SM5083 - BASICS OF PROGRAMMING

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Contents

Abstract—**This paper contains solution to problem no 5 of Examples III Section of Chapter III of Analytical Geometry by Hukum Chand. Links to Python codes are available below.**

Download python codes at

https://github.com/rsgirishkumar/SM5083/ASSIGNMENT2

1 Problem

The opposite vertices of a square are $\begin{pmatrix}0\\-1\end{pmatrix}, \begin{pmatrix}0\\3\end{pmatrix}$. Find the equations of four sides.

2 Solution

Let the given points are indicated as below

$$\mathbf{A} = \begin{pmatrix}0\\-1\end{pmatrix}, \mathbf{C} = \begin{pmatrix}0\\3\end{pmatrix}. \tag{2.0.1}$$

Let the unknown vertices are indicated as $\mathbf{B}, \mathbf{D}$. The step by step procedure involves

1) Find the diagonal **AC**.
2) Find the norm of **AC**.
3) Find the orthogonal of **AC** i.e. **BD** by using orthogonal matrix.
4) Find the midpoint of **AC**.
5) Using the norm of **BD**, find the vertices of **BD**.
6) Form the equations of lines using vertices.

Step-1: Diagonal **AC**

$$\mathbf{AC} = \begin{pmatrix}0-0\\3+1\end{pmatrix} = \begin{pmatrix}0\\4\end{pmatrix} \tag{2.0.2}$$

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Step-2: Norm of **AC**

$$\|\mathbf{AC}\| = \sqrt{0+4^2} = 4 \tag{2.0.3}$$

Step-3: Orthogonal of **AC**. i.e, **BD**.
Consider an 2x2 orthogonal matrix O be $\begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix}$.

$$\mathbf{BD} = AC * O = \begin{pmatrix}0 & 4\end{pmatrix}\begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix} = \begin{pmatrix}-4\\0\end{pmatrix} \tag{2.0.4}$$

Step-4: Midpoint of **AC** or **BD**.

$$MidpointM = \begin{pmatrix}0\\\frac{(3-1)}{2}\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix} \tag{2.0.5}$$

Step-5: Vertices of **BD**. Taking counter-clockwise and norm = 4,

$$B = \begin{pmatrix}x\_midpoint - \frac{norm}{2}\\y\_midpoint\end{pmatrix} = \begin{pmatrix}0-2\\1\end{pmatrix} = \begin{pmatrix}-2\\1\end{pmatrix}$$
$$D = \begin{pmatrix}x\_midpoint + \frac{norm}{2}\\y\_midpoint\end{pmatrix} = \begin{pmatrix}0+2\\1\end{pmatrix} = \begin{pmatrix}2\\1\end{pmatrix} \tag{2.0.6}$$

Step-6: Line Equations. Coordinates are

$$\mathbf{A} = \begin{pmatrix}0\\-1\end{pmatrix}, \mathbf{B} = \begin{pmatrix}-2\\1\end{pmatrix}, \mathbf{C} = \begin{pmatrix}0\\3\end{pmatrix}, \mathbf{D} = \begin{pmatrix}2\\1\end{pmatrix}. \tag{2.0.7}$$

When two coordinates $\begin{pmatrix}X_1\\Y_1\end{pmatrix}\begin{pmatrix}X_2\\Y_2\end{pmatrix}$ are given, then the line equation is given by $\frac{(Y-Y1)}{(Y2-Y1)} = \frac{(X-X1)}{(X2-X1)}$
and on comparison with the form ax+by+c=0,
$a = Y_2 - Y_1, b = X_1 - X_2$,
$c = (-(Y_2 - Y_1) * X_1) + ((X_2 - X_1) * Y_1)$.
In Matrix form, $PT_1 = \begin{pmatrix}X_1\\Y_1\end{pmatrix}, PT_2 = \begin{pmatrix}X_2\\Y_2\end{pmatrix}$ and vector
$DV_1 = \mathbf{PT_2} - \mathbf{PT_1} = \begin{pmatrix}X_2 - X_1\\Y_2 - Y_1\end{pmatrix}$
then, $a = DV_1[1]$,
$b = -DV_1[0]$,
$c = \begin{pmatrix}PT_1[1] & -PT_1[0]\end{pmatrix}\begin{pmatrix}DV_1.T\end{pmatrix}$

