Distributed Charging Control of Electric Vehicles Using Online Learning

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***Abstract*—We propose an algorithm for distributed charging control of electric vehicles (EVs) using online learning and online convex optimization. Many distributed charging control algorithms in the literature implicitly assume fast two-way communication between the distribution company and EV customers. This assumption is impractical at present and also raises security and privacy concerns. Our algorithm does not use this assumption; however, at the expense of slower convergence to the optimal solution and by relaxing the sense of optimality. The proposed algorithm requires one-way communication, which is implemented through the distribution company publishing the pricing profiles for the previous days. We provide convergence results for the algorithm and illustrate the results through numerical examples.**

***Index Terms*—Charging control, demand response, online learning, regret minimization.**

I. Introduction

Demand response (DR) (for space constraints, we refer the reader to [4], [7], [11], [22], [25] and the references therein for a general introduction) provides a new mechanism to accomplish the existing functionalities for the power systems. This technical note focuses on the flexible load capability offered by electric vehicles (EVs) owned by residential customers. Large-scale integration of EVs may impose a significant burden on the grid, leading to effects such as creation of new peaks, peak load amplification [15] and voltage deviations [18]. To cope with these issues, many algorithms have been proposed to schedule the charging of EVs, e.g., [5], [8], [20], [21], [27].

We are particularly interested in the algorithms such as those proposed in [9], [10], [19], [20], which lead to the convergence of the total load profile to a desired one (for instance, a valley-filling profile) through appropriate price signals transmitted to the customers. Many of these existing algorithms have analytical convergence guarantees and do not require the customers to share their charging constraints with the distribution company. On the other hand, they require a series of messages to be exchanged among the distribution company and the customers regarding possible price profiles and desired charging profiles in response. As the available power supply and the customer requirements for charging their EVs change from day to day, these messages need to be exchanged daily to calculate the charging profiles. Since these message exchanges need to be completed before the EVs can begin charging, the algorithms, thus, implicitly assume the presence of a communication infrastructure and protocols that can support low-latency two-way communication between the distribution company and the EV customers. Such infrastructure and protocols have not yet been deployed extensively [2]. Furthermore, the transmitted data carry information about the constraints faced by the individual customer, which raises privacy and security concerns. Motivated by these issues, we propose an online learning and online convex optimization based distributed charging control algorithm. This algorithm requires only one-way communication from the distribution company to the customers. Furthermore, the communication carries information about the pricing profiles of *previous* days.

In our formulation, we model the distribution company and every EV customer as an individual decision maker who wishes to optimize his own utility function. For the distribution company, the payoff is maximized if the total load profile over a day is valley-filling [9], [10]. For the EV customer, the utility function is maximized if the cost to charge the EV over a day is minimized. By designing a suitable pricing policy, the distribution company aims at ensuring that the aggregate charging profile adopted by the customers is valley-filling.

Our distributed charging control algorithm is based on an online learning and online convex optimization framework. The online learning framework has now assumed tremendous popularity in the online convex optimization and machine learning communities (see e.g., [6], [13], [24], [28] and the references therein). We use a regret minimization algorithm [23] in the online learning framework. The regret minimization algorithm uses *regret* as the performance measure and provides an iterative way for every decision maker to update its policy such that, at convergence, the policy is optimal in a suitably defined sense.

Some relevant references that apply regret minimization to DR are [14], [16], [26]. In [16], real-time electricity pricing strategies for DR are designed using regret minimization. However, the focus of that work is on optimizing the utility function for the distribution company, and the customer behavior is assumed to be such that the load change is linear in the price variation. The objective of [14] is to design pricing policies for customers with price responsive loads. The exact demand function of the customer is assumed to be unknown to the pricing policy maker. In [14], the distribution company is the only decision maker, whereas in our work, the distribution company and all the individual EV customers are decision makers. In [26], regret minimization is used to learn the charging behavior of the EV customers. The price responsiveness for a community of customers is captured through a conditional random field model. The regret minimization algorithm is adopted to learn the parameters of the model.

Our contributions are two-fold. First, we present a regret minimization based distributed algorithm for charging control of EVs that requires only one-way communication. Second, we allow heterogeneous utility functions for the various EV owners, as would be the case when customers vary in the elasticity of shifting their loads in response to prices.

The remainder of the technical note is organized as follows. Section II presents the problem formulation. The basic framework and

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main results are presented in Section III. Section IV extends the basic framework to consider inelastic customers. Some numerical examples can be found in Section V. Section VI concludes the technical note.

