# AP® Physics C
# 1974 Scoring Guidelines

1974 C

SP

PHYSICS C

SECTION II, MECHANICS

Time—45 minutes

3 Questions

Mech. 1.

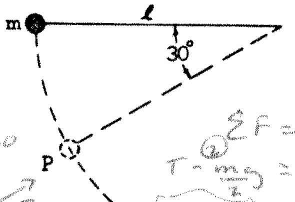
A pendulum consisting of a small heavy ball of mass m at the end of a string of length $\ell$ is released from a horizontal position. When the ball is at point P, the string forms an angle of $30^0$ with the horizontal as shown above.

(a) In the space below, draw a force diagram showing all of the forces acting on the ball at P. Identify each force clearly.

(b) Determine the speed of the ball at P.

(c) Determine the tension in the string when the ball is at P.

(d) Determine the tangential acceleration of the ball at P.




STANDARD SOLUTION:-

(a) [3 pts] as in B-Exam

(b) [3 pts] as in B-Exam $\quad V = \sqrt{g\ell}$

(c) [6 pts] as in B-Exam $\quad T = \frac{3}{2}mg$

(d) [3 pts] $F_T = ma_T$ [1 pt]

$mg\cos\theta = ma_T$ [1 pt]

$a_T = g\cos\theta$ [1 pt]

(c) $mg\sin 30$ T mg

① $\Sigma F = ma_c$

② $T - \frac{mg}{2} = m\frac{v^2}{R}$ ②

① $\boxed{T - \frac{mg}{2}} = m\frac{\ell g}{\ell}$

$T = \frac{3}{2}mg$

A student Solution [Score = 9 pts]:-

(a) T the tension in the string

mg the force of gravity ③pts

(b) $T\cos 60^\circ$ $60^\circ$ T mg

$T\sin 60^\circ$ $h$ $30^\circ$ $\ell$

$\frac{h}{\ell} = .5$

$h = .5\ell$

$\boxed{u = \sqrt{g\ell}}$ ③pts

$V = mgh$

$T = \frac{1}{2}mu_0$

$V_0 + T_0 = V_h + T_h$

$mg\ell = mg(.5\ell) + \frac{1}{2}mu^2$

$2g\ell = g\ell + u^2$

(c) $T\sin 60^\circ = mg$ $60^\circ$ T mg $\boxed{T = 2mg}$ ⓪pts

(d) $30^\circ$ $60^\circ$ 3mg $30^\circ$ $60^\circ$ $30^\circ$ mg F

$F = mg\frac{\sqrt{3}}{2}$

$F = ma$

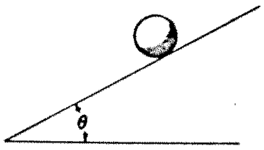
$\boxed{a = g\frac{\sqrt{3}}{2}}$ ③pts

Mech. 2.

The moment of inertia of a uniform solid sphere (mass M, radius R) about a diameter is $\frac{2}{5}MR^2$. The sphere is placed on an inclined plane (angle $\theta$) as shown above and released from rest.

(a) Determine the minimum coefficient of friction $\mu$ between the sphere and plane with which the sphere will roll down the incline without slipping.

(b) If $\mu$ were zero, would the speed of the sphere at the bottom be greater, smaller, or the same as in part (a)? Explain your answer.




STANDARD SOLUTION:

(a) [10 pts]

METHOD 1:-

$f = \mu N = \mu Mg\cos\theta$ [2 pts]

$\tau = I\alpha$ [2 pts]; $a = R\alpha$ [1 pt]

$fR = \frac{2}{5}MR^2(a/R)$; $f = \frac{2}{5}Ma$ [1 pt]

$\therefore a = \frac{5}{2}\mu g\cos\theta$

$\Sigma F = Ma$ [2 pts]

$Mg\sin\theta - \mu Mg\cos\theta = M\frac{5}{2}\mu g\cos\theta$ [1 pt]

