AP® Physics C
1990 Scoring Guidelines

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SOLUTIONS

1990 Physics C — Distribution of points

Mech 1.

(a) 4 points

For an expression of Newton's Law, $\mathbf{F}_{net} = m\mathbf{a}$ — 1 point

$\mathbf{F}_{net} = -k\mathbf{v}$ — 1 point

For calculating the correct acceleration corresponding to the initial velocity:

$m\mathbf{a}_0 = -k\mathbf{v}_0$

$\mathbf{a}_0 = -\dfrac{k\mathbf{v}_0}{m}$ — 1 point

For the correct direction, indicated by a negative sign or the words "to the left" — 1 point

(b) 6 points

For the recognition that $a = \dfrac{dv}{dt}$ — 1 point

$\dfrac{dv}{dt} = -\dfrac{kv}{m}$ — 1 point

$\dfrac{dv}{v} = -\dfrac{k}{m}\,dt$

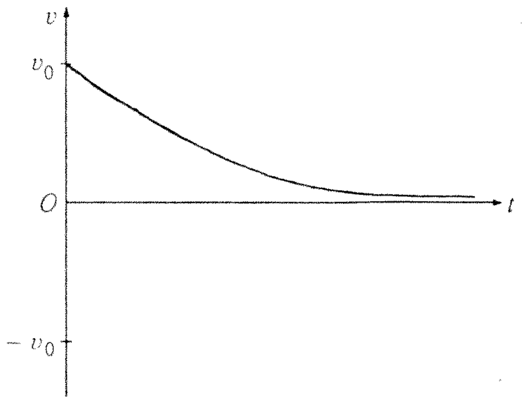
Upon integration,

$\ln v = -\dfrac{k}{m}t + \ln C$

$v = Ce^{-kt/m}$

$v = v_0$ at $t = 0$ gives $C = v_0$

$v = v_0 e^{-kt/m}$ — 1 point



For showing $v = v_0$ at $t = 0$ — 1 point

For a curve reasonably representative of exponential decrease — 1 point

For the curve asymptotically approaching zero as t approaches infinity — 1 point

Distribution of points

(c) 4 points

$v = \dfrac{dx}{dt} = v_0 e^{-kt/m}$ — 1 point

$dx = v_0 e^{-kt/m}\, dt$

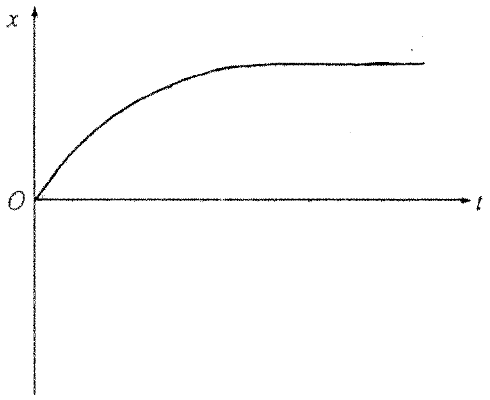
Upon integration,

$x = v_0\left(-\dfrac{m}{k}\right)e^{-kt/m} + C$

$x = 0$ at $t = 0$ gives $C = \dfrac{mv_0}{k}$

$x = \dfrac{mv_0}{k}\left(1 - e^{-kt/m}\right)$ — 1 point

(These points were awarded if a correct integration was performed using an incorrect expression for v)



For a reasonable concave-down curve — 1 point

For $x = 0$ at $t = 0$ and approaching an asymptote as t approaches infinity — 1 point

(d) 1 point

For $t \longrightarrow \infty$, $e^{-kt/m} \longrightarrow 0$

$x = \dfrac{mv_0}{k}$ — 1 point

1990 Physics C | Distribution of points

Mech 2.

