Homework 4

Statistical Inference, Fall 1401

1- Identify each of the following statements as true or false. Provide an explanation to justify each of your answers.

a. By resampling the population with replacement, a bootstrap distribution is created.

- ✓ False, the bootstrap distribution is created by resampling with replacement from the original sample.

b. Suppose that ANOVA at a 5% significance level rejects the null hypothesis that the means of the four groups are the same. Hence, the pairwise analysis will identify at least one pair of significantly different means.

- ✓ True

c. A smaller sample size is recommended when generating resampled samples.

- ✓ False, the resamples should be the same sample size as the original sample for it to be a representative distribution.

d. The difference between each pair of observations is the starting point for a paired analysis. Then we use these differences to do inferences.

- ✓ True

e. There is a reverse relation between variance within an ANOVA group and the amount of noise (variance).

- ✓ False, there is a correlation between variance within an ANOVA group and the amount of noise (variance).

f. In a test, we randomly sample 30 items from Amazon and note the price for each. Then we visit local markets and collect the price of each of those same 30 items. This is a paired t-test.

- ✓ True

g. An inference is made based on the difference between two observations in a paired analysis.

- ✓ True

h. In order to prevent Type-II errors, you should assign a large amount of β.

- ✓ False, if you care more about the type II error, you must assign a large amount of α.

2- A statistical inference class at a certain university has 500 students. The scores of 10 students were selected at random and are demonstrated in the following table.

a. Calculate the mean and standard deviation of the sample.

b. Calculate the margin of error.

c. Construct a 90 % confidence interval for the mean score of all the students in this class.

d. Interpret the calculated confidence interval.

76	84	69	92	58
89	73	97	85	77

a) $\bar{x} = \frac{\sum_{i=1}^{10} x_i}{n} = \frac{76+84+69+92+58+89+73+97+85+77}{10} = \frac{800}{10} = 80$

$$s = \sqrt{\frac{\sum_{i=1}^{10}(x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{(76-80)^2 + (84-80)^2 + \cdots + (77-80)^2}{9}} = 11.709$$

b) Margin of error: $df = n - 1 = 9.\ t_{df}^{*} \frac{s}{\sqrt{n}} = t_9^{*} \frac{11.709}{\sqrt{10}} = 1.8331 \times 3.702 = 6.787$

c) CI = $\bar{x} \pm t_{df}^{*} \frac{s}{\sqrt{n}}$ = 80 $\pm$ 6.787 = (73.213, 86.787)

d) We are 90% confident that the mean score of all the students' statistical inference class is somewhere between 73.213 and 86.787.

3- New York is known as "the city that never sleeps". A random sample of 25 New Yorkers were asked how much sleep they get per night. Statistical summaries of these data are shown below. The point estimate suggests New Yorkers sleep less than 8 hours a night on average. Is the result statistically significant?

n	$\bar{x}$	s	min	max
25	7.73	0.77	6.17	9.78

a. Write the hypotheses in symbols and in words.

b. Check conditions, then calculate the test statistic, T, and the associated degrees of freedom.

c. Find and interpret the p-value in this context.

d. What is the conclusion of the hypothesis test?

e. If you were to construct a 90% confidence interval that corresponded to this hypothesis test, would you expect 8 hours to be in the interval?

a) $H_0: \mu = 8$: New Yorkers sleep 8 hours per night on average
$H_A: \mu \neq 8$: New Yorkers sleep less or more than 8 hours per night on average
b) Independence: The sample is random. The min/max suggest there are no concerning outliers. $T = -1.75. df = 25 - 1 = 24$
c) P-value = 0.093, If in fact the true population mean of the amount New Yorkers sleep per night was 8 hours, the probability of getting a random sample of 25 New Yorkers where the average amount of sleep is 7.73 hours per night or less (or 8.27 hours or more) is 0.093.
d) Since p-value is greater than 0.05, do not reject H_0. The data do not provide strong evidence that New Yorkers sleep more or less than 8 hours per night on average.
e) No, since the p-value is smaller than $1 - 0.90 = 0.10$.

4- An experiment is performed to determine whether intensive tutoring (covering a great deal of material in a fixed amount of time is statistically different from (is more/less effective) than the paced tutoring (covering less material in the same amount of time). Two randomly chosen groups are tutored separately and then administered proficiency tests. Use the significance level $\alpha = 0.05$.

