SM5083 - BASICS OF PROGRAMMING

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CONTENTS

Abstract—**This paper contains solution to problem no 9(i) of Examples II Section of Analytical Geometry by Hukum Chand. Links to Python codes are available below.**

Download python codes at

https://github.com/rsgirishkumar/SM5083/ASSIGNMENT1

1 Problem

Find the area of the quadrilateral formed by the points

$$\mathbf{A} = \begin{pmatrix}1\\1\end{pmatrix} \mathbf{B} = \begin{pmatrix}3\\5\end{pmatrix} \mathbf{C} = \begin{pmatrix}-2\\4\end{pmatrix} \mathbf{D} = \begin{pmatrix}-1\\-5\end{pmatrix} \tag{1.0.1}$$

2 Solution

Let the given points are indicated as below

$$\mathbf{A} = \begin{pmatrix}1\\1\end{pmatrix} \mathbf{B} = \begin{pmatrix}3\\5\end{pmatrix} \tag{2.0.1}$$

$$\mathbf{C} = \begin{pmatrix}-2\\4\end{pmatrix} \mathbf{D} = \begin{pmatrix}-1\\-5\end{pmatrix} \tag{2.0.2}$$

$$\tag{2.0.3}$$

Step1: Let us check whether the given points form a quadrilateral or not. This can be ascertained by collinearity check of any two sets of points i.e. either $(\mathbf{A}, \mathbf{B}, \mathbf{C}) or (\mathbf{A}, \mathbf{C}, \mathbf{D}) or (\mathbf{B}, \mathbf{C}, \mathbf{D}) or (\mathbf{A}, \mathbf{B}, \mathbf{D})$

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2.1 *Collinearity Check*

Collinearity Check of points A, B, D i.e.

$$\mathbf{A} = \begin{pmatrix}1\\1\end{pmatrix} \tag{2.1.1}$$

$$\mathbf{B} = \begin{pmatrix}3\\5\end{pmatrix} \tag{2.1.2}$$

$$\mathbf{D} = \begin{pmatrix}-1\\-5\end{pmatrix} \tag{2.1.3}$$

$$\tag{2.1.4}$$

Method-I:
Collinearity can be checked by forming an equation from any two points and on substituting the third point, if it satisfies the equation then all the three points are said to be collinear.
or
Method-II:
By using if any triangle is formed and if the area of triangle formed by the points is > 0 then they are non-collinear

Using Method-I:
Here lets form an equation by using A and B, i.e. $\overrightarrow{\mathbf{AB}}$

The equation of a line formed by points $\begin{pmatrix}x_1\\y_1\end{pmatrix} and \begin{pmatrix}x_2\\y_2\end{pmatrix}$ is given by

$$\frac{(y - y_1)}{(y_2 - y_1)} = \frac{x - x_1}{x_2 - x_1} \tag{2.1.5}$$

By using the above equation, the equation of line formed by the points $\mathbf{A} = \begin{pmatrix}1\\1\end{pmatrix} and \mathbf{B} = \begin{pmatrix}3\\5\end{pmatrix}$ is as given below.

$$\Rightarrow \frac{y-1}{4} = \frac{x-1}{2}$$
$$\Rightarrow y-1 = 2x-2 \tag{2.1.6}$$
$$\Rightarrow 2x - y = 1$$

Now Lets substitute point $\mathbf{D} = \begin{pmatrix} -1 \\ -5 \end{pmatrix}$ in Equation no 2.1.6

LHS:

$$\Rightarrow 2*(-1)-(-5) = 3and \tag{2.1.7}$$
$$\mathbf{RHS} = 1 \tag{2.1.8}$$

Here **LHS≠RHS.**

In Vector approach, If the rank of a matrix formed by the vectors of points $\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}, \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}, \begin{pmatrix} x_3 \\ y_3 \end{pmatrix}$ is **1 or 2** for any 3 X 3 matrix then the points are said to be **collinear**. If $\rho\left(matrix\right)$=3 then the points are non collinear.

$$\Rightarrow \rho \begin{pmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{pmatrix} = 3 \quad (2.1.9)$$

Here the vectors are as below.

$$\overrightarrow{\mathbf{AB}} = \overrightarrow{\mathbf{B}} - \overrightarrow{\mathbf{A}} \Rightarrow \begin{pmatrix} 3 \\ 5 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 2 \\ 4 \end{pmatrix} \tag{2.1.10}$$

$$\overrightarrow{\mathbf{BD}} = \overrightarrow{\mathbf{D}} - \overrightarrow{\mathbf{B}} \Rightarrow \begin{pmatrix} -1 \\ -5 \end{pmatrix} - \begin{pmatrix} 3 \\ 5 \end{pmatrix} \Rightarrow \begin{pmatrix} -4 \\ -10 \end{pmatrix} \tag{2.1.11}$$

$$\overrightarrow{\mathbf{DA}} = \overrightarrow{\mathbf{A}} - \overrightarrow{\mathbf{D}} \Rightarrow \begin{pmatrix} 1 \\ 1 \end{pmatrix} - \begin{pmatrix} -1 \\ -5 \end{pmatrix} \Rightarrow \begin{pmatrix} 2 \\ 6 \end{pmatrix} \tag{2.1.12}$$

