *options functor*, denoted $\mathcal{O}_K$, and takes in a futures contract and gives back an option derived from the futures contact at strike price $K$

$$\mathcal{O}_K : F \to \mathcal{O}_K(F) \tag{1}$$

where $F$ is a futures contract. In addition to this assignment, we also introduce the *ETF functor*, denoted **ETF**. This takes in a collection of futures contracts and returns an object which contains in futures in relevant proportions

$$\mathbf{ETF} : \{F_j\} \to \sum_j p_j F_j \tag{2}$$

where $\sum_j p_j = 1$ and $0 < p_j < 1$.[2] Now the construction of the option from the ETF built over a given collection of futures contracts can be conceptualized as

$$\begin{tikzcd}
& \mathbf{ETF}(\{F_j\}) \arrow[dr, "\mathcal{O}_K"] & \\
\{F_j\} \arrow[ur, "\mathbf{ETF}"] \arrow[rr, "\mathcal{O}_K \circ \mathbf{ETF}"] & & \mathcal{O}_K(\mathbf{ETF}(\{F_j\}))
\end{tikzcd} \tag{3}$$

[1] These things are called *partial functions* in the vernacular.
[2] This is called a *convex combination*.

This construction immediately suggests an alternative–we just need to alternate $\mathcal{O}_K$ and **ETF**. This suggests that there are two ways to form the final instrument

$$
\begin{tikzcd}
\{F_j\} \arrow[r, "\mathbf{ETF}"] \arrow[d, "\mathcal{O}_{K_j}"] & \mathbf{ETF}(\{F_j\}) \arrow[d, "\mathcal{O}_K"] \\
\{\mathcal{O}_{K_j}(F_j)\} \arrow[r, "\mathbf{ETF}"] & \mathcal{O}_K \circ \mathbf{ETF}(\{F_j\}) \arrow[d, Rightarrow, "?"] \\
 & \mathbf{ETF}(\{\mathcal{O}_{K_j}(F_j)\})
\end{tikzcd}
\tag{4}
$$

which essentially means we ask the question: are there strike prices $\{K_j\}$ such that $\mathcal{O}_K \circ \mathbf{ETF} = \mathbf{ETF} \circ \{\mathcal{O}_{K_j}\}$? Using our definition of **ETF**, this question is equivalent to whether

$$\mathcal{O}_K\left(\sum_j p_j F_j\right) = \sum_j p_j \mathcal{O}_{K_j}(F_j) \tag{5}$$

holds. But this is the same thing as asking whether $\mathcal{O}$ is linear for a choice of a tuple $\langle K, K_1, ..., K_N\rangle$.[3] This observation will form the backbone of our construction.

# 3 Proposed Method

Notice that the key problem presents as the need to determine the set of strike prices for the options derived from the futures along the bottom legs of the diagram in (4). We propose a method for determining the collection of strike prices. Once we have these, we will construct the time series associated with the derived financial object along the bottom leg of the diagram, and compare it to the observed one along the top leg. If the portfolio is an accurate measure of the portfolio of the underlying futures, then we should observe that the object tracks the moving average in both cases, but the volatility should be tighter (so that price is better defined).

To do this, we will assume that there is a transformation on the price of the instruments so that in this transformed coordinate,[4] the price follows the usual stochastic differential equation

$$dp_j(t) = \mu_j dt + \sigma_j dW(t) \tag{6}$$

where $\mu_j, \sigma_j$ are constants, $dW \sim \mathcal{N}(0, dt)$ is an infinitesimal Wiener process, and $dt$ is the time step. We'll take the price along the bottom leg to be $dp_1$ and along the top leg $dp_2$. Now these processes will be said to *track each other* if $\langle p_1(t)\rangle \approx \langle p_2(t)\rangle$, which is equivalent to the claim that $\mu_1 \approx \mu_2$.[5] Because both objects are derived from an underlying collection of futures contracts, it is natural to claim that both branches ought to track each other. The key idea is that *they do*, but since the strike prices along the bottom leg are unknown, the moving average $\mu_1 = \mu_1(K_1, ..., K_N)$ depends on the strike prices as *parameters*. We therefore can *impose* that the two prices track each other and *solve* for those $\{K_j\}$ such that

$$\mu_1(K_1, ..., K_N) \approx \mu_2 \tag{7}$$

We want to sketch a method for determining the set of solutions for this equation. We propose to do this numerically. Inevitably, if there are sensible solutions, there likely will be many of them. At this point, we will need to add additional constraints in order to reduce the set of solutions so that the options chosen make sense economically. For now, we leave that question open and allow all solutions to the problem. We suppose that we can pull time series data for the option derived from the **ETF** at a fixed strike price $K$. In the case that we can pull the options of the futures themselves, we could just write a script that repeatedly pulls data, calculates the moving average as we move the strike price around, subject to the condition that we minimize $\delta\mu = |\mu_2 - \mu_1|$.

[3] There are a lot of details here we are sweeping under the rug; for example, we are tacitly treating the futures contracts as basis vectors in a vector space (or more generally, as the generators of some universal commutative algebra).

[4] Commonly, the transformation is via the log, since we work with log-returns. In that case, the price itself satisfies the general linear stochastic equation, $dP = \mu P dt + \sigma P dW$, where $dP$ is the (net) return at time $t$ and $P(t)$ is the current price of the instrument.

[5] It's easy to see this: $p_j(t) = \int^t dp_j(t')$ so $\langle p_j\rangle = \int^t \langle dp_j(t')\rangle = \mu_j t$, since $\langle dW(t)\rangle = 0$ for all $t$. Since the equivalence holds at every $t$, $\mu_1 \approx \mu_2$. We are neglecting the initial condition, assume it is zero–we implicitly assume that when the variables track each other, it is also because their initial conditions are approximately equal.