II. Problem Formulation

We consider a scenario in which N customers schedule the charging of their electric vehicles (EVs) daily. The charging needs to be completed over a day. Let there be T time slots in a day and denote the set of these time slots by $\mathcal{T} := \{1, \ldots, T\}$. Denote the set of EVs by $\mathcal{N} := \{1, \ldots, N\}$. Denote the base load on day k by $D^k(t) \in \mathbb{R},\ t \in \mathcal{T}$. We assume that this base load is unknown to the EV customers and to the distribution company at the beginning of the k-th day when the charging schedules are fixed. Furthermore, the base load may vary from day to day. Denote by $x_i^k(t) \in \mathbb{R}$ the charging rate of the i-th EV in the t-th time slot on the k-th day. The charging load profile of the i-th EV on the k-th day is denoted by a vector $x_i^k := \big(x_i^k(1), x_i^k(2), \ldots, x_i^k(T)\big)$. The aggregate charging profile of the EV customers is described by a vector $x^k := (x_1^k, \ldots, x_N^k)$. Let $x_i^{\text{low}}(t)$ and $x_i^{\text{up}}(t)$ denote the minimum and maximum charging rates possible for the i-th EV in the t-th time slot and let S_i denote the desired total charge for the i-th EV at the end of the k-th day; thus $S_i = \sum_{t \in \mathcal{T}} x_i^k(t)$, for every k. The total load as seen by the distribution company is the sum of the base load and the charging rates adopted by the EVs.

The objective of the distribution company is to achieve a total load profile that is valley-filling while ensuring that both the inflexible base load and the schedulable EVs are supplied with the required amount of energy. Thus, it wishes to obtain the aggregated charging profile x^k, for every $k \in \mathbb{N}_{>0}$, that solves

$$\begin{aligned} &\underset{x^k}{\text{minimize}} \ \ c_u^k(x^k) \\ &\text{subject to} \ \ x_i^{\text{low}}(t) \le x_i^k(t) \le x_i^{\text{up}}(t),\ t \in \mathcal{T}, i \in \mathcal{N}, \\ &\qquad\qquad \sum_{t \in \mathcal{T}} x_i^k(t) = S_i,\ i \in \mathcal{N}, \end{aligned} \tag{1}$$

where, following [9], [10], the distribution company cost function c_u^k is chosen as

$$c_u^k(x^k) := \sum_{t \in \mathcal{T}} \Big(D^k(t) + \sum_{i \in \mathcal{N}} x_i^k(t)\Big)^2. \tag{2}$$

To incentivize the customers to choose charging profiles that in aggregate minimize the cost (2), the distribution company designs suitable pricing profiles for the energy being supplied to the EVs. Every EV customer fixes the charging schedule at the beginning of the day based on the information about its own constraints and any information provided by the distribution company. A *price-sensitive* EV customer seeks to minimize the total cost of charging by suitably shaping its charging schedule. Thus, the optimization problem for each such customer $i,\ i \in \mathcal{N}$ is to design a charging profile $x_i^k,\ k \in \mathbb{N}_{>0}$ that solves

$$\begin{aligned} &\underset{x_i^k}{\text{minimize}} \ \ c_i^k(x_i^k, x_{j \ne i}^k) \\ &\text{subject to} \ \ x_i^{\text{low}}(t) \le x_i^k(t) \le x_i^{\text{up}}(t),\ t \in \mathcal{T}, \\ &\qquad\qquad \sum_{t \in \mathcal{T}} x_i^k(t) = S_i, \end{aligned} \tag{3}$$

where $c_i^k(x_i^k, x_{j \ne i}^k)$ is a convex function in x_i^k and $x_{j \ne i}^k := x_j^k$ for $j \in \mathcal{N},\ j \ne i$. For simplicity of notation, we will often suppress the dependency of c_i^k on $x_{j \ne i}^k$ and write it as $c_i^k(x_i^k)$. Since the charging cost is possibly a function of the base load D^k and other customer's charging profiles, $c_i^k(x_i^k)$ inherits these features as well. For $i \in \mathcal{N}$, define the set of feasible charging profiles as

$$\mathcal{F}_i := \Big\{ x_i^k \in \mathbb{R}^T \mid x_i^{\text{low}}(t) \le x_i^k(t) \le x_i^{\text{up}}(t), \ t \in \mathcal{T}, \sum_{t \in \mathcal{T}} x_i^k(t) = S_i \Big\}. \tag{4}$$

The information flow is as follows. The distribution company monitors the total load and publishes the price profile for the previous day as realized according to a fixed and known pricing policy. The customers decide on the charging schedules for the next day with access to these pricing profiles for the previous days. No other communication occurs between the distribution company and the customers, or among the customers.

III. Online Learning Framework

We now adopt a regret minimization framework to solve both problem (1) and problem (3). Let L be a λ-strongly convex function with respect to a given norm $\|\cdot\|$. Let $D_L(\cdot,\cdot)$ denote the Bregman divergence[1] [23] with respect to L. Let $\|\cdot\|_*$ denote the norm that is dual to $\|\cdot\|$. Let ∇L denote the gradient of L and ∇L^{-1} denote the inverse mapping of ∇L.

Customer perspective: In the regret minimization framework, the notion of regret is used to measure the performance of an online algorithm [24], [28]. For customer $i \in \mathcal{N}$, the customer regret R_i after K days is defined as

$$R_i(K, x_i^k) := \sum_{k=1}^{K} c_i^k(x_i^k) - \min_{x_i \in \mathcal{F}_i} \sum_{k=1}^{K} c_i^k(x_i). \tag{5}$$

We define x_i^* as

$$x_i^* := \arg\min_{x_i \in \mathcal{F}_i} \sum_{k=1}^{K} c_i^k(x_i). \tag{6}$$

Note that the solution x_i^* in (6) may not be unique. Since a price-sensitive EV customer only seeks to minimize the total cost of charging, it does not matter which minimizing argument x_i^* the customer selects.

