

PRACTICE EXAM 1 #1: 2:30pm–3:45am March –, 20–. (100 points) The exam is open book, open notes, and students are permitted to use a calculator and/or computer. Students are expected to complete the exam individually and are not permitted to communicate in any format with others during the exam. It is advised that students show all work and attempt each question to maximize the score.

1. (20 points) Chris is an accurate typist but occasionally makes errors. The number of errors that Chris makes on a report has the following distribution:

| | | | | |
|-------------|-----|---|-----|-----|
| Errors | 0 | 1 | 2 | 3 |
| Probability | 0.6 | ? | 0.1 | 0.1 |

(a - 5 pts) What is the probability that Chris makes exactly one error on a randomly selected report?

(b - 5 pts) What is the probability that Chris makes at least one error on a randomly selected report?

(c - 5 pts) What is the mean of the distribution of the number of errors in a report typed by Chris?

(d - 5 pts) What is the standard deviation of the distribution of the number of errors in a report typed by Chris? _____

2. (20 points) Suppose that a random sample of size 9 will be taken from a population which is Normally distributed with mean 50 and standard deviation 6.

(a - 10 pts) What is the mean and standard deviation of the distribution of the sample mean?

_____, _____

(b - 10 pts) What is the probability that the sample mean will be less than 53? _____

3. (20 points) According to genetic theory, the seed color in the second generation of a certain cross of sweet pea plants should be yellow or green in a 3:1 ratio. That is, each plant has probability $3/4$ of having yellow seeds, and the seed colors of separate plants are independent.

(a - 8 pts) What is the probability that exactly 3 out of 5 of these plants have yellow seeds? _____

(b - 4 pts) What is the mean number of yellow-seed plants when 192 plants of this type are grown?

(c - 8 pts) Use the central limit theorem to approximate the probability of obtaining at least 149 yellow-seed plants when 192 plants are grown? (Correct use of the continuity correction is worth 1 point.)

4. (20 points) Data were collected on pollution in a river around a chemical plant. Regulators are interested in the amount of a pollution in the river (in parts per million).

(a - 10 pts) A scientist collects the following 9 independent observations of pollution levels from the river, in parts per million (ppm):

8.1, 10.6, 7.5, 14.8, 11.0, 7.5, 11.5, 12.2, 11.7

Assuming that the observations come from a normal population with a known standard deviation $\sigma = 3$ ppm, compute a 93% confidence interval for the true pollution level in the river.

Round the values to at least 3 decimal places. (_____, _____)

(b - 10 pts) Assuming that the observations come from a normal population with a known standard deviation $\sigma = 3$ ppm, find the sample size needed to have a margin of error of at most 1 with 93% confidence? _____

5. (20 points) Consider a test of a hypothesis about the math SAT scores of Kentucky high school students based on a simple random sample of 400 students. The hypotheses are

$$H_0 : \mu = 500$$

$$H_a : \mu > 500$$

Assume that the population standard deviation is known to be $\sigma = 100$. The test rejects H_0 at the 5% level of significance when $z \geq 1.645$, where

$$z = \frac{\bar{x} - 500}{100/\sqrt{400}}.$$

Is this test sufficiently sensitive to usually detect an increase of 15 points in the population mean SAT score? Answer this question by calculating the power of the test against the alternative $\mu = 515$. _____