Track Reconstruction - I

Dr. Mohammed Salim. M

Department of Physics, TKM College of Arts and Science, Kollam

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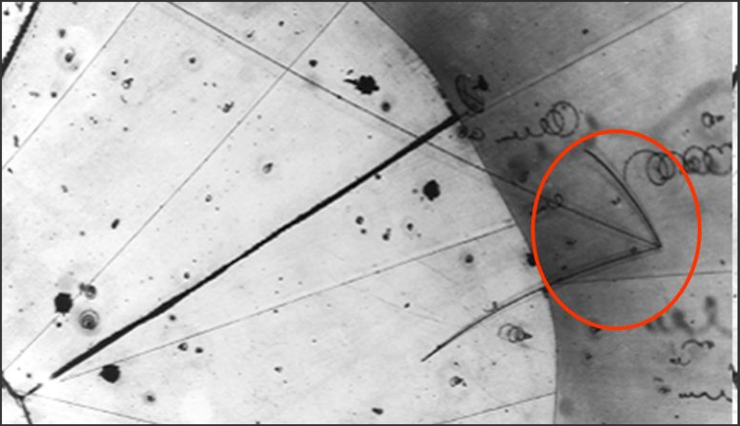
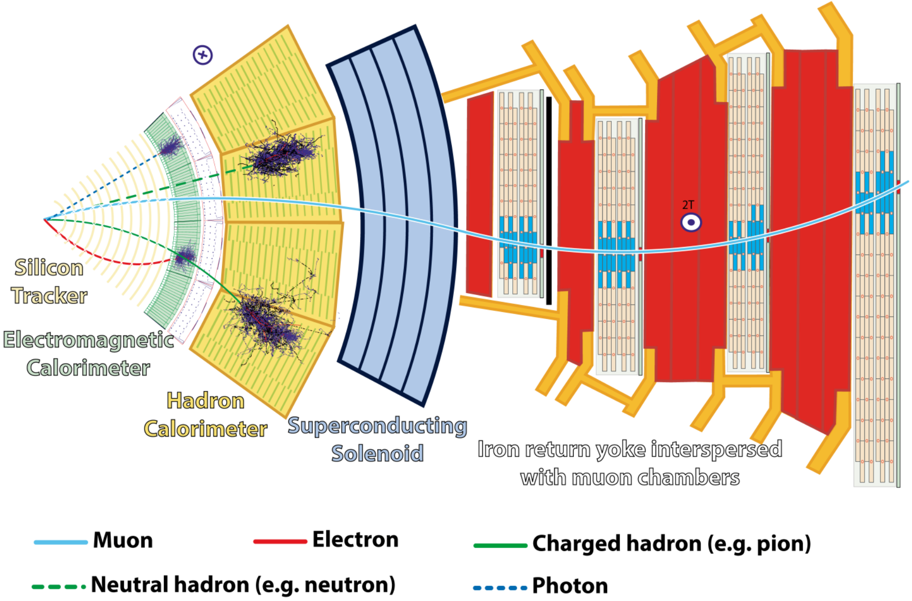


First neutrino observation Nov. 13, 1970in the Zero Gradient Synchrotron's 12-foot H bubble chamber. The invisible neutrino strikes a proton where three particle tracks originate (lower right). The neutrino turns into a mu-meson, the long center track (extending up and left). The short track is the proton. The third track (extending down and left) is a pi-meson created by the collision. *Argonne National Laboratory*


2T
Silicon Tracker
Electromagnetic Calorimeter
Hadron Calorimeter
Superconducting Solenoid
Iron return yoke interspersed with muon chambers
Muon
Electron
Charged hadron (e.g. pion)
Neutral hadron (e.g. neutron)
Photon

CMS

What are Particle Detectors?

