Revisiting Efficiency and Searching

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Table of Contents

Random Access Data

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Random Access Data

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In contrast, a *random access* data structure provides *constant* access time to any arbitrary item of that collection. Our string type is random accessing, since indexing is constant time. Our strings can only hold characters though — we don't currently have a type to store arbitrary data.

The `tuple` type

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```
tupA = tuple(1, 2, 3)
tupB = ("hello", 12, LL(1, 2, 3))
oneElemTup = (7,)
```

[1] In cases where ambiguity exists, a `tuple` literal can be forced by including a comma after the last value.

Recall sequence operations

All of our sequence operations are also available on `tuples`, and so we now recall them and note their algorithmic complexity.

Syntax	Operation	Complexity
`s[i]`	Indexing	$O(1)$
`s[start:end:skip]`	Slicing	$O(k)$
`s1 + s2`	Concatenation	$O(k)$
`s*c`	Sequence Repetition	$O(k)$
`s1==s2`	Equality	$O(min(m, n))$[2]
`len(s)`	Length	$O(1)$

The above k all represent the length of the resultant sequence.

[2]Here m and n represent the lengths of both sequences respectively.

Recursion over `tuples`

As `tuples` represent a sequence of arbitrary data, we could operate on them similarly to how we operate on our `LLists`. Consider writing the function `sumTuple` and how we might write it.

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Again, our natural base case is when the `tuple` is empty, how do we check that?

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```
def sumTuple(t):
  if t == tuple():
    return 0
```

We can construct an empty `tuple` simply by calling the `tuple` function with no arguments. To check if our parameter is empty we can compare equality with the empty `tuple`.

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How can we calculate the "rest" of a `tuple` for our recursive call?

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```
def sumTuple(t):
  if t == tuple():
    return 0
  return t[0] + sumTuple(t[1:])
```

Slicing can be used to construct a `tuple` that represents the "rest" of our parameter. However, this is a $O(n)$ operation! This means our `sumTuple` function is $O(n^2)$.

More efficient recursion over `tuples`

Since slicing is an expensive operation to perform on `tuple` just to ignore one element we would rather not perform it to act as our "`rest`" operation. What can we do instead?

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More efficient recursion over `tuples`

Since slicing is an expensive operation to perform on `tuple` just to ignore one element we would rather not perform it to act as our "`rest`" operation. What can we do instead?

Use indexing! The entire point of random access data is that *any* element can be accessed in constant time, not just the first! As such, our recursion doesn't need the first item to be the one we're looking at it just needs to know *which* element we're looking at!

Solution — write a helper function that takes a second parameter, which is the index of our current element. Then the problem simply becomes recursion on a natural number, with the `tuple` simply being a parameter that is along for the ride.

Better `sumTuple`

```
def sumTuple(t):
  def sumTupleHelper(i):
    if i >= len(t):
      return 0
    return t[i] + sumTupleHelper(i+1)
  return sumTupleHelper(0)
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Now, our `sumTuple` function is linear as it should be, as opposed to quadratic!

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This same issue has existed with most of the functions we've written that operated on strings, because we used slicing. As such, now that we are aware of the time complexities of our sequence operations we should be careful not to use unnecessarily expensive operations!

Practice with `tuples`

Practice problem:Write a function `isSorted` that takes a single `tuple` parameter and returns `True` if the elements of that `tuple` are sorted in either non-decreasing *or* non-increasing order, and returns `False` otherwise.

Table of Contents

Checking membership

Consider we want to write the function `contains` that has two parameters, first a value and second a `LList`, and returns `True` if the given value is contained within the `LList` and `False` otherwise. How would we achieve this?

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We would have to write a recursive function that compares each item of the `LList` with the given value. This is called *sequential search* because we search each item of our collection sequentially.

Sequential search on a `LList`

```
def contains(v, l):
  if isEmpty(l):
    return False
  if v == first(l):
    return True
  return contains(v, rest(l))
```

This function works perfectly fine, how many basic operations does it perform?

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It depends!

Analysis and variability

We've already discussed time complexity of our functions, however each time we've calculated it we've calculated the exact number of operations a function would take. What about when the exact number of operations a function would take depends on the given arguments?

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Consider how many operations the `contains` function executes if the element `v` is

- The first element of `l`
- The third element of `l`
- The middle element of `l`
- Not within `l`

Best, worst, and average case

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- The worst case for `contains` is when `v` is not an element of `l`, then we must compare it against every single element of `l`. As such a multiple of n operations are performed and the worst case is $O(n)$.

