SM5083 - BASICS OF PROGRAMMING

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Contents

***Abstract*—This paper contains solution to problem no 5 of Examples III Section of Chapter III of Analytical Geometry by Hukum Chand. Links to Python codes are available below.**

Download python codes at

https://github.com/rsgirishkumar/SM5083/ASSIGNMENT2

1 Problem

The opposite vertices of a square are $\begin{pmatrix} 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 3 \end{pmatrix}$. Find the equations of four sides.

2 Solution

Let the given points are indicated as below

$$\mathbf{A} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}. \tag{2.0.1}$$

Let the unknown vertices are indicated as $\mathbf{B}, \mathbf{D}$. The step by step procedure involves

1) By doing affine transformation steps i.e Translation and Rotation of the given vertices $\mathbf{A}$ and $\mathbf{C}$ those provide $\mathbf{A}'$ and $\mathbf{C}'$ for the ease of solution
2) Find the diagonal $\mathbf{AC}$.
3) Find the norm of $\mathbf{AC}$.
4) Find the points $\mathbf{B}$ and $\mathbf{D}$ by using inspection method and reverse affine transformation.
5) Form the equations of lines using vertices.

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2.1 AFFINE TRANSFORMATION

Translation

$$\mathbf{A} = \begin{pmatrix} 0 \\ -1 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$
$$\mathbf{C} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}. \tag{2.1.1}$$

Rotation The rotation is by 45^o clock wise. The rotation matrix is given by

$$\begin{pmatrix} cos(-45^o) & -sin(-45^o) \\ sin(-45^o) & cos(-45^o) \end{pmatrix}$$
$$\Rightarrow \mathbf{A}' = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2.1.2}$$
$$\Rightarrow \mathbf{C}' = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 0 \\ 4 \end{pmatrix} = \begin{pmatrix} 2\sqrt{2} \\ 2\sqrt{2} \end{pmatrix}.$$

Step-1: Diagonal $\mathbf{A}'\mathbf{C}'$

$$\mathbf{A}'\mathbf{C}' = \begin{pmatrix} 2\sqrt{2} - 0 \\ 2\sqrt{2} - 0 \end{pmatrix} = \begin{pmatrix} 2\sqrt{2} \\ 2\sqrt{2} \end{pmatrix} \tag{2.1.3}$$

The line equation of $\mathbf{A}'\mathbf{C}'$ is x-y=0 or simply x=y.
Step-2: Norm of $\mathbf{A}'\mathbf{C}'$

$$\|\mathbf{A}'\mathbf{C}'\| = \sqrt{(2\sqrt{2})^2 + (2\sqrt{2})^2} = 4 \tag{2.1.4}$$

Step-3: The angle between line $\mathbf{A}'\mathbf{B}'$ and $\mathbf{A}'\mathbf{C}'$ will be 45^o. Hence, the line equation will be y=0.The angle between line $\mathbf{A}'\mathbf{D}'$ and $\mathbf{A}'\mathbf{C}'$ will be 45^o. Hence, the line equation will be x=0.
Length of any side s= $\frac{\|\mathbf{A}'\mathbf{C}'\|}{\sqrt{2}} = 2\sqrt{2}$.
By inspection method, using length of a side, the vertices $\mathbf{B}'$ and $\mathbf{D}'$ can be found.

$$\mathbf{B}' = \begin{pmatrix} 0 + 2\sqrt{2} \\ 0 \end{pmatrix} = \begin{pmatrix} 2\sqrt{2} \\ 0 \end{pmatrix}$$
$$\mathbf{D}' = \begin{pmatrix} 0 \\ 0 + 2\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ 2\sqrt{2} \end{pmatrix} \tag{2.1.5}$$

Hence the vertices are

$$\mathbf{A}' = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B}' = \begin{pmatrix} 2\sqrt{2} \\ 0 \end{pmatrix} \mathbf{C}' = \begin{pmatrix} 2\sqrt{2} \\ 2\sqrt{2} \end{pmatrix} . \mathbf{D}' = \begin{pmatrix} 0 \\ 2\sqrt{2} \end{pmatrix}. \tag{2.1.6}$$

2.2 REVERSE AFFINE TRANSFORMATION

Rotation The rotation is by 45^o counter clock wise. The rotation matrix is given by

$$\begin{pmatrix} cos(45^o) & -sin(45^o) \\ sin(45^o) & cos(45^o) \end{pmatrix}$$

$$\Rightarrow \mathbf{A}' = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$
$$\Rightarrow \mathbf{B}' = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 2\sqrt{2} \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \tag{2.2.1}$$
$$\Rightarrow \mathbf{C}' = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 2\sqrt{2} \\ 2\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}.$$
$$\Rightarrow \mathbf{D}' = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 0 \\ 2\sqrt{2} \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$$

Translation

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$
$$\mathbf{B} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} + \begin{pmatrix} 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \tag{2.2.2}$$
$$\mathbf{C} = \begin{pmatrix} 0 \\ 4 \end{pmatrix} + \begin{pmatrix} 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}.$$
$$\mathbf{D} = \begin{pmatrix} -2 \\ 2 \end{pmatrix} + \begin{pmatrix} 0 \\ -1 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

