

SM5083 - BASICS OF PROGRAMMING

RS Girish - EE20RESCH14005*

CONTENTS

1	Problem	2
2	Solution	2
2.1	Collinearity Check	2
2.2	Area of a Quadrilateral . .	3

Abstract—This paper contains solution to problem no 9(i) of Examples II Section of Analytical Geometry by Hukum Chand. Links to Python codes are available below.

Download python codes at

<https://github.com/rsgirishkumar/SM5083/ASSIGNMENT1>

1 PROBLEM

Find the area of the quadrilateral formed by the points

$$\mathbf{A} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \mathbf{B} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} \mathbf{C} = \begin{pmatrix} -2 \\ 4 \end{pmatrix} \mathbf{D} = \begin{pmatrix} -1 \\ -5 \end{pmatrix} \quad (1.0.1)$$

2 SOLUTION

Let the given points are indicated as below

$$\mathbf{A} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \mathbf{B} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} \quad (2.0.1)$$

$$\mathbf{C} = \begin{pmatrix} -2 \\ 4 \end{pmatrix} \mathbf{D} = \begin{pmatrix} -1 \\ -5 \end{pmatrix} \quad (2.0.2)$$

$$(2.0.3)$$

Step1: Let us check whether the given points form a quadrilateral or not. This can be ascertained by collinearity check of any two sets of points i.e. either (A, B, C) or (A, C, D) or (B, C, D) or (A, B, D)

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: ee20resch14005@iith.ac.in. All content in this document is released under GNU GPL. Free and open source.

2.1 Collinearity Check

Collinearity Check of points A, B, D i.e.

$$\mathbf{A} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (2.1.1)$$

$$\mathbf{B} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} \quad (2.1.2)$$

$$\mathbf{D} = \begin{pmatrix} -1 \\ -5 \end{pmatrix} \quad (2.1.3)$$

$$(2.1.4)$$

Method-I:

Collinearity can be checked by forming an equation from any two points and on substituting the third point, if it satisfies the equation then all the three points are said to be collinear.

or

Method-II:

By using if any triangle is formed and if the area of triangle formed by the points is > 0 then they are non-collinear

Using Method-I:

Here lets form an equation by using A and B, i.e. \overrightarrow{AB}

The equation of a line formed by points $\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$ and $\begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$ is given by

$$\frac{(y - y_1)}{(y_2 - y_1)} = \frac{x - x_1}{x_2 - x_1} \quad (2.1.5)$$

By using the above equation, the equation of line formed by the points $\mathbf{A} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$ is as given below.

$$\begin{aligned}
&\Rightarrow \frac{y-1}{4} = \frac{x-1}{2} \\
&\Rightarrow y-1 = 2x-2 \\
&\Rightarrow 2x-y = 1
\end{aligned} \quad (2.1.6)$$

Now Lets substitute point $\mathbf{D} = \begin{pmatrix} -1 \\ -5 \end{pmatrix}$ in Equation no 2.1.6

LHS:

$$\Rightarrow 2 * (-1) - (-5) = 3 \text{ and} \quad (2.1.7)$$

$$\text{RHS} = 1 \quad (2.1.8)$$

Here **LHS** \neq **RHS**.

In Vector approach,

$$\overrightarrow{\mathbf{AB}} = \overrightarrow{\mathbf{B}} - \overrightarrow{\mathbf{A}} \Rightarrow \begin{pmatrix} 3 \\ 5 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 2 \\ 4 \end{pmatrix} \quad (2.1.9)$$

$$\Rightarrow \overrightarrow{\mathbf{AB}} \neq \lambda \mathbf{D} \quad (2.1.10)$$

\therefore point \mathbf{D} is not \parallel to $\overrightarrow{\mathbf{AB}}$

\therefore point \mathbf{D} has a projection onto $\overrightarrow{\mathbf{AB}}$,
it does not lie on $\overrightarrow{\mathbf{AB}}$

Let us examine the lines generated by the given points in the Figure below:

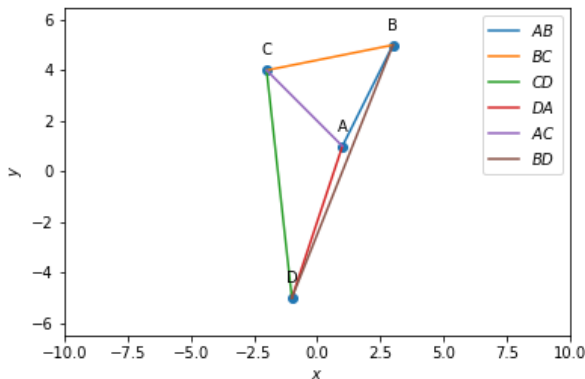


Fig. 0: Quadrilateral ABCD

The above figure clearly depicts the other set of

points are not collinear. Hence the points given form a Quadrilateral.

Using Method-II:

Area of a Triangle formed by points $\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}, \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}, \begin{pmatrix} x_3 \\ y_3 \end{pmatrix}$ is given by

$$\frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \quad (2.1.11)$$

Area of Triangle formed by

$$\mathbf{A} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} -1 \\ -5 \end{pmatrix} \quad (2.1.12)$$

$$(2.1.13)$$

is given by

$$\Delta \mathbf{ABD} = \frac{1}{2} \begin{vmatrix} 1 & 1 & 1 \\ 3 & 5 & 1 \\ -1 & -5 & 1 \end{vmatrix} = -2 \quad (2.1.14)$$

\therefore Area of $\Delta \mathbf{ABD} \neq 0$, the points are non collinear and all other points show a distinct area between the points, it can be ascertained that the points form a Quadrilateral.

2.2 Area of a Quadrilateral

Area of $\square \mathbf{ABCD}$ =
Area of Triangle $\Delta \mathbf{ACD}$ +
Area of Triangle $\Delta \mathbf{ABC}$

\therefore this does not fall into category of Square, Parallelogram or Rhombus, Trapezium.

Area of Triangle formed by $\mathbf{A} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$ is given by

$$\Delta \mathbf{ABD} = \frac{1}{2} \begin{vmatrix} 1 & 1 & 1 \\ 3 & 5 & 1 \\ -2 & 4 & 1 \end{vmatrix} = 9 \quad (2.2.1)$$

Area of Triangle formed by $\mathbf{A} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} -1 \\ -5 \end{pmatrix}$ is given by

$$\Delta \mathbf{ACD} = \frac{1}{2} \begin{vmatrix} 1 & 1 & 1 \\ -2 & 4 & 1 \\ -1 & -5 & 1 \end{vmatrix} = 11.5 \quad (2.2.2)$$

The area of quadrilateral ABCD = 9+11.5 = 20.5
--