$\mu(\frac{5}{2}+1)\cos\theta = \sin\theta$

$\mu = \frac{2}{7}\tan\theta$ [1 pt]

METHOD 2:

$f = \mu N = \mu Mg\cos\theta$ [2 pts]

$\Sigma\tau_P = I_P\alpha$ [1 pt]; $a = R\alpha$ [1 pt]

$I_P = I_{cm} + MR^2 = \frac{7}{5}MR^2$ [1 pt]

$a = \frac{5}{7}g\sin\theta$ [1 pt]

$\Sigma F = ma$ [2 pts]

$Mg\sin\theta - \mu Mg\cos\theta = M\frac{5}{7}g\sin\theta$ [1 pt]

$\mu = \frac{2}{7}\tan\theta$ [1 pt]

METHOD 3:-

used Conservation of Energy:

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

VERY RARE!

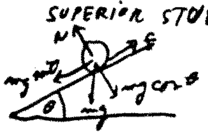
MECH 2 (con't)

$\mu = 0$
$\Sigma F = ma$
$mg\sin\theta = ma$

$\mu = \min$
$a = \frac{5}{2}\frac{f}{m} = \frac{5}{2}\mu g\cos\theta$
$= \frac{5}{2}\cdot\frac{2}{7}\tan\theta\, g\cos\theta$
$= \frac{5}{7} g\sin\theta$
$= \boxed{\frac{5}{7}\ \text{sliding}}$

(b) [5 pts]
i) Energy is the same at the bottom in both cases. [2 pts]
ii) If $\mu = 0$, there is no energy of rotation [1 pt.]
iii) Thus, more energy available for translation and velocity is greater when $\mu = 0$ than when $\mu > 0$. [2 pts]

SUPERIOR STUDENT SOLUTION, PART (a):




$\Sigma F_y = 0;\ N = mg\cos\theta$ ①

$\Sigma F_x = ma;\ ma = mg\sin\theta - f$ ②

$ma = mg\sin\theta - \mu N$

④ $ma = mg\sin\theta - \mu mg\cos\theta$ combine ①+②

$\tau = r\times F = rF\sin 90^\circ = Rf$
$\tau = I\alpha = \frac{2}{5}mR^2\left(\frac{a}{R}\right) = \frac{2}{5}mRa$
These torques are equal

$Rf = \frac{2}{5}mRa$

$a = \frac{5}{2}\frac{f}{m}$ ③

$\cancel{m}\left(\frac{5}{2}\frac{\mu \cancel{m}g\cos\theta}{\cancel{m}}\right) = \cancel{m}g\sin\theta - \mu\cancel{m}g\cos\theta$ plug ③ into ④

$\mu\left(\frac{5}{2}\cos\theta + \cos\theta\right) = \sin\theta$

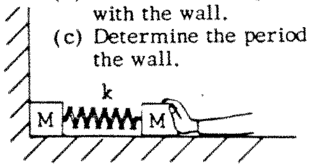
$\mu = \frac{2}{7}\frac{\sin\theta}{\cos\theta}$

$\boxed{\mu = \frac{2}{7}\tan\theta}$

Mech. 3.

A system consists of two blocks, each of mass M, connected by a spring of force constant k. The system is initially shoved against a wall so that the spring is compressed a distance D from its original uncompressed length. The floor is frictionless. The system is now released with no initial velocity.

(a) Determine the maximum speed of the right-hand block.
(b) Determine the speed of the center of mass of the system when the left-hand block is no longer in contact with the wall.
(c) Determine the period of oscillation for the system when the left-hand block is no longer in contact with the wall.




STANDARD SOLUTION:

(a) [5 pts]
Conservation of energy [2 pts]

$\frac{1}{2}kD^2 = \frac{1}{2}Mv^2$ [2 pts]

$v = \sqrt{\frac{kD^2}{M}} = D\sqrt{\frac{k}{M}}$ [1 pt]

(b) [5 pts]
Conservation of momentum [2 pts]

$(2M)v_{CM} = Mv + M(0)$ [1 pt]

$v_{CM} = \frac{1}{2}v$ [1 pt]

$v_{CM} = \frac{D}{2}\sqrt{\frac{k}{M}}$ [1 pt]

(c) [5 pts]
$T = 2\pi\sqrt{\frac{m}{k}}$ [2 pts]

But $m$ = reduced mass $= \frac{M}{2}$ [2 pts]

$T = 2\pi\sqrt{\frac{M}{2k}}$ [1 pt]

STUDENT VARIATIONS:-

(a) Usually no problem.

(b) Attempts to use conservation of energy resulting in $v_{CM} = D\sqrt{\frac{k}{2M}}$

(c) Failure to recognize need for reduced mass to define system motion.

Many who could not perform a math analysis could see that maximum velocity happens when spring is unstretched and that left block leaves the wall when spring reaches this unstretched point [+2 pts].

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PHYSICS C

SECTION II, ELECTRICITY AND MAGNETISM

E & M I.

Two concentric conducting spherical shells of radii a and b have charges of equal magnitude and opposite sign as shown above.

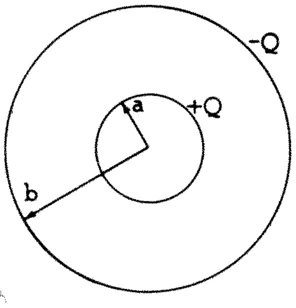
(a) Determine the electric field at a distance r from the center. Give separate expressions for $r < a$, $a < r < b$, and $r > b$.

(b) Determine the electric potential at $r = b$ and at $r = a$, taking the potential equal to zero at infinity.

STANDARD SOLUTION:-

(a) METHOD 1:

[8 pts] By symmetry, $\vec{E}$ is radial and a function of $r$ only.




Gauss' Law: $\oint \vec{E}\cdot d\vec{a} = 4\pi k\, q_{enclosed}$ [2 pts]

For a sphere of radius $r$, surface area $= 4\pi r^2$

Then $E \cdot 4\pi r^2 = 4\pi\, q_{enclosed}$ or $E = k\, q_{enclosed}/r^2$.

For $r < a$, $q_{enclosed} = 0$; $\therefore E = 0$ [2 pts]

For $a < r < b$, $q_{enclosed} = Q$; $\therefore E = k\frac{Q}{r^2}$ [2 pts] $= \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$

For $r > b$, $q_{enclosed} = 0$; $\therefore E = 0$ [2 pts.]

METHOD 2:-

A uniform spherical shell of charge acts like a point charge at its center as far as the field outside is concerned, but produces zero field inside. [2 pts]

$\therefore$ For $r < a$ (inside both shells) $E = 0$ [2 pts]

For $a < r < b$ (same as point charge $+Q$) $E = k\frac{Q}{r^2}$ [2 pts]

For $r > b$ (same as a charge $+Q - Q = 0$ at center) $E = 0$ [2 pts]

(b) METHOD 1:

[7 pts] By definition: $V(r) = \int_r^\infty \vec{E}\cdot d\vec{l} = \int_r^\infty E\,dr$ [1 pt]

For $r > b$, $E = 0$, $\therefore V(b) = \int 0\, dr = 0$ [2 pt]

Then for $a < r < b$, $E = k\frac{Q}{r^2}$, $\therefore$

$V(a) = \int_a^\infty E\,dr = \int_a^b \frac{kQ}{r^2}\,dr = kQ\left[\frac{1}{a} - \frac{1}{b}\right]$ [3 pts]

METHOD 2:

Potential of a spherical shell of radius R is: $V = \frac{kQ}{r}$ for $r > R$ + $V = \frac{kQ}{R}$ for $r \le R$. [2 pts]

So $V(b) = \frac{kQ}{b} + \frac{k(-Q)}{b} = 0$ [2 pts]

while:

$V(a) = \frac{kQ}{a} + \frac{k(-Q)}{b} = kQ\left[\frac{1}{a} - \frac{1}{b}\right]$ [3 pts]

COMMENTARY:- Many students missed the area of a sphere! A number did not try Gauss' Law in (a) - but "all" knew the field inside a conductor is zero.

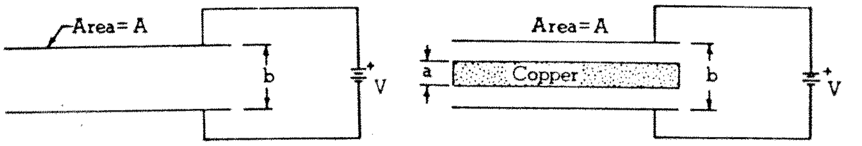
E & M 2.

A parallel-plate capacitor with spacing b and area A is connected to a battery of voltage V as shown above. Initially the space between the plates is empty. Make the following determinations in terms of the given symbols.

(a) Determine the electric field between the plates.
(b) Determine the charge stored on each capacitor plate.

A copper slab of thickness a is now inserted midway between the plates as shown below.

(c) Determine the electric field in the spaces above and below the slab.
(d) Determine the ratio of capacitances $\frac{C_{\text{with copper}}}{C_{\text{original}}}$ when the slab is inserted.




STANDARD SOLUTION:-

METHOD I (FOLLOWED BY 3 OF 8&7 EXAMINER)

(a) $V = \int \vec{E}\cdot d\vec{l} = Eb$

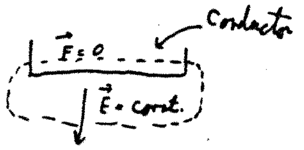
$\therefore\ E = V/b$;

with the fieldlines either airface

$\oint \vec{E}\cdot d\vec{s} = q/\epsilon_0 = 0$,

$\therefore\ \vec{E}$ is constant & $\perp$ to surface of the plates.

(b) $\oint \vec{E}\cdot d\vec{s} = q/\epsilon_0$




$\oint \vec{E}\cdot d\vec{s} = EA = q/\epsilon_0$

$q = \epsilon_0 EA = \epsilon_0 VA/b$

(c) $V = \int \vec{E}\cdot d\vec{l}$ but $\vec{E} = 0$ inside copper

$\therefore\ V = E(b-a)$

$E = V/(b-a)$

(d) From (b) & (c):

$C_{Cu} = \epsilon_0 AV/(b-a)$

$C_{air} = \epsilon_0 AV/b$

$\therefore\ \frac{C_{Cu}}{C_{air}} = \frac{b}{b-a}$

METHOD 2 (ONE USED BY STUDENTS):-

(a) $E = V/b$ [2 pts]

$C_{air} = \frac{\epsilon_0 A}{b}$ [2 pts]

$Q = CV$ [2 pts]

(b) $Q = \frac{\epsilon_0 VA}{b}$ [2 pts]

Copper slab in:

(c) $C_{Cu} = \frac{\epsilon_0 A}{b-a}$ or $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{\epsilon_0 A}{b-a}$ [2 pts]

(d) Ratio $= \frac{b}{b-a}$ [3 pts]

A STUDENT SOLUTION (SCORED 8 pts)

(a) $\mathcal{E} = \sigma/2\epsilon_0$ $\therefore V = \frac{\sigma b}{2\epsilon_0}$ $\boxed{\mathcal{E} = \frac{V}{b}}$ 2 pts

(b) $Q = \sigma A$ $\sigma = \frac{2\epsilon_0 V}{b}$ $\boxed{Q = \frac{2\epsilon_0 VA}{b}}$ 4 pts

