Chapter 6: Introduction to Inference

MATH 560-01
Statistical Data Analysis

February 8, 2021

These slides are based on material from *Introduction to the Practice of Statistics* by David S. Moore, George P. McCabe, and Bruce A. Craig, 9th edition.

Sections

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- There are two main types of methods used for statistical inference: **confidence estimation** and **tests of significance**.

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- In this chapter, we begin to consider a statistical problem: how to make inferences about μ (the mean of a population) based on a random sample of observed data from that population when the true value of μ is unknown.
- There are two main types of methods used for statistical inference: **confidence estimation** and **tests of significance**.
- In this chapter, we make the unrealistic assumption that σ (the standard deviation of the population) is known. In Chapter 7, we deal with the more realistic setting when we have to adjust for not knowing σ.

6.1 Estimating with Confidence

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- compute *confidence intervals for the population mean* μ when the population standard deviation σ is known
- correctly interpret the meaning of confidence intervals
- in the setting when the population standard deviation is known, determine the *sample size* needed so that the margin of error for the confidence interval is no more than a specified value m

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- We will develop an interval of the form:

$$\text{estimate} \pm \text{margin of error}.$$

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- From Section 5.2: If the population has a $N(\mu, \sigma)$ distribution, then the sampling distribution is $\bar{x}$ follows a $N\left(\mu, \dfrac{\sigma}{\sqrt{n}}\right)$.
- CLT from Section 5.2: If the population has mean μ and standard deviation σ, then the sampling distribution of $\bar{x}$ is approximately $N\left(\mu, \dfrac{\sigma}{\sqrt{n}}\right)$ when n is large.

6.1 Estimating with Confidence

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- A level C confidence interval for a parameter is an interval computed from the sample data by a method that has probability C of producing an interval containing the true value of the parameter.
- To determine the margin of error for a sampling distribution which is normal or approximately normal, we need to find m such that

$$P(\text{estimate} - m < \text{parameter} < \text{estimate} + m) = C$$

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- Let z^* be the value such that $P(-z^* < Z < z^*) = C$ for a standard normal random variable Z.
- The following table gives z^* for some commonly used confidence levels.

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- The following table gives z^* for some commonly used confidence levels.

C	90%	95%	99%
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- If the conditions are satisfied for $\bar{x}$ to be normal (or approximately normal), then

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

follows (or approximately follows) a $N(0, 1)$ distribution.

6.1 Estimating with Confidence

So, m can be determined as follows.

$$\begin{aligned} P(-z^* < Z < z^*) &= C \\ P\left(-z^* < \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} < z^*\right) &= C \end{aligned}$$

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\end{aligned}$$

6.1 Estimating with Confidence

Confidence Interval for μ when σ is known
and the population is normal

So $m = z^* \frac{\sigma}{\sqrt{n}}$ and a level C confidence interval for μ is

$$\bar{x} \pm z^* \frac{\sigma}{\sqrt{n}}.$$

The confidence level for the interval is approximately correct when n is large even when the population is not normal.

6.1 Estimating with Confidence

Example 6A: Suppose you want to estimate the mean SATM score for all high school seniors in California. Only about 40% of California students typically take the SAT so students who choose to take the SAT are not representive of all California seniors. At considerable expense and effort, you give the test to a simple random sample of 500 California high school seniors. The mean score for your sample is $\bar{x} = 461$. If we assume the standard deviation for this population is 100, find
(a) a 95% confidence interval for the mean score in the population.
(b) a 92% confidence interval for the mean score in the population.

6.1 Estimating with Confidence

- *Answer:*

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Interpreting Confidence Intervals

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- In Example 6A(a), the correct interpretation is that, before we observe the $n = 500$ values in the SRS, the probability that the true value of μ will fall between the two random endpoints of the confidence interval is 0.95.

6.1 Estimating with Confidence

Interpreting Confidence Intervals

- It is important to interpret the confidence interval correctly.
- In Example 6A(a), the correct interpretation is that, before we observe the $n = 500$ values in the SRS, the probability that the true value of μ will fall between the two random endpoints of the confidence interval is 0.95.
- A common incorrect interpretation is that the probability that μ is in the interval $(452.235, 469.765)$ is 0.95. Since μ is fixed, the probability is either 0 (if μ is not in the interval) or 1 (if μ is in the interval) after we observe the sample.

6.1 Estimating with Confidence

How confidence intervals behave

The following changes will reduce the margin of error $m = z^* \frac{\sigma}{\sqrt{n}}$ of a confidence interval for μ.

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How confidence intervals behave

The following changes will reduce the margin of error $m = z^* \frac{\sigma}{\sqrt{n}}$ of a confidence interval for μ.

- Decreasing the confidence level C
- Increasing the sample size n
- A smaller population standard deviation σ.

