

Optimal Battery Dispatch to minimize battery ageing

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ABSTRACT

Renewable energy systems are intermittent in nature. As renewables proliferate our energy generation, we need more batteries to balance out the system and store energy. One problem with batteries is the capacity decrease overtime. This report looks at solutions to optimize aging for a fleet of batteries with different lifetimes while meeting power demand and other constraints. The problem was simplified into a convex program. We were able to achieve a 33% improvement in cumulative battery capacity over the unoptimized case. In addition to that we were also able to derive control heuristics to extend our findings to any battery fleet and recommendations to consider while designing a battery fleet for deployment.

INTRODUCTION

Motivation and Background

Decreasing battery aging is becoming more important as the percentage of renewable energy sources that require battery storage increases in the grid. A report by Navigant Research suggests that more than $\frac{3}{5}$ th of total electricity in California will come from renewable energy sources by 2030^[9]. California mandate requires 50% of the total electricity to come by renewable energy sources by 2030^[8]. This shows that renewable energy is the future. For effective and efficient incorporation of renewables into the grid, battery storage plays a pivotal role as storage technologies also need to be efficient.

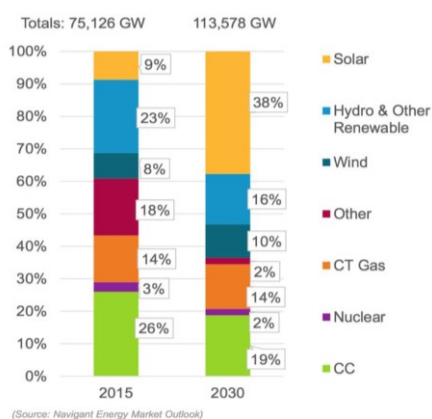


Figure 1: Market Report California, Navigant Research

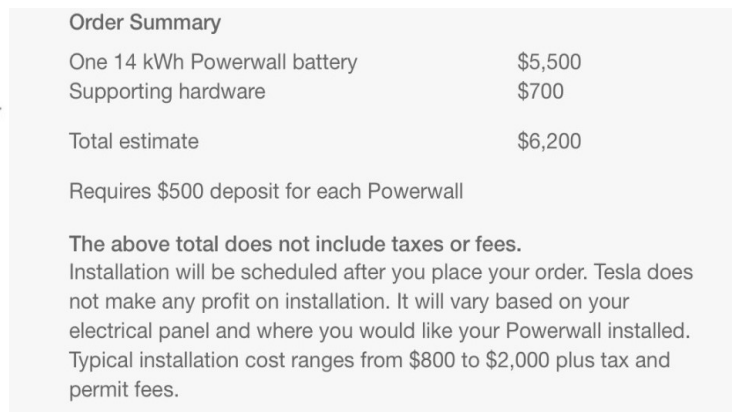


Figure 2: Cost of Tesla Powerwall, Tesla Website

A typical Tesla battery 14kWh powerwall costs \$6200 plus tax^[7]. Therefore, battery costs for new batteries are very high. In order to minimize battery replacement costs, we need to make sure that

batteries have a longer lifetime. This can be done by optimizing the dispatch from each battery depending on its state of health (or stage of life) and the maximum capacity.

For a set of n batteries, some of them are new and some are at the end of their life cycles. There is a capacity loss associated with each battery and it increases with time. Figure 3 represents the capacity loss of a BMW battery^[5]. It is important to optimize dispatch from each battery while satisfying the total power demand at each point. Optimal battery dispatch ensures minimal capacity loss, hence minimal aging of batteries.

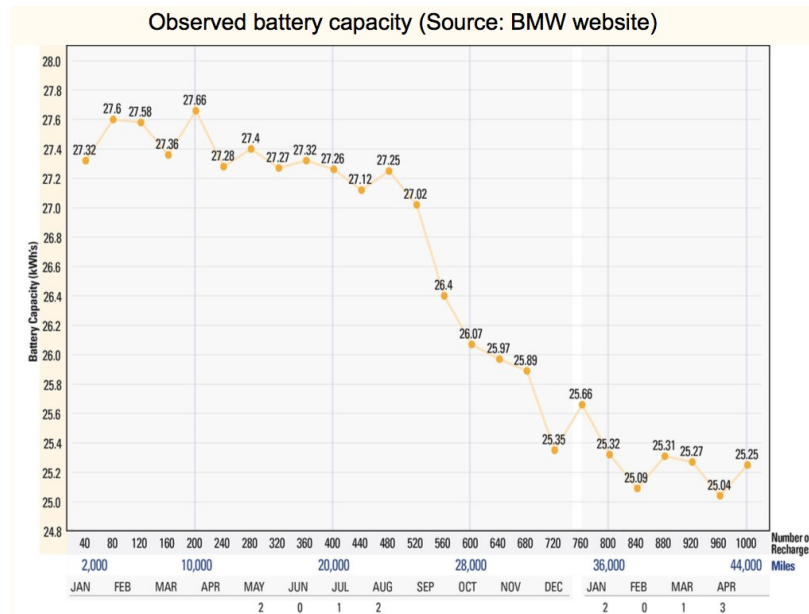


Figure 3: Battery capacity vs number of cycles

Relevant Literature

We began by studying the formulation given to us by eLum^[2]. It helped us to understand the formulation to minimize the capacity loss. We reviewed several papers on battery ageing to further understand the problem. Figure 4^[3] shows the graph of capacity retention vs throughput by Groot *et al.*