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- The worst case for `contains` is when `v` is not an element of `l`, then we must compare it against every single element of `l`. As such a multiple of n operations are performed and the worst case is $O(n)$.
- The average case can be calculated by calculating the work done in each case summing that up and dividing by the number of cases. This comes out to $O(n)$ as well.

Being pessimistic

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The average case is quite often more realistic, and more relevant for how functions are often used though.

Assume in this class if the complexity of a function is given, or asked for, then it is referring to the worst case complexity unless otherwise specified.

Membership of a `tuple`

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In the general case, no! No matter what you try and do in the worst case you'll have to look at every single element of the `tuple` in order to determine it does not contain the element you're looking for.

Writing `contains` for a `tuple`

```
def tupleContains(v, t):
  def tcHelper(i):
    if i >= len(t):
      return False
    if t[i] == v
      return True
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Is there ever a time that searching for an element could perform better than linearly? Not for the general case...

A slightly different problem

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We have random access, yes we can!

Minimizing examined values

Consider we are trying to determine if the value 1023 exists in a sorted `tuple` that contains 5000 elements. Remembering that we can only look at one element at a time, which element guarantees it will maximize the amount of information we gain in the worst case?

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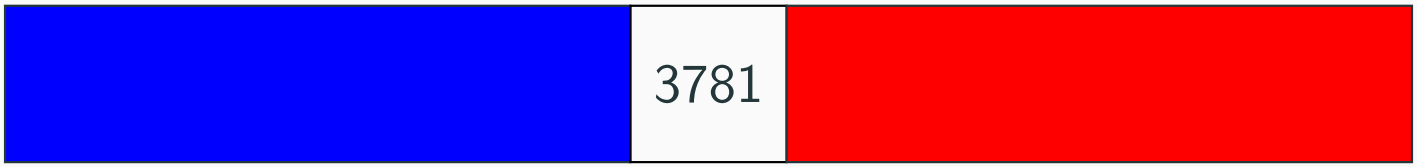

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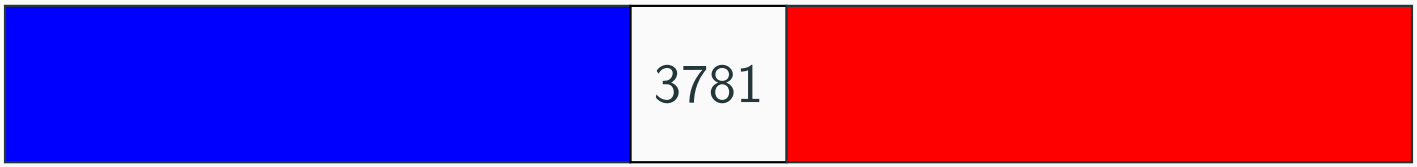

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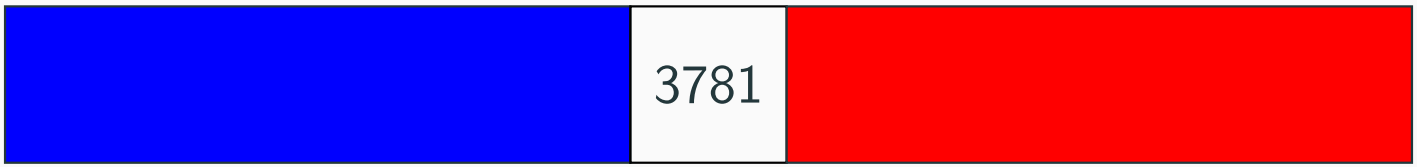

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The values in the `tuple` that appear *after* index 2500 are guaranteed not to be the value we're looking for. As such, we can eliminate that entire half of our `tuple`!

Binary search

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Repeating this process of halving until we find our element or we've ran out of values is the algorithm known as *binary search*.

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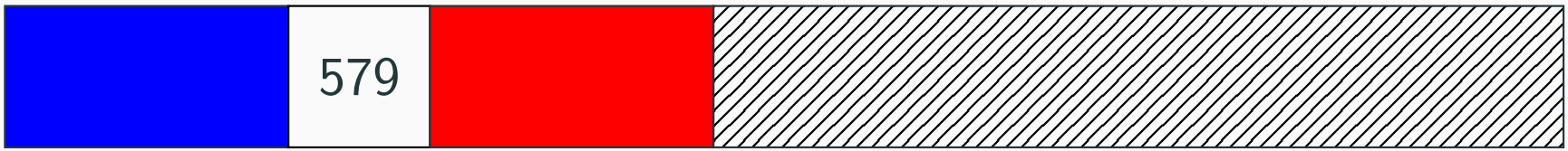

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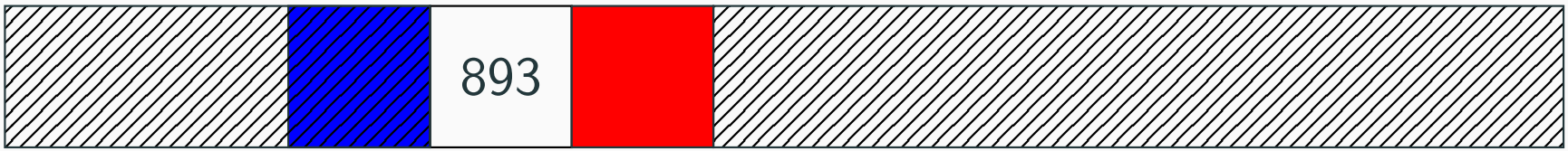

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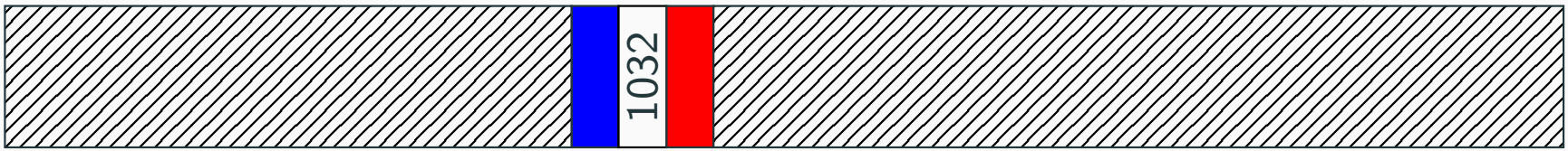

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Then the algorithm we need to implement simply is the one that updates these values accordingly.

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```

Now, if the element is larger than the one we're looking for then so is every value with an index larger than `mid`. As such, we update *hi* to be $mid - 1$

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      ...
```

