

SM5083 - BASICS OF PROGRAMMING

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Abstract—This paper contains solution to problem no 5 of Examples III Section of Chapter III of Analytical Geometry by Hukum Chand. Links to Python codes are available below.

Download python codes at

<https://github.com/rsgirishkumar/SM5083/ASSIGNMENT2>

1 PROBLEM

The opposite vertices of a square are $\begin{pmatrix} 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 3 \end{pmatrix}$.
Find the equations of four sides.

2 SOLUTION

Let the given points are indicated as below

$$\begin{aligned} \mathbf{A} &= \begin{pmatrix} 0 \\ -1 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \\ \mathbf{C} &= \begin{pmatrix} 0 \\ 3 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}. \end{aligned} \quad (2.0.1)$$

Let the unknown vertices are indicated as **B, D**. The step by step procedure involves

- 1) Find the diagonal **AC**.
- 2) Find the norm of **AC**.
- 3) Find the orthogonal of **AC** i.e. **BD** by using orthogonal matrix.
- 4) Find the midpoint of **AC**.
- 5) Using the norm of **BD**, find the vertices of **BD**.
- 6) Form the equations of lines using vertices.

Step-1: Diagonal **AC**

$$\mathbf{AC} = \begin{pmatrix} 0-0 \\ 4-0 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \end{pmatrix} \quad (2.0.2)$$

Step-2: Norm of **AC**

$$\|\mathbf{AC}\| = \sqrt{0+4^2} = 4 \quad (2.0.3)$$

Step-3: Orthogonal of **AC**. i.e, **BD**.

Consider an 2x2 orthogonal matrix **O** be $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$.

$$\mathbf{BD} = \mathbf{AC} * \mathbf{O} = \begin{pmatrix} 0 & 4 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} -4 \\ 0 \end{pmatrix} \quad (2.0.4)$$

Step-4: Midpoint of **AC** or **BD**.

$$\text{Midpoint } \mathbf{M} = \begin{pmatrix} 0 \\ \frac{0+4}{2} \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \quad (2.0.5)$$

Step-5: Vertices of **BD.** Taking counter-clockwise and norm = 4,

$$\begin{aligned} \mathbf{B} &= \begin{pmatrix} \mathbf{M}[0] - \frac{\text{norm}}{2} \\ \mathbf{M}[1] \end{pmatrix} = \begin{pmatrix} 0-2 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \end{pmatrix} \\ \mathbf{D} &= \begin{pmatrix} \mathbf{M}[0] + \frac{\text{norm}}{2} \\ \mathbf{M}[1] \end{pmatrix} = \begin{pmatrix} 0+2 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}. \end{aligned} \quad (2.0.6)$$

Step-6: Line Equations. Coordinates are

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}. \quad (2.0.7)$$

When two coordinates $\begin{pmatrix} X_1 \\ Y_1 \end{pmatrix} \begin{pmatrix} X_2 \\ Y_2 \end{pmatrix}$ are given, then the line equation is given by

$$\frac{(Y - Y_1)}{(Y_2 - Y_1)} = \frac{(X - X_1)}{(X_2 - X_1)} \quad (2.0.8)$$

and on comparison with the form $ax+by+c=0$,

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$$\begin{aligned} a &= Y_2 - Y_1, \\ b &= X_1 - X_2, \end{aligned} \quad (2.0.9)$$

$$c = -(Y_2 - Y_1) * X_1 + ((X_2 - X_1) * Y_1).$$

In Matrix form, $\mathbf{PT}_1 = \begin{pmatrix} X_1 \\ Y_1 \end{pmatrix}$, $\mathbf{PT}_2 = \begin{pmatrix} X_2 \\ Y_2 \end{pmatrix}$ and vector $\mathbf{DV}_1 = \mathbf{PT}_2 - \mathbf{PT}_1 = \begin{pmatrix} X_2 - X_1 \\ Y_2 - Y_1 \end{pmatrix}$ then,

$$\begin{aligned} a &= \mathbf{DV}_1[1], \\ b &= -\mathbf{DV}_1[0], \\ c &= (\mathbf{PT}_1[1] \quad -\mathbf{PT}_1[0]) (\mathbf{DV}_1.T) \end{aligned} \quad (2.0.10)$$

