Equivalent circuit models (ECM): A dynamic programming formulation

The purpose of this document is to elucidate the governing electrochemical-thermal equations used in the equivalent circuit model (ECM) for both continuous and dynamic programming (DP) frameworks. The current I is the controllable input. SOC, terminal voltage (consisting of two state variables V_1, V_2), and temperature (core and surface) are state variables. SOC is the current focus of implementation. The optimization function used is the max charge problem. Parameter values are taken from a SOC value of 0.5, and temperature value of 25 °C. We first introduce three equivalent circuit models that are representative of how they will be sequentially implemented.

Dynamic programming approach

The dynamic programming (DP) problem involves converting the optimization problem into a discrete problem. We evoke Bellman's Principal of Optimality, i.e.,:

Let V_k represent the value function for the max charge problem at each time step $k \in \{0, 1, .., N-1\}$:

$$\boxed{V_k = \min_{I_k} \left\{ \left(z_k - z_{target}\right)^2 + V_{k+1} \right\} + V_N} \tag{1}$$

$$V_N = \left(z_N - z_{target}\right)^2$$

We solve this objective by identifying optimal control I_k^* for all k. We utilize a forward finite difference method for discretization of the continuous state dynamics, with $dt = 1$. We now break down the dynamic programming formulation of each model.

Model 1: OCV-R

Model description

This model is the simplest circuit model using a voltage source V_{oc} and a resistor R_0. Since there are no current collectors, we have a single state: the state-of-charge of the battery, hereafter denoted as z. V_{oc} depends on z, and for the purpose of this report we use the data from an A123 2300 mAh battery.

Dynamics

First we understand the state equation for SOC evolution as a function of the current:

$$\frac{dz}{dt} = \frac{I}{C_{\text{batt}}}$$

We now convert the single continuous state equation to the dynamic form.

$$\frac{z_{k+1} - z_k}{dt} = \frac{I_k}{dt}$$

$$\boxed{z_{k+1} = \frac{I_k}{C_{\text{batt}}} \cdot dt + z_k} \tag{2}$$

State space

Our state space for z goes from $z_{\min}$ to $z_{\max}$, which are determined from the battery chemical parameters and safe operating limits.

Constraints

We present the constraints for the state and control separately:

$$z_{min} \leq z_k \leq z_{max}$$

$$V_{t,min} \leq V_{t,k} \leq V_{t,max}$$

$$I_{min} \leq I_k \leq I_{max}$$

We rewrite the state constraint given the time-stepping dynamics:

$$z_{min} \leq z_{k+1} - \frac{I_k}{C_{\text{batt}}} \cdot dt \leq z_{max}$$

We convert this form into a bound on the control I_k:

$$C_{\text{batt}} \cdot \frac{z_{k+1} - z_{\max}}{dt} \leq I_k \leq C_{\text{batt}} \cdot \frac{z_{k+1} - z_{\min}}{dt}$$

We now rewrite the terminal voltage constraint also as a bound on I_k:

$$V_{t,min} \leq V_{oc} + I_k R_0 \leq V_{t,max}$$

$$\frac{V_{t,min} - V_{oc}}{R_0} \leq I_k \leq \frac{V_{t,max} - V_{oc}}{R_0}$$

Combining the constraints on I_k, we can tighten the bounds with the following expression:

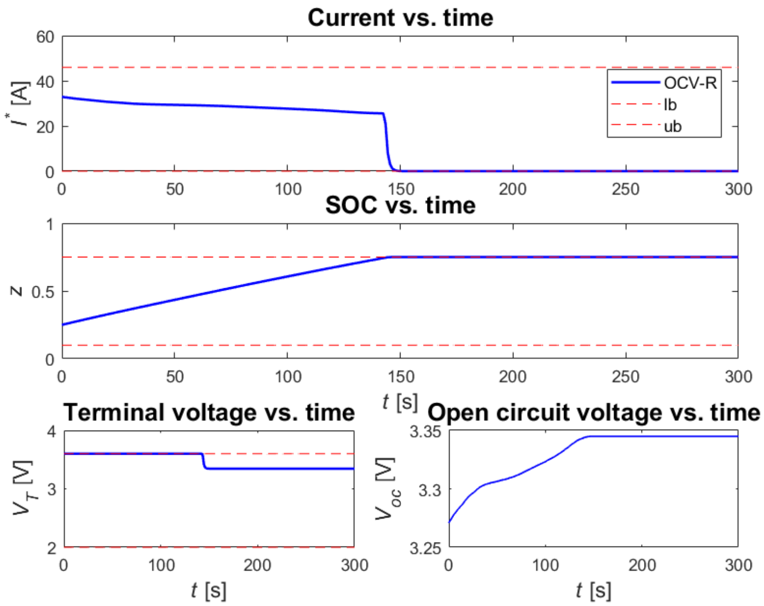
$$\max\left\{C_{\text{batt}} \cdot \tfrac{z_{k+1} - z_{\max}}{dt}, \tfrac{V_{min} - V_{oc}}{R_0}, I_{\min}\right\} \leq I_k \leq \min\left\{C_{\text{batt}} \cdot \tfrac{z_{k+1} - z_{\min}}{dt}, \tfrac{V_{max} - V_{oc}}{R_0}, I_{\max}\right\} \tag{3}$$

For notation purposes, hereafter we will note the state of charge lower/upper bounds as z_{lb}, z_{ub} and the terminal voltage lower/upper bounds as $V_{t,lb}, V_{t,ub}$ respectively. We can rewrite Equation 3 with an easier framework:

$$\boxed{\max\left\{z_{lb}, V_{t,lb}, I_{\min}\right\} \leq I_k \leq \min\left\{z_{ub}, V_{t,ub}, I_{\max}\right\}} \tag{4}$$

Solution

We present the solution to the max charge problem with a target SOC of 0.75 and initial SOC of 0.25.



Model 2: OCV-R-RC

Model description

This model adds an RC pair to the OCV-R model. The capacitor acts as a current collector and contributes a time-varying voltage V_1. We now have two states and one control.

Dynamics

The SOC dynamics remain the same. We now describe the state dynamics for the capacitor voltage with the continuous equation:

$$C_1 \frac{dV_1}{dt} = -\frac{V_1}{R_1} + I$$

We use the finite difference method to convert the continuous form to a dynamic one.

$$C_1 \frac{V_{1,k+1} - V_{1,k}}{dt} = -\frac{V_{1,k}}{R_1} + I_k$$

Rewriting,

$$\boxed{V_{1,k+1} = V_{1,k}\left(1 - \frac{dt}{R_1 \cdot C_1}\right) + \frac{I_k \cdot dt}{C_1}} \tag{5}$$

State space

We described the state space for z in the OCV-R model. We consider the minimum voltage for the capacitor. We assume we charge from rest, and as we are only interested in the charging problem, we take $\boxed{V_{1,\min} = 0}$. The maximum capacitor voltage comes from the steady-state solution to: $C_1 \cdot 0 = -\frac{V_1}{R_1} + I$. Solving, we get $\boxed{V_{1,\max} = I_{\max} \cdot R_0}$

Constraints

We retain the SOC constraints z_{lb}, z_{ub} from before, in addition to the current constraints $I_{\min}, I_{\max}$. The terminal voltage constraint only changes slightly:

$$V_{t,min} \leq V_{oc} + V_{1,k} + I_k R_0 \leq V_{t,max}$$

$$\frac{V_{t,min} - V_{oc} - V_{1,k}}{R_0} \leq I_k \leq \frac{V_{t,max} - V_{oc} - V_{1,k}}{R_0}$$

We use this updated framework for $V_{1,lb}, V_{1,ub}$. Our control constraints remain the same as described in Equation $\boxed{4}$.

Solution

We present the solution to the max charge problem with a target SOC of 0.75 and initial SOC of 0.25.