In words, the regret R_i measures the difference between the performance of the charging profile generated by an online algorithm and the performance that is obtained by the suboptimal (because it is day-to-day invariant) charging profile x_i^*.

We adopt the optimistic mirror descent (OMD) algorithm [23] to generate the charging profile update which minimizes the regret (5). On each day, the regret minimization algorithm generates the charging profile update without knowing the current objective function (and its gradient). Specifically, the OMD algorithm iteratively applies the updates as follows. Initialize $x_i^1 = h_i^1 = \text{argmin}_h\ L_i(h)$,

$$\begin{aligned} h_i^{k+1} &= \underset{h_i \in \mathcal{F}_i}{\text{argmin}}\ \eta_i h_i^\top \nabla c_i^k(x_i^k) + D_{L_i}(h_i, h_i^k), \\ x_i^{k+1} &= \underset{x_i \in \mathcal{F}_i}{\text{argmin}}\ \eta_i x_i^\top M_i^{k+1} + D_{L_i}(x_i, h_i^{k+1}), \end{aligned} \tag{7}$$

where $\eta_i \in \mathbb{R}_{>0}$ is an algorithm parameter that specifies the learning rate of the algorithm, h_i^k is an intermediate update of the charging profile, and M_i^{k+1} is the prediction of the gradient of the cost function c_i^k. In this technical note, for a vector $h_i^k \in \mathbb{R}^T$, $L_i(h_i^k)$ is set to $L_i(h_i^k) = \frac{\|h_i^k\|^2}{2}$. Intuitively, the iteration (7) updates the charging

[1]Given a strictly convex function L, the Bregman divergence is defined by $D_L(x,h) := L(x) - L(h) - \nabla L(h)^\top (x - h)$.

profile in the negative gradient direction and projects it onto the set of feasible charging profiles. Note that on day $k+1$, customers use the gradient of the cost function $\nabla c_i^k(x_i^k)$ as realized on the previous day k to perform the charging profile updates.

Company perspective: The distribution company incentivizes the customers to choose charging profiles that lead to desired aggregate charging profile x^k on the k-th day. The set of the aggregated feasible charging profiles is denoted by $\mathcal{F} := \mathcal{F}_1 \times \mathcal{F}_2 \times \ldots \times \mathcal{F}_N$. The distribution company's regret after K days is given by

$$R_u(K, x^k) := \sum_{k=1}^{K} c_u^k(x^k) - \min_{x\in\mathcal{F}} \sum_{k=1}^{K} c_u^k(x). \tag{8}$$

We define x^* as

$$x^* := \operatorname*{argmin}_{x\in\mathcal{F}} \sum_{k=1}^{K} c_u^k(x). \tag{9}$$

The OMD algorithm generates the charging profile update which minimizes the regret (8) as follows. Initialize $x^1 = h_u^1 = \operatorname{argmin}_h L_u(h)$, and then calculate

$$\begin{aligned} h_u^{k+1} &= \operatorname*{argmin}_{h_u\in\mathcal{F}} \eta_u h_u^\top \nabla c_u^k(x_u^k) + D_{L_u}(h_u, h_u^k), \\ x^{k+1} &= \operatorname*{argmin}_{x\in\mathcal{F}} \eta_u x^\top M_u^{k+1} + D_{L_u}(x, h_u^{k+1}), \end{aligned} \tag{10}$$

where $\eta_u \in \mathbb{R}_{>0}$ is an algorithm parameter that denotes the learning rate for the company, h_u^k is an intermediate update of the aggregated charging profile, and M_u^{k+1} is the prediction of the gradient of the cost function c_u^k. Once again, for a vector $h_u^k \in \mathbb{R}^{NT}$, $L_u(h_u^k)$ is set to $L_u(h_u^k) = \frac{\|h_u^k\|^2}{2}$.

A. Convergence Results

The following results summarize the convergence of the charging profile updates generated by the OMD algorithm. The proof of Proposition III.1 follows from that of [23, Lemma 3]. All other proofs can be found in the Appendix.

Proposition III.1. (Convergence of Regret): For every $x_i^* \in \mathcal{F}_i$, the iteration (7) converges in the sense that

$$R_i(K, x_i^k) \le \frac{1}{\eta_i} P_i + \frac{\eta_i}{2} \sum_{k=1}^{K} \|\nabla c_i^k(x_i^k) - M_i^k\|_*^2, \tag{11}$$

where

$$\begin{aligned} R_i(K, x_i^k) &:= \sum_{k=1}^{K} c_i^k(x_i^k) - \sum_{k=1}^{K} c_i^k(x_i^*), \\ P_i &:= \max_{x_i\in\mathcal{F}_i} L_i(x_i) - \min_{x_i\in\mathcal{F}_i} L_i(x_i). \end{aligned} \tag{12}$$

In particular, if η_i is chosen as $O(1/\sqrt{K})$, then the average regret, i.e., $R_i(K)/K$, converges to zero as $K \to \infty$.

