

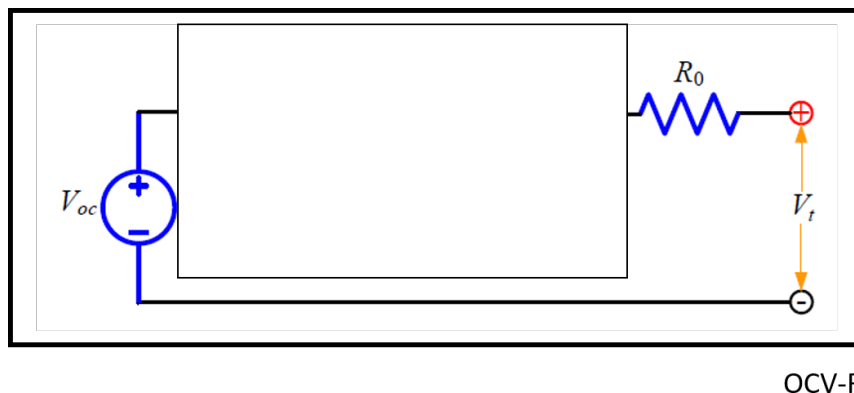
Continuous and DP formulation of three ECT models

The purpose of this document is to elucidate the governing electrochemical-thermal equations used in the equivalent circuit model (ECM) for both continuous and dynamic programming (DP) frameworks. The current I is the controllable input. SOC, terminal voltage (consisting of two state variables V_1, V_2), and temperature (core and surface) are state variables. SOC is the current focus of implementation. The optimization function used is the max charge problem. Parameter values are taken from a SOC value of 0.5, and temperature value of 25 °C. We first introduce three equivalent circuit models that are representative of how they will be sequentially implemented.

Introducing the three models

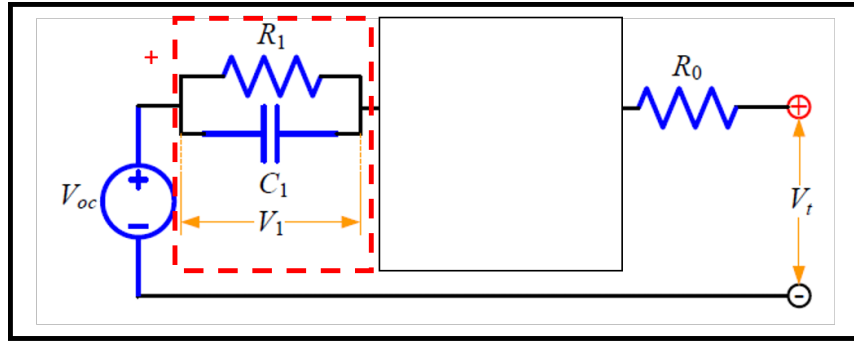
Model 1

Model 1 is OCV-R, as described below. Here we make the assumptions that the terminal voltage is dependent on the open circuit voltage V_{oc} and the drop across resistor R_0 . V_{oc} is a function of the state of charge (SOC), which is dynamic with the current.



Model 2

Model 2 is OCV-R-RC, as described below. Now a RC couple is included in the equivalent circuit model, so there are voltage dynamics for V_1 . The state dynamics are first order and are thus solved.

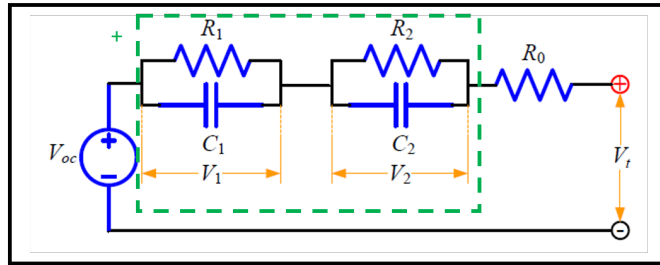
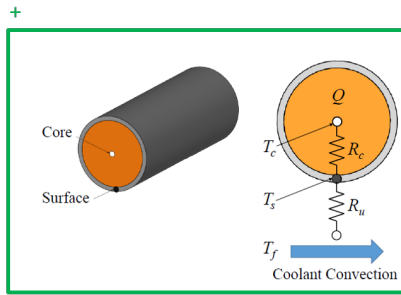


OCV-R

OCV-R-RC

Model 3

Model 3 is OCV-R-RC-RC with thermal input, as described below. Another RC couple is included and now we track the voltage dynamics for V_2 . The diagram below includes the thermal model diagram, which approximates an internal core resistance R_c and an external resistance R_u . The state dynamics are a bit more complex so model is a true benchmark for the DP performance when compared with the computation time for the continuous solution.



OCV-R

OCV-R-RC-RC-Thermal

Continuous time approach

The continuous time approach involves solving the following optimization problem without discretized intervals of time, i.e.,:

$$\max_{I_{valid}} \int_{t_o}^{t_{max}} I dt \quad (1)$$

We present the governing constraints, equations, and initial/terminal conditions for each model below.

