ASSIGNMENT 1 - EE5600

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Contents

***Abstract*—This paper contains solution to problem no 17 of Lines and Planes section. Links to Python codes are available below.**

Download python codes using

https://github.com/rsgirishkumar/Assignment1/codes/

1 Problem

Find m if

$$\begin{aligned}\begin{pmatrix}2 & 3\end{pmatrix}\mathbf{x} &= 11\\ \begin{pmatrix}2 & -4\end{pmatrix}\mathbf{x} &= -24\\ \begin{pmatrix}m & -1\end{pmatrix}\mathbf{x} &= -3\end{aligned} \tag{1.0.1}$$

2 Solution

Given, the system of equations in matrix equation format are as below

$$\begin{pmatrix}2 & 3\\ 2 & -4\\ m & -1\end{pmatrix}\mathbf{x} = \begin{pmatrix}11\\ -24\\ -3\end{pmatrix}$$

Step1: Assuming the system of equations are consistent,Since there is an unknown in equations, m is to found first. To find m, use

$$\begin{pmatrix}2 & 3\\ 2 & -4\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}11\\ -24\end{pmatrix}$$

To Find the solution,use ratio of determinants methods for x and y by forming a 2x2 matrix as above, i.e. Ax=B format.

Since the system of equations are assumed consistent, the x and y values should satisfy the third equation i.e.

$$\begin{pmatrix}m & -1\end{pmatrix}\mathbf{x} = -3$$

As per ratio of determinants,

$$\mathbf{x} = \frac{\begin{vmatrix}11 & 3\\ -24 & -4\end{vmatrix}}{\begin{vmatrix}2 & 3\\ 2 & -4\end{vmatrix}} = \frac{-44+72}{-8-6} = \frac{28}{-14} = -2$$

$$\mathbf{y} = \frac{\begin{vmatrix}2 & 11\\ 2 & -24\end{vmatrix}}{\begin{vmatrix}2 & 3\\ 2 & -4\end{vmatrix}} = \frac{-48-22}{-8-6} = \frac{70}{14} = 5$$

$$\begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}-2\\ 5\end{pmatrix}$$

On back-substituting the values in 3rd equation i.e.

$$\begin{pmatrix}m & -1\end{pmatrix}\mathbf{x} = -3 \tag{2.0.1}$$

The equation can be re-written as

$$\begin{pmatrix}m & -1\end{pmatrix}\begin{pmatrix}-2\\ 5\end{pmatrix} = -3$$

$$\implies m = -1 \tag{2.0.2}$$

Step2:Check the consistency of equations by using the rank of augumented matrix,M=[A'b] and N=matrix [A] as below.

$$N = \begin{pmatrix}2 & 3\\ 2 & -4\\ -1 & -1\end{pmatrix}$$

$$M = \begin{pmatrix}2 & 3 & 11\\ 2 & -4 & -24\end{pmatrix}$$

since rank of matrix N =2 and M =2, the system of equations are consistent.
Also,rank(N) = 2 and no of rows in N i.e.m =3, since rank(N) != m there exists an infinite number of solutions. Of which, the intersection point of any two equations is one of the solutions. From above, for these equations

$$\begin{pmatrix} 2 & 3 \\ 2 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 11 \\ -24 \end{pmatrix}$$

intersection point is

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$$

The third line equation can be substituted with intersection point for verification of solution.

$$\begin{pmatrix} -1 & -1 \end{pmatrix} * \begin{pmatrix} -2 \\ 5 \end{pmatrix} = -3$$

Since, the intersection point satisfies the equation, it is one of the solution of all the three equations i.e.

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$$

The same can be verified from the plot of vectors as below.

Step3:The vectors of equations are plotted on 2D axes by taking intersecting points on x and y axes respectively.Intersecting point is given in code.

https://github.com/rsgirishkumar/Assignment1/codes/assignment1_solution.py

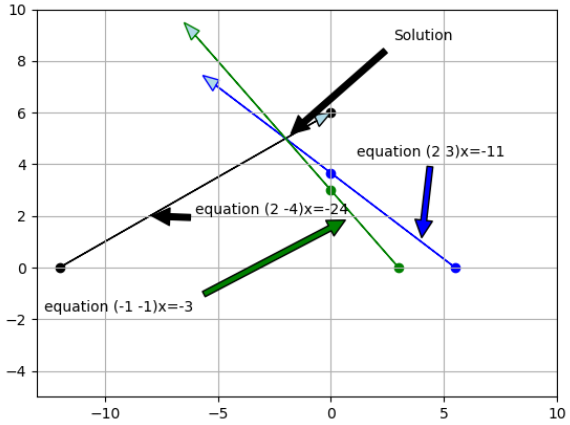

Fig. 0: Three lines intersecting at a point.

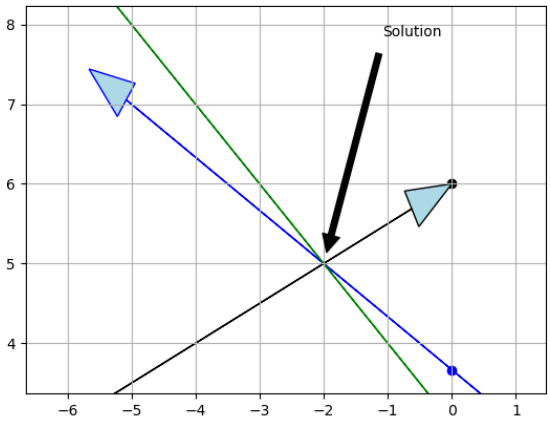

Fig. 0: A Clear view.