

ASSIGNMENT 1 - EE5600

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Abstract—This paper contains solution to problem no 17 of Lines and Planes section. Links to Python codes are available below.

Download python codes using

<https://github.com/rsgirishkumar/Assignment1/codes/>

As per ratio of determinants,

$$x = \frac{\begin{vmatrix} 11 & 3 \\ -24 & -4 \end{vmatrix}}{\begin{vmatrix} 2 & 3 \\ 2 & -4 \end{vmatrix}} = \frac{-44 + 72}{-8 - 6} = \frac{28}{-14} = -2$$

$$y = \frac{\begin{vmatrix} 2 & 11 \\ 2 & -24 \end{vmatrix}}{\begin{vmatrix} 2 & 3 \\ 2 & -4 \end{vmatrix}} = \frac{-48 - 22}{-8 - 6} = \frac{70}{14} = 5$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$$

On back-substituting the values in 3rd equation i.e.

$$(m - 1)x = -3 \quad (2.0.1)$$

The equation can be re-written as

$$(m - 1)\begin{pmatrix} -2 \\ 5 \end{pmatrix} = -3$$

$$\Rightarrow m = -1 \quad (2.0.2)$$

1 PROBLEM

Find m if

$$\begin{aligned} \begin{pmatrix} 2 & 3 \end{pmatrix} \mathbf{x} &= 11 \\ \begin{pmatrix} 2 & -4 \end{pmatrix} \mathbf{x} &= -24 \\ \begin{pmatrix} m & -1 \end{pmatrix} \mathbf{x} &= -3 \end{aligned} \quad (1.0.1)$$

2 SOLUTION

Step1: Assuming the system of equations are consistent, Since there is an unknown in equations, m is to found first. To find m , use

$$\begin{pmatrix} 2 & 3 \\ 2 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 11 \\ -24 \end{pmatrix}$$

To Find the solution, use ratio of determinants methods for x and y by forming a 2×2 matrix as above, i.e. $Ax=B$ format.

Since the system of equations are assumed consistent, the x and y values should satisfy the third equation i.e.

$$(m - 1)x = -3$$

Step2: Check the consistency of equations by using the rank of augmented matrix, $M=[A'b]$ and $N=\text{matrix } [A]$ as below.

$$N = \begin{pmatrix} 2 & 3 \\ 2 & -4 \\ -1 & -1 \end{pmatrix}$$

$$M = \begin{pmatrix} 2 & 3 & 11 \\ 2 & -4 & -24 \end{pmatrix}$$

since rank of matrix $N = 2$ and $M = 2$, the system of equations are consistent.

Also, $\text{rank}(N) = 2$ and no of rows in N i.e. $m = 3$, since $\text{rank}(N) \neq m$ there exists an infinite number of solutions. Of which, the intersection point of any two equations is one of the solutions. From above,

for these equations

$$\begin{pmatrix} 2 & 3 \\ 2 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 11 \\ -24 \end{pmatrix}$$

intersection point is

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$$

The third line equation can be substituted with intersection point for verification of solution.

$$\begin{pmatrix} -1 & -1 \end{pmatrix} * \begin{pmatrix} -2 \\ 5 \end{pmatrix} = -3$$

Since, the intersection point satisfies the equation, it is one of the solution of all the three equations i.e.

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$$

The same can be verified from the plot of vectors as below.

Step3:The vectors of equations are plotted on 2D axes by taking intersecting points on x and y axes respectively. Intersecting point is given in code.

https://github.com/rsgirishkumar/Assignment1/codes/assignment1_solution.py

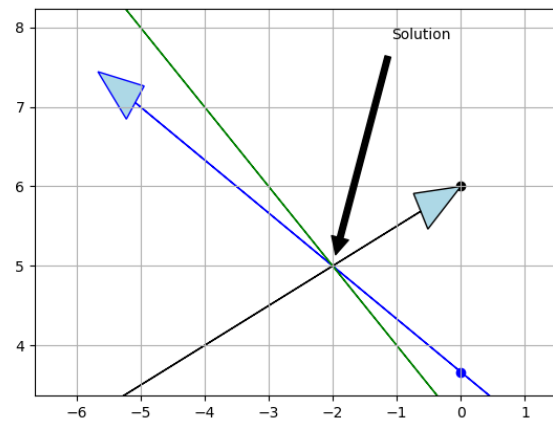


Fig. 0: A Clear view.

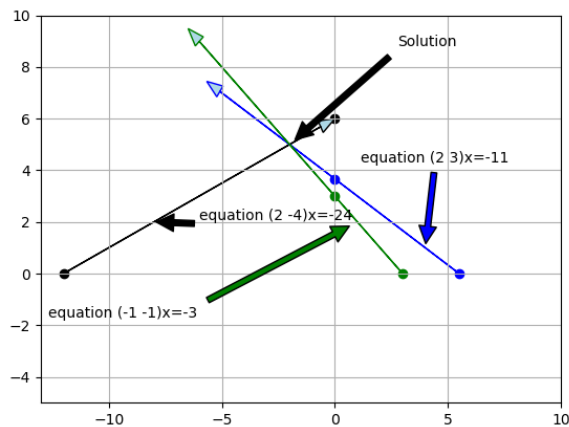


Fig. 0: Three lines intersecting at a point.