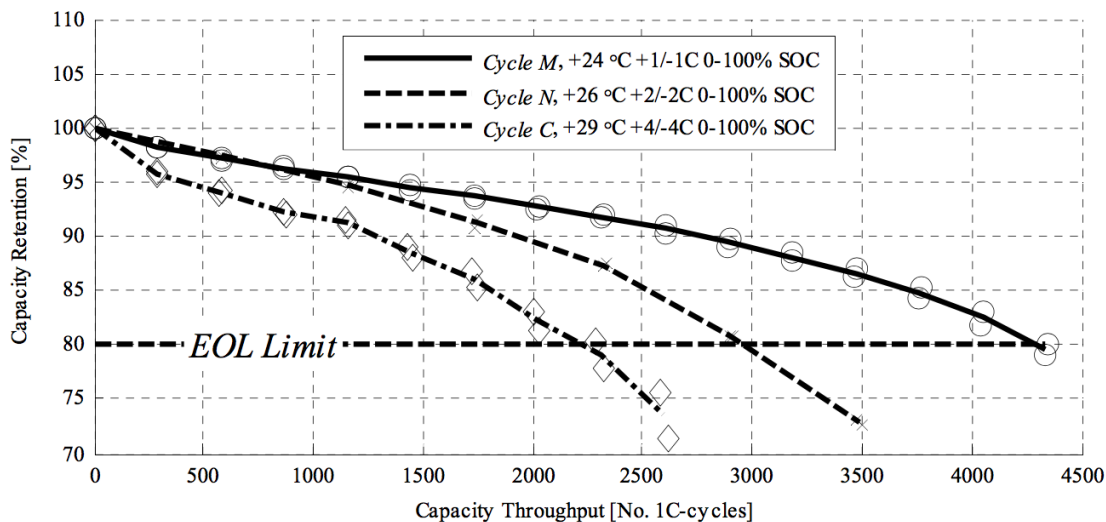


Figure 4: Capacity Retention vs Throughput (Number of cycles) for Li-ion battery under different conditions.

Each battery has an end of life (EOL) limit which signifies the capacity below which a battery cannot be used. It depends on the type of battery, but ranges from one to five years for Li-ion batteries depending on usage^[6]. In order to delve deeper into the field of capacity loss, we examined at several papers that identify capacity loss and predict models for estimating capacity loss. Various papers give estimates of the capacity determination of batteries with time. Most of the models are based on Li-ion batteries as they are the ones which are most commonly used to estimate the ageing parameters of batteries.

According to a paper by Zhong *et al.* on cycle-life model for LiFePO₄ batteries^[1], capacity loss is given by the Arrhenius model. The commercially available 2.2 Ah, 26650 cylindrical cells were purchased from A123 Systems in which the material chemistry is composed of LiFePO₄ cathode and carbon anode.

$$q_k^{loss}(t) = B \exp\left(-\frac{E_a}{RT_k}\right) (Ah_k(t))^z$$

This shows us that capacity loss is a function of temperature, the Amp hours of the battery and the pre-exponential factor B. Capacity loss increases with increase in temperature, increase in B and increase in Watt hours of the battery. This can be seen in Figure 5 and Figure 6^[1].

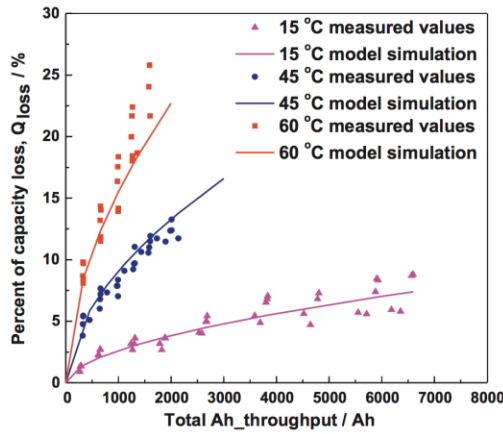


Figure 5: Capacity loss vs Total Ah for different Temperatures.

