SM5083 - BASICS OF PROGRAMMING

RS Girish - EE20RESCH14005*

Contents

***Abstract*—This paper contains solution to problem no 9(i) of Examples II Section of Analytical Geometry by Hukum Chand. Links to Python codes are available below.**

Download python codes at

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https://github.com/rsgirishkumar/SM5083/
    ASSIGNMENT1
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1 Problem

Find the area of the quadrilateral formed by the points

$$\mathbf{A} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} -1 \\ -5 \end{pmatrix}. \quad (1.0.1)$$

2 Solution

Let the given points are indicated as below

$$\mathbf{A} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} -1 \\ -5 \end{pmatrix}. \quad (2.0.1)$$

Step1: Let us check whether the given points form a quadrilateral or not. This can be ascertained by collinearity check of any two sets of points i.e. either (**A**, **B**, **C**) or (**A**, **C**, **D**) or (**B**, **C**, **D**) or (**A**, **B**, **D**).

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: ee20resch14005@iith.ac.in. All content in this document is released under GNU GPL. Free and open source.

2.1 Collinearity Check

Collinearity Check of points **A**, **B**, **D** i.e.

$$\mathbf{A} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} -1 \\ -5 \end{pmatrix} \quad (2.1.1)$$

In Vector approach, If the rank of a matrix formed by the vectors of points $\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$, $\begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$, $\begin{pmatrix} x_3 \\ y_3 \end{pmatrix}$ is **1 or 2** for any 3 X 3 matrix then the points are said to be **collinear**. If $\rho\left(matrix\right)$=3 then the points are non collinear.

$$\Rightarrow \rho \begin{pmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{pmatrix} = 3 \quad (2.1.2)$$

Here the vectors are as below.

$$\mathbf{AB} = (\mathbf{B} - \mathbf{A}) = \begin{pmatrix} 3 \\ 5 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \quad (2.1.3)$$

$$\mathbf{BD} = (\mathbf{D} - \mathbf{B}) = \begin{pmatrix} -1 \\ -5 \end{pmatrix} - \begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} -4 \\ -10 \end{pmatrix} \quad (2.1.4)$$

$$\mathbf{DA} = (\mathbf{A} - \mathbf{D}) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} - \begin{pmatrix} -1 \\ -5 \end{pmatrix} = \begin{pmatrix} 2 \\ 6 \end{pmatrix} \quad (2.1.5)$$

The matrix formed by vectors is

$$\begin{pmatrix} 2 & 4 & 1 \\ -4 & -10 & 1 \\ 2 & 6 & 1 \end{pmatrix}$$

Rank of the matrix: By reduction.

$$\Rightarrow \begin{pmatrix} 2 & 4 & 1 \\ -4 & -10 & 1 \\ 2 & 6 & 1 \end{pmatrix} \xleftrightarrow{R_2 \leftrightarrow R_3} \begin{pmatrix} 2 & 4 & 1 \\ 2 & 6 & 1 \\ -4 & -10 & 1 \end{pmatrix} \xleftrightarrow{R_3 \leftrightarrow R_1 + R_2 + R_3}$$

$$\begin{pmatrix} 2 & 4 & 1 \\ 2 & 6 & 1 \\ 0 & 0 & 3 \end{pmatrix} \xleftrightarrow{R_2 \leftrightarrow R_2 - R_1} \begin{pmatrix} 2 & 4 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} \quad (2.1.6)$$

The number of non-zero rows in the matrix = 3.
$\Rightarrow \rho\left(matrix\right)$ = 3.

Here the AB, BD, DA are not collinear.

Let us examine the lines generated by the given points in the Figure below:

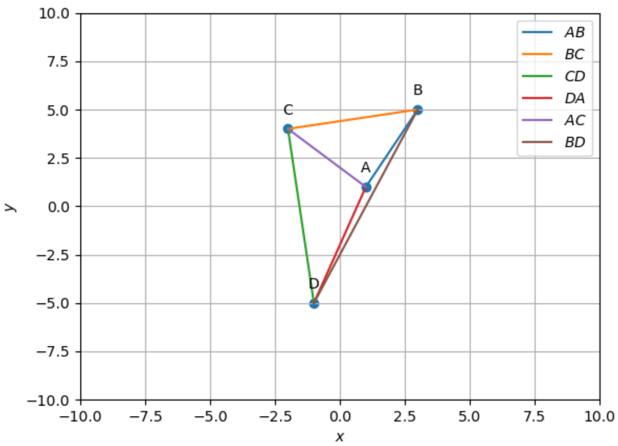

Fig. 0: Quadrilateral ABCD

The above figure clearly depicts the other set of points are not collinear. Hence the points given form a Quadrilateral.

2.2 *Area of a Quadrilateral*

$$Areaof\square ABCD = Areaof\triangle ACD + Areaof\triangle ABC$$

$\because$ this does not fall into category of Square, Parallelogram or Rhombus, Trapezium.

Area of a $\triangle ABC$ formed by points $\mathbf{A} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$, $\mathbf{C} = \begin{pmatrix} x_3 \\ y_3 \end{pmatrix}$ is given by

$$\frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ \mathbf{A} & \mathbf{B} & \mathbf{C} \end{vmatrix} \tag{2.2.1}$$

Area of $\triangle ABC$ formed by $\mathbf{A} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$,$\mathbf{B}$=$\begin{pmatrix} 3 \\ 5 \end{pmatrix}$,$\mathbf{C}$=$\begin{pmatrix} -2 \\ 4 \end{pmatrix}$ is given by

$$\triangle ABC = \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ 1 & 3 & -2 \\ 1 & 5 & 4 \end{vmatrix} = 9 \tag{2.2.2}$$

Area of $\triangle ACD$ formed by $\mathbf{A}$=$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$,$\mathbf{C}$=$\begin{pmatrix} -2 \\ 4 \end{pmatrix}$,$\mathbf{D}$=$\begin{pmatrix} -1 \\ -5 \end{pmatrix}$ is given by

$$\triangle ACD = \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ 1 & -2 & -1 \\ 1 & 4 & -5 \end{vmatrix} = 11.5 \tag{2.2.3}$$

The area of quadrilateral ABCD = 9+11.5 = 20.5