Assignment - 1

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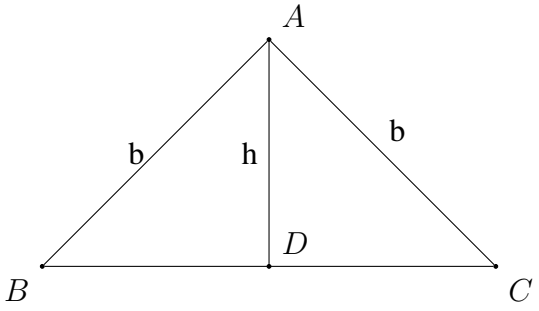
PROBLEM

An isoceles triangle has the extremities of its base at $(2,5)$ and $(-2,2)$. Find the two possible positions of the vertex if its area is 25 sq.units



SOLUTION

Let the vertex B be $(2,5)$ and vertex C be $(-2,2)$.

Let the other vertex A be (x,y).

Let us find the distance of BC

$$\mathbf{B} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$$

$$\|\mathbf{B} - \mathbf{C}\| = \left\| \begin{pmatrix} 2 \\ 5 \end{pmatrix} - \begin{pmatrix} -2 \\ 2 \end{pmatrix} \right\| = \sqrt{4^2 + 3^2} = 5$$

Given, the area of the triangle= 25 sq.units

$$\frac{1}{2} * BC * AD = 25$$

$$\frac{1}{2} * 5 * h = 25$$

$$AD = h = 10$$

Since ABC is an Isoceles triangle,

$$AB = AC$$

$$AB^2 = AC^2$$

$$(x-2)^2 + (y-5)^2 = (x+2)^2 + (y-2)^2$$

$$4x - 4y + 4 = -4x - 10y + 25$$

$$8x + 6y = 21 \tag{1}$$

Point D is the Midpoint of BC

$$D = \left(\frac{2-2}{2}, \frac{5+2}{2} \right)$$

$$D = (0, 3.5)$$

We know that $AD = h = 10$

$$AD^2 = 100$$

$$x^2 + (y - 3.5)^2 = 100 \tag{2}$$

From (1), $x = \dfrac{21 - 6y}{8}$. Subs x in equation (2)...

$$\left(\frac{21 - 6y}{8} \right)^2 + (y - 3.5)^2 = 100$$

Solving the above equation,

$$1.5625y^2 - 10.9375y - 80.86 = 0$$

$$y = 11.5, -4.5$$

When $y = 11.5$, $x = \dfrac{21 - 6(11.5)}{8} = -6$

When $y = 4.5$, $x = \dfrac{21 + 6(4.5)}{8} = 6$

Therefore, The two possible vertex are $(-6, 11.5)$ and $(6, -4.5)$