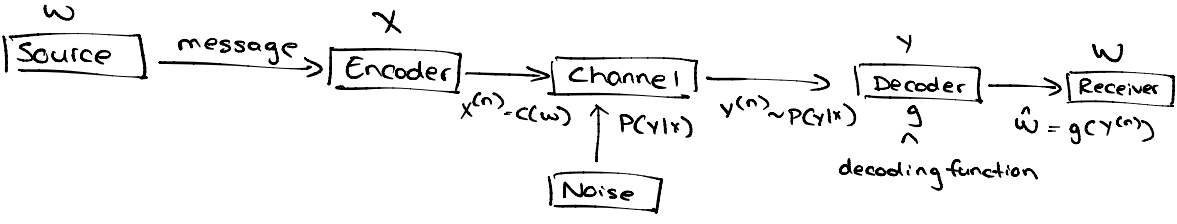
Lecture

Channel

Def: a channel is $(X, p(y|x), Y)$



Discrete: If X, Y are discrete

Memoryless: If Y_i is $\perp\!\!\!\perp$ of $x^{(i-1)}, y^{(i-1)}$

No Feedback: Input is not adapted based on what was sent.

- Send w

↓ encode (binary)

$X^{(n)} = \{X_1(w), X_2(w) \dots X_n(w)\} \in X^{(n)}$

$X_i(w) \in [0,1]$

↓ transmit

$y^{(n)}$

↓ decode

$\hat{w} = g(y^{(n)})$, hope $\hat{w} = w$, else, error

Key Q:

What is the fastest* rate at which a fixed channel can reliably transmit messages?

Capacity

Reliability:

Def: Probability of error:

(i) conditional on message sent: $Pr(\hat{w} \neq w | w = w) = \lambda^{(n)}(w)$

(ii) max probs over all messages: $\max\{\lambda^{(n)}(w)\}$

(iii) average prob of error:

$P_e^{(n)} = \frac{1}{|w|} \sum_{w \in w} \lambda^{(n)}(w) <= Pr(\hat{w} \neq w)$ if $w \sim$ uniformly

Def: rate

The rate of a code: $(|w|, n)$ $R = \frac{\log(|w|)}{n} = \frac{\text{unit of info}}{n}$

Why? Channel can only send r bits / second

Why the log?

Recall, if $w \sim \text{uniformly}(|w|)$

$H[w] = \log(|w|)$

Capacity

"Operational" capacity C

is longest achievable rate R

"information" capacity C

$C = \max\{I[Y;X]\}$

Thm: Given a discrete, memoryless channel

1. If $R < C$ the R is achievable
2. If R is achievable then $R \leq C$

(vice versa)

$C = \max_{P_X} \{I[X;Y]\}$

Channel Coding Thm.
(Shannon, 1978)