Distribution of points

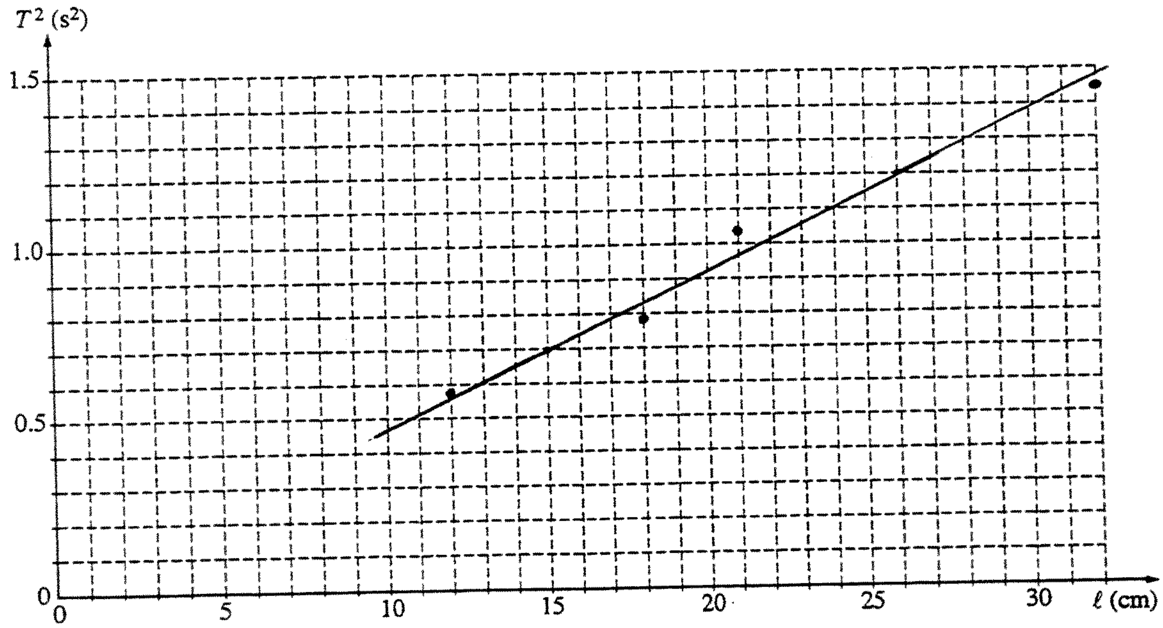
Mech. 1 (15 points)

(a) 4 points

ℓ (cm)	t_{10} (s)	T (s)	T^2 (s^2)
12	7.62	0.762	0.581
18	8.89	0.889	0.790
21	10.09	1.009	1.018
32	12.08	1.208	1.459

For correctly computing the period of each pendulum using the time for 10 oscillations — 1 point
For correctly computing the square of each period — 1 point
For expressing the periods and the squares of the periods to two or three decimal places — 1 point
For completing the data table — 1 point

(b) 3 points



For correctly plotting the square of the period as a function of the pendulum length — 1 point
For drawing a single straight line through the data points (could go through (0,0) but did not have to do so) — 1 point
For drawing this line to approximate a best fit with about an equal number of data points above and below the line — 1 point

Mech. 1 (continued)

(c) 4 points

For computing the slope of the straight line — 1 point

slope $= \dfrac{\Delta T^2}{\Delta \ell}$

Credit was awarded for a calculated value that was consistent with the line drawn. A typical correct value was 4.50 s^2/m. Most values were between 4 and 5 s^2/m.

For recognizing that the motion of each pendulum is simple harmonic or for using the equation for the period of a simple pendulum — 1 point

$T = 2\pi\sqrt{\dfrac{\ell}{g}}$

For using this relationship and the slope from the graph to determine g_{exp} — 1 point

For example, the relation between T^2 and ℓ is given by $T^2 = \dfrac{4\pi^2}{g}\ell$, so slope $= \dfrac{4\pi^2}{g_{exp}}$

$g_{exp} = \dfrac{4\pi^2}{\text{slope}}$

Credit was awarded for a calculated value that was consistent with the slope determined above. For example, for a slope = 4.50 s^2/m, g_{exp} = 8.77 m/s^2.

