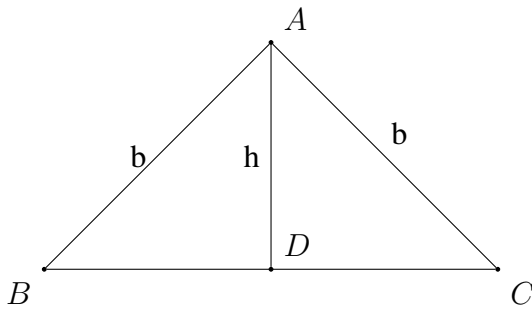


Assignment - 1

Prasanna Kumar R - SM21MTECH14001

PROBLEM

An isosceles triangle has the extremities of its base at $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ 2 \end{pmatrix}$. Find the two possible positions of the vertex if its area is 25 sq.units



SOLUTION

Let the vertex B be $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$ and vertex C be $\begin{pmatrix} -2 \\ 2 \end{pmatrix}$.

Let the other vertex A be $\begin{pmatrix} x \\ y \end{pmatrix}$.

Let us find the distance of BC

$$\mathbf{B} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$$

$$\|\mathbf{B} - \mathbf{C}\| = \left\| \begin{pmatrix} 2 \\ 5 \end{pmatrix} - \begin{pmatrix} -2 \\ 2 \end{pmatrix} \right\| = \sqrt{4^2 + 3^2} = 5$$

Given, the area of the triangle = 25 sq.units

We know area of a \triangle with the vertices A, B and C can be given by:

$$\Delta = \frac{1}{2} \begin{vmatrix} 1 & 1 & 1 \\ A & B & C \end{vmatrix}$$

$$\Delta ABC = \frac{1}{2} \begin{vmatrix} 1 & 1 & 1 \\ x & 2 & -2 \\ y & 5 & 2 \end{vmatrix}$$

Expanding along the first row,

$$\frac{1}{2} [1(4 + 10) - 1(2x + 2y) + 1(5x - 2y)] = 25$$

$$\frac{1}{2} [14 - (2x + 2y) + (5x - 2y)] = 25$$

$$-2x - 2y + 5x - 2y = 50 - 14$$

$$\implies 3x - 4y = 36 \quad (1)$$

Since ABC is an Isosceles triangle,
AD is the perpendicular bisector of BC

$$(\mathbf{A} - \mathbf{D})^T (\mathbf{B} - \mathbf{C}) = 0 \quad (2)$$

Also,

$$\mathbf{D} = \frac{1}{2} (\mathbf{A} + \mathbf{B})$$

$$\mathbf{D} = \frac{1}{2} \left[\begin{pmatrix} 2 \\ 5 \end{pmatrix} + \begin{pmatrix} -2 \\ 2 \end{pmatrix} \right]$$

$$\mathbf{D} = \begin{pmatrix} 0 \\ 3.5 \end{pmatrix}$$

Now,

$$(\mathbf{A} - \mathbf{D}) = \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 0 \\ 3.5 \end{pmatrix}$$

$$(\mathbf{A} - \mathbf{D}) = \begin{pmatrix} x \\ y - 3.5 \end{pmatrix}$$

$$(\mathbf{B} - \mathbf{C}) = \begin{pmatrix} 2 \\ 5 \end{pmatrix} - \begin{pmatrix} -2 \\ 2 \end{pmatrix}$$

$$(\mathbf{B} - \mathbf{C}) = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$$

By (2),

$$\begin{pmatrix} x \\ y - 3.5 \end{pmatrix}^T \begin{pmatrix} 4 \\ 3 \end{pmatrix} = 0$$

$$4x + 3(y - 3.5) = 0$$

$$4x + 3y = 10.5$$

Multiply by 2,

$$\implies 8x + 6y = 21 \quad (3)$$

By interchanging the columns, the area of triangle can also be expressed as,

$$\Delta ABC = \frac{1}{2} \begin{vmatrix} 1 & 1 & 1 \\ x & -2 & 2 \\ y & 2 & 5 \end{vmatrix}$$

Expanding along the first row,

$$\frac{1}{2} [1(-10 - 4) - 1(5x - 2y) + 1(2x + 2y)] = 25$$

$$\frac{1}{2} [-14 - 1(5x - 2y) + 1(2x + 2y)] = 25$$

$$-5x + 2y + 2x + 2y = 64$$

$$\implies -3x + 4y = 64 \quad (4)$$

Let us consider the matrices representation of equations (1) and (3),

$$\begin{bmatrix} 3 & -4 \\ 8 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 36 \\ 21 \end{bmatrix}$$

$$\mathbf{AX} = \mathbf{B}$$

$$\mathbf{X} = \mathbf{A}^{-1}\mathbf{B}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 8 & 6 \end{bmatrix}^{-1} \begin{bmatrix} 36 \\ 21 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{50} \begin{bmatrix} 6 & 4 \\ -8 & 3 \end{bmatrix} \begin{bmatrix} 36 \\ 21 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{50} \begin{bmatrix} 300 \\ -225 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ -4.5 \end{bmatrix} \quad (5)$$

Let us consider the matrices representation of equations (4) and (3),

$$\begin{bmatrix} -3 & 4 \\ 8 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 64 \\ 21 \end{bmatrix}$$

$$\mathbf{AX} = \mathbf{B}$$

$$\mathbf{X} = \mathbf{A}^{-1}\mathbf{B}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 & 4 \\ 8 & 6 \end{bmatrix}^{-1} \begin{bmatrix} 64 \\ 21 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-50} \begin{bmatrix} 300 \\ -575 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -6 \\ 11.5 \end{bmatrix} \quad (6)$$

\therefore From equations (5) and (6), **The two possible vertex are (6, -4.5) and (-6, 11.5)**