- **Particle detectors** are devices used to track, identify, and measure properties of subatomic particles.
- They operate based on different physical interactions, such as ionization, scintillation, and Cherenkov radiation.
- Common types include:
  - **Scintillation Detectors**: Convert particle energy into light.
  - **Semiconductor Detectors**: Use silicon sensors for precise tracking.
  - **Gas Detectors**: Detect ionization in gases (e.g., Geiger-Müller, TPCs, RPCs, GEMs).
  - **Calorimeters**: Measure energy deposition of particles.
- Used in particle physics experiments, medical imaging, space research, and radiation monitoring.

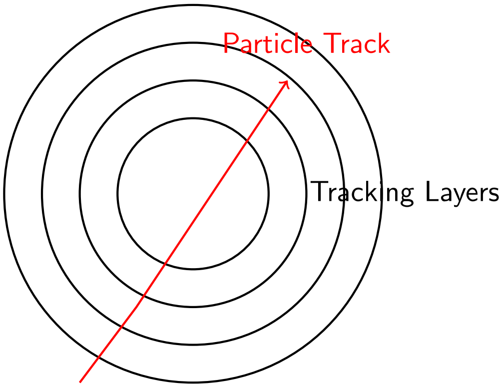
Why are Particle Detectors Important?

- **Fundamental Research**: - Used in experiments like the Large Hadron Collider (LHC) to study fundamental particles and forces.
- **Medical Applications**: - PET (Positron Emission Tomography) scanners use particle detection to diagnose diseases.
- **Space Exploration**: - Cosmic ray detectors help understand high-energy astrophysical phenomena.
- **Radiation Safety**: - Used in nuclear power plants and environmental monitoring to detect radiation levels.
- **Industrial Security Applications**: - X-ray imaging, cargo scanning, and non-destructive material testing.

What are Tracking Detectors?

- Tracking detectors are designed to measure the trajectory of charged particles.
- They operate by detecting the passage of particles through multiple sensor layers.
- Common tracking detectors:
  - **Silicon Strip Detectors (SSD)** – Used in precision tracking.
  - **Time Projection Chambers (TPC)** – Records ionization trails.
  - **Drift Chambers** – Measures charged particle positions using wire grids.
- Essential for reconstructing particle momenta and identifying interaction vertices.

Why Do We Arrange Detector Layers Sequentially?

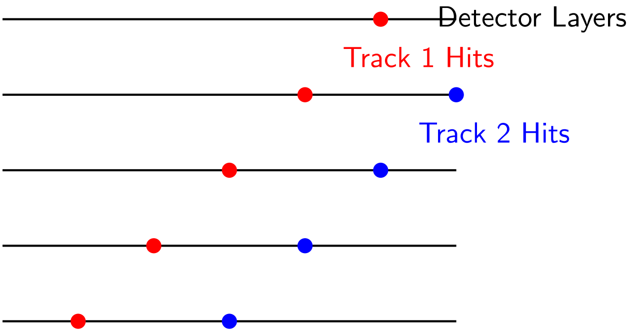


- **Multiple layers** allow precise tracking of a particle's trajectory.
- Each layer provides a position measurement, reducing uncertainty.
- Helps differentiate real tracks from noise and background signals.

From Raw Hits to Tracks

- In tracking detectors, charged particles leave *discrete hits* in different layers.
- These hits must be grouped (clustered) to reconstruct particle *trajectories*.
- Challenges in clustering:
  - Background noise and false hits.
  - Multiple overlapping tracks.
  - Detector inefficiencies and missing hits.
- Effective **track-finding algorithms** are required to identify real trajectories.

Clustering Hits: Pattern Recognition



- **Pattern recognition** involves grouping spatially correlated hits.
- Tracks are identified as *coherent structures* across layers.

Common Track-Finding Algorithms

- **Hough Transform**:
  - Converts hit coordinates into parameter space.
  - Detects straight-line patterns in tracking layers.
- **Kalman Filtering**:
  - Probabilistic approach to refine track fits.
  - Iteratively updates track parameters based on new hits.
- **Cellular Automaton**:
  - Uses local rules to grow track candidates from neighboring hits.
  - Efficient for high-multiplicity environments.
- **Combinatorial Track Finding**:
  - Iterates over possible hit combinations to find best-matching trajectories.

Introduction to Hough Transform

- The **Hough Transform** is a *pattern recognition technique* used to detect geometric shapes, especially lines.
- Originally used in **image processing**, it is widely applied in **particle track reconstruction**.
- It converts a set of points **(x, y)** into a parameter space (θ, ρ).
- Tracks appear as peaks in **Hough space**, simplifying track identification.

Mathematical Formulation

- A straight-line equation in Cartesian coordinates:

$$y = mx + c$$

- In Hough Transform, the equation is rewritten in polar form:

$$\rho = x \cos\theta + y \sin\theta$$

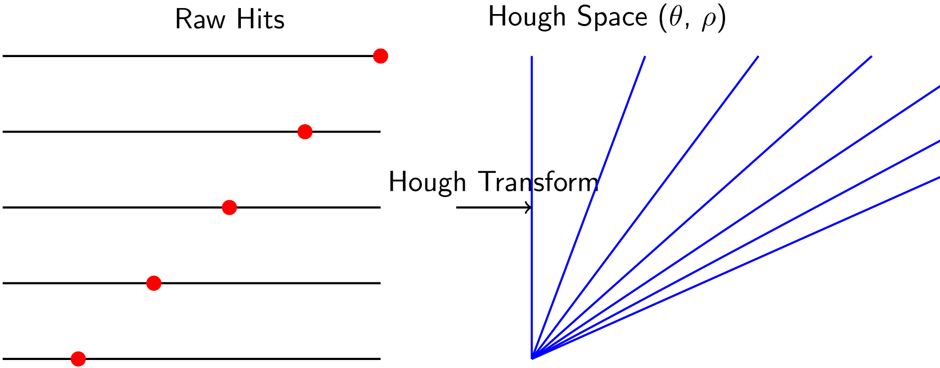
  where:
  - ρ is the perpendicular distance from the origin.
  - θ is the angle of the normal to the line.
- Each point **(x, y)** in detector space maps to a curve in Hough space.

Hough Transform for Track Reconstruction

- **Step 1**: Each hit in the detector is transformed into a curve in Hough space.
- **Step 2**: If multiple hit curves *intersect*, they define a candidate track.
- **Step 3**: The highest peaks in the (θ, ρ) accumulator space correspond to the best track candidates.

Each track corresponds to a peak in Hough space!

Hough Transform Visualization



- Track hits appear as peaks in Hough space.
- The most significant peak corresponds to the best-fit track.

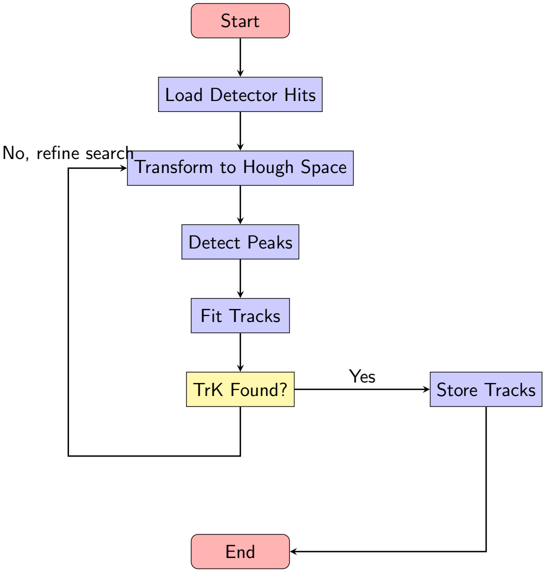
Algorithm

1. Extract hit positions (x, y) from detector layers.
2. Apply the Hough Transform:

$$\rho = x \cos\theta + y \sin\theta$$

3. Accumulate votes in a (θ, ρ) space.
4. Identify peaks corresponding to potential tracks.
5. Fit lines to the selected tracks.

