AP® Physics C
1991 Scoring Guidelines

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Distribution of points

Mech 1.

(a) 3 points

For some statement of conservation of momentum — 1 point

For a correct equation:

$mv_0 = 3mv$ — 1 point

$v = v_0/3$ — 1 point

(b) 5 points

For some statement of conservation of energy — 1 point

For correct equation containing potential and kinetic energy:

$U_i + K_i = U_f + K_f$ — 1 point

For correct substitution of all terms:

$$0 + \frac{1}{2}(3m)\left(\frac{v_0}{3}\right)^2 = 3mgr + K_f$$ — 2 points

(If only some terms correct, one point was deducted for each incorrect term, up to two points)

$K_f = \frac{mv_0{}^2}{6} - 3mgr$ or equivalent expression — 1 point

(Full credit was awarded if student solved for the final velocity instead of the kinetic energy)

(c) 7 points

$U_i + K_i = U_f + K_f$

For correct K_i: $K_i = \frac{1}{2}(3m)\left(\frac{v_{min}}{3}\right)^2$ — 1 point

For correct U_f: $U_f = 3mg(2r)$ — 1 point

$$0 + \frac{1}{2}(3m)\left(\frac{v_{min}}{3}\right)^2 = 3mg(2r) + \frac{1}{2}(3m)v_{top}^2$$

Force equation must be used to solve for v_{top}:

$F_{normal} + F_{gravity} = F_{centripetal}$

For recognition that in the limiting case $F_{normal} = 0$ — 1 point

$3mg = 3m\frac{v_{top}^2}{r}$ — 1 point

$v_{top}^2 = rg$ or $v_{top} = \sqrt{rg}$ — 1 point

$\frac{1}{2}(3m)\left(\frac{v_{min}}{3}\right)^2 = 3mg(2r) + \frac{1}{2}(3m)(rg)$ — 1 point

$$\frac{mv_{min}^2}{6} = 6mgr + \frac{3}{2}mgr = \frac{15}{2}mgr$$

$v_{min} = 3\sqrt{5gr}$ — 1 point

Distribution of points

Mech 2.

(a) 3 points

For recognition that $\sum \text{torque} = 0$ or equivalent — 1 point

$m_2 g r_2 = m_1 g r_1$ (or $m_2 r_2 = m_1 r_1$) — 1 point

$$m_2 = \frac{m_1 r_1}{r_2} = \frac{(20\ \text{kg})(0.5\ \text{m})}{(1.5\ \text{m})}$$

$$m_2 = \frac{20}{3}\ \text{kg}$$ — 1 point

(b) and (c) 8 points

For a correct dynamical equation for the torque:
$\tau = I\alpha$ — 1 point

For a correct application of the torque equation for the two cylinders:
$Tr_1 = (45\ \text{kg}\cdot\text{m}^2)\alpha$ — 1 point

For Newton's second law:
$F = ma$ — 1 point

For a correct application of Newton's law for mass m_1:
$(20\ \text{kg})g - T = (20\ \text{kg})a$ — 1 point

For recognizing that parts (b) and (c) are coupled (i.e., attempting to solve simultaneous equations) — 1 point

$T = (20\ \text{kg})(g - a)$

$a = \alpha r$ — 1 point

$T = (20\ \text{kg})(g - \alpha r_1)$

Substituting into torque equation:

$(20\ \text{kg})(g - \alpha r_1) = (45\ \text{kg}\cdot\text{m}^2)\alpha$

$(20\ \text{kg})gr_1 - (20\ \text{kg})\alpha r_1{}^2 = (45\ \text{kg}\cdot\text{m}^2)\alpha$

$\alpha[45\ \text{kg}\cdot\text{m}^2 + (20\ \text{kg})r_1{}^2] = (20\ \text{kg})gr_1$

$\alpha = (20\ \text{kg})(9.8\ \text{m/s}^2)(0.5\ \text{m})\ /\ [45\ \text{kg}\cdot\text{m}^2 + (20\ \text{kg})(0.5\ \text{m})^2]$

$\alpha = 2.0\ \text{rad/s}^2$ — 1 point

$$T = (20\ \text{kg})\left[9.8\ \text{m/s}^2 - \left(2.0\ \frac{\text{rad}}{\text{s}^2}\right)(0.5\ \text{m})\right]$$

$T = 180\ \text{N}$ — 1 point

Mech 2. (continued)

