AP® Physics C
1983 Scoring Guidelines

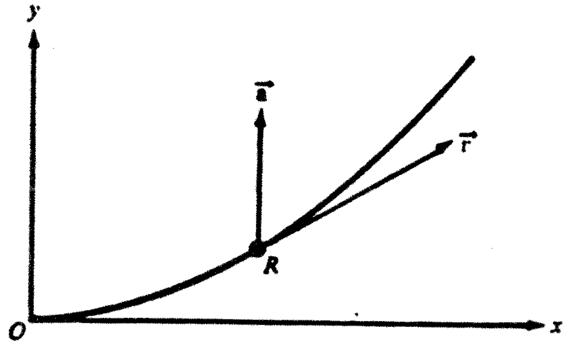
1983 Physics C Solutions

Mechanics

Mech. 1. (a) 9 points

i. The velocity vector is tangent to the path. — 1 point

Since v_x = constant, $a_x = 0$, and the acceleration vector has only a positive y-component as shown below. — 2 points



ii. The y-component of velocity is $v_y = \frac{dy}{dt}$ — 1 point

By the chain rule, $\frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt}$ — 1 point

Since $y = \frac{1}{2}x^2$, $\frac{dy}{dx} = x$ — 1 point

and $v_y = x\frac{dx}{dt} = Cx$ — 1 point

iii. The acceleration is given by $a_y = \frac{dv_y}{dt}$ — 1 point

so $a_y = C\frac{dx}{dt} = C^2$ — 1 point

Alternate solution for ii and iii: — *(Alternate points)*

ii. Since $y = \frac{1}{2}x^2$ and $x = Ct$, $y = \frac{1}{2}C^2t^2$ — (1 point)

Thus $v_y = \frac{dy}{dt} = C^2t$ — (2 points)

Since $Ct = x$, $v_y = Cx$ — (1 point)

iii. $a_y = \frac{dv_y}{dt} = \frac{d}{dt}(C^2t) = C^2$ — (2 points)

Distribution of points

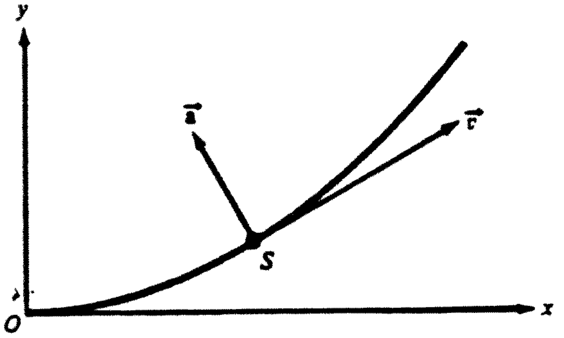
(b) 6 points

i. The speed is given by $v = \sqrt{v_x^2 + v_y^2}$ — 1 point

By the chain rule, $v_y = \frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt} = xv_x$ — 1 point

So $v = \sqrt{v_x^2(1+x^2)} = \sqrt{\frac{C^2}{1+x^2}(1+x^2)} = C$ — 1 point

ii. Again the velocity vector is tangent to the path. — 1 point
Since the speed is constant, there is no component of $\vec{a}$ along the path, so $\vec{a}$ is centripetal, perpendicular to $\vec{v}$ as shown. — 2 points



Mech. 2. (a) 4 points

On the block, the forces are weight $= m_2g$ — 1 point
and tension T — 1 point
On the disk, there are tension T — 1 point
and the pair: m_1g and supporting force — 1 point

(b) i. and ii. 8 points

On the disk, torque $= \Gamma = I\alpha = \frac{1}{2}m_1R^2\alpha$ — 1 point
with $\Gamma = T \cdot R$ and — 1 point
acceleration of the block $= a = R\alpha$, so $\alpha = a/R$ — 1 point
For the block, $F_{net} = m_2a$ — 1 point
with $F_{net} = m_2g - T$ — 1 point
Solving simultaneously gives — 1 point