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- When designing an experiment, a statistician can arrange to a sufficiently small margin of error at a specified confidence level by choosing n to be sufficiently large.

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So, we need n to be at least $\left(\frac{z^*\sigma}{m}\right)^2$.

6.1 Estimating with Confidence

Example 6B: Suppose you want to estimate the mean SATM score for all high school seniors in California. You give the test to a simple random sample of n California high school seniors. If we assume the standard deviation for this population is 100, find

(a) the sample size needed to have a margin of error of at most 10 with 95% confidence.

(b) the sample size needed to have a margin of error of at most 1 with 95% confidence.

(c) the sample size needed to have a margin of error of at most 1 with 92% confidence.

6.1 Estimating with Confidence

- *Answer*:

6.2 Tests of Significance

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- perform *one-sided* or *two-sided significance tests* involving the population mean μ in the setting where the population standard deviation σ is known by using the 4-step process
- correctly interpret the results of significance tests
- understand the connection between confidence intervals and signficance tests

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- A **significance test** is a formal procedure for comparing observed data with a hypothesis whose truth we want to assess.
- The hypothesis is a statement about the population parameter (in this section, μ).
- The results of a test are expressed in terms of a probability that measures how well the data and the hypothesis agree.

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General Steps of a Hypothesis Test

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4. State the conclusion based on the P-value.

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- **Alternative hypothesis** (denoted by H_a): the statement that we hope or suspect is true instead of H_0
- In this chapter, the null hypothesis is of the form $H_0 : \mu = \mu_0$.
- In this chapter, the alternative hypothesis is either **one-sided** ($H_a : \mu > \mu_0$ or $H_a : \mu < \mu_0$) or **two-sided** ($H_a : \mu \neq \mu_0$).

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- In this chapter, the test statistic is $z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$.

6.2 Tests of Significance

Step 3: Compute the P-value of the observed data.

- **Under the assumption that the null hypothesis** H_0 **is true**, the P-value is the probability that the test statistic would take a value as extreme or more extreme than that actually observed.

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- **Under the assumption that the null hypothesis** H_0 **is true**, the P-value is the probability that the test statistic would take a value as extreme or more extreme than that actually observed.
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- The smaller the P-value, the stronger the evidence against H_0.
- The P-value depends on the form of the alternative hypothesis.

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Step 3: Compute the P-value of the observed data.

- **Under the assumption that the null hypothesis H_0 is true**, the P-value is the probability that the test statistic would take a value as extreme or more extreme than that actually observed.
- The smaller the P-value, the stronger the evidence against H_0.
- The P-value depends on the form of the alternative hypothesis.
- In this chapter, if $\bar{x}$ satisfies the conditions to be normal or approximately normal, then the test statistic is $N(0,1)$ when H_0 is true.

6.2 Tests of Significance

Step 4: State the conclusion based on the P-value.

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- The **significance level** (denoted by α) is the value that is considered to be small.

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Step 4: State the conclusion based on the P-value.

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- The **significance level** (denoted by α) is the value that is considered to be small.
- If the P-value is less than or equal to α, then we conclude that there is statistically significant evidence based on the data to reject H_0 at level α.

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Step 4: State the conclusion based on the P-value.

- Finally, we must make a decision regarding the null hypothesis.
- We reject H_0 if the P-value is small, but prior to collecting the data, we must declare what we mean by small.
- The **significance level** (denoted by α) is the value that is considered to be small.
- If the P-value is less than or equal to α, then we conclude that there is statistically significant evidence based on the data to reject H_0 at level α.
- If the P-value is greater than α, then there is not sufficient evidence based on the data so we fail to reject H_0 at level α.

6.2 Tests of Significance

Two-sided test of significance for a population mean when σ is known

1. Test $H_0 : \mu = \mu_0$ vs. $H_a : \mu \neq \mu_0$

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2. Test statistic: $z = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}$
3. Compute the P-value by finding the area in both tails: $P(|Z| \geq |z|) = 2P(Z \geq |z|)$

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Two-sided test of significance for a population mean when σ is known

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3. Compute the P-value by finding the area in both tails: $P(|Z| \geq |z|) = 2P(Z \geq |z|)$
4. Reject H_0 if the P-value $\leq \alpha$

6.2 Tests of Significance

<u>One-sided (left-sided) test of significance for a population mean when σ is known</u>

1. Test $H_0 : \mu = \mu_0$ vs. $H_a : \mu < \mu_0$
2. Test statistic: $z = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}$
3. Compute the P-value by finding the area: $P(Z \le z)$
4. Reject H_0 if the P-value $\le \alpha$

6.2 Tests of Significance

One-sided (right-sided) test of significance for a population mean when σ is known

1. Test $H_0 : \mu = \mu_0$ vs. $H_a : \mu > \mu_0$
2. Test statistic: $z = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}$
3. Compute the P-value by finding the area: $P(Z \geq z)$
4. Reject H_0 if the P-value $\leq \alpha$

6.2 Tests of Significance

Example 6C: In discussing SATM scores, someone states "I think that if all high school seniors in California take the test, the mean score would be no more than 450." At considerable expense and effort, you give the test to a simple random sample of 500 California high school seniors. The mean score for your sample is $\bar{x} = 461$. Is this good evidence against this claim? At level $\alpha = .05$, assume that $\sigma = 100$ and perform a test of significance to assess this statement. Carefully state the hypotheses, calculate an appropriate test statistic, compute the P-value, and state your conclusion.