Flow Chart

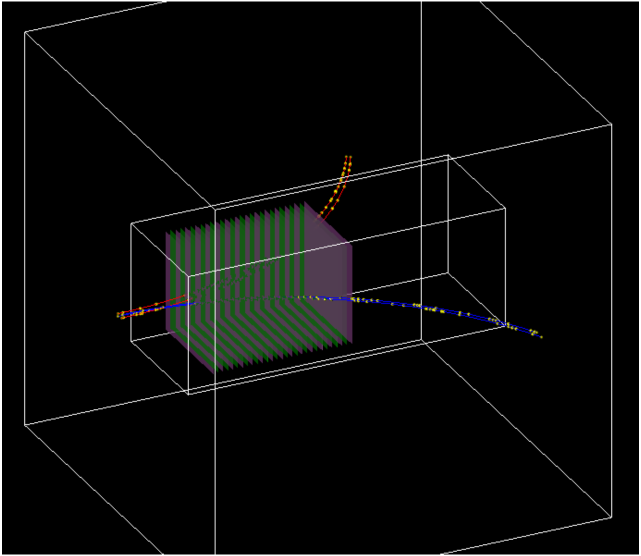


Start
Load Detector Hits
Transform to Hough Space
No, refine search
Detect Peaks
Fit Tracks
TrK Found?
Yes
Store Tracks
End

How Tracking Leads to Track Reconstruction

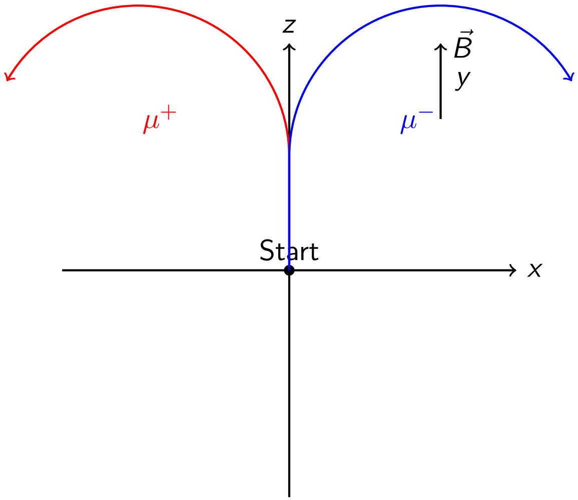
- By aligning detector hits, we can reconstruct a particle's **momentum** and **charge**.
- A curved track in a magnetic field reveals the **momentum** via the curvature radius.
- Track fitting algorithms like **Kalman filters** help refine trajectories.
- Multiple layers allow rejection of spurious hits, improving resolution.

Bending in Magnetic Field



Trajectory of both μ^+ and μ^- with an energy of 1 GeV ($\vec{\boldsymbol{B}} = \mathbf{1}\,\boldsymbol{Tesla}$)

Motion of μ^+ and μ^- in Magnetic Field



Equation of Motion in Magnetic Field

The motion of a charged particle in a magnetic field is governed by the Lorentz force:

$$\vec{F} = q(\vec{v} \times \vec{B}) \tag{1}$$

Since the force is always perpendicular to velocity, the motion is circular

in a uniform magnetic field.
Centripetal force balances Magnetic force.

$$\frac{mv^2}{R} = qvB \tag{2}$$

$$R = \frac{p}{qB} \tag{3}$$

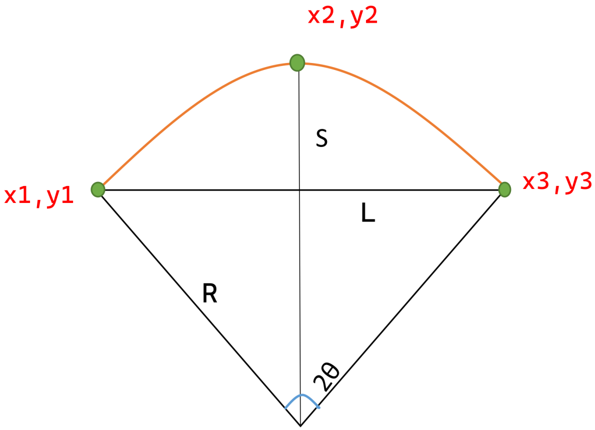
Equation of Motion in Magnetic Field (Cont.)