$a = \frac{2m_2g}{2m_2 + m_1}$ and — 1 point

$T = \frac{m_1m_2g}{2m_2 + m_1}$ — 1 point

Distribution of points

(b) iii. 3 points

By definition, angular momentum $L = I\omega$ — 1 point

Angular velocity $\omega = \alpha t$ — 1 point

Since $I = \frac{1}{2} m_1 R^2$ and $\alpha = \frac{a}{R}$,

$L = \frac{1}{2} m_1 R^2 \cdot \frac{a}{R} \cdot t = \frac{1}{2} m_1 a R t$

Substituting the expression for a from part (b) gives

$L = \dfrac{m_1 m_2 g R t}{2m_2 + m_1}$ — 1 point

Mech. 3. (a) 11 points

i. Energy is conserved as the particle slides down the sphere, so

$K + U =$ constant, — 2 points

where gravitational potential energy $U = mgh$. — 1 point

If h is measured from the center of this sphere,

$0 + mgR = K + mgR \cos\theta$ — 1 point

so that

$K = mgR\,(1 - \cos\theta)$. — 1 point

ii. The centripetal acceleration is

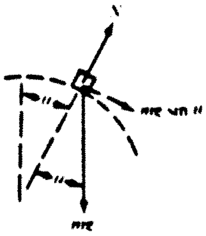
$a_c = \dfrac{v^2}{R}$ — 1 point

From part i:

$\frac{1}{2} m v^2 = mgR(1 - \cos\theta)$ — 1 point

so $a_c = \dfrac{v^2}{R} = 2g(1 - \cos\theta)$ — 1 point

iii. The only force with a tangential component is the gravitational force mg. As the diagram shows, its tangential component is $mg \sin\theta$



2 pt

By Newton's second law, $ma_T = mg \sin\theta$, so

$a_T = g \sin\theta$ — 1 point

(b) 4 points

The particle leaves the sphere when the normal force N has decreased to zero. — 2 points

At that point the radial component of the weight provides the centripetal acceleration, so

$mg \cos\theta = ma_c$ — 1 point

Therefore $g \cos\theta = a_c = 2g(1 - \cos\theta)$

and $\cos\theta = \frac{2}{3}$

or $\theta = \arccos \frac{2}{3}$ — 1 point

Electricity and Magnetism

	Distribution of points
E&M 1. (a) 6 points	
Gauss's Law states that	
$\Phi = \oint \vec{E} \cdot d\vec{A} = \frac{q(\text{enclosed})}{\epsilon_0}$	2 points
Apply it to a sphere of radius r ($a < r < b$). Since $\vec{E}$ is uniform and directed outward,	
$\oint \vec{E} \cdot d\vec{A} = E \cdot (\text{area of sphere}) = E \cdot 4\pi r^2$	2 points
The enclosed charge is Q.	1 point
Therefore $E = \frac{Q}{4\pi\epsilon_0 r^2}$	1 point
(b) 5 points	
The potential V_0 equals the potential difference between the spheres, which may be expressed as	
$\int \vec{E} \cdot d\vec{r}$	1 point
Therefore $V_0 = \frac{Q}{4\pi\epsilon_0} \int_a^b \frac{dr}{r^2}$	1 point
Evaluating the integral gives	
$V_0 = \frac{Q}{4\pi\epsilon_0} \left[-\frac{1}{r} \right]_a^b$	2 points
or $V_0 = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{a} - \frac{1}{b} \right) = \frac{Q}{4\pi\epsilon_0} \cdot \frac{b-a}{ab}$	1 point
(c) 4 points	
By definition	
$C = \frac{Q}{V_0}$	2 points
Substitution of the expression for V_0 from part (b) gives	
$C = \frac{Q}{\frac{Q}{4\pi\epsilon_0} \cdot \frac{b-a}{ab}} = \frac{4\pi\epsilon_0 ab}{b-a}$	2 points
E&M 2. (a) 4 points	
Initially there is no potential drop across the capacitor, so	
$\mathcal{E} = i_0 R$	2 points
and $\mathcal{E} = 10(10^{-6})(2 \times 10^6) = 20$ volts	2 points