(d) 3 points

For applicable kinematic equation(s):

$$\left.\begin{array}{l} v^2 = 2as \end{array}\right. \quad \text{OR} \quad \left.\begin{array}{c} s = \frac{1}{2}at^2 \\ \text{and} \\ v = at \end{array}\right\} \quad \text{OR} \quad \left.\begin{array}{c} \omega^2 = 2\alpha\theta \\ \text{and} \\ v = \omega r \end{array}\right\}$$ 1 point

$$a = \alpha r \qquad t = v/a \qquad v = r\sqrt{2\alpha\theta}$$

$$s = \frac{1}{2}a\frac{v^2}{a^2} = \frac{1}{2}\frac{v^2}{a} \qquad \theta = \frac{s}{r}$$

$$v^2 = 2as \qquad v = r\sqrt{\frac{2\alpha s}{r}}$$

$$a = \alpha r$$

$$v = \sqrt{2\alpha r_1 s}$$

For correct substitution: $v = \sqrt{2(2.0\ \text{rad/s}^2)(0.5\ \text{m})(1\ \text{m})}$ 1 point

$v = 1.4$ m/s 1 point

(Alternate solution) (Alternate Points)

Using conservation of energy:

$mgs = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$ (1 point)

$\omega = v/r$ (1 point)

$$mgs = \frac{1}{2}mv^2 + \frac{1}{2}I\frac{v^2}{r^2}$$

$$= \frac{1}{2}v^2\left(m + \frac{I}{r^2}\right)$$

$$v^2 = 2mgs/(m + I/r^2) = \frac{2(20\ \text{kg})(9.8\ \text{m/s}^2)(1\ \text{m})}{[20\ \text{kg} + (45\ \text{kg}\cdot\text{m}^2)/(0.5\ \text{m})^2)]}$$

$v = 1.4$ m/s (1 point)

For at least one answer with correct units, and no incorrect units 1 point

Distribution of points

Mech 3.

(a) 3 points

For a correct expression for the spring force:
$F_s = kD$ (either + or −) — 1 point

For a correct expression for the gravitational force:

$F_g = mg$ — 1 point

$kD = mg$

$k = mg/D$ — 1 point

If a student attempted to solve the problem using conservation of energy, which is not correct, one point was awarded for $\Delta U_{gravity} = \pm mgD$ and one for either $W = \Delta K$ or

$-\Delta U_{gravity} - \Delta U_{spring} = \Delta K$.

(b) 1 point

For any reasonable explanation — 1 point

Some examples:

When $v_{relative} = 0$, the separation is neither increasing or decreasing.

When $v_{relative} = 0$, K_{tot} is a minimum. Therefore U_{total} is a maximum, which occurs at maximum compression.

(c) 7 points

For some statement of conservation of momentum — 1 point

For correctly applying conservation of momentum between the initial state and the state of maximum compression with common speed V:

$mv_0 = 3mV$ — 1 point

For some statement of conservation of energy — 1 point

For the correct expression for spring potential energy, $\frac{1}{2}kx^2$ — 1 point

For correctly applying conservation of energy between the initial state and the state of maximum compression x:

$\frac{1}{2}mv_0{}^2 = \frac{1}{2}kx^2 + \frac{1}{2}(3m)V^2$ — 1 point

Mech 3. (continued)

For attempting to solve simultaneous equations — 1 point

$$V = \frac{v_0}{3}$$

$$\frac{1}{2}mv_0^2 = \frac{1}{2}kx^2 + \frac{1}{2}3m\left(\frac{v_0}{3}\right)^2$$

$$= \frac{1}{2}kx^2 + \frac{1}{2}m\frac{v_0^2}{3}$$

$$kx^2 = mv_0^2\left(1 - \frac{1}{3}\right) = \frac{2}{3}mv_0^2$$

$$x = v_0\sqrt{\frac{2m}{3k}} = v_0\sqrt{\frac{2m}{3}\ \frac{D}{mg}}$$

$$x = v_0\sqrt{\frac{2D}{3g}}$$ — 1 point

(d) 4 points

For correctly applying momentum conservation:

$mv_0 = mv_I + 2mv_{II}$ — 1 point

For correctly applying conservation of energy:

$\frac{1}{2}mv_0^2 = \frac{1}{2}mv_I^2 + \frac{1}{2}(2m)v_{II}^2$ — 1 point

(This point also awarded for any other equation applicable to elastic collisions)

For attempting to solve simultaneous equations — 1 point

$$v_I = v_0 - 2v_{II}$$

$$v_0^2 = (v_0 - 2v_{II})^2 + 2v_{II}^2$$

$$= v_0^2 - 4v_0v_{II} + 4v_{II}^2 + 2v_{II}^2$$

$$6v_{II}^2 = 4v_0v_{II}$$

$$v_{II} = 0 \text{ or } 6v_{II} = 4v_0$$

$v_{II} = 2/3\ v_0$ — 1 point

E & M 1.