(c) $\epsilon_c$ - copper $\boxed{\mathcal{E} = \frac{2V}{b-a}}$ $\left(V = \frac{\sigma \epsilon_c b}{2\epsilon_0}\right)$ 2 pts

(d) $C_0 = \frac{Q}{V} = \frac{2\epsilon_0 a}{b}$ $\therefore Q_c = \sigma_c a = \frac{4\epsilon_0 V}{b-a}$

$C_c = \frac{4\epsilon_0 a}{b-a}$ $\epsilon_c$ for copper is missing? 0 pts

$\frac{C_c}{C_0} = \frac{\frac{4\epsilon_0 a}{b-a}}{2\epsilon_0 a/b} = \frac{2b}{(b-a)} \rightarrow \boxed{\frac{2b}{\epsilon_c(b-a)}}$

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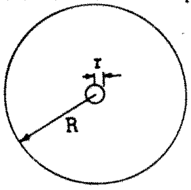
E & M 3.

A small circular loop of wire with radius r is placed at the center of a large circular loop of wire with radius R. The two loops lie in the same plane, and $r \ll R$. In the outer loop there is a sinusoidal current $I = I_0 \sin \omega t$, where t is time and $I_0$ and $\omega$ are constants. Find an expression for the induced emf in the inner loop.




STANDARD SOLUTION:

(i) [2 pts] The magnetic field in the region of the small loop is given by the Biot Savart Law as:

$$\vec{B}(t) = \frac{2\pi k' I(t)}{R}\hat{n} \text{ or } \frac{\mu_0 I(t)}{R}\hat{\theta}$$

(ii) [1 pt] or $B(t) = \frac{2\pi k'}{R} I_0 \sin \omega t = \frac{\mu_0 I_0 \sin \omega t}{2R}$

(iii) [4 pts] From the Faraday-Henry Law:

$$\mathcal{E}_{mf} = -\frac{d\Phi_B}{dt} \quad [2 pts]$$

$$d\Phi_B = A\, dB$$

$$\therefore \mathcal{E}_{mf} = -A\frac{dB}{dt} \quad [2 pts]$$

(iv) [6 pts] $\mathcal{E}_{mf} = -A\frac{d}{dt}\left[\frac{2\pi k'}{R} I_0 \sin \omega t\right]$

$$A = \pi r^2 \quad [2 pt]$$

$$\mathcal{E}_{mf} = -\pi r^2 \frac{d}{dt}\left[\frac{2\pi k'}{R} I_0 \sin \omega t\right] \quad [4 pts]$$

(v) [2 pts] $\mathcal{E}_{mf} = \left[-\frac{2\pi^2 r^2 k'}{R} I_0 \omega\right] \cos \omega t$

$$k' = \frac{\mu_0}{4\pi}$$

$$\mathcal{E} = -\frac{4\pi}{2}\frac{\mu_0 r^2}{R} I_0 \omega \cos \omega t$$

Student's response:

1) B at center:

Biot's Law, $d\vec{B} = k\frac{d\vec{l}i \times \hat{r}}{r^2}$

2) $d\vec{l}$ is always $\perp$ to $\hat{r}$ for center:

$\therefore dB = k\frac{dl\, i}{R^2}$, $B = \frac{k i}{r^2}\int dl$ ($i$ constant for given time.)

$$= \frac{ki}{R^2} 2\pi R = \frac{k i 2\pi}{R}$$

3) $\Phi$ of closed loop r:

B almost ~~constant~~ for small loop

$$\therefore \Phi = \left(\frac{k i 2\pi}{R}\right)(\pi r^2) = \frac{k 2\pi^2 r^2}{R} I_0 \sin \omega t$$

4) From Faraday's law

$$\mathcal{E}_{ind} = -\frac{d\Phi}{dt}$$

$$= \boxed{-\left(\frac{k 2\pi^2 r^2}{R}\right)(I_0)(\omega \cos \omega t)}$$

This is typical answer for 7-10% of all examinees.