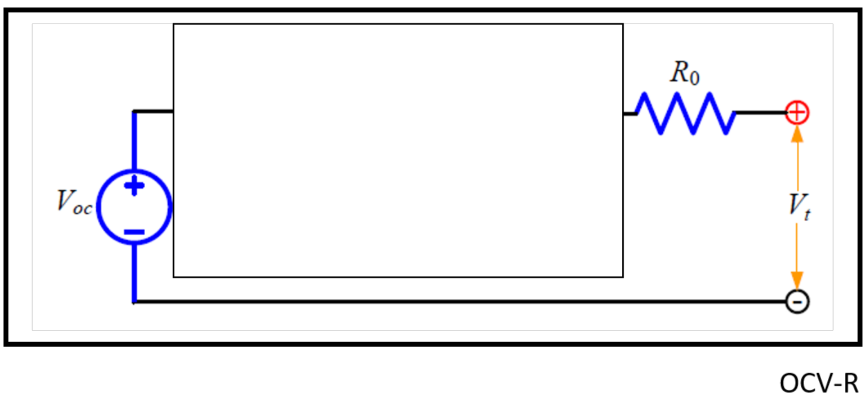
Continuous and DP formulation of three ECT models

The purpose of this document is to elucidate the governing electrochemical-thermal equations used in the equivalent circuit model (ECM) for both continuous and dynamic programming (DP) frameworks. The current I is the controllable input. SOC, terminal voltage (consisting of two state variables V_1, V_2), and temperature (core and surface) are state variables. SOC is the current focus of implementation. The optimization function used is the max charge problem. Parameter values are taken from a SOC value of 0.5, and temperature value of 25 °C. We first introduce three equivalent circuit models that are representative of how they will be sequentially implemented.

Introducing the three models

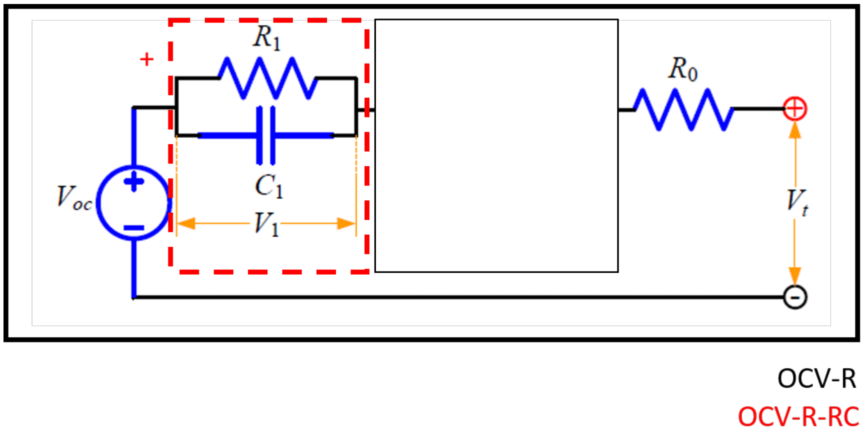
Model 1

Model 1 is OCV-R, as described below. Here we make the assumptions that the terminal voltage is dependent on the open circuit voltage V_{oc} and the drop across resistor R_0. V_{oc} is a function of the state of charge (SOC), which is dynamic with the current.



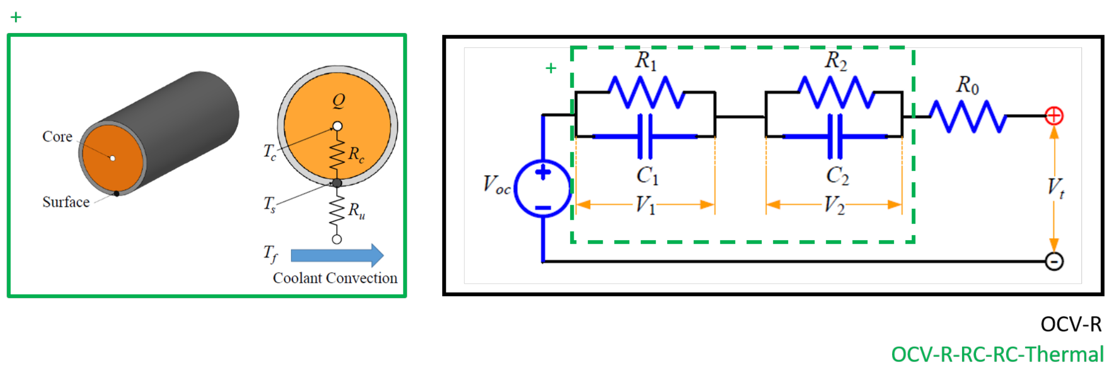
Model 2

Model 2 is OCV-R-RC, as described below. Now a RC couple is included in the equivalent circuit model, so there are voltage dynamics for V_1. The state dynamics are first order and are thus solved.



Model 3

Model 3 is OCV-R-RC-RC with thermal input, as described below. Another RC couple is included and now we track the voltage dynamics for V_2. The diagram below includes the thermal model diagram, which approximates an internal core resistance R_c and an external resistance R_u. The state dynamics are a bit more complex so model is a true benchmark for the DP performance when compared with the computation time for the continuous solution.



