ASSIGNMENT 2 - EE5600

RS Girish - EE20RESCH14005*

Contents

Abstract—**This paper contains solution to problem no 17 of 1.1 examples section. Links to Python codes are available below.**

Download python codes using

https://github.com/rsgirishkumar/Assignment2/codes/

I. Problem

Consider the experiment of tossing a coin. If the coin shows head, toss it again but if it shows tail, then throw a die. Find the conditional probability of the event that "the die shows a number greater than 4" given that "there is at least one tail".

II. Solution

Let

event A = tossing a coin,
event B = throwing a dice

for which

sample size of A = 2

$$\Rightarrow S1 = (heads, tails).$$

sample size of B = 6

$$\Rightarrow S2 = (1, 2, 3, 4, 5, 6).$$

P(A)=P(tails)

$$\Rightarrow P(A) = \frac{1}{2} \tag{1}$$

P(B)=P(dice showing number greater than 4)

$$\Rightarrow P(B) = \frac{2}{6} \Rightarrow (5, 6) \tag{2}$$

From Conditinal probability formula,

$$P(B/A) = P(A \cap B)/P(A) \tag{3}$$

Since both are independent events,

$$P(A \cap B) = P(A)P(B) \Rightarrow P(B/A) = P(B) \tag{4}$$

Therefore

P(dice throwing number greater than 4
given a tail when coin is tossed)

$$\Rightarrow P(B/A) = \frac{1}{3}$$

The above is true for one toss but for n tosses it will be a binomial distribution as below

$$P(A = k) = {}^{n}C_k * p^k * q^{n-k} \tag{5}$$

indicating
p = probability of tails in individual event
q = probability of heads in individual event
n = number of trails of event
and
k = number of success i.e favourable outcomes of tails
For n=500, assuming equi-probable outcomes and only 1 success.
Theorotically,

$$P(A) = \frac{1}{2^{500}} \approx 0$$

Using Binomial distribution,

$$P(A = 1) = {}^{500}C_1 * (0.5)^1 * (0.5)^{500-1} \tag{6}$$

$$\Rightarrow P(A = 1) = 500 * 0.5^{500}$$
$$\Rightarrow P(A = 1) \approx 0$$

The PDF is of no relevance as P(A) = 0 will be more and it can be depicted as below.

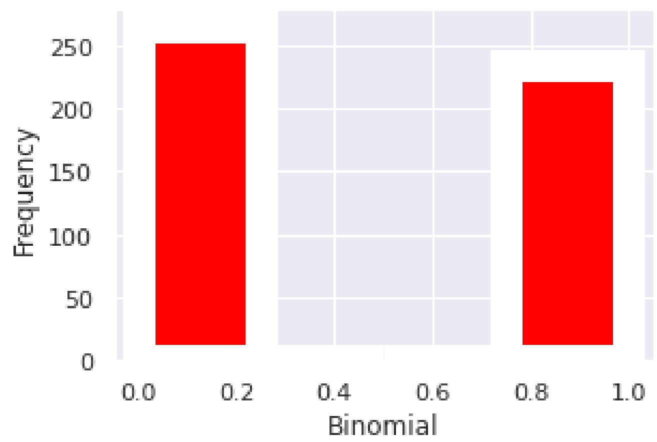

Fig. 0: Binomial Distriution of 500 tosses with 1 tail success.

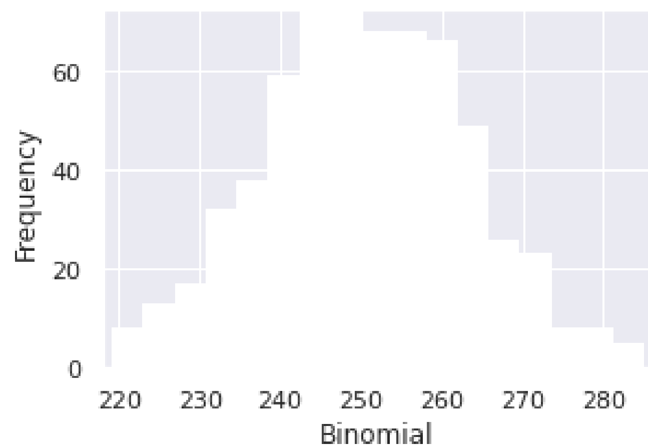

Fig. 0: Binomial Distriution of 500 tosses with many success.

If only 1 success is available then

$$P(B/(A=1)) = P(B) \Rightarrow = \frac{1}{3}$$

Here given condition is "atleast one tail".Then k varies from 1 to 500 but not 0.Theorotically,

$$P(A) = 1 - \frac{1}{2^{500}} \approx 1$$

Using Binomial distribution,

$$P(A > 0) = 1 - ({}^{500}C_{499} * (0.5)^1 * (0.5)^{499}) \quad (7)$$

$$\Rightarrow P(A > 0) = 1 - (500 * 0.5^{500})$$
$$\Rightarrow P(A > 0) \approx 1$$

The PDF can be depicted as below

When such success of tails are available as outcomes, the success of dice showing number greater than 4 also increases. This also follows a binomial distribution with total number of trials are number of successful outcomes i.e tails as outcome of coin tossing.Also, the event B is independent of event A.

Using Binomial distribution,

$$P(B > 0) = ({}^{499}C_k * (\frac{1}{3})^k * (\frac{2}{3})^{499-k}) \quad (8)$$

The PDF is given by

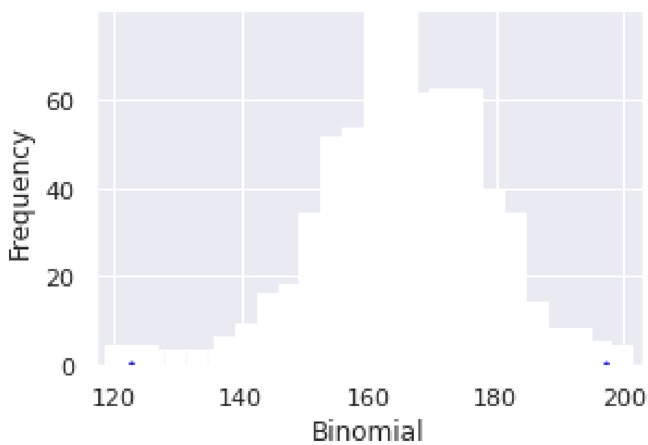

Fig. 0: Binomial Distriution of Dice with many success i.e number greater than 4.