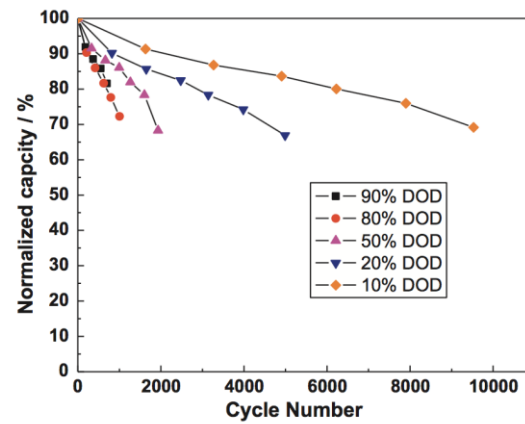


Figure 6: Normalized capacity vs number of cycles for different DOD.

The battery factor z is a constant and is taken to be 0.5 as given to us by eLum energy^[2] and independently validated by a paper on Li-ion batteries by Zhong *et al.* where the typical value is between 0.5 to 0.55^[1].

After getting the model for estimating the battery capacity loss, we wanted to find the values of various parameters like the minimum and maximum energy and power capacity of the battery. We used a paper by Moura *et al.* on stochastic control of smart home energy management^[3]. They used a Tesla battery which is essentially Li-ion battery. The data we collected from various papers and is shown in Table 1^[1, 2, 4].

Parameter Description	Symbol	Value	Unit
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Battery empirical Coefficient	B	30330	$(\text{Ah})^{-0.5}$
Activation Energy	E_a	31500	J/mol
Gas constant	R	8.314	J/Kmol
Temperature of battery k	T	298	K
Battery constant	z	0.5	-
Maximum battery energy	E_{\max}	76.5	kWh
Minimum battery energy	E_{\min}	17	kWh

Table 1: Parameter values used in model.

Focus of this Study

Focus of this study is to optimize battery aging while fulfilling demand constraints at each point in time.

TECHNICAL DESCRIPTION

We have made several underlying assumptions while formulating our model which helped us get rid of the complicated nature of the problem and make it a convex problem that is not computationally intensive and at the same time, helps understand the phenomena of battery ageing.

Assumptions:

1. Thermal effects are neglected: As seen from Figure 5, battery aging accelerates with increasing temperature. We have assumed the temperature to be constant at 298 K.
2. The pre-exponential factor B, is constant: This assumption is actually true for a particular range of discharge rates. If the discharge rates change drastically from the value specified, the value of B changes. However, for a particular set of discharge rates, the variation in B is less than 5% which makes the assumption of taking B to be a constant acceptable. Table 2 shows the value of B for different discharge rates. Here, C rates correspond to a current of 2A.

C - rates	C/2	C	2C	10C
B values	31,630	30,330	21,681	15,512

Table 2: B values for different C rates.

3. Power Demand is a sum of random sine curves: This simulates the power usage of a 24 hour period. If the code works for random sine demand, it will also work for actual demand. The function used to calculate power was:

$$P(d) = P_{\max} \sin(\Omega t + \theta) + 10^3 \text{ W}$$

A representative graph for the power demand equation is shown in Figure 7.

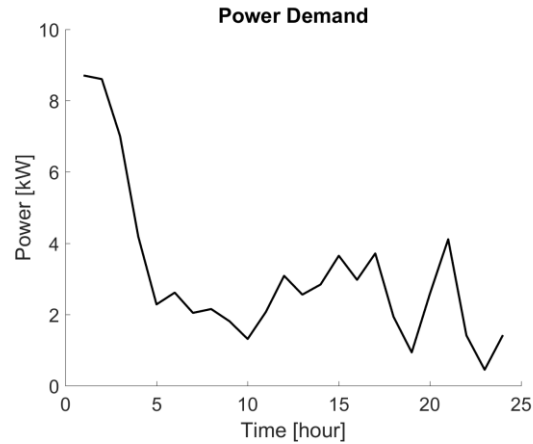


Figure 7: Power demand simulated by sine curve.

Notations:

Symbol	Description
$q_{(k,loss)}$	Battery capacity loss at each time step
$P_k(t)$	Power gained/supplied at time t
R	Universal Gas constant
$Wh_k(t)$	Total Watt hours until time t
$Ah_k(t)$	Total Amp hours until time t
Δt	Time step
$P_{dem}(t)$	Power demand for time t
$E_k(t)$	Energy of battery 'k' at time t
E_{min}	Minimum allowed battery energy
E_{max}	Maximum allowed battery energy

P_{\min}	Minimum battery power
P_{\max}	Maximum battery power
t	Time

Table 3: Notation and physical descriptions.**First Approach - Linear Programming:****Aging Model:**

From Arrhenius Power Law $q_k^{loss} = Bexp\left(-\frac{E_a}{RT_k}\right)(Ah_k(t))^z$

Formulation received by eLum:

$$\underset{\{e_1, e_2, \dots, e_n\}}{\text{minimize}} J_t(e_1, e_2, \dots, e_n) = \sum_{k=1}^N Bexp\left(-\frac{E_a}{RT_k}\right)\left(Ah_k(t) + \frac{e_k}{v_k}\right)^z$$

Constraints:

1. The sum of the energies supplied by the battery should be equal to the load.

$$\underset{\{e_1, e_2, \dots, e_n\}}{\text{minimize}} J_t(e_1, e_2, \dots, e_n) = \sum_{k=1}^N Bexp\left(-\frac{E_a}{RT_k}\right)\left(Ah_k(t) + \frac{e_k}{v_k}\right)^z$$

2. The energies of the batteries are non negative.

$$\text{For } k = 1, 2, \dots, N \quad e_k \geq 0$$

3. The battery cannot supply more than it's capacity and the current is always smaller than its maximum value. Hence the energy would be the minimum of both these constraints.

$$\text{For } k = 1, 2, \dots, N \quad e_k \leq \min\{c_k SOC_k, v_k i_k^{max} \Delta t\}$$

This problem is concave ($z=0.5$) with state variable as e_k and control variable i_k . This problem was solved by linearizing with respect to Amp-hours using Taylor Series. However, this is an inaccurate assumption since the non-linear nature of Ah is unaccounted for. The non-linear nature of Amp-hours are a key feature of the problem. Also, this formulation minimized cumulative capacity loss instead of minimizing capacity loss for each time interval, which is clearly incorrect.

Second Approach - Dynamic Programming

Hence we reformulated the problem converting the final equation as a function of Watt-hours, (Wh) instead of Amp-hours, and minimizing the capacity loss rate. This made the problem simpler because then we could directly describe power consumption and battery dynamics in terms of capacity and energy level of the batteries.

Converting the equation in terms of Wh:

$$q_k^{loss} = \frac{Bexp\left(-\frac{E_a}{RT_k}\right)(V_k * Ah_k(t))^z}{V_k^z}$$

Or,

$$q_k^{loss} = \beta_k (Wh_k(t))^z$$

Taking derivative of the equation with respect to t (to find out capacity loss rate),

$$\begin{aligned} \frac{dq_{k,loss}}{dt} &= \beta_k z (Wh_k)^{z-1} \frac{d(Wh(k))}{dt} \\ \frac{dq_{k,loss}}{dt} &= \beta_k z (Wh_k)^{z-1} |P_k| \end{aligned}$$

Objective function:

$$\underset{Wh_1, Wh_2, \dots, Wh_N, P_1, P_2, \dots, P_N}{\text{minimize}} \quad \frac{dq_{loss}}{dt} = \sum_{t=0}^T \sum_{k=1}^N \beta_k z (Wh_k)^{z-1} |P_k|$$

Constraints:

1. Sum of the power supplied/gained by each battery should be equal to the demand

$$P_{demand}(t) = \sum_{k=1}^N P_k(t)$$

2. Energy and Wh should be updated after each time interval

$$\begin{aligned} \text{For } k &= 1, 2, \dots, N \\ Wh_k(t + \Delta t) &= Wh_k(t) + \Delta t * |P_k(t)| \\ E_k(t + \Delta t) &= E_k(t) - \Delta t * |P_k(t)| \end{aligned}$$

3. There are lower and upper limits in energy and power supplied/gained

$$\begin{aligned} \text{For } k &= 1, 2, \dots, N \\ E_{min} &\leq E_k(t) \leq E_{max} \\ P_{min} &\leq P_k(t) \leq P_{max} \end{aligned}$$

This had state variables E_k and Wh_k with control variable being P_k . We solved this in the form of dynamic programming but it had a long run-time and formulation of u^* wasn't clear to us.

Third Approach - Convex Programming

To get over our problems with the long run time and formulation. We made another approximation to the system by assuming a step change in Watt-hours, i.e we hypothesized that the Watt-hours would be constant for the time scale of the optimization. This assumption as noted previously is imperfect but helped us get over our issues in addition to negating the need for scalability. As noted in Table 2.

Final formulation is same as written in the second approach.

Method and Algorithm:

We coded this on a MATLAB solver using the cvx tool. To completely cover our bases we ran the program for the following cases:

Case Number	Battery 1	Battery 2	Battery 3
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1	New (Low Wh)	New (Low Wh)	None
2	New (Low Wh)	Old (High Wh)	None
3	Old (High Wh)	Old (High Wh)	None
4	New (Low Wh)	New (Low Wh)	Old (High Wh)

Table 4: The different cases shown in the results.

We used these different cases to extract control heuristics which we felt could be useful for eLum. To check whether the program is truly scalable, we also ran the following simulations primarily to check for solver time:

Number of Days	Number of Batteries	Approximate Time to Run Code [sec]
14	2	105
	3	100
	5	110
	10	100
30	2	210
	3	230
	5	220

Table 5: Approximate run times for different numbers of batteries and days.

Lastly, to check whether our optimization is truly useful, we compared it with a base case at each step. The base case is an unoptimized case where there are no limits on the power or energy.

We assume that the power delivered/consumed by each battery is equal and simply a function of the power demand, $P_{\text{batt}} = P_{\text{demand}}/K$, where K represents the no. of batteries in the fleet.