For using appropriate units for the computed acceleration — 1 point

(d) 2 points

For correctly stating whether 9.8 m/s^2 is within ±4% of the experimental value g_{exp}. — 1 point

Credit was awarded if the answer was consistent with the value obtained for g_{exp}. For example, for g_{exp} = 8.77 m/s^2, the experimental value is not in agreement with g.

For justification, either by displaying the range of acceptable values within ±4% of the value of g_{exp} (e.g., approximately 8.42 m/s^2 to 9.12 m/s^2, for g_{exp} = 8.77 m/s^2), <u>OR</u> by computing the percent difference between the g_{exp} and 9.80 m/s^2 (e.g., $\dfrac{9.80 - 8.77}{8.77} \times 100 = 11.7\%$, which is greater than 4%, for g_{exp} = 8.77 m/s^2) — 1 point

(e) 2 points

For correctly computing the acceleration of the elevator — 1 point

From Newton's 2nd law for objects in the elevator:

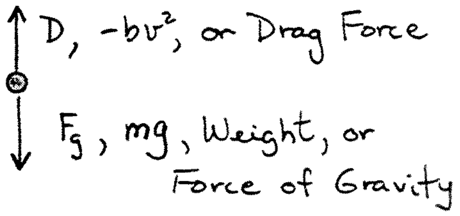
$mg - N = ma$, where $N = mg_{exp}$, so $a = g - g_{exp}$

For correctly stating whether the acceleration of the elevator is upward or downward — 1 point

Credit for magnitude and direction of acceleration given for answers consistent with g_{exp}. For example, for g_{exp} = 8.77 m/s^2, $a = 9.80 - 8.77 = 1.03\,m/s^2$, directed downward.

Mech. 2 (15 points)

(a) 3 points



For a vector arrow pointing downward — 1 point
For a vector arrow pointing upward — 1 point
For correct force labels on both vectors — 1 point
For any extra vectors drawn, deduct 1 point

(b) 3 points

For indicating that the acceleration decreases — 1 point
For a correct explanation that includes a correct mention of forces. — 2 points
For example, as the ball approaches terminal speed, the velocity increases, so the drag force increases and gets closer in magnitude to the gravitational force. The resultant force, which is the difference between the gravitational and drag forces, gets smaller, and since it is proportional to the acceleration, the acceleration decreases.

Partial credit of 1 point given for only a statement including a basic definition of terminal velocity (e.g., at terminal velocity v = constant, so a must decrease from 9.8 m/s^2 to zero)

(c) 2 points

For an expression for the resultant force on the ball — 1 point
$F = mg - bv^2$
Since $F = ma = m\dfrac{dv}{dt}$, then $m\dfrac{dv}{dt} = mg - bv^2$
For a correct differential equation — 1 point
$$\frac{dv}{dt} = g - \frac{b}{m}v^2$$

Students did not need to use the convention + and – for up and down, respectively, but they did have to be consistent in their sign notation for credit. The integral form of the differential equation was also acceptable.

Mech. 2 (continued)

(d) 3 points

For recognition that acceleration is zero at terminal speed	1 point
For setting the drag force equal to the gravitational force	1 point
$mg = bv_t^2$	
For a correct solution for v_t	1 point
$v_t = \sqrt{\dfrac{mg}{b}}$	

Full credit also given for writing answer only with no other work shown

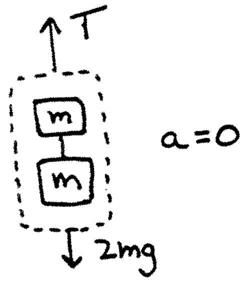
(e) 4 points

For a correct statement of work-energy, recognizing that the energy dissipated by the drag force is equal to the initial energy minus the final energy	1 point
For correct recognition of both initial potential energy ***mgh*** and final kinetic energy $\frac{1}{2}mv_t^2$	1 point
$\Delta E = mgh - \frac{1}{2}mv_t^2$	
For correct substitution of v_t from part (d)	1 point
$\Delta E = mgh - \frac{1}{2}m\left(\frac{mg}{b}\right)$	
For correct answer	1 point
$\Delta E = mg\left(h - \frac{m}{2b}\right)$	