By using the same, the line equation \mathbf{AB} for points $\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$ is as follows:

$$\begin{aligned} \mathbf{AB} &= \begin{pmatrix} -2 \\ 2 \end{pmatrix} \\ a &= 2, \quad b = 2, \\ c &= (0 \quad 0) \begin{pmatrix} -2 \\ 2 \end{pmatrix} = 0 \end{aligned} \quad (2.0.11)$$

Line equation for \mathbf{AB}

$$\Rightarrow 2x + 2y + 0 = 0 \text{ or } x = -y. \quad (2.0.12)$$

In vector form

$$\Rightarrow (1 \quad 1) \mathbf{X} = 0 \quad (2.0.13)$$

The line equation \mathbf{BC} for points $\mathbf{B} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$, $\mathbf{C} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$ is as follows:

$$\begin{aligned} \mathbf{BC} &= \begin{pmatrix} 2 \\ 2 \end{pmatrix} \\ a &= 2, \quad b = -2, \\ c &= (2 \quad 2) \begin{pmatrix} 2 \\ 2 \end{pmatrix} = 8 \end{aligned} \quad (2.0.14)$$

Line equation for \mathbf{BC}

$$\Rightarrow 2x - 2y + 8 = 0 \text{ or } x - y = -4. \quad (2.0.15)$$

In vector form

$$\Rightarrow (1 \quad -1) \mathbf{X} = -4 \quad (2.0.16)$$

The line equation \mathbf{CD} for points $\mathbf{C} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$, $\mathbf{D} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$ is as follows:

$$\begin{aligned} \mathbf{CD} &= \begin{pmatrix} 2 \\ -2 \end{pmatrix} \\ a &= -2, \quad b = -2, \\ c &= (4 \quad 0) \begin{pmatrix} 2 \\ -2 \end{pmatrix} = 8 \end{aligned} \quad (2.0.17)$$

Line equation for \mathbf{CD}

$$\Rightarrow -2x - 2y + 8 = 0 \text{ or } x + y = 4. \quad (2.0.18)$$

In vector form

$$\Rightarrow (1 \quad 1) \mathbf{X} = 4 \quad (2.0.19)$$

The line equation \mathbf{DA} for points $\mathbf{D} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$, $\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ is as follows:

$$\begin{aligned} \mathbf{DA} &= \begin{pmatrix} -2 \\ -2 \end{pmatrix} \\ a &= -2, \quad b = 2, \\ c &= (2 \quad -2) \begin{pmatrix} -2 \\ -2 \end{pmatrix} = 0 \end{aligned} \quad (2.0.20)$$

Line equation for \mathbf{DA}

$$\Rightarrow -2x + 2y + 0 = 0 \text{ or } x = y. \quad (2.0.21)$$

In vector form

$$\Rightarrow (1 \quad -1) \mathbf{X} = 0 \quad (2.0.22)$$

The plotted graph is shown as below.

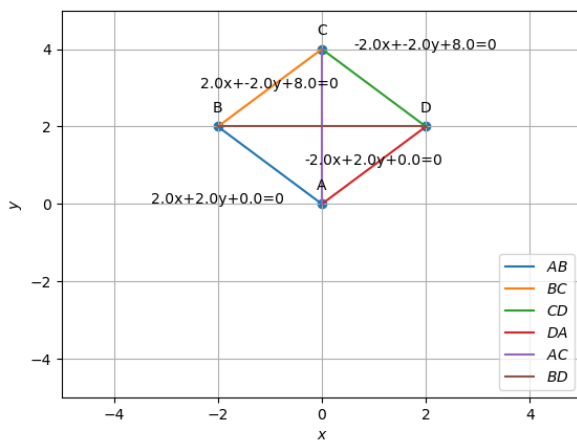


Fig. 6: Square ABCD