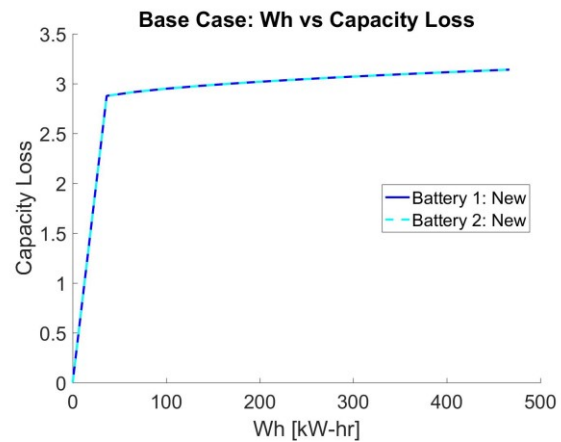
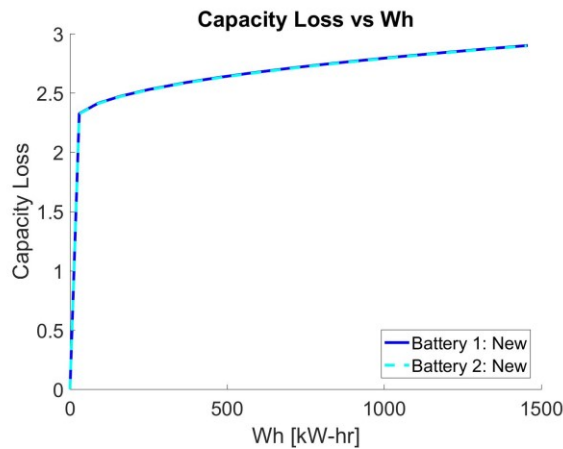
RESULTS

In this section, we take a look at each of the cases we mentioned in Table 4 in further detail. Note that the capacity loss is plotted as a percentage subtracted from 100%. While old and new batteries would not both start at 100% capacity, the difference in capacity loss is more apparent when all of the batteries start at the same capacity.

CASE 1: Two New Batteries

Starting Watt-hours for new batteries: 10 Wh

a



b

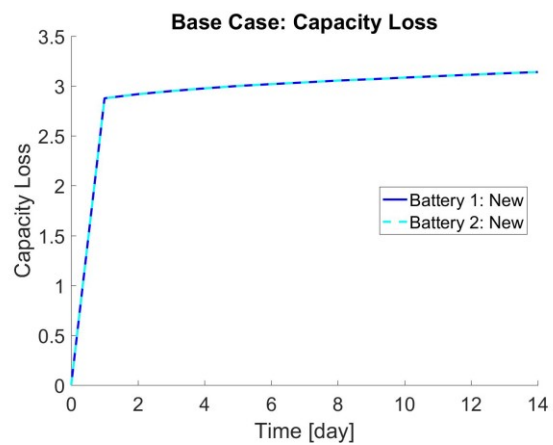
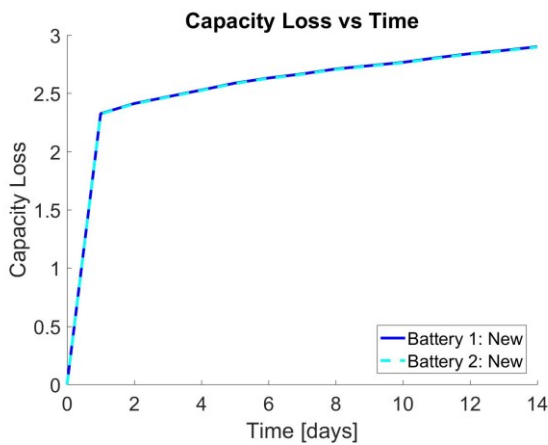


Figure 8: For the case of two batteries starting at 10 Wh, optimal vs base case for a) capacity vs day and b) capacity loss vs Watt-hour.

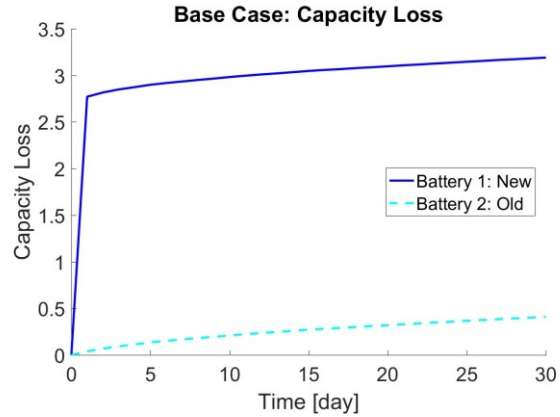
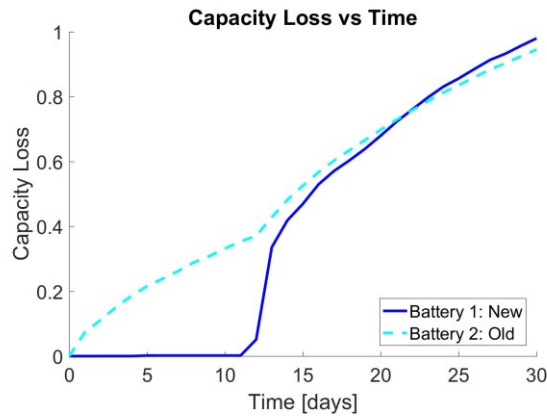
For two new batteries, the power demand is split equally causing both batteries to have the same capacity loss (Figure 8). The capacity loss for both batteries is 2.9% for the optimal case and 3.2% for the base case. The capacity loss is better in the optimal case despite the batteries reaching three times the Watt-hours as the base case over 14 days. In both cases, most capacity loss occurs on the first day. For both batteries, 2.3% is lost for the optimal case and 2.9% for the base case, on the first day.