Now, if the element is larger than the one we're looking for then so is every value with an index larger than `mid`. As such, we update *hi* to be $mid - 1$

Binary search on a `tuple`

```
def bsContains(v, t):
  def bsHelper(low, hi):
    if low > hi:
      return False
    mid = low + (hi-low)//2
    if t[mid] == v:
      return True
    if t[mid] > v:
      return bsHelper(low, mid-1)
    if t[mid] < v:
      ...
```

Finally, if the element is smaller than the one we're looking for then so is every value with an index smaller than `mid`. As such, we update *low* to be $mid + 1$.

Binary search on a `tuple`

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def bsContains(v, t):
  def bsHelper(low, hi):
    if low > hi:
      return False
    mid = low + (hi-low)//2
    if t[mid] == v:
      return True
    if t[mid] > v:
      return bsHelper(low, mid-1)
    if t[mid] < v:
      return bsHelper(mid+1, hi)
```

Finally, if the element is smaller than the one we're looking for then so is every value with an index smaller than `mid`. As such, we update *low* to be $mid + 1$.

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      return bsHelper(mid+1, hi)
```

Now `bsHelper` is complete. What initial values should `bsContains` call `bsHelper` with?

Binary search on a `tuple`

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def bsContains(v, t):
  def bsHelper(low, hi):
    if low > hi:
      return False
    mid = low + (hi-low)//2
    if t[mid] == v:
      return True
    if t[mid] > v:
      return bsHelper(low, mid-1)
    if t[mid] < v:
      return bsHelper(mid+1, hi)
  return bsHelper(0, len(t)-1)
```

In the initial call we want to consider that the value might be anywhere within the entire `tuple`! So we choose 0 for low and `len(t)-1` as hi.

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In each recursive application `bsHelper` halves the number of elements in our `tuple` that *may* contain `v`. How many times can we divide a number by two before we reach zero?

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$$log_2 n$$

Practice problem part 1

Practice problem: Write the function `findPairs` that has two parameters, the first being an arbitrary `tuple` of integers and the second being a target integer. Your function should return a `LList` of pairs where each pair is a `LList` of two integers. The pairs returned by your function should be all of the pairs of numbers in the given `tuple` that sum to `target`

For example, `findPairs((7, 3, 6, -2, 2), 5)` would return the `LList` `LL(LL(7, -2), LL(3, 2))`.

Once your function is complete determine its worst case time complexity.

Practice problem part 2

Practice problem: Write the function `findPairsSorted` that is the exact same as `findPairs` except that it assumes that the elements of its `tuple` parameter is sorted in non-decreasing order.

Try to come up with a solution that has a better worst case time complexity than your original `findPairs`, as this is possible given the new assumption.