Model 1

Equations

Let SOC hereafter be represented as z . This OCV model is taken from CE295 HW1 (Spring 2018):

$$\frac{dz(I, t)}{dt} = \frac{I(t)}{C_{batt}} \quad (2)$$

$$V_t(I, z, t) = V_{oc}(z) + I(t) \cdot R_0 \quad (3)$$

$$V_{oc}(z) = p_0 + p_1 \cdot z_{eq} + p_2 \cdot z_{eq}^2 + p_3 \cdot z_{eq}^3 + (p_1 + 2p_2 \cdot z_{eq} + 3p_3 \cdot z_{eq}^2) (z - z_{eq})^1 \quad (4)$$

Variable	Value	Units
C_{batt}	4320	[C] ^a
R_0	0.01	[Ω] ^b
p_0	3.471	[-] ^c
p_1	1.611	[-]
p_2	-2.629	[-]
p_3	1.7175	[-]
z_{eq}	0.5	[-]

^aEstimated 1200 mAh for battery

^bDetermined at $z_{eq} = 0.5$ and $T_{\infty} = 25^{\circ}C$ from Perez et. al 2015

^cCoefficient values taken from CE295 HW1

¹Note that V_{oc} is calculated by a first order Taylor series expansion about the equilibrium SOC value, z_{eq} .

Constraints

$$I_{\min} \leq I(t) \leq I_{\max}$$

$$z_{\min} \leq z(I, t) \leq z_{\max}$$

$$V_{\min} \leq V_t(I, z, t) \leq V_{\max}$$

Variable	Value	Units
I_{min}	0	[A]
I_{max}	46	[A]
z_{min}	0.25	[-]
z_{max}	0.75	[-]
V_{min}	2	[V]
V_{max}	3.6	[V]

Initial and Final Conditions

Variable	Value	Units
t_0	0	[sec]
t_{max}	1800	[sec]
$z(t_0)$	0.25	[-]
$z(t_{max})$	0.75	[-]

Model 2

We include all the equations in Model 1, with modifications made to the V_t term as described below.

Equations

$$\frac{dV_1(I, t)}{dt} = \frac{-V_1(I, t)}{R_1 \cdot C_1} + \frac{I(t)}{C_1} \quad (5)$$

$$V_t(I, z, t) = V_{oc}(z) + \textcolor{red}{V}_1(\textcolor{red}{I}, \textcolor{red}{t}) + I(t) \cdot R_0 \quad (6)$$

Variable	Value	Units
R_1	0.01	[Ω]
C_1	2500	[F]

Constraints

No change

Initial and Final Conditions

No change

Model 3

We include all the equations in Model 1 and 2, with modifications made to the V_t term as described below. In addition, thermal dynamics are now included.

Equations

$$\frac{dT_c(t)}{dt} = \frac{T_s(t) - T_c(t)}{R_c \cdot C_c} + \frac{I(t)|V_{oc}(z) - V_t(I, z, t)|}{C_c} \quad (7)$$

$$\frac{dT_s(t)}{dt} = \frac{T_\infty - T_s(t)}{R_u \cdot C_s} - \frac{T_s(t) - T_c(t)}{R_c \cdot C_s} \quad (8)$$

$$\frac{dV_2(I, t)}{dt} = \frac{-V_2(I, t)}{R_2 \cdot C_2} + \frac{I(t)}{C_2} \quad (9)$$

$$V_t(I, z, t) = V_{oc}(z) + V_1(I, t) + \textcolor{violet}{V}_2(\textcolor{violet}{I}, t) + I(t) \cdot R_0 \quad (10)$$

Variable	Value	Units
R_c	1.94	[K/W]
C_c	62.7	[J/K]
R_u	3.08	[K/W]
C_s	4.5	[J/K]
R_2	0.02	[Ω]
C_2	5.5	[F]

Constraints

$$T_{\min} \leq T_c(t) \leq T_{\max}$$

Variable	Value	Units
T_{min}	5	[C]
T_{max}	45	[C]

Initial and Final Conditions

Variable	Value	Units
$T_c(0)$	25	[C]
$T_s(0)$	25	[C]

Dynamic programming approach

The dynamic programming (DP) problem involves converting the optimization problem presented earlier in Equation [1](#) into a discrete problem. This can be done using Bellman's Principal of Optimality, i.e.,:

Let V_k represent the charge accumulation from time step k to total time N :

$$V_k(I_k) = \max_{I_k, z_k, V_{t,k}, T_{c,k}} \{ \sum_k I_k + V(k+1) \} \quad \forall k \in \{t_0..N-1\} \quad (11)$$

This conversion allows us to identify the optimal control I_k^* for all k . When introducing the complex thermal dynamics, this approach may be advantageous computationally, but this is yet to be proven. We present the governing constraints, system dynamics, and initial/terminal conditions for each model below. For the parameter values, please reference the "continuous time approach" section. Consider $dt = 1$.