CASE 2: One New and One Old Battery

Starting Watt-hours for old battery: 50 kWh

Starting Watt-hours for new battery: 10 Wh

a



b

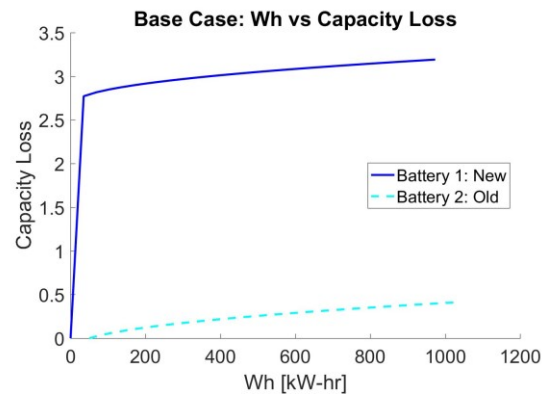
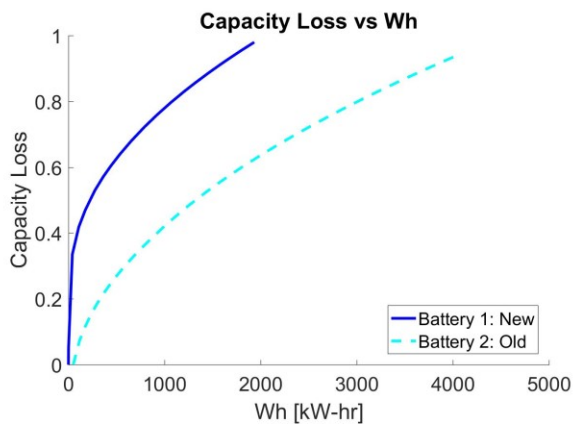


Figure 9: For the case of one battery starting at 10 Wh and the other at 50 kWh, optimal vs base case for a) capacity vs day and b) capacity loss vs Watt-hour.

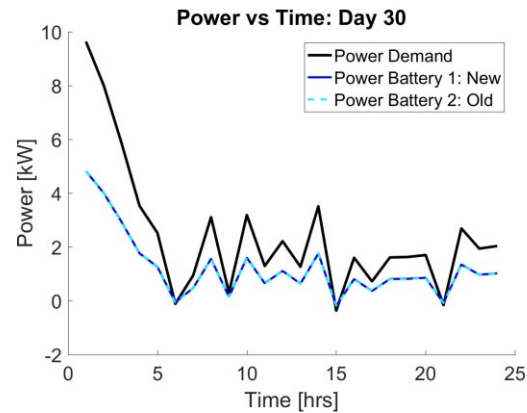
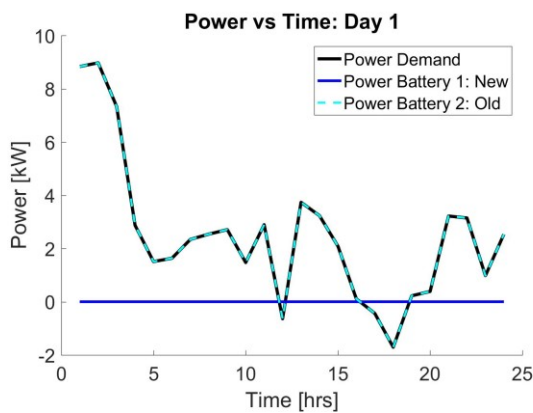