Similarly, for every $x^* \in \mathcal{F}$, the iteration (10) converges in the sense that

$$R_u(K, x^k) \le \frac{1}{\eta_u} P_u + \frac{\eta_u}{2} \sum_{k=1}^{K} \|\nabla c_u^k(x^k) - M_u^k\|_*^2, \tag{13}$$

where

$$\begin{aligned} R_u(K, x^k) &:= \sum_{k=1}^{K} c_u^k(x^k) - \sum_{k=1}^{K} c_u^k(x^*), \\ P_u &:= \max_{x\in\mathcal{F}} L_u(x) - \min_{x\in\mathcal{F}} L_u(x). \end{aligned} \tag{14}$$

In particular, if η_u is chosen as $O(1/\sqrt{K})$, then the average regret, i.e., $R_u(K)/K$, converges to zero as $K \to \infty$.

The results (11) and (13) guarantee that, as the number of days increases, the average performance of the charging profiles generated by the OMD algorithm approaches the performance that is obtained by the optimal day-to-day invariant charging profiles x^* and $x_i^*,\ i \in \mathcal{N}$, respectively. In Section IV-A, we compare the performance of the charging profile generated by the OMD algorithm with the performance that is obtained by the solutions of problems (1) and (3). However, the convergence guarantees obtained in Section IV-A are weaker.

B. Design of the Pricing Function

There are no guarantees that the solutions $x_i^*,\ i \in \mathcal{N}$ that minimize $\sum_{k=1}^{K} c_i^k(x_i)$ will also minimize $\sum_{k=1}^{K} c_u^k(x)$. In fact, after some algebraic manipulation of the updates (7) and (10), we observe that the choice of c_i^k as

$$c_i^k(x_i^k) = \left(\sum_{j=1}^{N} x_j^k + D^k\right)^\top x_i^k \tag{15}$$

does not lead to the charging profiles $(x_1^*, \ldots, x_N^*)$ that reduce the regret of the distribution company to zero. We now propose a choice of c_i^k to ensure that when each customer minimizes its regret, the aggregated charging profile that is realized minimizes the distribution company's regret.

Proposition III.2: If c_i^k is chosen as

$$c_i^k(x_i^k) = \left(0.5x_i^k + \sum_{j\neq i}^{N} x_j^k + D^k\right)^\top x_i^k, \quad i \in \mathcal{N}, \tag{16}$$

the customers adopt the iteration (7), $\eta_u = \frac{1}{2}\eta_i$, and $\eta_i = \eta_j$ for all $i, j \in \mathcal{N}$, then the average regret of the distribution company as defined in (8) converges to zero as the total number of days goes to infinity.

Remark III.1: To update the charging profile on day k, the i-th customer needs to know the quantity $2x_i^{k-1} + \sum_{j\neq i} x_j^{k-1} + D^{k-1}$ or $\sum_j x_j^{k-1} + D^{k-1}$ depending on whether the pricing function (15) or (16) is adopted. The distribution company can simply publish the total load information for the previous day to supply this information.

IV. Extensions

The basic framework presented above can be extended in various directions. In the sequel, we assume that the distribution company selects the cost function (16) for each EV customer and sets $\eta_u = \frac{1}{2}\eta_i,\ i \in \mathcal{N}$ (where $\eta_i = \eta_j,\ i, j \in \mathcal{N}$).

A. Regret With Respect to the Optimal Charging Profiles

The regrets defined in (5) and (8) measure the difference between the performance of the charging profiles generated by our algorithm and the performance that is obtained by the charging profiles x^* and $x_i^*,\ i \in \mathcal{N}$ that are the solutions of the related optimization problems (6) and (9), respectively. We can instead consider the original optimization problems (1) and (3) to define *tracking regret* after K days as

$$R_u^{\text{tracking}}(K, x^k) := \sum_{k=1}^{K} c_u^k(x^k) - \min_{x^k\in\mathcal{F},\forall k\in\mathcal{K}} \sum_{k=1}^{K} c_u^k(x^k), \tag{17}$$

where $\mathcal{K} := \{1, \ldots, K\}$. We define the set $\{x^{k*},\ k \in \mathcal{K}\}$ as

$$\left\{x^{k*} \in \mathbb{R}^{NT},\ k \in \mathcal{K} \mid x^{k*} = \arg\min_{x^k\in\mathcal{F},\forall k\in\mathcal{K}} \sum_{k=1}^{K} c_u^k(x^k)\right\}. \tag{18}$$

This notion of tracking regret characterizes the difference between the cumulative cost of the charging profiles generated by our algorithm and the cumulative cost of executing the optimal charging profiles that can be calculated only in hindsight. For comparison, the regrets (5) and (8) can be seen as *static regrets*.

Theorem IV.1: For every $x^{k*} \in \mathcal{F}$, the OMD algorithm yields that,

$$\begin{aligned} R_u^{\text{tracking}}(K, x^k) &\leq \frac{1}{\eta_u}\left[L_u(h_u^{K+1}) - L_u(h_u^1)\right] \\ &+ \frac{1}{\eta_u}\left[\nabla L_u(h_u^{K+1})^\top (x^{K+1*} - h_u^{K+1}) - \nabla L_u(h_u^1)^\top (x^{1*} - h_u^1)\right] \\ &+ \frac{1}{\eta_u}\max_{k\in\mathcal{K}} \|\nabla L_u(h_u^k)\| \sum_{k=1}^{K} \|x^{k*} - x^{k+1*}\| \\ &+ \frac{\eta_u}{2}\sum_{k=1}^{K} \|\nabla c_u^k(x^k) - M_u^k\|_*^2. \end{aligned} \tag{19}$$

In particular, if $\eta_u = O(1/\sqrt{K})$, then the tracking regret is order $O\big(\sqrt{K}[1 + \sum_{k=1}^{K} \|x^{k*} - x^{k+1*}\|]\big)$.

A comparison of the regret bounds (13) and (19) is of interest. If $\eta_u = O(\sqrt{K})$, in (13), the first term $\frac{1}{\eta_u}P_u$ and the second term $\frac{\eta_u}{2}\sum_{k=1}^{K} \|\nabla c_u^k(x^k) - M_u^k\|_*^2$ are order $O(\sqrt{K})$. In (19), the first two terms measure the difference between the initial iterate h_u^1 and the final iterate h_u^{K+1}, the difference between the final iterate h_u^{K+1} and the optimal solution x^{K+1*}, and the difference between the iterate h_u^1 and the optimal solution x^{1*}. The first two terms are again order $O(\sqrt{K})$. The last term in (19), $\frac{\eta_u}{2}\sum_{k=1}^{K} \|\nabla c_u^k(x^k) - M_u^k\|_*^2$, also appears in (13) and is also order $O(\sqrt{K})$. The third term increases as K increases and is order $O\big(\sqrt{K}\sum_{k=1}^{K} \|x^{k*} - x^{k+1*}\|\big)$. Because of the presence of the third term, the regret bound in (19) increases as a function of how much the optimal decision x_i^{k*} varies from one day to the next. In particular, if the optimal solution remains the same from one day to the next, then $\sum_{k=1}^{K} \|x^{k*} - x^{k+1*}\| = 0$ and the tracking regret R_u^{tracking} is the order $O(\sqrt{K})$. On the other hand, if the optimal solution varies significantly from one day to the next, then the tracking regret R_u^{tracking} will be order $O\big(\sqrt{K}[1 + \sum_{k=1}^{K} \|x^{k*} - x^{k+1*}\|]\big)$, and the average tracking regret will not necessarily converge to zero.

If the distribution company has a perfect prediction of the gradient of the cost function, i.e., $\nabla c_u^k(x^k) = M_u^k$ for all k, then the last term in (19) vanishes and the distribution company can set $\eta \to \infty$ to ensure that the regret bound is zero, i.e., $R_u^{\text{tracking}} \leq 0$. In this case, the cumulative cost function value of the charging profiles $x^k,\ k = 1, \ldots, K$ generated by the online algorithm (10) and the one generated by the elements in the set $\{x^{k*},\ k \in \mathcal{K}\}$ are identical. Note that the elements in the set $\{x^{k*},\ k \in \mathcal{K}\}$ solve problem (1) exactly.

B. Presence of Inelastic Customers

The discussion so far assumed that all customers were rational in the sense that they wanted to choose their charging profile to solve (3). Furthermore, they were elastic in scheduling their charging (within the constraints pre-specified by $x_i^{\text{low}}(t)$, $x_i^{\text{up}}(t)$, and S_i). We now assume that some customers are either irrational or inelastic and they do not optimize their schedules to solve (3). Suppose that N_l out of N customers are inelastic. Denote the set of inelastic customers by $\mathcal{N}_l$. For every inelastic customer $i \in \mathcal{N}_l$, we assume that its charging profile remains the same from day to day and is not updated to minimize (3). Equivalently, for inelastic customers, the cost function c_i^k can be selected as $c_i^k \equiv r$, for all k, where r is an arbitrary constant.

Since the inelastic customers do not carry out any predictions, we set all customers' predictions to zeros, i.e., $M_i^k = 0,\ i \in \mathcal{N},\ k \in \mathbb{N}_{>0}$. The update of the charging profile for inelastic customer $i \in \mathcal{N}_l$ can thus be written as follows. Initialize $x_i^1 = h_i^1 = \text{argmin}_h\ L_i(h)$, and then calculate

$$\begin{aligned} h_i^{k+1} &= \underset{h_i\in\mathcal{F}_i}{\text{argmin}}\ D_{L_i}(h_i, h_i^k), \\ x_i^{k+1} &= \underset{x_i\in\mathcal{F}_i}{\text{argmin}}\ D_{L_i}(x_i, h_i^{k+1}), \end{aligned} \tag{20}$$

where ϵ_i^k is an error term. For instance, for the cost function (16), the error ϵ_i^k is equal to the total load $\sum_j x_j^k + D^k$ on the k-th day. This error term quantifies the inconsistency between the updates as desired by the distribution company for each customer to execute and the updates actually carried out by the inelastic customer. Denote by ϵ^k the vector of all such terms, i.e., $\epsilon^k := (\epsilon_1^k, \ldots, \epsilon_N^k)$.