<u>*Alternate partial solution*</u> (for maximum credit of 2 points)	(*Alternate points*)
For a correct integral for work	(*1 point*)
$W = \int P\,dt$ <u>OR</u> $W = \int F\,dx$	
For correct substitutions for P or F	(*1 point*)
$W = \int bv^3\,dt$ <u>OR</u> $W = \int bv^2\,dx$ <u>OR</u> $W = \int kv^2\,dx$	

Mech. 3 (15 points)

(a) 2 points



For a correct free-body diagram OR recognition that $a = 0$ OR correct use of Newton's second law — 1 point

$\Sigma F = ma$
$T - 2mg = 0$

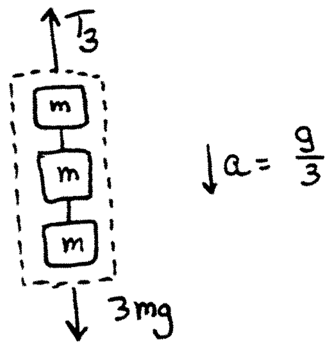
For the correct answer — 1 point

$T = 2mg$

Full credit was also awarded for writing the correct answer with no other work shown.

(b)

i. 2 points



$\Sigma F = ma$

For correct substitutions into Newton's second law — 1 point

$$3mg - T_3 = 3m\left(\frac{g}{3}\right)$$

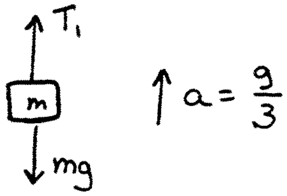
For a correct solution for T_3 — 1 point

$T_3 = 2mg$

Mech. 3 (continued)

(b) (continued)

ii. 2 points



$\Sigma F = ma$

For correct substitutions into Newton's second law — 1 point

$T_1 - mg = m\left(\frac{g}{3}\right)$

For a correct solution for T_1 — 1 point

$T_1 = \frac{4}{3}mg$

iii. 4 points

For $\tau = I_1\alpha$ — 1 point

For $\alpha = \frac{a}{R_1} = \frac{g}{3R_1}$ — 1 point

For $\tau = (T_3 - T_1)R_1$ — 1 point

For correct substitutions into $\tau = I_1\alpha$ and solution for I_1 — 1 point

$\left(2mg - \frac{4}{3}mg\right)R_1 = I_1\left(\frac{g}{3R_1}\right)$

$I_1 = 2mR_1^2$

Alternate Solution — (*alternate points*)

Use conservation of energy, $\Delta E = \Delta K + \Delta U = 0$

For $\Delta K = -\Delta U$ — (*1 point*)

For $\Delta K = \frac{1}{2}mv^2 + \frac{1}{2}(3m)v^2 + \frac{1}{2}I_1\omega^2$, where $\omega = \frac{v}{R_1}$ — (*1 point*)

For $\Delta U = mgh - 3mgh = -2mgh$, where $h = \frac{v^2}{2a} = \frac{3v^2}{2g}$ — (*1 point*)

For correct substitutions and solution for I_1 — (*1 point*)

$I_1 = 2mR_1^2$

Mech. 3 (continued)

(c)

i. 2 points

For recognition that the speed of the cord or the tangential speed of the pulleys is the same for both pulleys — 1 point

$\omega_1 R_1 = \omega_2 R_2 = \omega_2 (2R_1)$

For the correct answer — 1 point

$\omega_2 = \dfrac{\omega_1}{2}$

ii. 1 point

For correct substitutions in $L = I\omega$ and correct solution — 1 point

$L_2 = (16I_1)\left(\dfrac{\omega_1}{2}\right)$

$L_2 = 8I_1\omega_1$

iii. 2 points

For a correct expression for the kinetic energy as the sum of the kinetic energies of the two pulleys — 1 point

$K = \dfrac{1}{2} I_1\omega_1^2 + \dfrac{1}{2} I_2\omega_2^2$

For correct substitutions and solution — 1 point

$K = \dfrac{1}{2} I_1\omega_1^2 + \dfrac{1}{2}(16I_1)\left(\dfrac{\omega_1}{2}\right)^2$

$K = \dfrac{5}{2} I_1\omega_1^2$

E&M. 1 (15 points)

(a) 4 points

Since brightness is proportional to the power dissipated by a bulb, the answer may be found by solving the circuit to determine the power dissipated by each bulb. For example,

$\frac{1}{R_p} = \frac{1}{12\ \Omega} + \frac{1}{6\ \Omega} = \frac{3}{12\ \Omega}$, where R_p is the resistance of the parallel combination of resistors

$R_p = 4\Omega$

$$I_A = \frac{\mathcal{E}}{R_A + R_p} = \frac{42\ \text{V}}{10\ \Omega + 4\ \Omega} = 3\ \text{A}$$

$$I_B = \frac{V_p}{R_B} = \frac{I_A R_p}{R_B} = \frac{(3\ \text{A})(4\ \Omega)}{12\ \Omega} = 1\ \text{A}$$

$$I_C = \frac{V_p}{R_C} = \frac{I_A R_p}{R_C} = \frac{3 \cdot 4(3\ \text{A})(4\ \Omega)}{6\ \Omega} = 2\ \text{A}$$

$$P_A = I_A{}^2 R_A = (3\ \text{A})^2(10\ \Omega) = 90\ \text{W}$$

$$P_B = I_B{}^2 R_B = (1\ \text{A})^2(12\ \Omega) = 12\ \text{W}$$

$$P_C = I_C{}^2 R_C = (2\ \text{A})^2(6\ \Omega) = 24\ \text{W}$$

For correct ordering, i.e., bulb *A* is brighter than bulb *C*, which is brighter than bulb *B* — 3 points

(Partial credit of 1 point given for incorrect answer but with an indication that bulb *A* is brightest or that bulb *C* is brighter than bulb *B*.)

For a correct explanation, which can be by a quantitative solution for the currents and powers as above, or by a qualitative approach that notes that all the current in the circuit flows through bulb *A*, then branches in such a way that bulb *C* receives more current than bulb *B*. — 1 point

(b)

i. 3 points

Immediately after the switch is closed there is no current in the inductor so the circuit consists of resistors *A* and *B* in series with the source of emf.

For $I_C = 0$ — 1 point

For recognition that $I_A = I_B$ and they are nonzero — 1 point

For correct numerical answers for I_A and I_B, i.e., $I_A = I_B = \frac{42\ \text{V}}{10\ \Omega + 12\ \Omega} = 1.91\ \text{A}$ — 1 point

E&M. 1 (continued)

(b) (continued)

ii. 3 points

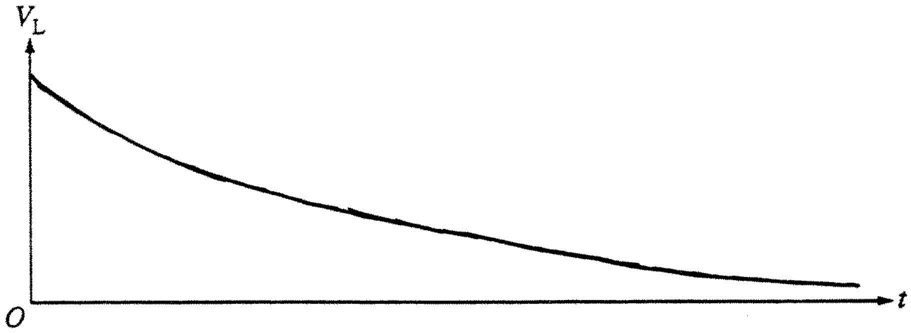
A long time after the switch is closed, the potential difference across the inductor is zero, so the circuit is essentially the same as in part (a)

For recognizing that $V_L = 0$	1 point
For correct currents, the same as in part (a), i.e., $I_A = 3$ A, $I_B = 1$ A, $I \;\; = 2$ A	2 points

(If currents not computed in part (a), they could be computed here.)