6.2 Tests of Significance

- *Answer*:

6.2 Tests of Significance

- There is a connection between two-sided significance tests and confidence intervals.
- In this chapter, a level α two-sided significance test rejects a hypothesis $H_0 : \mu = \mu_0$ exactly when the value μ_0 falls outside a level $1 - \alpha$ confidence interval for μ.

6.2 Tests of Significance

- **Example 6D**: A population follows a Normal distribution with mean μ and standard deviation $\sigma = 2$. A SRS of size $n = 8$ is taken from this population to perform a two-sided test of the hypothesis $H_0 : \mu = 5$. The observed sample mean is $\bar{x} = 4$.
  (a) Should we reject H_0 at level $\alpha = .01$?
  (b) Find a 99% confidence interval for μ.

6.2 Tests of Significance

- **Example 6D**: A population follows a Normal distribution with mean μ and standard deviation $\sigma = 2$. A SRS of size $n = 8$ is taken from this population to perform a two-sided test of the hypothesis $H_0 : \mu = 5$. The observed sample mean is $\bar{x} = 4$.
  (a) Should we reject H_0 at level $\alpha = .01$?
  (b) Find a 99% confidence interval for μ.
- *Answer:*

6.3 Use and Abuse of Tests

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6.3 Use and Abuse of Tests

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- Statistical significance is different from practice significance. (When n is large, even tiny deviations from the null hypothesis will be statistically significant.)
- Failing to reject H_0 is not the same as accepting H_0.
- Tests are only valid when the experimental design is good and the model assumptions are reasonable.

6.4 Power and Inference as a Decision

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- understand the two *types of error* that can occur in a significance test

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- given the true value of the population mean μ in the setting where the population standard deviation σ is known, compute the *power* of significance tests for the population mean

6.4 Power and Inference as a Decision

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- Significance tests control the Type I error (fix it to be α).
- The **power** of a test to detect a particular alternative is the probability that a level α test will reject H_0 when the particular alternative value of the parameter is true; this is $1 - P$(Type II error for the particular alternative value).

6.4 Power and Inference as a Decision

Four Possible Results of a Decision in a Significance Test

Reality About H_0	Decision: Fail to reject H_0	Decision: Reject H_0
H_0 is actually True	Correct Decision	Type I Error
H_0 is actually False	Type II Error	Correct Decision

6.4 Power and Inference as a Decision

Rejection Region for Testing $H_0 : \mu = \mu_0$

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- Under $H_a : \mu \neq \mu_0$, the rejection region has the form $(-\infty, \mu_0 - z^* \frac{\sigma}{\sqrt{n}}] \cup [\mu_0 + z^* \frac{\sigma}{\sqrt{n}}, \infty)$ where z^* is determined from $P(Z > z^*) = \frac{\alpha}{2}$.

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- Under $H_a : \mu < \mu_0$, the rejection region has the form $(-\infty, \mu_0 - z^* \frac{\sigma}{\sqrt{n}}]$ where z^* is determined from $P(Z < z^*) = \alpha$.
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6.4 Power and Inference as a Decision

- Here is a demonstration of how to obtain the rejection region for testing $H_0 : \mu = \mu_0$ versus $H_a : \mu \neq \mu_0$.

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- $z > z^* \Rightarrow \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} > z^* \Rightarrow \bar{x} > \mu_0 + z^* \dfrac{\sigma}{\sqrt{n}}$
- So, H_0 is rejected if $\bar{x}$ is in the set $(-\infty, \mu_0 - z^* \frac{\sigma}{\sqrt{n}}] \cup [\mu_0 + z^* \frac{\sigma}{\sqrt{n}}, \infty)$

6.4 Power and Inference as a Decision

Example 6E: A population follows a Normal distribution with mean μ and standard deviation $\sigma = 2$. A SRS of size $n = 8$ is taken from this population to perform a two-sided test of the hypothesis $H_0 : \mu = 5$ at level $\alpha = .01$.

(a) What is the probability of a Type I error for this test?
(b) What is the rejection region for this test?
(c) What is the power of the test when the true value of the population mean is $\mu = 6$?
(d) What is the power of the test when the true value of the population mean is $\mu = 7$?

6.4 Power and Inference as a Decision

- *Answer*: