# SM5083 - BASICS OF PROGRAMMING

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*Abstract*—**This paper contains solution to problem no 5 of Examples III Section of Chapter III of Analytical Geometry by Hukum Chand. Links to Python codes are available below.**

Download python codes at

```
https://github.com/rsgirishkumar/SM5083/
    ASSIGNMENT2
```

## 1 Problem

The opposite vertices of a square are $\begin{pmatrix} 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 3 \end{pmatrix}$. Find the equations of four sides.

## 2 Solution

Let the given points are indicated as below

$$\mathbf{A} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}. \tag{2.0.1}$$

Let the unknown vertices are indicated as $\mathbf{B}, \mathbf{D}$. The step by step procedure involves

1) Find the diagonal **AC**.
2) Find the norm of **AC**.
3) Find the orthogonal of **AC** i.e. **BD** by using orthogonal matrix.
4) Find the midpoint of **AC**.
5) Using the norm of **BD**, find the vertices of **BD**.
6) Form the equations of lines using vertices.

**Step-1:** Diagonal **AC**

$$\mathbf{AC} = \begin{pmatrix} 0-0 \\ 3+1 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \end{pmatrix} \tag{2.0.2}$$

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**Step-2:** Norm of **AC**

$$\|\mathbf{AC}\| = \sqrt{0+4^2} = 4 \tag{2.0.3}$$

**Step-3:** Orthogonal of **AC**. i.e, **BD**.
Consider an 2x2 orthogonal matrix O be $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$.

$$\mathbf{BD} = AC * O = \begin{pmatrix} 0 & 4 \end{pmatrix} * \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} -4 \\ 0 \end{pmatrix} \tag{2.0.4}$$

**Step-4:** Midpoint of **AC** or **BD**.

$$MidpointM = \begin{pmatrix} 0 \\ \frac{(3-1)}{2} \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.5}$$

**Step-5:** Vertices of **BD**. Taking counter-clockwise and norm = 4,

$$B = \begin{pmatrix} x\_midpoint - \frac{norm}{2} \\ y\_midpoint \end{pmatrix} = \begin{pmatrix} 0-2 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$
$$D = \begin{pmatrix} x\_midpoint + \frac{norm}{2} \\ y\_midpoint \end{pmatrix} = \begin{pmatrix} 0+2 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \tag{2.0.6}$$

**Step-6:** Line Equations. Coordinates are

$$\mathbf{A} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}. \tag{2.0.7}$$

When two coordinates $\begin{pmatrix} X_1 \\ Y_1 \end{pmatrix}\begin{pmatrix} X_2 \\ Y_2 \end{pmatrix}$ are given, then the line equation is given by $\frac{(Y-Y1)}{(Y2-Y1)} = \frac{(X-X1)}{(X2-X1)}$
and on comparison with the form ax+by+c=0,
a= $Y_2 - Y_1, b = X_1 - X_2$,
$c = (-(Y_2 - Y_1) * X_1) + ((X_2 - X_1) * Y_1)$.
In Matrix form, $PT_1 = \begin{pmatrix} X_1 \\ Y_1 \end{pmatrix}, PT_2 = \begin{pmatrix} X_2 \\ Y_2] \end{pmatrix}$ and vector
$DV_1 = \mathbf{PT_2} - \mathbf{PT_1} = \begin{pmatrix} X_2 - X_1 \\ Y_2 - Y_1 \end{pmatrix}$
then, $a = DV_1[1]$,
$b = -DV_1[0]$,
$c = \begin{pmatrix} PT_1[1] & -PT_1[0] \end{pmatrix} * \begin{pmatrix} DV1.T \end{pmatrix}$

