Homework 3

Statistical Inference, Fall 1401

Question 1:

a. 98 % CI: (777 700, 823 000)
b. 95 % CI: (780 400, 819 600)
c. 90 % CI: (783 551, 816 440)
d. 50 % CI: (777 700, 823 000)
e. XX% Confidence Interval interpretation: if multiple samples of size n are taken and confidence intervals are calculated for each sample, XX% of those intervals will include the true population mean($\boldsymbol{\mu}$).
   Check this out for better understanding: https://rpsychologist.com/d3/ci/

f. For increasing the probability of including true population mean, wider confidence interval is required.
g.

$$Margin\ of\ Error < 5000$$
$$\frac{z^* \sigma}{\sqrt{n}} < 5000$$
$$2.57 * \frac{90\,000}{5000} < \sqrt{n} \rightarrow n > 2139.9$$
$$minimum\ houses\ required = 2140$$

h. Results would be four times larger.

$$2140 \times 4 = 8560$$

Question 2:

The hypothesis should always be about the population mean(μ), not the sample mean. The null hypothesis should have an equal sign and the alternative hypothesis should be about the null hypothesized value, not the observed sample mean.

Correction:

H_0: $\mu = 7\ hours$
H_A: $\mu > 7\ hours$

The one-side test indicate that we are only interested in showing that 7 is an underestimate. Here the interest is in only one direction, so a one-sided test seems must appropriate. If we would also be interested if the data show strong evidence that 7 was an overestimate, then the test should be two-sided.

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Question 3:

a.

$Hypothesis$:

$$H_0: \mu = 5$$
$$H_A: \mu < 5$$

For testing hypothesis we use t-test:

$$t = \frac{4.6 - 5}{\frac{2.2}{\sqrt{20}}} = -0.81$$

using t distribution, we calculate p value: 0.21

$$\alpha < p\ value \rightarrow we\ fail\ to\ reject\ Null\ hypothesis$$

b. 95% CI = (3.57, 5.63)
   Since the 95% confidence interval includes 5, the mayor's claim cannot be rejected.
c. Both CI and Hypothesis test cannot reject the mayors claim and they agree. Nevertheless, the hypothesis test gives us a p-value, and we can get a better sense what data mean.

Question 4:

a. Conditions are satisfied:
   The data is not skewed
   We have 52 observations which are more than 30
   The sample is independent
b.

   Null hypothesis(H_0):
   The difference between the suggested mean by the study and the mean suggested by the doctor is by chance.

   Alternative hypothesis(H_A):
   This difference between these two suggested value for mean is not by chance.

c.

$$z^* = 2.3263$$
$$SE = \frac{s}{\sqrt{n}} = 0.0946$$

$$\bar{x} \pm z^* \times SE = 98.2846 \pm 2.3263 \times 0.0946$$

$$CI = (98.0645, 98.5047)$$

d.

$$H_0: \mu = 98.6$$
$$H_A: \mu \neq 98.6$$
$$SE = \frac{s}{\sqrt{n}} = \frac{0.6824}{\sqrt{52}} = 0.09463$$

$$p_{value} = \Pr[\bar{x} < 98.2846 \vee \bar{x} > 98.9154 | H_0]$$
$$= \Pr[z < -3.33 \vee z > 3.33] = 0.0008$$
$$p_{value} < \alpha = 0.02$$

Hence, we should reject the null hypothesis.

Question 5

a)

