ASSIGNMENT 1 - EE5600

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Contents

Abstract—**This paper contains solution to problem no 17 of Lines and Planes section. Links to Python codes are available below.**

Download python codes using

https://github.com/rsgirishkumar/Assignment1/codes/

1 Problem

Find m if

$$\begin{pmatrix} 2 & 3 \end{pmatrix}\mathbf{x} = 11 \tag{1.0.1}$$

$$\begin{pmatrix} 2 & -4 \end{pmatrix}\mathbf{x} = -24 \tag{1.0.2}$$

$$\begin{pmatrix} m & -1 \end{pmatrix}\mathbf{x} = -3 \tag{1.0.3}$$

2 Solution

Given, the system of equations in matrix equation format are as below

$$\begin{pmatrix} 2 & 3 \\ 2 & -4 \\ m & -1 \end{pmatrix}\mathbf{x} = \begin{pmatrix} 11 \\ -24 \\ -3 \end{pmatrix} \tag{2.0.1}$$

Step1: Assuming the system of equations are consistent, lets reduce the augmented matrix [A'b], to find the value of m.

$$\begin{pmatrix} 2 & 3 & 11 \\ 2 & -4 & -24 \\ m & -1 & -3 \end{pmatrix}$$

$$R2- > R2 - R1$$

$$\begin{pmatrix} 2 & 3 & 11 \\ 0 & -7 & -35 \\ m & -1 & -3 \end{pmatrix}$$

$$R3- > 2R3 + R1$$

$$\begin{pmatrix} 2 & 3 & 11 \\ 0 & -7 & -35 \\ 2m+2 & 1 & 5 \end{pmatrix}$$

$$R3- > R2 + 7R3$$

$$\begin{pmatrix} 2 & 3 & 11 \\ 0 & -7 & -35 \\ 14m+14 & 0 & 0 \end{pmatrix}$$

Since the system of equations are assumed consistent,

$$14m + 14 = 0 \tag{2.0.2}$$

$$\Rightarrow m = -1 \tag{2.0.3}$$

So the system of equations can be re-written as

$$\begin{pmatrix} 2 & 3 \end{pmatrix}\mathbf{x} = 11 \tag{2.0.4}$$

$$\begin{pmatrix} 2 & -4 \end{pmatrix}\mathbf{x} = -24 \tag{2.0.5}$$

$$\begin{pmatrix} -1 & -1 \end{pmatrix}\mathbf{x} = -3 \tag{2.0.6}$$

Step2:Check the consistency of equations by using the rank of augumented matrix,M=[A'b] and

N=matrix [A] as below.

$$N = \begin{pmatrix} 2 & 3 \\ 2 & -4 \\ -1 & -1 \end{pmatrix}$$

$$M = \begin{pmatrix} 2 & 3 & 11 \\ 2 & -4 & -24 \\ -1 & -1 & -3 \end{pmatrix}$$

Calculating the rank of matrix M:

$$\begin{pmatrix} 2 & 3 & 11 \\ 2 & -4 & -24 \\ -1 & -1 & -3 \end{pmatrix}$$

$$R2- > R2 - R1$$

$$\begin{pmatrix} 2 & 3 & 11 \\ 0 & -7 & -35 \\ -1 & -1 & -3 \end{pmatrix}$$

$$R3- > 2R3 + R1$$

$$\begin{pmatrix} 2 & 3 & 11 \\ 0 & -7 & -35 \\ 0 & 1 & 5 \end{pmatrix}$$

$$R3- > R2 + 7R3$$

$$\begin{pmatrix} 2 & 3 & 11 \\ 0 & -7 & -35 \\ 0 & 0 & 0 \end{pmatrix}$$

No of non zero rows = 2.
Hence Rank of matrix(M) =2.
Calculating the rank of matrix N:

$$\begin{pmatrix} 2 & 3 \\ 2 & -4 \\ -1 & -1 \end{pmatrix}$$

$$R2- > R2 - R1$$

$$\begin{pmatrix} 2 & 3 \\ 0 & -7 \\ -1 & -1 \end{pmatrix}$$

$$R3- > 2R3 + R1$$

$$\begin{pmatrix} 2 & 3 \\ 0 & -7 \\ 0 & 1 \end{pmatrix}$$

$$R3- > R2 + 7R3$$

$$\begin{pmatrix} 2 & 3 \\ 0 & -7 \\ 0 & 0 \end{pmatrix}$$

No of non zero rows = 2.
Hence Rank of matrix(N) =2.