Model 1

Dynamics

$$z_{k+1} = \frac{I_k}{C_{\text{batt}}} \cdot dt + z_k \quad \forall k \in \{t_0..N-1\} \quad (12)$$

Constraints

$$\max \left\{ C_{\text{batt}} \cdot \frac{z_{k+1} - z_{\text{max}}}{dt}, \frac{V_{\text{min}} - V_{oc}}{R_0}, I_{\text{min}} \right\} \leq I_k \leq \min \left\{ C_{\text{batt}} \cdot \frac{z_{k+1} - z_{\text{min}}}{dt}, \frac{V_{\text{max}} - V_{oc}}{R_0}, I_{\text{max}} \right\} \quad (13)$$

Initial and Final Conditions

We add the terminal cost constraint: $V_N = 0$

Model 2

Dynamics

$$V_{1,k+1} = V_{1,k} \left(1 - \frac{dt}{R_1 \cdot C_1} \right) + \frac{I_k \cdot dt}{C_1} \quad (14)$$

Constraints

For simplicity, we define the following quantities:

•

$$C_{\text{batt}} \cdot \frac{z_{k+1} - z_{\text{max}}}{dt} = z_{lb}$$

•

$$C_{\text{batt}} \cdot \frac{z_{k+1} - z_{\text{min}}}{dt} = z_{ub}$$

•

$$\frac{C_1}{C_1 \cdot R_0 + dt} \left\{ V_{\text{min}} - V_{oc} - V_{1,k} \left(1 - \frac{dt}{R_1 \cdot C_1} \right) \right\} = V_{1,lb}$$

•

$$\frac{C_1}{C_1 \cdot R_0 + dt} \left\{ V_{\text{max}} - V_{oc} - V_{1,k} \left(1 - \frac{dt}{R_1 \cdot C_1} \right) \right\} = V_{1,ub}$$

Using this new notation, we arrive at the expression below:

$$\max \{z_{lb}, \textcolor{red}{V}_{1,lb}, I_{\min}\} \leq I_k \leq \min \{z_{ub}, \textcolor{red}{V}_{1,ub}, I_{\max}\} \quad (15)$$

Initial and Final Conditions

No change from continuous model

Model 3

Dynamics

$$V_{2,k+1} = V_{2,k} \left(1 - \frac{dt}{R_2 \cdot C_2}\right) + \frac{I_k \cdot dt}{C_2} \quad (16)$$

$$T_{c,k+1} = \left\{ \frac{T_{s,k} + T_{c,k} \cdot (R_c \cdot C_c - 1) + R_c \cdot I_k (V_{1,k} + V_{2,k} + R_0 \cdot I_k)}{R_c \cdot C_c} \right\} dt \quad (17)$$

$$T_{s,k+1} = \left\{ \frac{R_c \cdot T_{\infty} - T_{s,k} \cdot (R_c + R_u - \frac{R_c R_u C_s}{dt}) + R_u \cdot T_{c,k}}{R_u \cdot R_c \cdot C_s} \right\} dt \quad (18)$$

Constraints

For simplicity, we define the following quantities (in addition to those defined in Model 2):

- $$\frac{C_1 C_2 R_0}{(C_1 + C_2) R_0 dt + C_1 C_2 R_0} \left\{ V_{\min} - V_{oc} - \sum_{i \in [1,2]} V_{i,k} \left(1 - \frac{dt}{R_i \cdot C_i}\right) \right\} = \textcolor{blue}{V}_{t,lb}$$
- $$\frac{C_1 C_2 R_0}{(C_1 + C_2) R_0 dt + C_1 C_2 R_0} \left\{ V_{\max} - V_{oc} - \sum_{i \in [1,2]} V_{i,k} \left(1 - \frac{dt}{R_i \cdot C_i}\right) \right\} = \textcolor{blue}{V}_{t,ub}$$
- $$\frac{R_c C_c \left(\frac{T_{c,\min}}{dt}\right) - T_{s,k} - T_{c,k} (R_c C_c - 1)}{R_c} = \hat{\textcolor{blue}{T}}_{c,lb}$$
- $$\frac{R_c C_c \left(\frac{T_{c,\max}}{dt}\right) - T_{s,k} - T_{c,k} (R_c C_c - 1)}{R_c} = \hat{\textcolor{blue}{T}}_{c,ub}$$
- $$\frac{-(V_{1,k} + V_{2,k}) + \sqrt{(V_{1,k} + V_{2,k})^2 - 4R_0 \hat{\textcolor{blue}{T}}_{c,lb}}}{2R_0} = \textcolor{blue}{T}_{c,lb}$$
- $$\frac{-(V_{1,k} + V_{2,k}) + \sqrt{(V_{1,k} + V_{2,k})^2 - 4R_0 \hat{\textcolor{blue}{T}}_{c,ub}}}{2R_0} = \textcolor{blue}{T}_{c,ub}$$

Using this new notation, we arrive at the expression below:

$$\max \{z_{lb}, \textcolor{green}{V}_{t,lb}, \textcolor{green}{T}_{c,lb}, I_{\min}\} \leq I_k \leq \min \{z_{ub}, \textcolor{green}{V}_{t,ub}, \textcolor{green}{T}_{c,ub}, I_{\max}\} \quad (19)$$

Initial and Final Conditions

No change from continuous model