Continuous time approach

The continuous time approach involves solving the following optimization problem without discretized intervals of time, i.e.,:

$$\max_{I_{valid}} \quad \int_{t_o}^{t_{max}} I dt \tag{1}$$

We present the governing constraints, equations, and initial/terminal conditions for each model below.

Model 1

Equations

Let SOC hereafter be represented as z. This OCV model is taken from CE295 HW1 (Spring 2018):

$$\frac{dz(I,t)}{dt} = \frac{I(t)}{C_{\text{batt}}} \tag{2}$$

$$V_t(I,z,t) = V_{oc}(z) + I(t) \cdot R_0 \tag{3}$$

$$V_{oc}(z) = p_0 + p_1 \cdot z_{eq} + p_2 \cdot z_{eq}^2 + p_3 \cdot z_{eq}^3 + \left(p_1 + 2p_2 \cdot z_{eq} + 3p_3 \cdot z_{eq}^2\right)(z - z_{eq})^1 \tag{4}$$

Variable	**Value**	**Units**
C_{batt}	4320	[C][a]
R_0	0.01	[Ω][b]
p_0	3.471	[-][c]
p_1	1.611	[-]
p_2	-2.629	[-]
p_3	1.7175	[-]
z_{eq}	0.5	[-]

[a]Estimated 1200 mAh for battery

[b]Determined at $z_{eq} = 0.5$ and $T_\infty = 25°C$ from Perez et. al 2015

[c]Coefficient values taken from CE295 HW1

[1]Note that V_{oc} is calculated by a first order Taylor series expansion about the equilibrium SOC value, z_{eq}.

Constraints

$$I_{\min} \leq I(t) \leq I_{\max}$$

$$z_{\min} \leq z(I,t) \leq z_{\max}$$

$$V_{\min} \leq V_t(I,z,t) \leq V_{\max}$$

Variable	Value	Units
I_{min}	0	[A]
I_{max}	46	[A]
z_{min}	0.25	[-]
z_{max}	0.75	[-]
V_{min}	2	[V]
V_{max}	3.6	[V]

Initial and Final Conditions

Variable	Value	Units
N	1800	[sec]
$z(0)$	0.25	[-]
$z(N)$	0.75	[-]

Model 2

We include all the equations in Model 1, with modifications made to the V_t term as described below.

Equations

$$\frac{dV_1(I,t)}{dt} = \frac{-V_1(I,t)}{R_1 \cdot C_1} + \frac{I(t)}{C_1} \tag{5}$$

$$V_t(I,z,t) = V_{oc}(z) + V_1(I,t) + I(t) \cdot R_0 \tag{6}$$

Variable	Value	Units
R_1	0.01	[Ω]
C_1	2500	[F]

Constraints

No change

Initial and Final Conditions

No change

Model 3

We include all the equations in Model 1 and 2, with modifications made to the V_t term as described below. In addition, thermal dynamics are now included.

Equations

$$\frac{dT_c(t)}{dt} = \frac{T_s(t) - T_c(t)}{R_c \cdot C_c} + \frac{I(t)|V_{oc}(z) - V_t(I, z, t)|}{C_c} \tag{7}$$

$$\frac{dT_s(t)}{dt} = \frac{T_\infty - T_s(t)}{R_u \cdot C_s} - \frac{T_s(t) - T_c(t)}{R_c \cdot C_s} \tag{8}$$

$$\frac{dV_2(I, t)}{dt} = \frac{-V_2(I, t)}{R_2 \cdot C_2} + \frac{I(t)}{C_2} \tag{9}$$

$$V_t(I, z, t) = V_{oc}(z) + V_1(I, t) + V_2(I, t) + I(t) \cdot R_0 \tag{10}$$

Variable	Value	Units
R_c	1.94	[K/W]
C_c	62.7	[J/K]
R_u	3.08	[K/W]
C_s	4.5	[J/K]
R_2	0.02	[Ω]
C_2	5.5	[F]

Constraints

$$T_{\min} \leq T_c(t) \leq T_{\max}$$

Variable	Value	Units
T_{min}	26	[C]
T_{max}	36	[C]

Initial and Final Conditions

Variable	Value	Units
$T_c(0)$	25	[C]
$T_s(0)$	25	[C]

Dynamic programming approach

The dynamic programming (DP) problem involves converting the optimization problem presented earlier in Equation $\boxed{1}$ into a discrete problem. This can be done using Bellman's Principal of Optimality, i.e.,:
Let V_k represent the charge accumulation from time step k to total time N. We define control variable I_k as u_k $\forall k$ and state variables $z_k, V_{t,k}, T_{c,k}$ as x_k $\forall k$:

$$V_k(u_k) = \max_{u_k, x_k} \left\{ \sum_k u_k + V(k+1) \right\} \quad \forall k \in \left\{ t_0..N-1 \right\} \tag{11}$$

This conversion allows us to identify the optimal control u_k^* for all k. When introducing the complex thermal dynamics, this approach may be advantangeous computationally, but this is yet to be proven. We present the governing constraints, system dynamics, and initial/terminal conditions for each model below. For the parameter values, please reference the "continuous time approach" section. Consider $dt = 1$.