Due to the presence of the inelastic customers, the ability of the aggregated solution to be valley-filling and hence to minimize the cost function in problem (1) is decreased. The performance loss is given in the following result.

Theorem IV.2: Assume that there are N EV customers out of which N_l are inelastic customers. If $\eta_u = \frac{1}{2}\eta_i,\ i \in \mathcal{N}$ and $\eta_i = \eta_j,\ i, j \in \mathcal{N}$, then for every $x^* \in \mathcal{F}$, the regret of the distribution company is bounded as

$$\begin{aligned} R_u(K, x^k) &\leq \frac{1}{\eta_u}P_u + \frac{\eta_u}{2}\sum_{k=1}^{K} \left\|\left(\nabla c_u^k(x^k) + 2\epsilon^k\right)\right\|_*^2 \\ &\quad + 2K\sum_{i\in\mathcal{N}_l} \|F_i\|\|\epsilon_i\|, \end{aligned} \tag{21}$$

where

$$\begin{aligned} P_u &:= \max_{x\in\mathcal{F}} L_u(x) - \min_{x\in\mathcal{F}} L_u(x), \\ \|F_i\| &:= \max_{x,y\in\mathcal{F}_i} \|x - y\|, \quad \|\epsilon_i\| := \max_k \|\epsilon_i^k\|,\ i \in \mathcal{N}_l. \end{aligned} \tag{22}$$

In (21), the average regret converges to a constant, i.e., $\lim_{K\to\infty} R_u(K)/K = \sum_{i\in\mathcal{N}_l} \|F_i\|\|\epsilon_i\|$. The size of this constant depends on the error terms $\epsilon_i,\ i \in \mathcal{N}_l$ and the charging constraints of the inelastic customers. For inelastic customer $i \in \mathcal{N}_l$, the error term $\|\epsilon_i\|$ can be bounded as

$$\begin{aligned} \|\epsilon_i\| &= \max_k \left\|\left(\sum_{i\in\mathcal{N}} x_i^k + D^k\right)\right\| \leq \max_k \left(\sum_{i\in\mathcal{N}} \|x_i^k\| + \|D^k\|\right) \\ &\leq \sum_{i\in\mathcal{N}} \|x_i^{\text{up}}\| + \max_k \|D^k\|. \end{aligned} \tag{23}$$

Assume that the Euclidean ball B is the smallest Euclidean ball containing the set of feasible charging schedules $\mathcal{F}_i$ and r is the radius of the ball B. Then, the term $\|F_i\|$ can be further bounded by $2r$. Since the set $\mathcal{F}_i$ is a polytope, algorithms such as those proposed in [17] can be adopted to compute this bound efficiently.

Notice that if $N_l = 0$, (21) reduces to (13) (without the prediction, i.e., $M_u^k = 0$ for all $k = 1, \ldots, K$). In general, the presence of inelastic customers degrades the solution quality for the distribution company. To improve the ability of the aggregated solution to be valley-filling in the presence of inelastic customers, the distribution company can consider using some loads that can be controlled completely. We now discuss this option.

C. Controllable Customers

On the other end of the spectrum from customers that are completely inelastic are customers that are under the complete control of the distribution company. Such customers adopt the charging profiles assigned by the company. The charging constraints of these customers are also known and controlled by the distribution company. In practice, the distribution company can offer a contract to a subset of customers offering a special price to be such a controllable load. This contract-based direct load control has been implemented by many distribution companies, e.g., [1], [3].

To introduce such directly controlled customers in our formulation, we allow the charging constraints of the controllable customers to be relaxed (thus enlarging the set of feasible charging profiles for the controllable customers). Denote the set of controllable customers by $\mathcal{N}_c$. On day k, for the controllable customer $i \in \mathcal{N}_c$, the set of the relaxed feasible charging profiles $\tilde{\mathcal{F}}_i$ is defined as

$$\tilde{\mathcal{F}}_i := \Bigg\{ \tilde{x}_i^k \in \mathbb{R}^T \mid \tilde{x}_i^{\text{low}}(t) \le \tilde{x}_i^k(t) \le \tilde{x}_i^{\text{up}}(t), \quad t \in \mathcal{T}, \sum_{t\in\mathcal{T}} a_i \tilde{x}_i^k(t) = \tilde{S}_i \Bigg\}, \tag{24}$$

where $a_i = \{0, 1\}$, $\tilde{x}_i^k$ is the feasible charging profile in the set $\tilde{\mathcal{F}}_i$, and $\tilde{x}_i^{\text{low}}$, $\tilde{x}_i^{\text{up}}$, $\tilde{S}_i$, the relaxed minimum charging rate, the relaxed maximum charging rate, and the relaxed total charge, respectively. The aggregated relaxed charging profile of the EV customers is described by a vector $\tilde{x}^k$ which consists of all elements in the set

$$\left\{\tilde{x}_i^k \in \mathbb{R}^T \mid i \in \mathcal{N}_c\right\} \bigcup \left\{x_i^k \in \mathbb{R}^T \mid i \in \mathcal{N}_l,\ i \in \mathcal{N}_p\right\}, \tag{25}$$

where $\mathcal{N}_p$ denotes the set of the price-sensitive customers.