2.3 LINE EQUATIONS

Coordinates are

$$\mathbf{A} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}. \tag{2.3.1}$$

When two coordinates $\begin{pmatrix} X_1 \\ Y_1 \end{pmatrix}, \begin{pmatrix} X_2 \\ Y_2 \end{pmatrix}$ are given, then the line equation is given by

$$\frac{(Y - Y1)}{(Y2 - Y1)} = \frac{(X - X1)}{(X2 - X1)} \tag{2.3.2}$$

and on comparison with the form ax+by+c=0,

$$a = Y_2 - Y_1,$$
$$b = X_1 - X_2, \tag{2.3.3}$$
$$c = (-(Y_2 - Y_1) * X_1) + ((X_2 - X_1) * Y_1)$$

.

In Matrix form, $\mathbf{PT_1} = \begin{pmatrix} X_1 \\ Y_1 \end{pmatrix}, \mathbf{PT_2} = \begin{pmatrix} X_2 \\ Y_2 \end{pmatrix}$ and vector $\mathbf{DV_1} = \mathbf{PT_2} - \mathbf{PT_1} = \begin{pmatrix} X_2 - X_1 \\ Y_2 - Y_1 \end{pmatrix}$

then,

$$a = \mathbf{DV_1[1]},$$
$$b = -\mathbf{DV_1[0]}, \tag{2.3.4}$$
$$c = \begin{pmatrix} \mathbf{PT_1[1]} & -\mathbf{PT_1[0]} \end{pmatrix} \begin{pmatrix} \mathbf{DV_1}.T \end{pmatrix}$$

By using the same, the line equation **AB** for points $\mathbf{A} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$ is as follows:

$$\mathbf{AB} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$$
$$a = 2,\ b = 2, \tag{2.3.5}$$
$$c = \begin{pmatrix} -1 & 0 \end{pmatrix} \begin{pmatrix} -2 \\ 2 \end{pmatrix} = 2$$

Line equation for **AB**

$$\Rightarrow\ 2x + 2y + 2 = 0\ or\ x + y = -1. \tag{2.3.6}$$

In vector form

$$\Rightarrow \begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{X} = -1 \tag{2.3.7}$$

The line equation **BC** for points $\mathbf{B} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$ is as follows:

$$\mathbf{BC} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$
$$a = 2,\ b = -2, \tag{2.3.8}$$
$$c = \begin{pmatrix} 1 & 2 \end{pmatrix} \begin{pmatrix} 2 \\ 2 \end{pmatrix} = 6$$

Line equation for **BC**

$$\Rightarrow\ 2x - 2y + 6 = 0\ or\ x - y = -3. \tag{2.3.9}$$

In vector form

$$\Rightarrow \begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{X} = -3 \tag{2.3.10}$$

The line equation **CD** for points $\mathbf{C} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ is as follows:

$$\mathbf{CD} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$$
$$a = -2,\ b = -2, \tag{2.3.11}$$
$$c = \begin{pmatrix} 3 & 0 \end{pmatrix} \begin{pmatrix} 2 \\ -2 \end{pmatrix} = 6$$

Line equation for **CD**

$$\Rightarrow \quad -2x - 2y + 6 = 0 \ or \ x + y = 3. \tag{2.3.12}$$

In vector form

$$\Rightarrow \begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{X} = 3 \tag{2.3.13}$$

The line equation **DA** for points $\mathbf{D} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \mathbf{A} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$ is as follows:

$$\mathbf{DA} = \begin{pmatrix} -2 \\ -2 \end{pmatrix}$$
$$a = -2,\ b = 2, \tag{2.3.14}$$
$$c = \begin{pmatrix} 1 & -2 \end{pmatrix} \begin{pmatrix} -2 \\ -2 \end{pmatrix} = 2$$

Line equation for **DA**

$$\Rightarrow \quad -2x + 2y + 2 = 0 \ or \ x - y = 1. \tag{2.3.15}$$

In vector form

$$\Rightarrow \begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{X} = 1 \tag{2.3.16}$$

.
The plotted graph is shown as below.

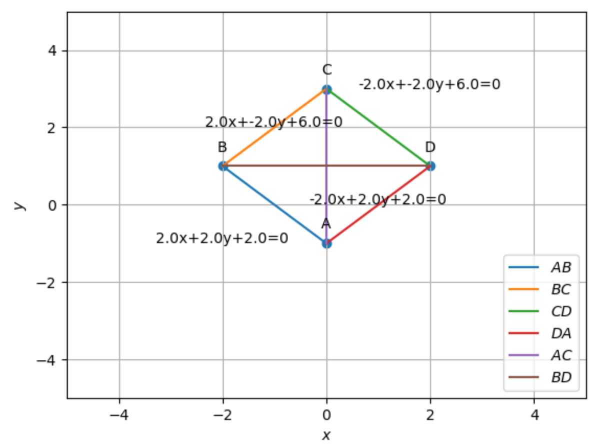

Fig. 5: Square ABCD