Since rank of matrix N =2 and M =2, the system of equations are consistent.But,rank(N) = 2 is not equal to total no of rows in matrix(N) i.e.m =3.Since rank(N) != m there exist infinite number of solutions. Of which, the intersection point of any two equations is one of the solutions. As per ratio of determinants,

$$\mathbf{x} = \frac{\begin{vmatrix} 11 & 3 \\ -24 & -4 \end{vmatrix}}{\begin{vmatrix} 2 & 3 \\ 2 & -4 \end{vmatrix}} = \frac{-44 + 72}{-8 - 6} = \frac{28}{-14} = -2$$

$$\mathbf{y} = \frac{\begin{vmatrix} 2 & 11 \\ 2 & -24 \end{vmatrix}}{\begin{vmatrix} 2 & 3 \\ 2 & -4 \end{vmatrix}} = \frac{-48 - 22}{-8 - 6} = \frac{70}{14} = 5$$

The solution is

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ 5 \end{pmatrix} \tag{2.0.7}$$

The 1.0.3 equation can be substituted with intersection point for verification of solution.

$$\begin{pmatrix} -1 & -1 \end{pmatrix} * \begin{pmatrix} -2 \\ 5 \end{pmatrix} = -3$$

Since, the intersection point satisfies the equation, it is one of the solution of all the three equations. The same can be verified from the plot of vectors as below.

Step3:The vectors of equations are plotted on 2D axes by taking intersecting points on x and y axes respectively.Intersecting point is given in code.

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https://github.com/rsgirishkumar/Assignment1/
    codes/assignment1_solution.py
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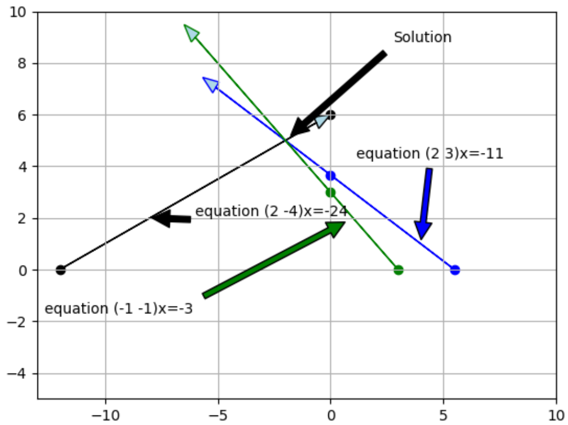

Fig. 0: Three lines intersecting at a point.

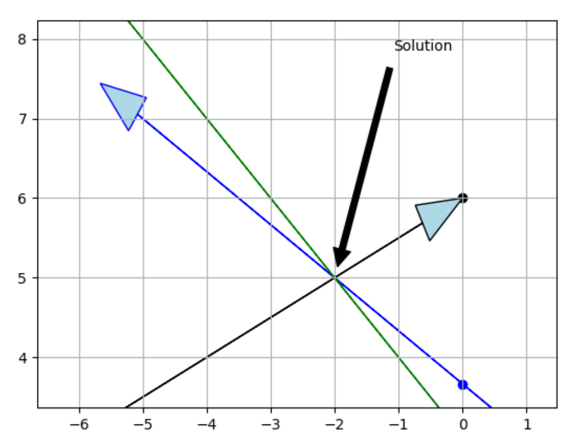

Fig. 0: A Clear view.