By using the same, the line equation AB for points A = $\begin{pmatrix}0\\-1\end{pmatrix}$,B=$\begin{pmatrix}-2\\1\end{pmatrix}$ is as follows:

$$\begin{aligned}\mathbf{AB} &= \begin{pmatrix}-2\\2\end{pmatrix}\\ a = 2,\ b &= 2,\\ c = \begin{pmatrix}-1 & 0\end{pmatrix} * \begin{pmatrix}-2\\2\end{pmatrix} &= 2\\ Line\ equation\ for\ \mathbf{AB}\\ \Rightarrow\ 2x + 2y + 2 = 0\ or\ x + y &= -1.\\ In\ vectorform\\ \Rightarrow \begin{pmatrix}1 & 1\end{pmatrix} x &= -1\end{aligned} \tag{2.0.8}$$

The line equation BC for points B = $\begin{pmatrix}-2\\1\end{pmatrix}$,C=$\begin{pmatrix}0\\3\end{pmatrix}$ is as follows:

$$\begin{aligned}\mathbf{BC} &= \begin{pmatrix}2\\2\end{pmatrix}\\ a = 2,\ b &= -2,\\ c = \begin{pmatrix}1 & 2\end{pmatrix} * \begin{pmatrix}2\\2\end{pmatrix} &= 6\\ Line\ equation\ for\ \mathbf{BC}\\ \Rightarrow\ 2x - 2y + 6 = 0\ or\ x - y &= -3.\\ In\ vectorform\\ \Rightarrow \begin{pmatrix}1 & -1\end{pmatrix} x &= -3\end{aligned} \tag{2.0.9}$$

The line equation CD for points C=$\begin{pmatrix}0\\3\end{pmatrix}$,D = $\begin{pmatrix}2\\1\end{pmatrix}$ is as follows:

$$\begin{aligned}\mathbf{CD} &= \begin{pmatrix}2\\-2\end{pmatrix}\\ a = -2,\ b &= -2,\\ c = \begin{pmatrix}3 & 0\end{pmatrix} * \begin{pmatrix}2\\-2\end{pmatrix} &= 6\\ Line\ equation\ for\ \mathbf{CD}\\ \Rightarrow\ -2x - 2y + 6 = 0\ or\ x + y &= 3.\\ In\ vectorform\\ \Rightarrow \begin{pmatrix}1 & 1\end{pmatrix} x &= 3\end{aligned} \tag{2.0.10}$$

The line equation DA for points D = $\begin{pmatrix}2\\1\end{pmatrix}$, A = $\begin{pmatrix}0\\-1\end{pmatrix}$ is as follows:

$$\begin{aligned}\mathbf{DA} &= \begin{pmatrix}-2\\-2\end{pmatrix}\\ a = -2,\ b &= 2,\\ c = \begin{pmatrix}1 & -2\end{pmatrix} * \begin{pmatrix}-2\\-2\end{pmatrix} &= 2\\ Line\ equation\ for\ \mathbf{DA}\\ \Rightarrow\ -2x + 2y + 2 = 0\ or\ x - y &= 1.\\ In\ vectorform\\ \Rightarrow \begin{pmatrix}1 & -1\end{pmatrix} x &= 1\end{aligned} \tag{2.0.11}$$

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The plotted graph is shown as below.

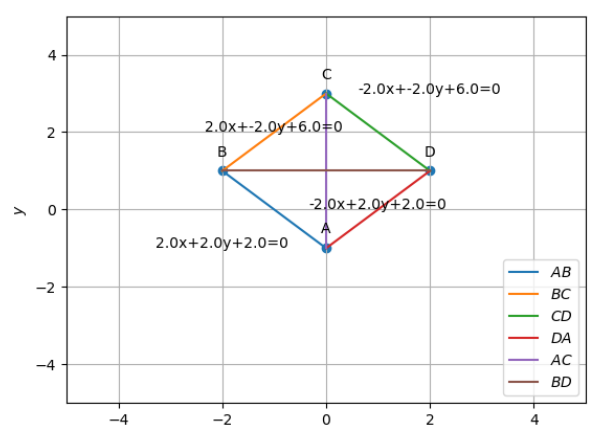


Fig. 6: Square ABCD