Error Propagation

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Abstract

This is just a very brief introduction of Error propagation.
Error Analysis is very important subject for experimentalist point of view. Presentation data from Simulation or Experiment without its analysis is of no use. Here we will show how to derive the formula for error calculation of very simple mathematical form like ratios of two independent variable, multiplication of two independent variable, addition of two independent variables etc.

Case I: when f(x,y) is of following form

$$f(x,y) = \frac{x}{y}$$

Case II: when f(x,y) is of following form

$$f(x,y) = x \times y$$

Case III: when f(x,y) is of following form

$$f(x,y) = x + y$$

Derivation for error expression for any mathematical form start with **General Propagation Formula** which states that **Uncertainties typically add as the root-sum-of-squares** ("RSS"; can be derived with calculus)

$$\sigma_f = \sqrt{(\frac{\partial f}{\partial x})^2\sigma_x^2 + (\frac{\partial f}{\partial y})^2\sigma_y^2} \tag{1}$$

where

$$\sigma_x = \sqrt{x}$$
$$\sigma_y = \sqrt{y}$$

Now for Case I : when f(x,y) is of form

$$f(x,y) = \frac{x}{y}$$

$$\frac{\partial f}{\partial x} = \frac{1}{y}$$

and

$$\frac{\partial f}{\partial y} = \frac{-x}{y^2}$$

Now we have all the values require for (1). On substituting these in we get

$$\frac{\sigma_f}{f} = \sqrt{(\frac{\sigma_x}{x})^2 + (\frac{\sigma_y}{y})^2} \tag{2}$$

On further simplification we get

$$\sigma_f = \frac{x}{y}\sqrt{\frac{1}{x} + \frac{1}{y}}$$

For Case II : when f(x,y) is of form

$$f(x,y) = x \times y$$

$$\frac{\partial f}{\partial x} = y$$

$$\frac{\partial f}{\partial y} = x$$

Substituting in (1) we get

$$\frac{\sigma_f}{f} = \sqrt{(\frac{\sigma_x}{x})^2 + (\frac{\sigma_y}{y})^2} \tag{3}$$

On further simplification we get

$$\sigma_f = \frac{x}{y}\sqrt{\frac{1}{x} + \frac{1}{y}}$$

For Case II : when f(x,y) is of form

$$f(x, y) = x + y$$

$$\frac{\partial f}{\partial x} = 1$$

$$\frac{\partial f}{\partial y} = 1$$

Substituting in (1) we get

$$\sigma_f = \sqrt{\sigma_x^2 + \sigma_y^2} \tag{4}$$

or

$$\sigma_f = \sqrt{x + y}$$

A point worth noting, the error formula for Case I and Case II are same