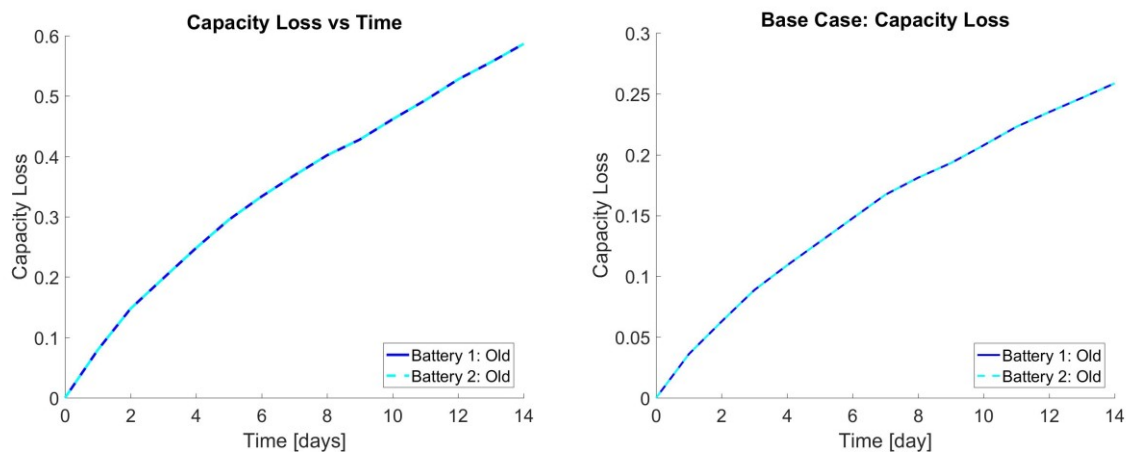
Figure 10: The power demand vs power for each battery comparing day 1 to day 30, for one new and one old battery.

For one new and one old battery, the power demand is supplied by the old battery for about the first two weeks and then split equally (Figure 9, 10). The capacity losses for both batteries in the optimal case are about 1%, despite the new battery only being used for half of the time. The new battery loses 0.35% in the first few days it is used. For the base case, the new battery loses capacity in the same was as Case 1, 2.7% in the first day and 3.2% overall. The old battery loses 0.4% overall. Again, the capacity loss is better in the optimal case despite the batteries reaching two to four times the Watt-hours as the base case over 14 days.

CASE 3: Two Old Batteries

Starting Watt-hours for old batteries: 50 kWh

a



b

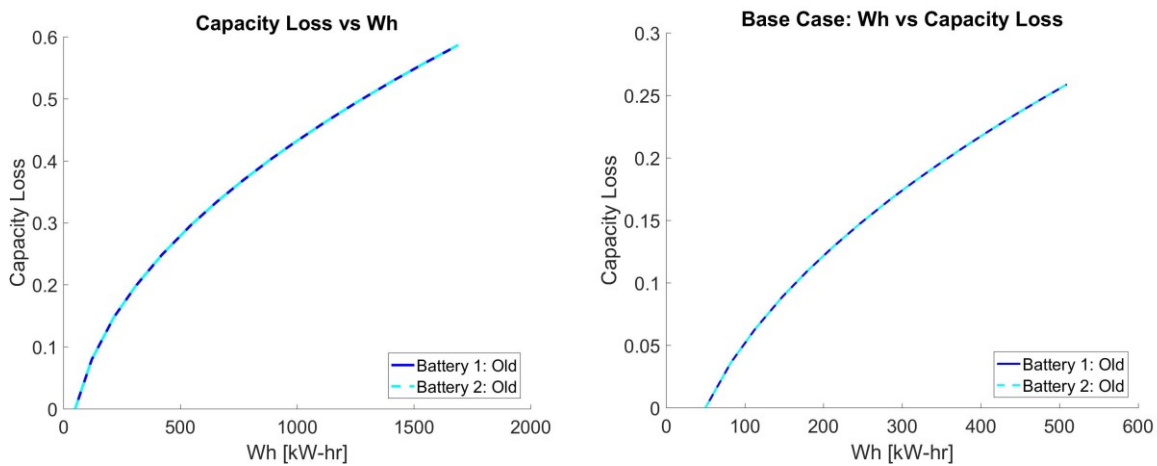


Figure 11: For the case of two batteries starting at 50 kWh, optimal vs base case for a) capacity vs day and b) capacity loss vs Watt-hour.

For two old batteries, the power demand is split equally causing both batteries to have the same capacity loss (Figure 11). The capacity loss for both batteries is 0.6% for the optimal case and 0.26% for the base case. The capacity loss is worse in the optimal case, but the batteries reach three times the Watt-hours as the base case over 14 days.