(a) 3 points

For a correct expression for the electric field magnitude for a point charge:

$E = \frac{kQ}{r^2}$ or $\frac{kQ}{a^2}$ — 1 point

For recognition that the electric field is a vector — 1 point

$E = 0$ — 1 point

E & M 1. (continued)

(b) 3 points

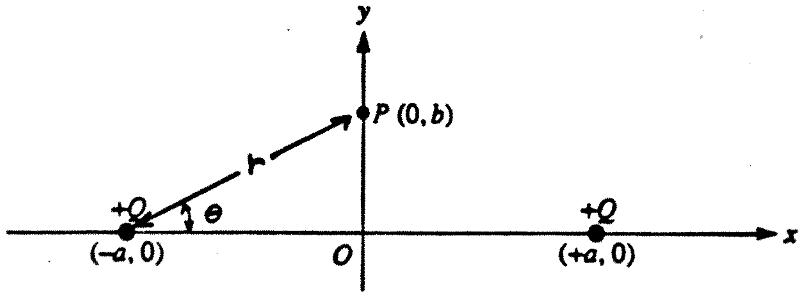
For a correct expression for the electric potential for a point charge:

$V = \frac{kQ}{r}$ or $\frac{kQ}{a}$ — 1 point

For recognition that the potential is a scalar — 1 point

$V = \frac{2kQ}{a}$ — 1 point

(c) 3 points



From figure $r = \sqrt{a^2 + b^2}$

For the magnitude of E due to one of the charges:

$E = \frac{kQ}{a^2 + b^2}$ — 1 point

For recognizing the need to find the components of E — 1 point

The x-components cancel, so only the y-components need be calculated

$$E_y = \frac{kQ}{a^2 + b^2} \sin\theta = \frac{kQ}{a^2 + b^2} \frac{b}{\sqrt{a^2 + b^2}} = \frac{kQb}{(a^2 + b^2)^{3/2}}$$

The y-components add:

$E_P = \frac{2kQb}{(a^2 + b^2)^{3/2}}$ — 1 point

(d) 1 point

For indicating that the particle will move toward the origin or that it will oscillate about the origin — 1 point

(e) 4 points

For indicating that the particle will move away from the origin — 1 point

For some statement of conservation of energy — 1 point

Distribution of points

E & M 1. (continued)

For an expression for the initial potential energy of the particle:

$U = \frac{2kQq}{a}$ or qV — 1 point

$\frac{1}{2}mv_\infty^2 = \frac{2kQq}{a}$

$v_\infty = 2\sqrt{\frac{kQq}{ma}}$ — 1 point

(Alternate solution) — (Alternate Points)

For indicating that the particle will move away from the origin — (1 point)

$\frac{1}{2}mv_\infty^2 = W_{net}$ or $\int F\,dy$ — (1 point)

For correctly setting up the integral:

$\int F\,dy = 2kQq\int_0^\infty \frac{y\,dy}{(a^2 + y^2)^{3/2}}$ — (1 point)

$u = a^2 + y^2 \longrightarrow du = 2y\,dy$

$\int F\,dy = kQq \int_{a^2}^{\infty} \frac{du}{u^{3/2}} = kQq(-2)\,\frac{1}{u^{1/2}}\Bigg|_{a^2}^{\infty} = \frac{2kQq}{a}$

$\frac{1}{2}mv_\infty^2 = \frac{2kQq}{a}$

$v_\infty = 2\sqrt{\frac{kQq}{ma}}$ — (1 point)

(f) 1 point
For indicating that the particle will move back toward the origin or that it will oscillate about the origin — 1 point

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E & M 2.

(a) 1 point
Since the inductor prevents any sudden change in current:

$I_1 = 0$ — 1 point

(b) 3 points
The voltage across the inductor is $L\frac{dI}{dt}$, which is zero in the steady state condition.

$\mathcal{E} = IR_{tot}$ — 2 points

$50\text{ V} = I(100\ \Omega + 150\ \Omega)$

$I = 0.2\text{ A}$ — 1 point

Distribution of points

E & M 2. (continued)

(Alternate solution) (Alternate Points)

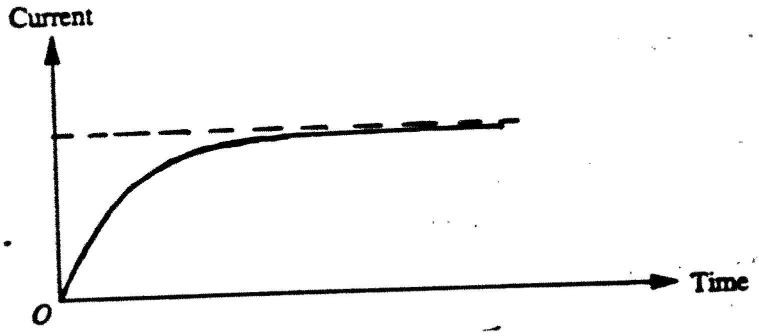
For an RL circuit with zero initial current:

$$I = \frac{\mathcal{E}}{R}\left(1 - e^{-Rt/L}\right)$$

As $t \longrightarrow \infty$, $I = \mathcal{E}/R$ (2 points)

$$I = \frac{50\ \text{V}}{250\ \Omega} = 0.2\ \text{A}$$ (1 point)