(a) 4 points

For use of conservation of energy	1 point
$mgH = \frac{1}{2}mv_0^2$	
For correct expression for potential energy	1 point
For correct expression for kinetic energy	1 point
$H = \frac{v_0^2}{2g}$	1 point
Alternate solution	(Alternate points)
$a = -g \sin \theta$	(1 point)
$v^2 - v_0^2 = 2ad$ (or equivalent relevant kinematics)	(1 point)
(d is the distance traveled up the incline)	
$d = H/\sin \theta$	(1 point)
$v^2 - v_0^2 = -2(g \sin \theta)(H/\sin \theta) = -2gH$	
$v = 0$ at highest point	
$H = \frac{v_0^2}{2g}$	(1 point)

(b) 5 points

For relating kinetic energy K, potential energy U, and work done by frictional force W_f:

$K = W_f + U$	1 point
$W_f = F_f d$	1 point
$F_f = \mu mg \cos \theta$	1 point
$d = h/\sin \theta$	1 point
Therefore, $W_f = (\mu mg \cos \theta)(h/\sin \theta)$	
$\frac{1}{2}mv_0^2 = (\mu mg \cos \theta)(h/\sin \theta) + mgh$	
$= mgh(\mu \cot \theta + 1)$	
$h = \frac{v_0^2}{2g(\mu \cot \theta + 1)} = \frac{H}{\mu \cot \theta + 1}$	1 point

Distribution of points

Alternate solution (Alternate points)

$F = mg \sin\theta + \mu mg \cos\theta$ (one point for each term) (2 points)

$a = -g(\sin\theta + \mu\cos\theta)$ (1 point)

$v^2 - v_0{}^2 = 2ad$

$d = h/\sin\theta$ (1 point)

$-v_0{}^2 = -2g(\sin\theta + \mu\cos\theta)\ h/\sin\theta$

$$h = \frac{v_0{}^2 \sin\theta}{2g(\sin\theta + \mu\cos\theta)} = \frac{H\sin\theta}{\sin\theta + \mu\cos\theta} = \frac{H}{1 + \mu\cot\theta}$$ (1 point)

(c) 4 points
For including both translational and rotational kinetic energy in an equation for conservation of energy:

$K_{trans} + K_{rot} = mgh'$ 1 point

$K_{rot} = \frac{1}{2}I\omega^2$ 1 point

$I = mR^2$ 1 point

$$\left.\begin{aligned} \omega &= \frac{v}{R} \\ \frac{1}{2}mv_0{}^2 + \frac{1}{2}(mR^2)\left(\frac{v_0}{R}\right)^2 &= mgh' \\ \frac{1}{2}mv_0{}^2 + \frac{1}{2}mv_0{}^2 &= mgh' \\ h' = \frac{v_0{}^2}{g} &= 2H \end{aligned}\right\}$$ 1 point

Alternate solution (Alternate points)

For an expression relating torque to angular acceleration:

$\sum\tau = I\alpha$ (1 point)

Taking the torque about the point of contact with the incline:

$(-mg)(R)(\sin\theta) = I'\alpha$

$I' = I + mR^2 = 2mR^2$ (1 point)

$\alpha = \frac{a}{R}$ (1 point)

(This point was awarded only if some indication was present that the point of contact was used as the reference point.)

 Distribution of points

$-mgR\ \sin\ \theta = 2mR^2\frac{a}{R}$

$a = \frac{-g\ \sin\ \theta}{2}$

$v^2 - v_0{}^2 = 2ad$

$-v_0{}^2 = -g\ \sin\ \theta\ \frac{h'}{\sin\ \theta} = -gh'$

$h' = \frac{v_0{}^2}{g} = 2H$ (1 point)

(d) 2 points

Rotational kinetic energy does not change. Therefore,

$\frac{1}{2}mv_0{}^2 = mgh''$ 1 point

$h'' = \frac{v_0{}^2}{2g} = H$ 1 point

Full credit was awarded for merely saying that the answer is the same as in part (a).

Mech 3.

(a) 2 points

$F = k\Delta x$ 1 point

$\Delta x = \frac{mg}{k} = \frac{(8\ \text{kg})(9.8\ \text{m/s}^2)}{(1{,}000\ \text{N/m})} = 0.078$ m <u>or</u> 0.08 m 1 point

(b) 3 points

For any one of the following equations:

$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ <u>or</u> $\omega = \sqrt{\frac{k}{m}}$ <u>or</u> $T = 2\pi\sqrt{\frac{m}{k}}$ 1 point

For correct substitution in any of the above:

$f = \frac{1}{2\pi}\sqrt{\frac{1{,}000\ \text{N/m}}{8\ \text{kg}}}$ <u>or</u> $\omega = \sqrt{\frac{1{,}000\ \text{N/m}}{8\ \text{kg}}}$ <u>or</u> $T = 2\pi\sqrt{\frac{8\ \text{kg}}{1{,}000\ \text{N/m}}}$ 1 point

$f = 1.8\ \text{s}^{-1}$ <u>or</u> $\omega = 11\ \text{s}^{-1}$ 1 point

(The third point was <u>not</u> awarded for $T = 0.56$ s)

 Distribution of points

(c) 2 points

If the spring pulls the 5-kg block downward such that $a > g$, the 5-kg block moves faster than the 3-kg block can fall, and they will lose contact. Therefore

$a_{max} = g = 9.8\ m/s^2$ — 2 points

Mathematically:

For 3-kg block: $3g - F_c = 3a$, where F_c is the contact force

$F_c = 0$ when the blocks lose contact, so

$a_{max} = g$

(d) 3 points

Maximum acceleration occurs at the extremes of motion, when $x = A$, the amplitude.

$a_{max} = \frac{kA}{m}$ or $\omega^2 A$ — 1 point

For using a_{max} obtained in part (c) — 1 point

$$A = \frac{ma_{max}}{k} = \frac{(8\ kg)(9.8\ m/s^2)}{(1{,}000\ N/m)} \quad \underline{or} \quad \frac{a_{max}}{\omega^2} = \frac{9.8\ m/s^2}{(11.2\ s^{-1})^2}$$

$A = 0.078$ m (or answer consistent with part (c)) — 1 point

(e) 4 points

For use of conservation of energy, $K_{max} = U_{max}$ — 1 point

$K_{max} = \frac{1}{2}mv_{max}^2$ — 1 point

$U_{max} = \frac{1}{2}kA^2$ — 1 point

$v_{max}^2 = \frac{kA^2}{m}$

$$v_{max} = \sqrt{\frac{(1{,}000\ N/m)(0.078\ m)^2}{8\ kg}}$$

$v_{max} = 0.87\ m/s$ — 1 point

Alternate Solution — (Alternate points)

$v_{max} = \omega A$ <u>or</u> $\frac{a_{max}}{\omega}$, for simple harmonic motion — (2 points)

$v_{max} = (11.2\ s^{-1})(0.078\ m)$ <u>or</u> $\frac{9.8\ m/s^2}{11.2\ s^{-1}}$ — (1 point)

$v_{max} = 0.87\ m/s$ <u>or</u> $0.88\ m/s$ — (1 point)

1 point was awarded for correct units in at least three answers — 1 point

1990 Physics C

Distribution of points

E & M 1.

(a) 5 points

For an expression of Gauss's Law — 1 point

$$\int \mathbf{E} \cdot d\mathbf{A} = \frac{q_{enc}}{\epsilon_0}$$

Volume of a sphere $= \frac{4}{3}\pi r^3$ — 1 point

$$q_{enc} = \frac{Q\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} = \frac{Qr^3}{R^3}$$ — 1 point

(One of these last two points was awarded for an erroneous calculation which indicated a recognition that the enclosed change depends on r)

Surface area of a sphere $= 4\pi r^2$ — 1 point

$$E4\pi r^2 = \frac{Qr^3}{\epsilon_0 R^3}$$

$$E = \frac{Qr}{4\pi\epsilon_0 R^3}$$ — 1 point

(b) 2 points

$q_{enc} = Q$ — 1 point

$$E4\pi r^2 = \frac{Q}{\epsilon_0}$$

$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$ — 1 point

(Second point was not awarded if R was used instead of r)

(c) 2 points

$2R < r < 3R$ is inside the conductor, therefore:
$E = 0$ — 2 points

(Some students continued to treat this shaded area as an insulator. For an attempt to solve part (c) in this manner, evidenced by the use of $r^3 - R^3$, 1 point was awarded)

Distribution of points

(d) 4 points

Since $E = 0$ for r inside the conductor, total charge enclosed by such a sphere must be zero.

