SM5083 - BASICS OF PROGRAMMING

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Abstract—**This paper contains solution to problem no 5 of Examples III Section of Chapter III of Analytical Geometry by Hukum Chand. Links to Python codes are available below.**

Download python codes at

https://github.com/rsgirishkumar/SM5083/ASSIGNMENT2

1 Problem

The opposite vertices of a square are $\begin{pmatrix}0\\-1\end{pmatrix}, \begin{pmatrix}0\\3\end{pmatrix}$. Find the equations of four sides.

2 Solution

Let the given points are indicated as below

$$\mathbf{A} = \begin{pmatrix}0\\-1\end{pmatrix}, \mathbf{C} = \begin{pmatrix}0\\3\end{pmatrix}. \tag{2.0.1}$$

Let the unknown vertices are indicated as $\mathbf{B}, \mathbf{D}$. The step by step procedure involves

1) Find the diagonal **AC**.
2) Find the norm of **AC**.
3) Find the orthogonal of **AC** i.e. **BD** by using orthogonal matrix.
4) Find the midpoint of **AC**.
5) Using the norm of **BD**, find the vertices of **BD**.
6) Form the equations of lines using vertices.

Step-1: Diagonal **AC**

$$\mathbf{AC} = \begin{pmatrix}0-0\\3+1\end{pmatrix} = \begin{pmatrix}0\\4\end{pmatrix} \tag{2.0.2}$$

Step-2: Norm of **AC**

$$\|\mathbf{AC}\| = \sqrt{0+4^2} = 4 \tag{2.0.3}$$

Step-3: Orthogonal of **AC**. i.e, **BD**.

Consider an 2x2 orthogonal matrix **O** be $\begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix}$.

$$\mathbf{BD} = \mathbf{AC} * \mathbf{O} = \begin{pmatrix}0 & 4\end{pmatrix}\begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix} = \begin{pmatrix}-4\\0\end{pmatrix} \tag{2.0.4}$$

Step-4: Midpoint of **AC** or **BD**.

$$Midpoint\ \mathbf{M} = \begin{pmatrix}0\\\frac{(3-1)}{2}\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix} \tag{2.0.5}$$

Step-5: Vertices of **BD**. Taking counter-clockwise and norm = 4,

$$\begin{aligned}\mathbf{B} &= \begin{pmatrix}\mathbf{M[0]} - \frac{norm}{2}\\\mathbf{M[1]}\end{pmatrix} = \begin{pmatrix}0-2\\1\end{pmatrix} = \begin{pmatrix}-2\\1\end{pmatrix}\\ \mathbf{D} &= \begin{pmatrix}\mathbf{M[0]} + \frac{norm}{2}\\\mathbf{M[1]}\end{pmatrix} = \begin{pmatrix}0+2\\1\end{pmatrix} = \begin{pmatrix}2\\1\end{pmatrix}\end{aligned} \tag{2.0.6}$$

Step-6: Line Equations. Coordinates are

$$\mathbf{A} = \begin{pmatrix}0\\-1\end{pmatrix}, \mathbf{B} = \begin{pmatrix}-2\\1\end{pmatrix}, \mathbf{C} = \begin{pmatrix}0\\3\end{pmatrix}, \mathbf{D} = \begin{pmatrix}2\\1\end{pmatrix}. \tag{2.0.7}$$

When two coordinates $\begin{pmatrix}X_1\\Y_1\end{pmatrix}, \begin{pmatrix}X_2\\Y_2\end{pmatrix}$ are given, then the line equation is given by

$$\frac{(Y-Y1)}{(Y2-Y1)} = \frac{(X-X1)}{(X2-X1)} \tag{2.0.8}$$

and on comparison with the form ax+by+c=0,

$$\begin{aligned} a &= Y_2 - Y_1,\\ b &= X_1 - X_2,\\ c = (-(Y_2 - Y_1) * X_1&) + ((X_2 - X_1) * Y_1)\end{aligned} \tag{2.0.9}$$

.