Unit point: For expressing all currents in (b) in correct units of amperes	1 point

(c) 2 points



Attributes of correct curve:

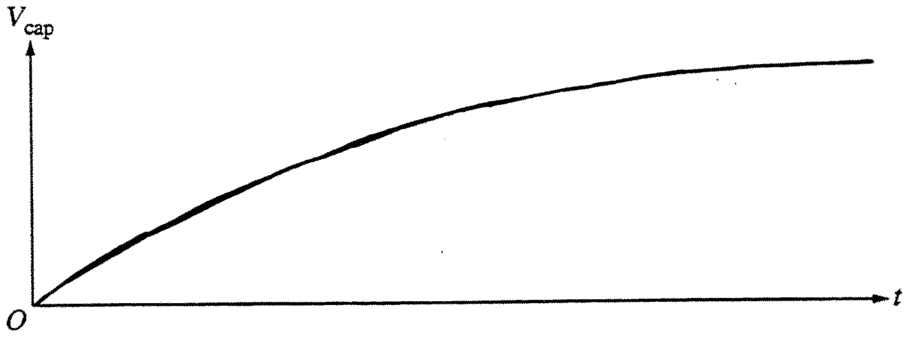
1. Starts at a nonzero but finite point on the vertical axis
2. Smooth
3. Concave upward
4. Has asymptote equal to zero

For a correct curve with all four attributes	2 points

Partial credit of 1 point for curve with flaws but at least two correct attributes

E&M. 1 (continued)

(d) 2 points



Attributes of correct curve:

1. Starts at zero
2. Smooth
3. Concave downward
4. Has finite but nonzero asymptote

For a correct curve with all four attributes — 2 points

Partial credit of 1 point for curve with flaws but at least two correct attributes

E&M. 2 (15 points)

(a)

i. 6 points

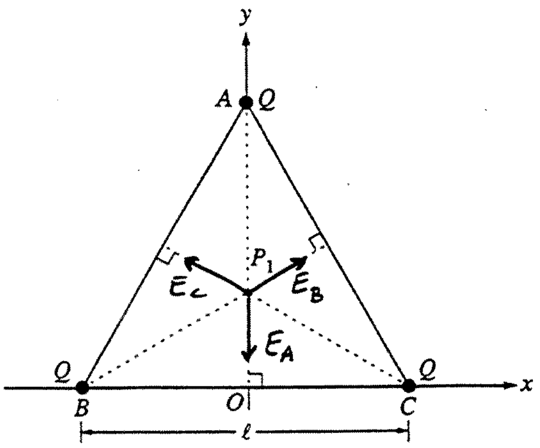


Figure 1

One point for each arrow drawn in the correct direction — 3 points

For not having all arrows approximately the same length, deduction of 1 point
For not having all arrows start at P_1, deduction of 1 point
For having one or more extra vectors, deduction of 1 point

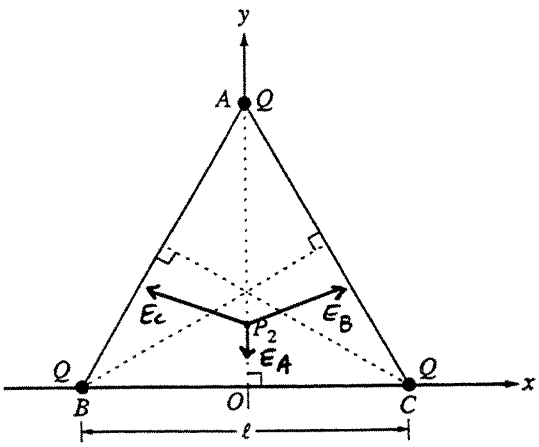


Figure 2

One point for each arrow drawn in the correct direction — 3 points

For not having lengths of arrows such that $E_C \approx E_B > E_A$, deduction of 1 point
For not having all arrows start at P_2, deduction of 1 point
For having one or more extra vectors, deduction of 1 point

E&M. 2 (continued)