$$\alpha = P(Reject\ H_0 | H_0) = P(|X - 50| > 10 | H_0)$$

$$P(X - 50 > 10\ OR\ X - 50 < -10 | H_0)$$

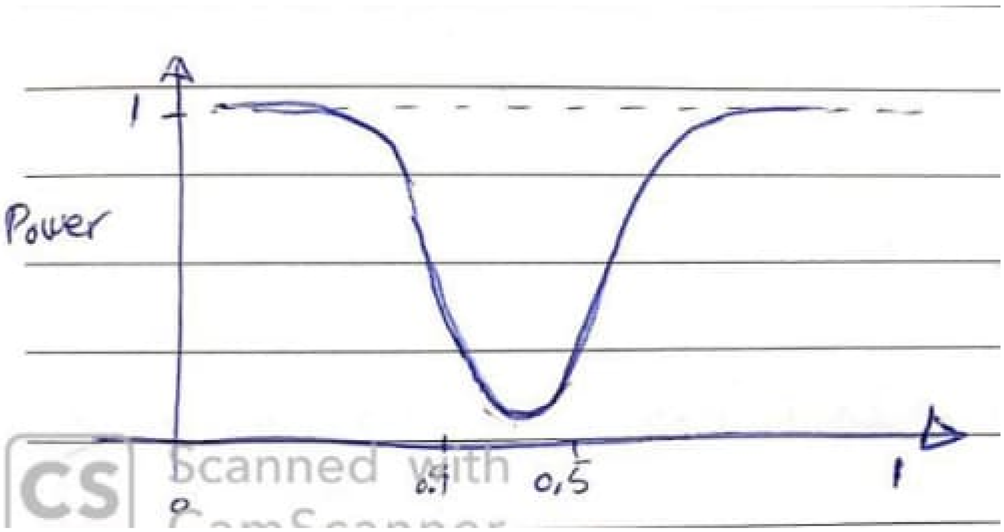
$$P(X > 60\ |H_0) + P(X < 40\ |H_0)$$

$$P\left(\frac{X - 100(0.5)}{\sqrt{100(0.5)(1-0.5)}} > \frac{60 - 100(0.5)}{\sqrt{100(0.5)(1-0.5)}}\right) +$$

$$P\left(\frac{X - 100(0.5)}{\sqrt{100(0.5)(1-0.5)}} < \frac{40 - 100(0.5)}{\sqrt{100(0.5)(1-0.5)}}\right)$$

$$\alpha \approx P(z > 2) + P(z < -2) = 0.045$$

b)



Question 6

a)

$$SE = \frac{5.6}{\sqrt{50}} = 0.792$$

$n > 30 \rightarrow z = \frac{25.9 - 28}{0.792} = -2.652 \rightarrow p - value = (Z < -2.652) = 0.004 < \alpha$

Based on sample information we can reject $H0$ in favor of HA.

b)

$$(Z \geq -1.645) = (\bar{x} \geq \bar{x}_{critical} | \mu = 28, SE = 0.792) = 0.95$$

We can find the value of $\bar{x}_{critical}$ by solving the following equation:

$-1.645 = Z_{critical} = \frac{\bar{x}_{critical} - \mu0}{SE} = \frac{\bar{x}_{critical} - 28}{0.792} \rightarrow \bar{x}_{critical} = 26.697$

$\beta = (\bar{x} > 26.697 | \mu = 27, SE = 0.792) = p\ (Z > 26.697 - 27\ /0.792\ ) = p(Z > -0.382) = 0.649$

c) No, because type II error will be made if we fail to reject $H0$, but in part (a) the pvalue is too small to reject $H0$.

Question 7

a)

$P(27) = 1 - \beta = 1 - p\left(Z > \frac{26.697-27}{0.792}\right) = 1 - p(Z > -0.382) = 1 - 0.649 = 0.351$

$P(26) = 1 - p\left(Z > \frac{26.697 - 26}{0.792}\right) = 1 - p(Z > 0.88) = 0.81$

$P(25) = 1 - p\left(Z > \frac{26.697 - 25}{0.792}\right) = 1 - p(Z > 2.142) = 0.984$

$P(24) = 1 - p\left(Z > \frac{26.697 - 24}{0.792}\right) = 1 - p(Z > 3.405) = 0.9997$

$P(23) = 1 - p\left(Z > \frac{26.697 - 23}{0.792}\right) = 1 - p(Z > 4.668) = 0.99999$

$P(22) = 1 - p\left(Z > \frac{26.697 - 22}{0.792}\right) = 1 - p(Z > 5.93) = 1$

As we consider the smaller values for μ, the difference between mean and the null-value increases and the type II error will decrease, so the power will increase.

b) As we change α to 0.01, $\bar{x}critical$ will decrease and the β will increase, so the power will decrease.

c) As we decrease the n to 20, the SE will increase and the β will increase, so the power will decrease