The matrix formed by vectors is

$$\begin{pmatrix} 2 & 4 & 1 \\ -4 & -10 & 1 \\ 2 & 6 & 1 \end{pmatrix} \tag{2.1.13}$$

Rank of the matrix: By reduction.

$$\Rightarrow \begin{pmatrix} 2 & 4 & 1 \\ -4 & -10 & 1 \\ 2 & 6 & 1 \end{pmatrix} \quad (2.1.14)$$

$$\xleftrightarrow{R_2 \leftrightarrow R_3}$$
$$\Rightarrow \begin{pmatrix} 2 & 4 & 1 \\ 2 & 6 & 1 \\ -4 & -10 & 1 \end{pmatrix} \quad (2.1.15)$$

$$\xleftrightarrow{R_3 \leftrightarrow R_1 + R_2 + R_3}$$
$$\Rightarrow \begin{pmatrix} 2 & 4 & 1 \\ 2 & 6 & 1 \\ 0 & 0 & 3 \end{pmatrix} \quad (2.1.16)$$

$$\xleftrightarrow{R_2 \leftrightarrow R_2 - R_1}$$
$$\Rightarrow \begin{pmatrix} 2 & 4 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} \quad (2.1.17)$$

The number of non-zero rows in the matrix = 3.
$\Rightarrow \rho\left(matrix\right) = 3$.
Here the $\overrightarrow{\mathbf{AB}}, \overrightarrow{\mathbf{BD}}, \overrightarrow{\mathbf{DA}} arenotcollinear.$

Let us examine the lines generated by the given points in the Figure below:

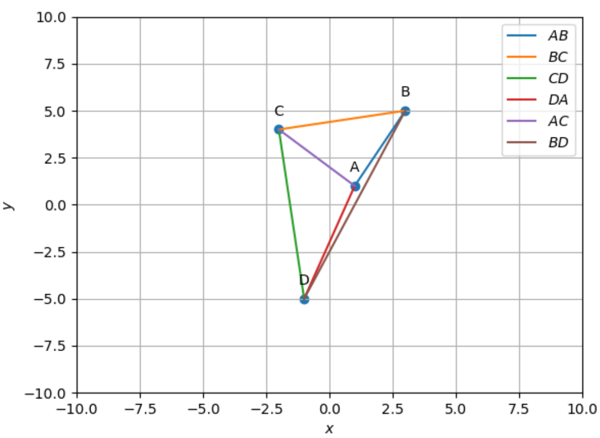

Fig. 0: Quadrilateral ABCD

The above figure clearly depicts the other set of points are not collinear. Hence the points given form a Quadrilateral.

Using Method-II:

Area of a Triangle formed by points $\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}, \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}, \begin{pmatrix} x_3 \\ y_3 \end{pmatrix}$ is given by

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \tag{2.1.18}$$

Area of Triangle formed by

$$\mathbf{A} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} -1 \\ -5 \end{pmatrix} \tag{2.1.19}$$

$$\tag{2.1.20}$$

is given by

$$\Delta\mathbf{ABD} = \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ 3 & 5 & 1 \\ -1 & -5 & 1 \end{vmatrix} = -2 \tag{2.1.21}$$

$\because$ Area of Δ**ABD**$\neq 0$, the points are non collinear and all other points show a distinct area between the points, it can be ascertained that the points form a Quadrilateral.

2.2 Area of a Quadrilateral

Area of $\square$**ABCD** =
Area of Triangle Δ**ACD** +
Area of Triangle Δ**ABC**

$\because$ this does not fall into category of Square, Parallelogram or Rhombus, Trapezium.

Area of Triangle formed by $\mathbf{A} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$,$\mathbf{B}$=$\begin{pmatrix} 3 \\ 5 \end{pmatrix}$,$\mathbf{C}$=$\begin{pmatrix} -2 \\ 4 \end{pmatrix}$ is given by

$$\Delta\mathbf{ABD} = \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ 3 & 5 & 1 \\ -2 & 4 & 1 \end{vmatrix} = 9 \tag{2.2.1}$$

Area of Triangle formed by $\mathbf{A}$=$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$,$\mathbf{C}$=$\begin{pmatrix} -2 \\ 4 \end{pmatrix}$,$\mathbf{D}$=$\begin{pmatrix} -1 \\ -5 \end{pmatrix}$ is given by

$$\Delta\mathbf{ACD} = \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ -2 & 4 & 1 \\ -1 & -5 & 1 \end{vmatrix} = 11.5 \tag{2.2.2}$$

$\boxed{\text{The area of quadrilateral ABCD = 9+11.5 = 20.5}}$