(a) (continued)

ii. 3 points

	Greater than at P_1	Less than at P_1	The same as at P_1
E_A		✓	
E_B	✓		
E_C	✓		

One point for having check mark or other indicator in each correct box — 3 points

(b) 1 point

For an indication that the x-components of the field vectors due to particles C and B cancel each other due to the symmetry created by having a vertex of the triangle on the y-axis — 1 point

(c) 3 points

For an indication that the potential is the sum of the potentials due to the individual charges — 1 point

$$V = \sum_i \frac{kQ_i}{r_i} = k\left(\frac{Q_A}{r_A} + \frac{Q_B}{r_B} + \frac{Q_C}{r_C}\right)$$

For recognition that the terms due to the particles at B and C are equal — 1 point

$$V = k\left(\frac{Q_A}{r_A} + \frac{2Q}{r_B}\right)$$

For correct substitutions for Q's and r's and correct answer — 1 point

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{\frac{\sqrt{3}\ell}{2} - y} + \frac{2Q}{\sqrt{\frac{\ell^2}{4} + y^2}}\right)$$, or equivalent

E&M. 2 (continued)

(d) 2 points

Since $E_y = -\frac{d}{dy}V(y)$, to find the y coordinates of the points on the y-axis at which the electric field is zero, take the derivative of the expression in part (c) with respect to y, set the expression equal to zero and solve for y.

For recognition that E is a derivative of V — 1 point

For recognition that $\frac{dV}{dy} = 0$ — 1 point

Distribution of points

E&M. 3 (15 points)

(a)

i. 3 points

For a correct statement of Gauss's law — 1 point

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\epsilon}$$

For expressing the permittivity of the oil in terms of the dielectric constant κ — 1 point

$\epsilon = \kappa\epsilon_0$

For a correct expression for the electric field in the oil — 1 point

$$E(2\pi rL) = \frac{Q}{\kappa\epsilon_0}$$

$$E = \frac{Q}{2\pi\kappa\epsilon_0 rL}$$

ii. 2 points

For a correct statement of Gauss's law in the space outside the outer shell — 1 point

$$\oint \mathbf{E} \cdot d\mathbf{A} = 0$$

For stating that the electric field is zero in this region — 1 point

$\mathbf{E} = 0$

(b)

i. 3 points

For an expression for the electric potential between the two shells — 1 point

$$\Delta V = V_b - V_a = \int_a^b E_r\, dr$$

For substituting the expression for the electric field between the shells — 1 point

$$\Delta V = \frac{Q}{2\pi\kappa\epsilon_0 L}\int_a^b \frac{dr}{r}$$

For a correct expression for the electric potential difference between the shells — 1 point

$$\Delta V = \frac{Q}{2\pi\kappa\epsilon_0 L}\ln\left(\frac{b}{a}\right)$$

E&M. 3 (continued)

(b) (continued)

ii. 2 points

For an expression for the capacitance in terms of Q and ΔV — 1 point

$$C = \frac{Q}{\Delta V}$$

Substituting the expression for ΔV from (b)i:

$$C = \frac{Q}{\frac{Q}{2\pi\kappa\epsilon_0 L}\ln\left(\frac{b}{a}\right)}$$

For a correct expression for the capacitance — 1 point

$$C = \frac{2\pi\kappa\epsilon_0 L}{\ln\left(\frac{b}{a}\right)}$$

(c)

i. 3 points

For a correct statement of Ampere's law — 1 point

$$\oint \mathbf{B} \cdot d\boldsymbol{\ell} = \mu_0 I$$

For substituting the current through the inner shell — 1 point

$$B(2\pi r) = \mu_0\left(\frac{\mathcal{E}}{R}\right)$$

For a correct expression for the magnetic field between the shells — 1 point

$$B = \frac{\mu_0 \mathcal{E}}{2\pi r R}$$

ii. 2 points

For the correct substitution of the total current through both shells in to Ampere's law — 1 point

$$B(2\pi r) = \mu_0\left(\frac{4\mathcal{E}}{R}\right)$$

For a correct expression for the magnetic field around the outer shell — 1 point

$$B = \frac{2\mu_0 \mathcal{E}}{\pi r R}$$