There are different ways to relax the set of feasible charging profiles for the controllable customers. For example, if we select $a_i = 0$ and $\tilde{S}_i = 0$, then the equality constraint is removed from the set of feasible charging profiles for the i-th customer, $i \in \mathcal{N}_c$, namely, the i-th controllable customer removes its daily total charging sum requirement. We can also relax the charging deadline by adjusting $\tilde{x}_i^{\text{low}}(t)$ and $\tilde{x}_i^{\text{up}}(t)$.

By enlarging the set of the feasible charging profiles, for controllable customer $i \in \mathcal{N}_c$, the cumulative cost function value of the iterates that are generated by the iteration (7) is a lower bound for the cumulative cost function value of the iterates that are obtained by using the same iteration (7) without relaxation. The distribution company can use the above mentioned set of the relaxed feasible charging profiles $\tilde{\mathcal{F}}_i$, $i \in \mathcal{N}_c$ and the controllable customers i, $i \in \mathcal{N}_c$ to compensate for the performance loss due to the presence of the inelastic customers.

V. Numerical Examples

Assume that there are 20 price-sensitive customers. A time slot representing an interval of 30 minutes is used. There are $T = 24$ time slots. The starting time is set to 8:00 pm. For simplicity, we consider that all EV customers charge their EVs from the 9th to the 16th time slots. The maximum and minimum charging rates are set to $x_i^{\text{up}}(t) = 2$ kW and $x_i^{\text{low}}(t) = 0$ kW, $i \in \mathcal{N}$, respectively, and the desired sum $S_i = 10$ kW, $i \in \mathcal{N}$. The simulation is carried out for total $K = 200$ days. The base load is assumed to switch between two base load profiles (see Fig. 1). We set the parameters $\eta_i = 0.005/\sqrt{K}$, $i \in \mathcal{N}$. The prediction is done by averaging the gradients of the cost functions for the previous days. We now examine the convergence of the static regret. Fig. 2 shows the trajectories of the regrets with and without prediction (i.e., $M_i^{k+1} = 0$) given the varying base load in Fig. 1. Fig. 2 shows that the average regrets converge to zero and the average regret with prediction converges faster than the one without any prediction. Fig. 3 shows the static regrets with and without prediction. Fig. 3 shows that the regrets are sublinear functions of the number of days. The results in Figs. 2 and 3 indicate that despite the fact that the base load is switching and the distribution company is not aware of this varying behavior of the base load, our algorithm still provides updates of the charging profiles such that the regret converges to zero. This means that in the long term, the average performance of the charging schedules x^k generated by the OMD algorithm approaches the performance obtained by x^*, where x^* solves (9).

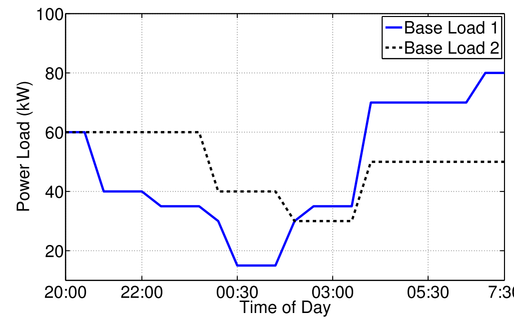

Fig. 1. Two different base load profiles. The actual base load is realized by switching between the two base load profiles from day to day.

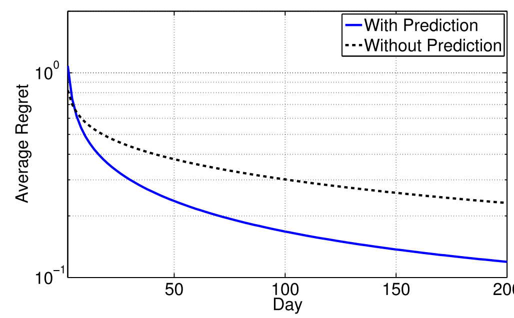

Fig. 2. Average regrets generated by OMD with and without prediction given the varying base load profile in Fig. 1.

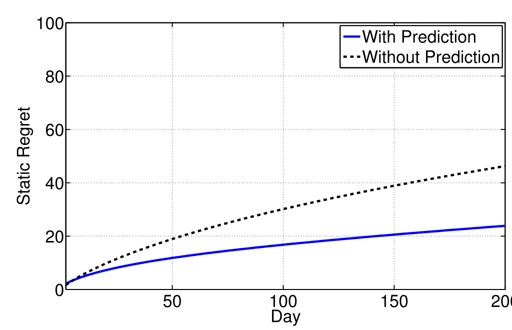

Fig. 3. Static regrets generated by OMD with and without prediction given the varying base load profile in Fig. 1.

VI. Conclusion

We have presented a novel framework for distributed charging control of EVs using online learning and online convex optimization. The proposed algorithm can be implemented without low-latency two-way communication between the distribution company and the EV customers, which fits in with the current communication infrastructure and protocols used in the smart grid.