Group	Method	n	$\bar{x}$	s
1	Intensive	12	46.31	6.44
2	Paced	10	42.79	7.52

Let μ_1 represent the population mean for the intensive tutoring group and μ_2 represent the population mean for the paced tutoring. The hypotheses are defined as follow as:

$$H_0: \mu_1 - \mu_2 = 0$$
$$H_A: \mu_1 - \mu_2 \neq 0$$

$$df = \min(n_1 - 1. n_2 - 1) = \min(11.9) = 9$$

$$t_9 = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{(6.44)^2}{12} + \frac{(7.52)^2}{10}}} = \frac{(46.31 - 42.79) - 0}{\sqrt{3.46 + 5.66}} = 1.166$$

$$p - value = 2 * pt(1.166. df = 9. lower.tail = FALSE) = 0.27 > 0.05$$

The null hypothesis cannot be rejected. This test has not provided statistically evidence that intensive tutoring is more/ less effective compared to the paced tutoring.

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5- In order to conduct medical research, a medical research group is seeking participants to complete short surveys about their medical history. One survey, for example, asks about a person's family history of cancer. As part of another survey, the respondent is asked what topics were discussed during their last hospital visit. According to our data, as people sign up, they complete approximately 4 surveys on average, and the standard deviation for the number of surveys is approximately 2.2. The research group wants to try a new interface that they think will encourage new enrollees to complete more surveys, where they will randomize each enrollee to either get the new interface or the current interface. How many new enrollees do they need for each interface to detect an effect size of 0.5 surveys per enrollee if the desired power level is 80% (α=0.05)?

$$H_0: \mu_{new} - \mu_{current} = 0$$

$$H_A: \mu_{new} - \mu_{current} > 0$$

$$\delta = 0.5, P(H_0\ rejection | H_A\ is\ True) = 0.8$$

$$\rightarrow\ P(\mu_{new} - \mu_{current} > 1.96 \times SE \mid \mu_{new} - \mu_{current} = 0.5) = 0.8$$

$$\rightarrow 1.64 \times SE - 0.5 = \ -0.84 \times SE \rightarrow SE = 0.178$$

$$SE = \sqrt{\frac{s^2}{n} + \frac{s^2}{n}} \rightarrow (0.178)^2 = 2 \times \frac{(2.2)^2}{n} \rightarrow\ n \approx 304$$

6- To study the effect of cigarette smoking on platelet aggregation researchers drew blood samples from 11 individuals before and after they smoked a cigarette and measured the percentage of blood platelet aggregation. Platelets are involved in the formation of blood clots and it is known that smokers suffer from disorders involving blood clots than do non-smokers. Test the null hypothesis that the means before and after are the same. Use significance level $\alpha = 0.05$.

Before	After	diff
25	27	2
25	29	4
27	37	10
44	56	12
30	46	16
67	82	15
53	57	4
52	61	9

53	80	27
60	59	-1
28	43	15

$\bar{x}_{diff} = 10.27273.\ s_{diff} = 7.97.\ n_{diff} = 11$, The hypotheses are defined as follow as:

$$H_0: \mu_{diff} = 0$$
$$H_A: \mu_{diff} \neq 0$$

T- statistic: $\frac{10.27273-0}{\frac{7.97}{\sqrt{11}}} = \frac{10.27273}{2.403} = 4.27,\ df = n-1 = 10$

$$p-value = 2 * pt(4.27.df = 10.lower.tail = FALSE) = 0.0016 < 0.05$$

Therefore, the null hypothesis should be rejected and the means of blood platelet aggregation before and after smoking are statistically difference.

7- The table below shows the heights of 24 men above 20 years of age from US, UK and India. We are interested in whether or not a significant difference exists between the mean heights of these three different countries. Use significance level $\alpha = 0.05$.

Country (US)	Country (UK)	Country (India)
180	185	170
183	181	183
172	180	180
178	179	175
169	164	181
179	173	183
178	180	176
180	178	167

a. Write the hypotheses for testing if the average height of three groups of men varies.
b. Calculate and conduct analysis using one way ANOVA and complete the table.

		DF	Sum SQ	Mean SQ	F-value
Group	Class				

Error	Residuals				
	Total				

c. What is the conclusion of the test?

$a)$ $H_0: \mu_{US} = \mu_{UK} = \mu_{India}$
$H_A: At\ least\ one\ pair\ of\ means\ are\ different\ from\ each\ other$

b)

	n	$mean$	sd
Country (US)	8	177.375	21.125
Country (UK)	8	177.5	40.85714
Country (India)	8	176.875	35.83929

		DF	**Sum SQ**	**Mean SQ**	**F-value**
Group	Class	2	1.75	0.875	0.0268
Error	Residuals	21	684.75	32.607	
	Total	23	686.5		

c) $p - value = pf(0.0268.2.21.lower.tail = FALSE)$ = 0.974 > 0.05

Thus, we cannot reject the null hypothesis and the heights of men above 20 years of age from these three countries are not statistically difference.

8- (R)

9- (R)