In Matrix form, $\mathbf{PT_1} = \begin{pmatrix}X_1\\Y_1\end{pmatrix}, \mathbf{PT_2} = \begin{pmatrix}X_2\\Y_2\end{pmatrix}$ and vector

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$\mathbf{DV_1} = \mathbf{PT_2} - \mathbf{PT_1} = \begin{pmatrix} X_2 - X_1 \\ Y_2 - Y_1 \end{pmatrix}$
then,

$$\begin{aligned} a &= \mathbf{DV_1[1]}, \\ b &= -\mathbf{DV_1[0]}, \\ c &= \begin{pmatrix} \mathbf{PT_1[1]} & -\mathbf{PT_1[0]} \end{pmatrix} \begin{pmatrix} \mathbf{DV_1}.T \end{pmatrix} \end{aligned} \tag{2.0.10}$$

By using the same, the line equation **AB** for points $\mathbf{A} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$ is as follows:

$$\begin{aligned} \mathbf{AB} &= \begin{pmatrix} -2 \\ 2 \end{pmatrix} \\ a = 2,\ b &= 2, \\ c = \begin{pmatrix} -1 & 0 \end{pmatrix} \begin{pmatrix} -2 \\ 2 \end{pmatrix} &= 2 \end{aligned} \tag{2.0.11}$$

Line equation for **AB**

$$\implies 2x + 2y + 2 = 0 \ or \ x + y = -1. \tag{2.0.12}$$

In vector form

$$\implies \begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{X} = -1 \tag{2.0.13}$$

The line equation **BC** for points $\mathbf{B} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$ is as follows:

$$\begin{aligned} \mathbf{BC} &= \begin{pmatrix} 2 \\ 2 \end{pmatrix} \\ a = 2,\ b &= -2, \\ c = \begin{pmatrix} 1 & 2 \end{pmatrix} \begin{pmatrix} 2 \\ 2 \end{pmatrix} &= 6 \end{aligned} \tag{2.0.14}$$

Line equation for **BC**

$$\implies 2x - 2y + 6 = 0 \ or \ x - y = -3. \tag{2.0.15}$$

In vector form

$$\implies \begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{X} = -3 \tag{2.0.16}$$

The line equation **CD** for points $\mathbf{C} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ is as follows:

$$\begin{aligned} \mathbf{CD} &= \begin{pmatrix} 2 \\ -2 \end{pmatrix} \\ a = -2,\ b &= -2, \\ c = \begin{pmatrix} 3 & 0 \end{pmatrix} \begin{pmatrix} 2 \\ -2 \end{pmatrix} &= 6 \end{aligned} \tag{2.0.17}$$

Line equation for **CD**

$$\implies -2x - 2y + 6 = 0 \ or \ x + y = 3. \tag{2.0.18}$$

In vector form

$$\implies \begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{X} = 3 \tag{2.0.19}$$

The line equation **DA** for points $\mathbf{D} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \mathbf{A} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$ is as follows:

$$\begin{aligned} \mathbf{DA} &= \begin{pmatrix} -2 \\ -2 \end{pmatrix} \\ a = -2,\ b &= 2, \\ c = \begin{pmatrix} 1 & -2 \end{pmatrix} \begin{pmatrix} -2 \\ -2 \end{pmatrix} &= 2 \end{aligned} \tag{2.0.20}$$

Line equation for **DA**

$$\implies -2x + 2y + 2 = 0 \ or \ x - y = 1. \tag{2.0.21}$$

In vector form

$$\implies \begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{X} = 1 \tag{2.0.22}$$

.
The plotted graph is shown as below.

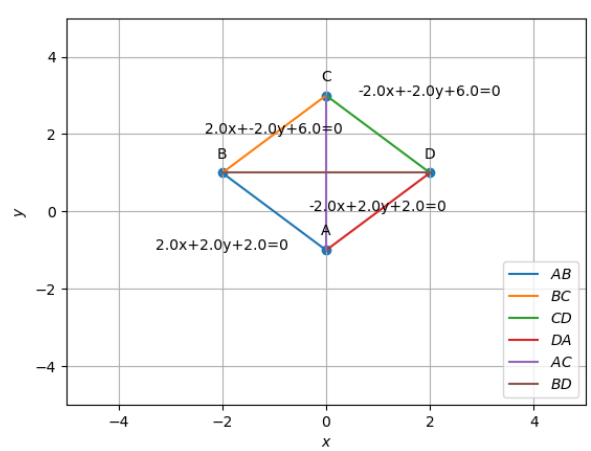

Fig. 6: Square ABCD