By using the same, the line equation AB for points $\mathbf{A} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$ is as follows:

$$\begin{aligned} \mathbf{AB} &= \begin{pmatrix} -2 \\ 2 \end{pmatrix} \\ a = 2,\ b &= 2, \\ c = \begin{pmatrix} -1 & 0 \end{pmatrix} \begin{pmatrix} -2 \\ 2 \end{pmatrix} &= 2 \\ \textit{Line equation for } & \mathbf{AB} \\ \implies 2x + 2y + 2 = 0 \ or \ x + y &= -1. \\ \textit{In vectorform} & \\ \implies \begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{X} &= -1 \end{aligned} \tag{2.0.8}$$

The line equation BC for points $\mathbf{B} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$ is as follows:

$$\begin{aligned} \mathbf{BC} &= \begin{pmatrix} 2 \\ 2 \end{pmatrix} \\ a = 2,\ b &= -2, \\ c = \begin{pmatrix} 1 & 2 \end{pmatrix} \begin{pmatrix} 2 \\ 2 \end{pmatrix} &= 6 \\ \textit{Line equation for } & \mathbf{BC} \\ \implies 2x - 2y + 6 = 0 \ or \ x - y &= -3. \\ \textit{In vectorform} & \\ \implies \begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{X} &= -3 \end{aligned} \tag{2.0.9}$$

The line equation CD for points $\mathbf{C} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ is as follows:

$$\begin{aligned} \mathbf{CD} &= \begin{pmatrix} 2 \\ -2 \end{pmatrix} \\ a = -2,\ b &= -2, \\ c = \begin{pmatrix} 3 & 0 \end{pmatrix} \begin{pmatrix} 2 \\ -2 \end{pmatrix} &= 6 \\ \textit{Line equation for } & \mathbf{CD} \\ \implies -2x - 2y + 6 = 0 \ or \ x + y &= 3. \\ \textit{In vectorform} & \\ \implies \begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{X} &= 3 \end{aligned} \tag{2.0.10}$$

The line equation DA for points $\mathbf{D} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \mathbf{A} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$ is as follows:

$$\begin{aligned} \mathbf{DA} &= \begin{pmatrix} -2 \\ -2 \end{pmatrix} \\ a = -2,\ b &= 2, \\ c = \begin{pmatrix} 1 & -2 \end{pmatrix} \begin{pmatrix} -2 \\ -2 \end{pmatrix} &= 2 \\ \textit{Line equation for } & \mathbf{DA} \\ \implies -2x + 2y + 2 = 0 \ or \ x - y &= 1. \\ \textit{In vectorform} & \\ \implies \begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{X} &= 1 \end{aligned} \tag{2.0.11}$$

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The plotted graph is shown as below.

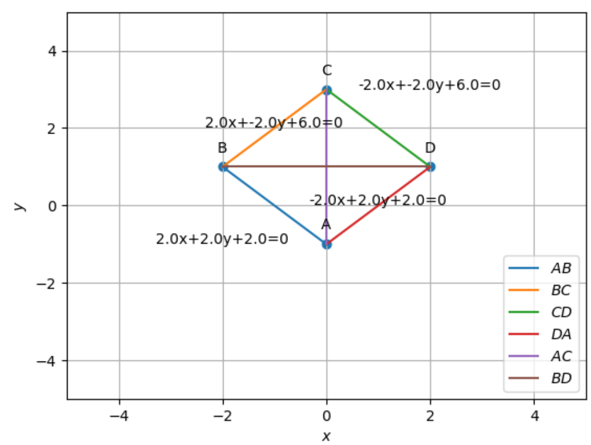

Fig. 6: Square ABCD