The radius of the trajectory is given by:

$$R = \frac{p}{qB} \tag{4}$$

where p is the momentum of the particle, q is its charge, and B is the magnetic field.

Momentum Determination



Sagitta Measurement

Momentum Measurement using Sagitta

$$2\theta = \frac{L}{R} \tag{5}$$

The sagitta s of a track in a uniform magnetic field is given by:

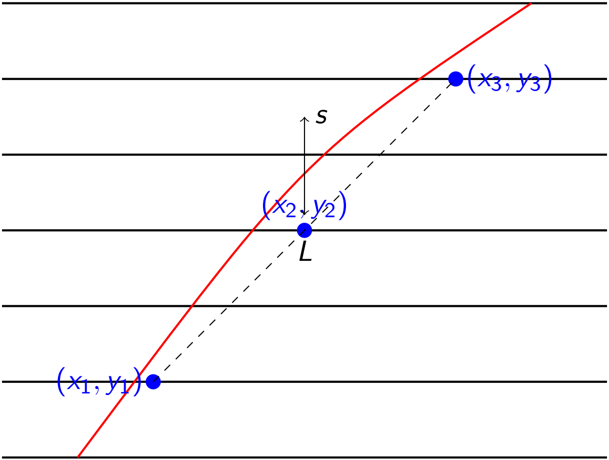
$$s = R(1 - \cos\theta)$$

$$s \approx R\frac{\theta^2}{2}$$

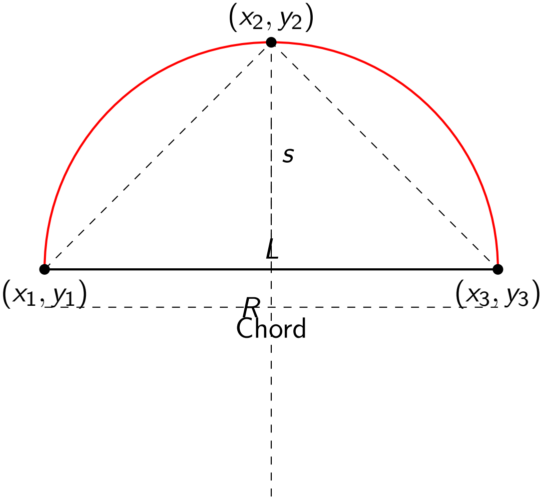
$$s \approx \frac{L^2}{8R}$$

$$s \approx \frac{L^2}{8}\frac{qB}{p} \tag{6}$$

Multi Layer Detector



Sagitta Measurement



Momentum Calculation using Sagitta

Using three nearby hits, we compute:

- Chord length:

$$L = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2} \tag{7}$$

- Sagitta approximation:

$$s = y_2 - \frac{y_1 + y_3}{2} \tag{8}$$

- Momentum estimation:

$$p = \frac{0.3BL^2}{8s} \tag{9}$$

Sagitta and Momentum Relation

The sagitta s is defined as the perpendicular distance from the midpoint of the chord to the arc:

$$s \approx \frac{L^2}{8R} \tag{10}$$

where L is the chord length and R is the bending radius.

Momentum Resolution Derivation

Taking the uncertainty in sagitta σ_s and propagating it to momentum uncertainty σ_p:

$$\frac{\sigma_p}{p} = \frac{\sigma_s}{s} = \frac{8R}{L^2}\sigma_s \tag{11}$$

This leads to the final relative momentum resolution formula.

Relative Momentum Resolution

The relative momentum resolution is given by:

$$\frac{\sigma_p}{p} = \frac{\sigma_s}{s} = \frac{8p\sigma_s}{0.3BL^2} \tag{12}$$

where:

- σ_p is the uncertainty in momentum,
- p is the momentum of the particle,
- σ_s is the uncertainty in sagitta measurement,
- s is the sagitta,
- B is the magnetic field,
- L is the length scale of measurement.