Appendix A
Proof of Proposition III.2

The update (7) yields

$$h_i^{k+1} = \operatorname*{argmin}_{h_i \in \mathcal{F}_i} \eta_i h_i^\top \left(\sum_i x_i^k + D^k \right) + \frac{\|h_i - h_i^k\|^2}{2}$$
$$x_i^{k+1} = \operatorname*{argmin}_{x_i \in \mathcal{F}_i} \eta_i x_i^\top M_i^{k+1} + \frac{\|x_i - h_i^{k+1}\|^2}{2}. \quad (26)$$

The update (10) yields

$$h_u^{k+1} = \operatorname*{argmin}_{h_u \in \mathcal{F}_u} \eta_u h_u^\top G_u + \frac{\|h_u - h_u^k\|^2}{2}$$
$$x^{k+1} = \operatorname*{argmin}_{x \in \mathcal{F}} \eta_u x^\top M_u^{k+1} + \frac{\|x - h_u^{k+1}\|^2}{2}, \quad (27)$$

where M_u^k is the prediction of

$$G_u := \begin{bmatrix} 2(D^k + \sum_{i\in\mathcal{N}} x_i^k) \\ 2(D^k + \sum_{i\in\mathcal{N}} x_i^k) \\ \vdots \\ 2(D^k + \sum_{i\in\mathcal{N}} x_i^k) \end{bmatrix}_{NT\times 1.} \quad (28)$$

By comparing (27) with (26) and substituting $\eta_u = \frac{1}{2}\eta_i,\ i \in \mathcal{N}$, $\eta_i = \eta_j,\ i,\ j \in \mathcal{N}$, we have that the updates (27) and (26) are identical. Since the updates coincide, the average regret of the distribution company converges to zero as $k \to \infty$.

Appendix B
Proof of Theorem IV.1

The proof technique that we use to derive the tracking regret bound in (19) is similar to the one used in [12, Theorem 4]. The main step is to bound the difference $c_u^k(x^k) - c_u^k(x^{k*})$ instead of the difference $c_u^k(x^k) - c_u^k(x^*)$ that is considered in the static regret (13). However, in [12], the authors derive the tracking regret bounds for a different regret minimization algorithm rather than OMD.

Following the proof of [23, Lemma 2], we have

$$c_u^k(x^k) - c_u^k(x^{k*}) \le (x^k - x^{k*})^\top \nabla c_u^k(x^k), \quad (29)$$

and

$$(x^k - x^{k*})^T \nabla c_u^k(x^k) \le \frac{\eta_u}{2} \|\nabla c_u^k(x^k) - M_u^k\|_*^2$$
$$+ \frac{1}{\eta_u}\big(D_{L_u}(x^{k*}, h_u^k) - D_{L_u}(x^{k*}, h_u^{k+1})\big). \quad (30)$$

Furthermore,

$$\begin{aligned} &D_{L_u}(x^{k*}, h_u^k) - D_{L_u}(x^{k*}, h_u^{k+1}) \\ &= L_u(x^{k*}) - L_u(h_u^k) - \nabla L_u(h_u^k)^\top (x^{k*} - h_u^k) \\ &- L_u(x^{k*}) + L_u(h_u^{k+1}) + \nabla L_u(h_u^{k+1})^\top (x^{k*} - h_u^{k+1}) \\ &= L_u(h_u^{k+1}) - L_u(h_u^k) + \nabla L_u(h_u^{k+1})^\top (x^{k+1*} - h_u^{k+1}) \\ &- \nabla L_u(h_u^k)^T (x^{k*} - h_u^k) - \nabla L_u(h_u^{k+1})^\top (x^{k+1*} - x^{k*}). \quad (31) \end{aligned}$$

The remainder of the proof is followed by summing over $k = 1, \ldots, K$ and collecting terms.

Appendix C
Proof of Theorem IV.2

Following the proof of [23, Lemma 2], we have

$$c_u^k(x^k) - c_u^k(x^*) \le (x^k - x^*)^\top \nabla c_u^k(x^k), \quad (32)$$

and

$$(x^k - x^*)^T \nabla c_u^k(x^k) \le \frac{\eta_u}{2} \|\nabla c_u^k(x^k)\|_*^2$$
$$+ \frac{1}{\eta_u}\big(D_{L_u}(x^*, h_u^k) - D_{L_u}(x^*, h_u^{k+1})\big). \quad (33)$$

For each $i \in \mathcal{N}_l$, the cost function c_i^k is selected as a constant function noted in Section IV-B. The corresponding gradient of the inelastic customer's cost function is zero, namely, for inelastic customer $i \in \mathcal{N}_l$,

$$\nabla c_i^k(x^k) = \left(D^k + \sum_{i\in\mathcal{N}} x_i^k \right) + \epsilon_i^k = 0, \quad (34)$$

where $\epsilon_i^k = -(D^k + \sum_{i\in\mathcal{N}} x_i^k)$. Move $\epsilon_i^k,\ i \in \mathcal{N}_l$ to the right hand side of the inequality in (33). The remainder of the proof is followed by summing over $k = 1, \ldots, K$ and collecting terms.

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