1 Formulation: Part I

Optimization equation

$$\min \sum_{k \in N} \sum_{i \in I} \Big(\alpha \ln Q_i(k) + \boxed{c(k) P_{0i}(k)} \Big) \tag{1}$$

α represents the cost associated with losing energy to capacity fade. Note that Q_i represents the percent capacity fade for node i at time step k. $c(k)$ represents the cost to generate electricity.
We want to know **whether implementing the boxed power cost term is appropriate** . We want our problem to include a cost for drawing power from the grid in addition to losing charge capacity, but are unsure if this is ambitious.

$$\ln Q_i(k) = \ln(B) - \frac{E_A}{RT_i(k)} + z \ln A_{h,i}(k) \quad \forall k \in 1..N, i \in 1..I \tag{2}$$

Linearized version of the percent capacity loss expression. Note that B, E_A, R, z are all constants based on battery kinetics. We note that T_i- the temperature of battery i- could be a constant but here we present the temperature dynamics as well. The red term represents the amp-hour throughput, which is defined below:

$$A_{h,i} = \underset{k-indexed}{\text{cycle number}} \cdot \underset{\frac{I_i \Delta t}{Q_{\max}}}{\text{DOD}} \cdot \underset{Q_{\max}}{\text{full cell capacity}} \tag{3}$$

Here are uncertain whether the **cycle number and depth-of-discharge are accurately represented**. Namely, the depth-of-discharge could be calculated relative to the current total capacity instead of the initial total capacity. If the expression is valid as written, then $A_{h,i}(k)$ is simply $I_i(k)\Delta t$. Now we describe the battery power dynamics:

$$E_i(k+1) = E_i(k) + p_{b,i}(k)\Delta t \quad \forall k \in 1..N, \forall i \in 1..I \tag{4}$$

$$p_{b,i}(k) = P_{0i}(k) - d_i(k) \quad \forall k \in 1..N, \forall i \in 1..I \tag{5}$$

$$\sum_{i \in I} P_{0i}(k) \leq G(k) \quad \forall k \in 1..N, \forall i \in 1..I \tag{6}$$

Note quickly here that $p_{b,i}(k)$ is the battery power available at node i at time step k, $P_{0i}(k)$ is the power delivered to node i at time step k, and $d_i(k)$ is the demand at node i at time step k. $G(k)$ is the total power available from the grid at time step k. Here we want to know whether **power can be returned** to the grid from the battery nodes.

$$E_{\min} \leq E_i(k) \leq E_{\max} \quad \forall k \in 1..N, i \in 1..I \tag{7}$$

2 Temperature dynamics

$$C\dot{T}_i(k) = h\left(T_i(k) - T_\infty\right) + R_B I_i(k)^2 \quad \forall k \in 1..N, i \in 1..I \tag{8}$$

$$T_i(k)\dot{T}_i(k)\Delta t = T_i(k+1) \quad \forall k \in 1..N, i \in 1..I \tag{9}$$

$$T_{\min} \leq T_i(k) \leq T_{\max} \quad \forall k \in 1..N, i \in 1..I \tag{10}$$

Beyond the general question of whether we want to implement temperature dynamics or not, we wanted to know here the **units of h and C**.

3 Formulation: Part II

$$p_{b,i}(k) = I_i(k)V_i(k) \quad \forall k \in 1..N, i \in 1..I \tag{11}$$

$$V_i(k) = V_{oc,i}(k) - I_i(k)R_B \quad \forall k \in 1..N, i \in 1..I \tag{12}$$

$$V_{oc,i}(k) = f(\mathrm{SOC}_i(k)) \quad \forall k \in 1..N, i \in 1..I \tag{13}$$

$$\mathrm{SOC}_i(k) = \frac{E_i(k)}{Q_{\max} V_{oc,i}(k)} \quad \forall k \in 1..N, i \in 1..I \tag{14}$$

Hopefully for these sets of equations the variables are self-illuminating. The main question here is from equation 14- $\boxed{\textbf{is the } \boldsymbol{Q} \textbf{ term } Q_{\mathbf{max}} \textbf{ or } Q_{\mathbf{cap,\ i}}}$, as defined below?

$$Q_{\text{cap, i}}(k) = Q_{\max}\left(1 - Q_i(k)\right) \quad \forall k \in 1..N, i \in 1..I \tag{15}$$

4 DP Formulation

Let $V(k)$ represent the cumulative capacity fade and power generation from time step k to total time N. We define control variables $I_i(k)$ as u_k $\forall i$ and state variables $T_i(k)$ as x_k $\forall i$:

$$V_k(x_k) = \min_{u_k, x_k} \left\{ \sum_{i \in I} \left(\alpha \underbrace{\ln(\mathrm{Q(k)})}_{f(u_k, x_k)} + c(k)P_{0i}(k) \right) + V(k+1) \right\} \quad \forall k \in 1..N \tag{16}$$

We finally establish the boundary condition:

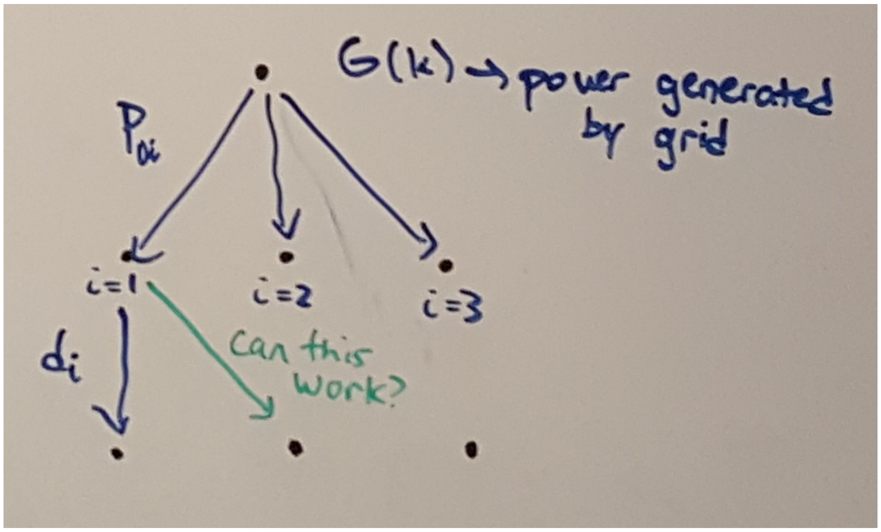
$$V(N+1) = 0$$

The only question here is whether $\boxed{T_i \textbf{ or } \mathbf{SOC}_i}$ represent the state variables.

5 Detailed questions

In summary, here are a detailed list of our questions:

- Is implementing the power cost term appropriate? Will it complicate our solution to try to minimize the overall cost to draw power from the grid in addition to minimizing cumulative aging?
- What are the units of α, h, and C?
- Is $Q_{\max}$ or $Q_{\text{cap},i}$ in the denominator for the depth of discharge expression? What about for the state of charge?
- Is P_{0i} strictly non-negative? (i.e. is power not returned to grid?)
- Will T_i or SOC_i be our state variable?
- Where can we get physical data for our constants and input data?
- What additional research can be done from literature? How can we boost that portion of our report?
- Is demand at each node i only met by the node it is connected to? Are there other configurations we can explore? Please reference the picture below.



We appreciate your time to read through this!