Distribution of points

(b) 5 points

Current i and charge Q are related by

$i = \frac{dQ}{dt}$ or $Q = \int i dt$ — 1 point

so that $Q = \int i_0 e^{\left(\frac{-t}{6}\right)} dt$ — 1 point

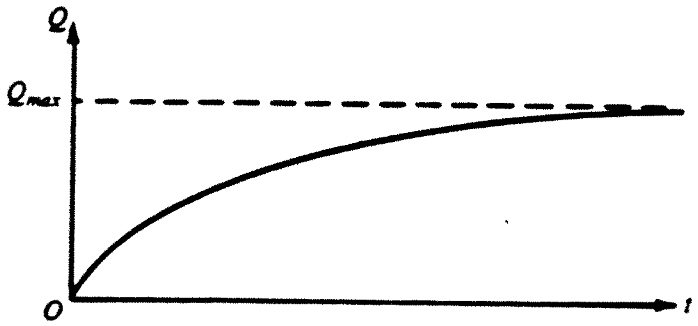
Evaluation of the antiderivative gives

$Q = -6 i_0 e^{\left(\frac{-t}{6}\right)} + C$ — 2 points

Since $Q = 0$ when $t = 0$, the constant of integration C must equal $6i_0$, and

$Q = 6i_0\left(1 - e^{\left(\frac{-t}{6}\right)}\right)$ — 1 point

(c) 3 points



$Q = 0$ at $t = 0$ — 1 point

As $t \rightarrow \infty$, $Q \rightarrow Q_{max}$ — 1 point

Exponential shape — 1 point

(d) 3 points

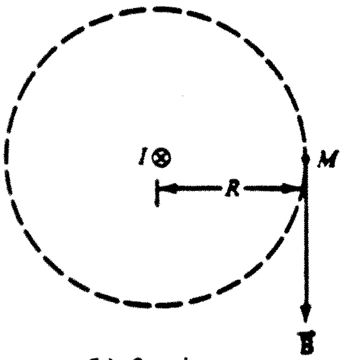
For a charging or discharging capacitor, the time constant is RC. — 1 point

Here, the numerical value of the time constant is 6 seconds, so

$C = \frac{6}{R} = \frac{6}{2 \times 10^6} = 3 \times 10^{-6}$ farads — 2 points

Distribution of points

E&M 3. (a) 6 points



The field at M is down. — 1 point

The path of integration is a circle around I. — 1 point

Ampere's Law states that $\oint \vec{B} \cdot d\vec{l} = \mu_0 I$ — 2 points

Applied to the circular path, it gives

$B \cdot 2\pi R = \mu_0 I$ — 1 point

so that $B = \dfrac{\mu_0 I}{2\pi R}$ — 1 point

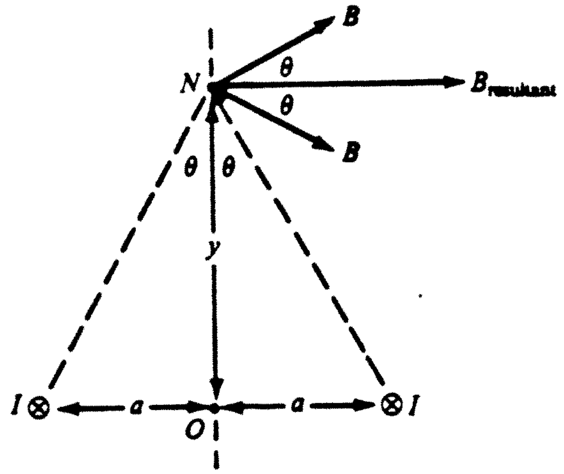
(b) 9 points

i. 3 points

At point O, the B field is zero — 2 points

because the field caused by the right current cancels the field caused by the left current. — 1 point

ii. 6 points



Each current contributes a field vector of magnitude

$B = \dfrac{\mu_0 I}{2\pi\sqrt{a^2 + y^2}}$ — 1 point

The vector sum of the two is directed to the right. — 1 point

$B_{resultant} = 2 \cdot B_x$ — 1 point

$= 2 \cdot B \cdot \cos\theta$ — 1 point

$= 2 \cdot B \cdot \left(\dfrac{y}{\sqrt{a^2 + y^2}}\right)$, giving the final result — 1 point

$B_{resultant} = \dfrac{\mu_0 I y}{\pi(a^2 + y^2)}$, directed to the right. — 1 point