For an indication that the total charge on the inside surface, q_i, has magnitude Q — 1 point

For indicating that this charge is negative — 1 point

$\sigma = \frac{q_i}{\text{Area}}$ — 1 point

For consistent use of formula for area — 1 point

$$\sigma = \frac{-Q}{4\pi(2R)^2} = \frac{-Q}{16\pi R^2}$$

(If student indicates that $q = 0$, 1 point is awarded for indicating $\sigma = 0$)

(e) 2 points

For $r > 3R$, E depends on total free charge enclosed, i.e., $Q + q$

Since $q_{enc} = 0$ inside the conductor, the net charge q_0 on the outside surface is:

$q_0 = Q + q$ — 1 point

For consistent use of formula for area — 1 point

$$\sigma = \frac{Q + q}{4\pi(3R)^2} = \frac{Q + q}{36\pi R^2}$$

E & M 2.

(a) 1 point

For a correct arrow (↑) or a statement indicating the correct direction — 1 point

(b) 3 points

Speed as particles enter region III equals speed at which they travel through region II in a straight line

$F_{elec} = QE$ — 1 point

$F_{mag} = QvB$ — 1 point

$QvB = QE$

$v = E/B$ — 1 point

(c) 3 points

Region III is, in effect, a mass spectrometer, in which equating centripetal force to magnetic force allows the determination of mass

$$F_{cen} = \frac{mv^2}{R}$$ 1 point

$$\frac{mv^2}{R} = QvB$$

$$m = \frac{QBR}{v}$$ 1 point

$$m = \frac{QB^2R}{E}$$ 1 point

(d) 4 points

The accelerating potential brings the particle to the speed at which it moves through region II.

Energy imparted by accelerating potential = QV 1 point

$$K = \frac{1}{2}mv^2$$ 1 point

$$QV = \frac{1}{2}mv^2$$

$$V = mv^2/2Q$$ 1 point

$$= \left(\frac{QB^2R}{E}\right)\left(\frac{E}{B}\right)^2 \quad 2Q$$

$$V = \frac{RE}{2}$$ 1 point

(e) 2 points

$$a = a_{cen} = \frac{v^2}{R}$$ 1 point

$$= \frac{1}{R}\left(\frac{E}{B}\right)^2$$

$$a = \frac{E^2}{RB^2}$$ 1 point

<u>Alternate solution</u> (Alternate Points)

$$a = \frac{QvB}{m}$$ (1 point)

$$= QB\left(\frac{E}{B}\right) \quad \left(\frac{QB^2R}{E}\right)$$

$$a = \frac{E^2}{RB^2}$$ (1 point)

Distribution of points

(f) 2 points

$$t = \frac{\text{distance}}{v}$$

$$= \frac{2\pi R/2}{v} = \frac{\pi R}{v}$$ 1 point

$$= \pi R \frac{E}{B}$$

$$t = \frac{\pi RB}{E}$$ 1 point

(For erroneous solutions, one point could be earned if π appeared in some sensible fashion that indicated an attempt to utilize the circular path)

E & M 3.

(a) 2 points

For arrows or words indicating a clockwise direction — 2 points

(b) 3 points

$F_{gravity} = Mg$ — 1 point

$F_{mag} = I\ell B$ — 1 point

$I = \frac{Mg}{\ell B}$ — 1 point

(c) 2 points

$\mathcal{E} = IR$ — 1 point

$\mathcal{E} = \frac{MgR}{\ell B}$ — 1 point

(d) 2 points

$\mathcal{E}_{ind} = d\phi/dt$ where ϕ is the magnetic flux — 1 point

$\mathcal{E}_{ind} = B\ell v$ — 1 point

(e) 3 points

$I' = \mathcal{E}_{net}/R$ — 1 point

$= (\mathcal{E} - \mathcal{E}_{ind})/R$ — 1 point

$= \frac{Mg}{B\ell} - \frac{B\ell v}{R}$ — 1 point

(Two of the three points were awarded for calculating the induced current and not the net current)

Distribution of points

(f) 3 points

Once the loop reaches terminal velocity, the net force on it is zero:

$$(M - \Delta m)g = I'\ell B$$ 1 point

$$= \left(\frac{Mg}{B\ell} - \frac{B\ell v}{R}\right)\ell B$$

$$= Mg - \frac{B^2\ell^2 v}{R}$$ 1 point

$$\Delta m = \frac{B^2\ell^2 v}{Rg}$$ 1 point