CASE 4: Two New and One Old

Starting Watt-hours for old battery: 50 kWh

Starting Watt-hours for new battery: 10 Wh

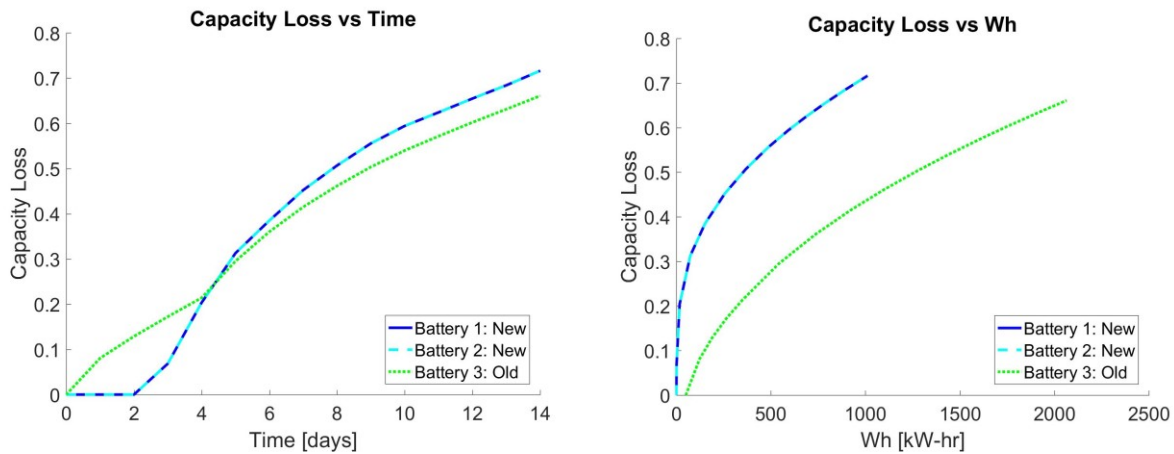


Figure 12: For the case of one battery starting at 10 Wh and two at 50 kWh, optimal vs base case for a) capacity vs day and b) capacity loss vs Watt-hour.

For two new batteries and one old battery, the power demand is supplied by the old battery for about the first two days and then split equally (Figure 12). The capacity losses for all of the batteries in the optimal case is between 0.6% and 0.7% over 14 days. The new batteries lose almost 0.1% more capacity than the old battery despite being used for half of the Watt-hours. This result matches the case with only one new battery and one old battery.

DISCUSSION

Comparison of the cases:

Some clear trends stand out when taking look at the different cases showed in the results. Each of these trends is discussed in detail in the subsequent paragraphs.

In a degenerate system, the older battery always starts supplying most of the power demanded of the fleet. In Case 2 especially we see that the fresh battery supplies negligible power for over the first 10 days before picking up the power supply. As the objective is to minimize capacity loss, the program is minimizing the power delivered by the battery until the watt hours increase, as seen in figure 9. Because the capacity loss is a function of power per square root of Wh, the program increases the Wh while minimizing the power delivered until a suitable value of the fraction. After that the power delivered is divided evenly between each battery regardless of their capacities. This behavior is consistently repeated across all the degenerate cases.

In the degenerate cases, it is very clear that the overall capacity loss of the system is being minimized. If we consider Case 2, we see that in the base case the total capacity loss is about 3% after 30 days. However if we compare that with the optimized case we see a total capacity loss of only 2% after 30

days. In other words the optimization resulted in a 33% improvement in cumulative capacity in only one month of operation.

Across all the cases, we see a gradual decrease in the capacity loss as time goes on. This is simply because as the batteries continue to provide power, the Wh continue to increase. Since the capacity loss equation is set up with Watt-hours in the denominator, after a point the incremental loss of capacity becomes too small. This nature of the curve matches excellently to Figure 5, thereby providing us with validation of the model and of our assumptions of switch Ah with Wh.

Another interesting result is in the case of fresh batteries. Across all the cases analysed, fresh batteries lose a most of their total capacity in the first few days of operation. This is understood as the initial Watt-hours of these batteries are low and any power supply increases the capacity loss function. What we also see is that having a system where both the batteries are fresh simultaneously can be a bad idea from a controls point of view. This is counterintuitive to the notion that the grid should always have new fresh batteries. What we realize when we take Case 2 & Case 4 in conjunction is that a grid should always have some old batteries to allow the new batteries to undergo a “breaking in” period.

Because the way the base case is set-up it does not differentiate between the charge & discharge power. We consider the discharge & charge power separately for the optimized case. This is why we see some difference in the Watt-hours for the two cases.

Lastly, when we take a look at the different solver times for different cases, we see that the program is completely scalable and can be used for a large battery fleet with no increase in computation burden when number of batteries is increased.

Control Heuristics:

Based on the aforementioned optimization we can extend our findings and make them control heuristics. The following rules emerge from our analysis:

- Older batteries will deliver more power in a degenerate fleet.
- If both batteries have a similar age (Watt-hours) they give out the same power.
- It is better to have a fleet of older batteries and let them run out to minimize total capacity loss.
- Never overhaul a battery fleet completely. Instead take off some old batteries and add new ones.
- If installing a new fleet: Install some old batteries with new ones to minimize total capacity loss.
- Eventually (1-2 months to be safe) all the batteries will provide the same power to the system.
- The optimization adds value for only the first few days of operation.

SUMMARY

To summarize, we built a convex optimization program over a proposed battery aging, capacity loss model to optimally manage the dispatch of batteries. We found out that in some cases it may be possible to achieve a 33% increase in capacity as compared to an unoptimized system over 30 days. We studied a range of extreme situations with two batteries. Using these simulations as references we came up with control heuristics that eLum should use while making battery decisions.

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