Model 1

Dynamics

$$z_{k+1} = \frac{I_k}{C_{\text{batt}}} \cdot dt + z[k] \quad \forall k \in \left\{ t_0..N-1 \right\} \tag{12}$$

Constraints

$$\max\left\{ C_{\text{batt}} \cdot \tfrac{z_{k+1} - z_{\max}}{dt}, \tfrac{V_{min} - V_{oc}}{R_0}, I_{\min} \right\} \leq I_k \leq \min\left\{ C_{\text{batt}} \cdot \tfrac{z_{k+1} - z_{\min}}{dt}, \tfrac{V_{max} - V_{oc}}{R_0}, I_{\max} \right\} \tag{13}$$

Initial and Final Conditions

We add the terminal cost constraint: $V_N = 0$

Model 2

Dynamics

$$V_{1,k+1} = V_{1,k}\left(1 - \tfrac{dt}{R_1 \cdot C_1}\right) + \frac{I_k \cdot dt}{C_1} \tag{14}$$

Constraints

For simplicity, we define the following quantities:

- $$C_{\text{batt}} \cdot \frac{z_{k+1} - z_{\max}}{dt} = z_{lb}$$
- $$C_{\text{batt}} \cdot \frac{z_{k+1} - z_{\min}}{dt} = z_{ub}$$
- $$\frac{C_1}{C_1 \cdot R_0 + dt} \left\{ V_{min} - V_{oc} - V_{1,k}\left(1 - \tfrac{dt}{R_1 \cdot C_1}\right) \right\} = V_{1,lb}$$

- $$\frac{C_1}{C_1 \cdot R_0 + dt}\left\{V_{max} - V_{oc} - V_{1,k}\left(1 - \frac{dt}{R_1 \cdot C_1}\right)\right\} = V_{1,ub}$$

Using this new notation, we arrive at the expression below:

$$\max\left\{z_{lb}, V_{1,lb}, I_{\min}\right\} \leq I_k \leq \min\left\{z_{ub}, V_{1,ub}, I_{\max}\right\} \tag{15}$$

Initial and Final Conditions

No change from continuous model

Model 3

Dynamics

$$V_{2,k+1} = V_{2,k}\left(1 - \frac{dt}{R_2 \cdot C_2}\right) + \frac{I_k \cdot dt}{C_2} \tag{16}$$

$$T_{c,k+1} = \left\{\frac{T_{s,k} + T_{c,k} \cdot \left(R_c \cdot C_c - 1\right) + R_c \cdot I_k \left(V_{1,k} + V_{2,k} + R_0 \cdot I_k\right)}{R_c \cdot C_c}\right\} dt \tag{17}$$

$$T_{s,k+1} = \left\{\frac{R_c \cdot T_{\infty} - T_{s,k} \cdot \left(R_c + R_u - \frac{R_c R_u C_s}{dt}\right) + R_u \cdot T_{c,k}}{R_u \cdot R_c \cdot C_s}\right\} dt \tag{18}$$

Constraints

For simplicity, we define the following quantities (in addition to those defined in Model 2):

- $$\frac{C_1 C_2 R_0}{\left(C_1 + C_2\right) R_0 dt + C_1 C_2 R_0}\left\{V_{min} - V_{oc} - \sum_{i \in [1,2]} V_{i,k}\left(1 - \frac{dt}{R_i \cdot C_i}\right)\right\} = V_{t,lb}$$

- $$\frac{C_1 C_2 R_0}{\left(C_1 + C_2\right) R_0 dt + C_1 C_2 R_0}\left\{V_{max} - V_{oc} - \sum_{i \in [1,2]} V_{i,k}\left(1 - \frac{dt}{R_i \cdot C_i}\right)\right\} = V_{t,ub}$$

- $$\frac{R_c C_c \left(\frac{T_{c,min}}{dt}\right) - T_{s,k} - T_{c,k}\left(R_c C_c - 1\right)}{R_c} = \hat{T}_{c,lb}$$

- $$\frac{R_c C_c \left(\frac{T_{c,max}}{dt}\right) - T_{s,k} - T_{c,k}\left(R_c C_c - 1\right)}{R_c} = \hat{T}_{c,ub}$$

- $$\frac{-\left(V_{1,k} + V_{2,k}\right) + \sqrt{\left(V_{1,k} + V_{2,k}\right)^2 - 4 R_0 \hat{T}_{c,lb}}}{2 R_0} = T_{c,lb}$$

- $$\frac{-\left(V_{1,k}+V_{2,k}\right)+\sqrt{\left(V_{1,k}+V_{2,k}\right)^{2}-4R_{0}\hat{T}_{c,ub}}}{2R_{0}}=T_{c,ub}$$

Using this new notation, we arrive at the expression below:

$$\max\left\{z_{lb},V_{t,lb},T_{c,lb},I_{\min}\right\}\leq I_{k}\leq\min\left\{z_{ub},V_{t,ub},T_{c,ub},I_{\max}\right\} \tag{19}$$

Initial and Final Conditions

No change from continuous model