**<u>Optimal Charging Problem Formulation</u>**

UC Berkeley
03/08/2018

<u>Optimal Control Problem</u>

$$\min_{I(t),x(t),t_f} \quad J$$

Subject to

Model Dynamics

Input Constraints

State Constraints

Side Reaction Constraints

Time Constraints

Initial Conditions

Terminal Conditions

<u>Possible Objective Functions</u>

Minimum Time

$$J = \int_{t_0}^{t_f} 1dt$$

Minimum Time and Temperature Rise

$$J = \beta \int_{t_0}^{t_f} 1dt + (1-\beta)\left(T\left(t_f\right) - T(t_0)\right)$$

Minimum Time and Side Reaction Overpotential (If $\eta_s(t)$ is allowed to go below 0V)

$$J = \beta \int_{t_0}^{t_f} 1dt + (1-\beta) \int_{t_0}^{t_f} -[\eta_s(t)]^- dt$$

where $[\eta_s(t)]^- = \min\{0, \eta_s(t)\}$

<u>Possible Model Dynamics</u>

Single Particle Model with Electrolyte and Temperature Dynamics

Solid Phase Concentration States

$$c_s^{\pm}(r,t)$$

Electrolyte Phase Concentration States

$$c_e^{\{-,sep,+\}}(x,t)$$

Temperature States

$$T(t)$$

Inputs

$$I(t)$$

Outputs

$$SOC(t), V(t), \eta_s(t)$$

<u>Possible Constraints</u>

Input Constraints

$$I_{min} \leq I(t) \leq I_{max}$$

State Constraints

$$\theta_{min}^{\pm} \leq \frac{c_s^{\pm}(r,t)}{c_{s,max}^{\pm}} \leq \theta_{max}^{\pm}$$

$$c_{e,min} \leq c_e^{\{-,sep,+\}}(x,t) \leq c_{e,max}$$

$$T_{min} \leq T(t) \leq T_{max}$$

Side Reaction Constraints

$$\eta_s(t) \geq 0$$

Time Constraints

$$t_0 \leq t_f \leq t_{max}$$

<u>Initial and Terminal Conditions</u>

Initial Conditions

$$c_s^{\pm}(r,t_0) = c_{s,0}^{\pm}$$

$$c_e^{\{-,sep,+\}}(x,t_0) = c_{e,0}^{\{-,sep,+\}}$$

$$T(t_0) = T_0$$

$$SOC(t_0) = SOC_0$$

Terminal Conditions

$$SOC(t_f) = SOC_f$$