The purpose of this document is to elucidate the governing electrochemical-thermal equations used in the equivalent circuit model (ECM) with a dynamic programming (DP) framework. The current is the controllable input. SOC, terminal voltage (consisting of two state variables V_1, V_2), and temperature (core and surface) are state variables. SOC is the current focus of implementation. The optimization function used is the max charge problem. Parameter values are taken from a SOC value of 0.5, and temperature value of 25 °C.

1 Constraints and State dynamics

$$I_{\min} \leq I_k \leq I_{\max} \quad \forall k \in 1..N$$

$$SOC_{\min} \leq SOC_k \leq SOC_{\max}$$

$$V_{\min} \leq V_{t,k} \leq V_{\max}$$

$$T_{\min} \leq T_{c,k}, T_{s,k} \leq T_{\max}$$

$$\frac{dSOC_k}{dt} = \frac{I_k}{C_{\text{batt}}} \tag{1}$$

$$\frac{dV_{i,k}}{dt} = \frac{-V_{i,k}}{R_i C_i} + \frac{I_k}{C_i} \quad \forall i \in 1,2 \tag{2}$$

$$V_{t,k} = V_{oc}(SOC_k) + V_{1,k} + V_{2,k} + I_k R_o{}^1 \tag{3}$$

$$\frac{dT_{c,k}}{dt} = \frac{T_{s,k} - T_{c,k}}{R_c C_c} + \frac{I_k |V_{oc}(SOC_k) - V_{t,k}|}{C_c} \tag{4}$$

$$\frac{dT_{s,k}}{dt} = \frac{T_\infty - T_{s,k}}{R_u C_s} - \frac{T_{s,k} - T_{c,k}}{R_c C_s} \tag{5}$$

2 Current Implementation

$$\max\left\{C_{\text{batt}} \cdot \tfrac{SOC_{k+1} - SOC_{\max}}{dt}, I_{\min}\right\} \leq I_k \leq \min\left\{C_{\text{batt}} \cdot \tfrac{SOC_{k+1} - SOC_{\min}}{dt}, I_{\max}\right\}$$

3 DP Formulation

Let V_k represent the charge accumulation from time step k to total time N. Note that the maximum charge (with fixed time horizon) objective function is equivalent to the minimum time problem. We define control variable I_k as u_k $\forall k$ and state variables SOC_k as x_k $\forall k$:

$$V_k(u_k) = \max_{u_k, x_k} \left\{u_k + V(k+1)\right\} \quad \forall k \in 1..N \tag{6}$$

We finally establish the boundary conditions:

$$V_{N+1} = 0$$

$$x_{N+1} = x_{\max}$$

[1] If one uses the Taylor approximation about some equilibrium value z_{eq}, we get this expression: $V_{oc} = p_0 + p_1 z_{eq} + p_2 z_{eq}^2 + p_3 z_{eq}^3 + \left(p_1 + 2p_2 z_{eq} + 3p_3 z_{eq}^2\right)(z - z_{eq})$, where z represents the SOC value at any time k