

SM5083 - BASICS OF PROGRAMMING

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CONTENTS

Abstract—This paper contains solution to problem no 5 of Examples III Section of Chapter III of Analytical Geometry by Hukum Chand. Links to Python codes are available below.

Download python codes at

<https://github.com/rsgirishkumar/SM5083/ASSIGNMENT2>

1 PROBLEM

The opposite vertices of a square are $\begin{pmatrix} 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 3 \end{pmatrix}$.
Find the equations of four sides.

2 SOLUTION

Let the given points are indicated as below

$$\mathbf{A} = \begin{pmatrix} 0 \\ -1 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}. \quad (2.0.1)$$

Let the unknown vertices are indicated as **B, D**. The step by step procedure involves

- 1) Find the diagonal **AC**.
- 2) Find the norm of **AC**.
- 3) Find the orthogonal of **AC** i.e. **BD** by using orthogonal matrix.
- 4) Find the midpoint of **AC**.
- 5) Using the norm of **BD**, find the vertices of **BD**.
- 6) Form the equations of lines using vertices.

Step-1: Diagonal AC

$$\mathbf{AC} = \begin{pmatrix} 0-0 \\ 4-0 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \end{pmatrix} \quad (2.0.2)$$

Step-2: Norm of AC

$$\|\mathbf{AC}\| = \sqrt{0^2 + 4^2} = 4 \quad (2.0.3)$$

Step-3: Orthogonal of AC. i.e. BD.

Consider an 2x2 orthogonal matrix O be $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$.

$$\mathbf{BD} = \mathbf{AC} * \mathbf{O} = \begin{pmatrix} 0 & 4 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} -4 \\ 0 \end{pmatrix} \quad (2.0.4)$$

Step-4: Midpoint of AC or BD.

$$\text{Midpoint } M = \begin{pmatrix} 0 \\ \frac{4-0}{2} \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \quad (2.0.5)$$

Step-5: Vertices of **BD**. Taking counter-clockwise and norm = 4,

$$\begin{aligned} B &= \begin{pmatrix} x_{\text{midpoint}} - \frac{\text{norm}}{2} \\ y_{\text{midpoint}} \end{pmatrix} = \begin{pmatrix} 0-2 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \end{pmatrix} \\ D &= \begin{pmatrix} x_{\text{midpoint}} + \frac{\text{norm}}{2} \\ y_{\text{midpoint}} \end{pmatrix} = \begin{pmatrix} 0+2 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \end{aligned} \quad (2.0.6)$$

Step-6: Line Equations. Coordinates are

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}. \quad (2.0.7)$$

When two coordinates $\begin{pmatrix} X_1 \\ Y_1 \end{pmatrix} \begin{pmatrix} X_2 \\ Y_2 \end{pmatrix}$ are given, then the

line equation is given by $\frac{(Y-Y_1)}{(Y_2-Y_1)} = \frac{(X-X_1)}{(X_2-X_1)}$ and on comparison with the form $ax+by+c=0$,

$$\begin{aligned} a &= Y_2 - Y_1, b = X_1 - X_2, \\ c &= -(Y_2 - Y_1) * X_1 + ((X_2 - X_1) * Y_1). \end{aligned}$$

In Matrix form, $PT_1 = \begin{pmatrix} X_1 \\ Y_1 \end{pmatrix}, PT_2 = \begin{pmatrix} X_2 \\ Y_2 \end{pmatrix}$ and vector

$$DV_1 = PT_2 - PT_1 = \begin{pmatrix} X_2 - X_1 \\ Y_2 - Y_1 \end{pmatrix}$$

then, $a = DV_1[1]$,

$b = -DV_1[0]$,

$$c = (PT_1[1] - PT_1[0]) * (DV_1.T)$$

By using the same, the line equation AB for points $\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$ is as follows:

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$$\begin{aligned}
 \mathbf{AB} &= \begin{pmatrix} -2 \\ 2 \end{pmatrix} \\
 a &= 2, \quad b = 2, \\
 c &= \begin{pmatrix} 0 & 0 \end{pmatrix} \begin{pmatrix} -2 \\ 2 \end{pmatrix} = 0 \\
 \text{Line equation for } \mathbf{AB} \\
 \Rightarrow 2x + 2y + 0 &= 0 \text{ or } x = -y. \\
 \text{In vector form} \\
 \Rightarrow \begin{pmatrix} 1 & 1 \end{pmatrix} x &= 0
 \end{aligned} \tag{2.0.8}$$

The line equation BC for points $B = \begin{pmatrix} -2 \\ 2 \end{pmatrix}, C = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$ is as follows:

$$\begin{aligned}
 \mathbf{BC} &= \begin{pmatrix} 2 \\ 2 \end{pmatrix} \\
 a &= 2, \quad b = -2, \\
 c &= \begin{pmatrix} 2 & 2 \end{pmatrix} \begin{pmatrix} 2 \\ 2 \end{pmatrix} = 8 \\
 \text{Line equation for } \mathbf{BC} \\
 \Rightarrow 2x - 2y + 8 &= 0 \text{ or } x - y = -4. \\
 \text{In vector form} \\
 \Rightarrow \begin{pmatrix} 1 & -1 \end{pmatrix} x &= -4
 \end{aligned} \tag{2.0.9}$$

The line equation CD for points $C = \begin{pmatrix} 0 \\ 4 \end{pmatrix}, D = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$ is as follows:

$$\begin{aligned}
 \mathbf{CD} &= \begin{pmatrix} 2 \\ -2 \end{pmatrix} \\
 a &= -2, \quad b = -2, \\
 c &= \begin{pmatrix} 4 & 0 \end{pmatrix} \begin{pmatrix} 2 \\ -2 \end{pmatrix} = 8 \\
 \text{Line equation for } \mathbf{CD} \\
 \Rightarrow -2x - 2y + 8 &= 0 \text{ or } x + y = 4. \\
 \text{In vector form} \\
 \Rightarrow \begin{pmatrix} 1 & 1 \end{pmatrix} x &= 4
 \end{aligned} \tag{2.0.10}$$

The line equation DA for points $D = \begin{pmatrix} 2 \\ 2 \end{pmatrix}, A = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ is as follows:

$= \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ is as follows:

$$\begin{aligned}
 \mathbf{DA} &= \begin{pmatrix} -2 \\ -2 \end{pmatrix} \\
 a &= -2, \quad b = 2, \\
 c &= \begin{pmatrix} 2 & -2 \end{pmatrix} \begin{pmatrix} -2 \\ -2 \end{pmatrix} = 0 \\
 \text{Line equation for } \mathbf{DA} \\
 \Rightarrow -2x + 2y + 0 &= 0 \text{ or } x = y. \\
 \text{In vector form} \\
 \Rightarrow \begin{pmatrix} 1 & -1 \end{pmatrix} x &= 0
 \end{aligned} \tag{2.0.11}$$

The plotted graph is shown as below.

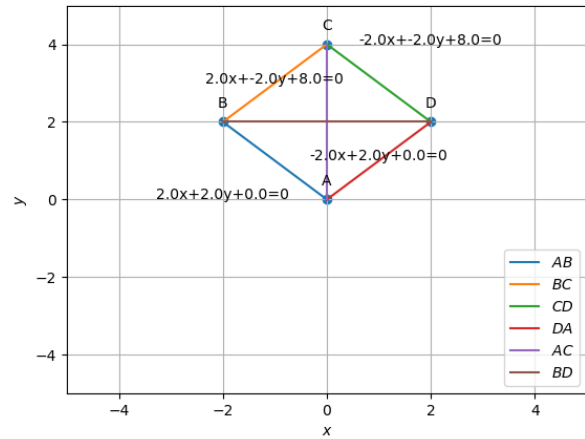


Fig. 6: Square ABCD