

SM5083 - BASICS OF PROGRAMMING

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Abstract—This paper contains solution to problem no 9(i) of Examples II Section of Analytical Geometry by Hukum Chand. Links to Python codes are available below.

Download python codes at

<https://github.com/rsgirishkumar/SM5083/ASSIGNMENT1>

1 PROBLEM

Find the area of the quadrilateral formed by the points

$$\mathbf{A} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \mathbf{B} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} \mathbf{C} = \begin{pmatrix} -2 \\ 4 \end{pmatrix} \mathbf{D} = \begin{pmatrix} -1 \\ -5 \end{pmatrix} \quad (1.0.1)$$

2 SOLUTION

Let the given points are indicated as below

$$\begin{aligned} \mathbf{A} &= \begin{pmatrix} 1 \\ 1 \end{pmatrix} \mathbf{B} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} \\ \mathbf{C} &= \begin{pmatrix} -2 \\ 4 \end{pmatrix} \mathbf{D} = \begin{pmatrix} -1 \\ -5 \end{pmatrix} \end{aligned} \quad (2.0.1)$$

Step1: Let us check whether the given points form a quadrilateral or not. This can be ascertained by collinearity check of any two sets of points i.e. either $(\mathbf{A}, \mathbf{B}, \mathbf{C})$ or $(\mathbf{A}, \mathbf{C}, \mathbf{D})$ or $(\mathbf{B}, \mathbf{C}, \mathbf{D})$ or $(\mathbf{A}, \mathbf{B}, \mathbf{D})$

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2.1 Collinearity Check

Collinearity Check of points A, B, D i.e.

$$\mathbf{A} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} -1 \\ -5 \end{pmatrix} \quad (2.1.1)$$

In Vector approach, If the rank of a matrix formed by the vectors of points $\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}, \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}, \begin{pmatrix} x_3 \\ y_3 \end{pmatrix}$ is **1 or 2** for any 3 X 3 matrix then the points are said to be **collinear**. If $\rho(\text{matrix})=3$ then the points are non collinear.

$$\Rightarrow \rho \begin{pmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{pmatrix} = 3 \quad (2.1.2)$$

Here the vectors are as below.

$$\mathbf{AB} = \mathbf{B} - \mathbf{A} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \quad (2.1.3)$$

$$\mathbf{BD} = \mathbf{D} - \mathbf{B} = \begin{pmatrix} -1 \\ -5 \end{pmatrix} - \begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} -4 \\ -10 \end{pmatrix} \quad (2.1.4)$$

$$\mathbf{DA} = \mathbf{A} - \mathbf{D} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} - \begin{pmatrix} -1 \\ -5 \end{pmatrix} = \begin{pmatrix} 2 \\ 6 \end{pmatrix} \quad (2.1.5)$$

The matrix formed by vectors is

$$\begin{pmatrix} 2 & 4 & 1 \\ -4 & -10 & 1 \\ 2 & 6 & 1 \end{pmatrix}$$

Rank of the matrix: By reduction.

$$\begin{aligned} \Rightarrow \begin{pmatrix} 2 & 4 & 1 \\ -4 & -10 & 1 \\ 2 & 6 & 1 \end{pmatrix} &\xleftrightarrow{R_2 \leftrightarrow R_3} \begin{pmatrix} 2 & 4 & 1 \\ 2 & 6 & 1 \\ -4 & -10 & 1 \end{pmatrix} \xleftrightarrow{R_3 \leftrightarrow R_1 + R_2 + R_3} \\ &\begin{pmatrix} 2 & 4 & 1 \\ 2 & 6 & 1 \\ 0 & 0 & 3 \end{pmatrix} \xleftrightarrow{R_2 \leftrightarrow R_2 - R_1} \begin{pmatrix} 2 & 4 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} \end{aligned} \quad (2.1.6)$$

The number of non-zero rows in the matrix = 3.
 $\Rightarrow \rho(\text{matrix}) = 3$.

Here the **AB**, **BD**, **DA** are not collinear.

is given by

Let us examine the lines generated by the given points in the Figure below:

$$\Delta \mathbf{ACD} = \frac{1}{2} \begin{vmatrix} 1 & 1 & 1 \\ -2 & 4 & 1 \\ -1 & -5 & 1 \end{vmatrix} = 11.5 \quad (2.2.3)$$

The area of quadrilateral ABCD = 9 + 11.5 = 20.5

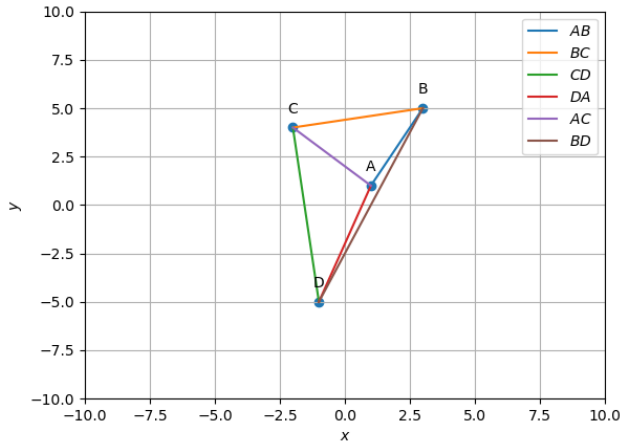


Fig. 0: Quadrilateral ABCD

The above figure clearly depicts the other set of points are not collinear. Hence the points given form a Quadrilateral.

2.2 Area of a Quadrilateral

Area of $\square \mathbf{ABCD} =$

Area of Triangle $\Delta \mathbf{ACD} +$

Area of Triangle $\Delta \mathbf{ABC}$

\therefore this does not fall into category of Square, Parallelogram or Rhombus, Trapezium.

Area of a Triangle formed by points $\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}, \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}, \begin{pmatrix} x_3 \\ y_3 \end{pmatrix}$ is given by

$$\frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \quad (2.2.1)$$

Area of Triangle formed by $\mathbf{A} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$ is given by

$$\Delta \mathbf{ABD} = \frac{1}{2} \begin{vmatrix} 1 & 1 & 1 \\ 3 & 5 & 1 \\ -2 & 4 & 1 \end{vmatrix} = 9 \quad (2.2.2)$$

Area of Triangle formed by $\mathbf{A} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} -1 \\ -5 \end{pmatrix}$