(c) 3 points



For beginning the graph at the origin — 1 point

For a monotonically increasing curve — 1 point

For an asymptotic approach to a constant value as $t \longrightarrow \infty$ — 1 point

(d) 2 points
For an expression for the energy stored in an inductor: — 1 point

$$U = \frac{1}{2}LI^2$$

$$U = \frac{1}{2}(1.0\ \text{H})(0.2\ \text{A})^2$$

$$U = 0.02\ \text{J}$$ — 1 point

(e) 2 points
Since the current in an inductor does not change abruptly, it is equal to the steady-state current calculated in part (b) — 2 points
$I = 0.2$ A

(One point was awarded for either of these equations:
$-L\frac{dI}{dt} - IR = 0$, $I = I_0\, e^{-Rt/L}$)

Distribution of points

E & M 2. (continued)

(f) 2 points

$V_L = \left|L\frac{dI}{dt}\right| = IR$

For using correct value of R: $R = 150\ \Omega$ — 1 point

$V_L = (0.2\text{ A})(150\ \Omega)$

$V_L = 30\text{ V}$ — 1 point

(g) 2 points

For indicating that the stored energy is dissipated in the resistor or becomes thermal energy — 2 points

(One point was awarded for stating that the stored energy is used to keep the current flowing.)

E & M 3.

(a) 3 points

$\mathcal{E} = -\frac{d\phi}{dt}$ — 1 point

$\phi = \int \mathbf{B}\cdot d\mathbf{A}$ — 1 point

$\mathcal{E} = -B\frac{dA}{dt}$

$\mathcal{E} = -B\ell v_0$ (+ or − acceptable) — 1 point

(b) 5 points

$\mathbf{F} = I\boldsymbol{\ell} \times \mathbf{B}$ or $F = I\ell B$ — 1 point

$I = \left|\frac{\mathcal{E}}{R}\right|$ — 1 point

For using $\mathcal{E}$ calculated in part (a) — 1 point

$F = \left|\frac{\mathcal{E}}{R}\right|\ell B = \frac{B\ell v_0}{R}\ell B$

$F = \frac{v_0 B^2 \ell^2}{R}$ (+ or − acceptable) — 1 point

For indicating that the force is opposite the direction of the velocity (including minus sign in above expression is sufficient) — 1 point

E & M 3. (continued)

(c) 5 points

For an expression of Newton's second law:

$F = ma$ — 1 point

Using expression for F from part (b), with a generic velocity:

$a = \frac{vB^2\ell^2}{mR}$ (+ or − acceptable) — 1 point

For indicating that the acceleration is opposite the direction of the velocity (including minus sign above or in differential equation below is sufficient) — 1 point

For a correct differential equation:

$\frac{dv}{dt} = -v\,\frac{B^2\ell^2}{mR}$ — 1 point

$$\frac{dv}{v} = -\frac{B^2\ell^2}{mR}\,dt$$

$$\ln v\Big|_{v_0}^{v} = -\frac{B^2\ell^2}{mR}\,t$$

$$\ln\frac{v}{v_0} = -\frac{B^2\ell^2}{mR}\,t$$

$v = v_0 e^{-B^2\ell^2 t/mR}$ — 1 point

(Alternate solution) — (Alternate Points)

For a statement of conservation of energy — (1 point)

For placing I^2R term on the correct side of the energy equation — (1 point)

For using expression for $\mathcal{E}$ from part (a) with a generic velocity:

$I = \mathcal{E}/R = \frac{B\ell v}{R}$ — (1 point)

For a correct integral equation:

$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + \int\frac{B^2\ell^2v^2}{R^2}R\,dt$ — (1 point)

Try $v = Ce^{-Dt}$

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mC^2e^{-2Dt} + \frac{B^2\ell^2}{R}C^2\int e^{-2Dt}\,dt$$

$$\int e^{-2Dt}\,dt = -\frac{1}{2D}\,e^{-2Dt} + K_0$$

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mC^2e^{-2Dt} - \frac{B^2\ell^2}{R}\,C^2\left(\frac{1}{2D}\right)e^{-2Dt} + K_0$$

Since this equation must hold for any time t:

$K_0 = \frac{1}{2}mv_0^2$ and $\frac{1}{2}mC^2 = \frac{B^2\ell^2}{R}\,C^2\left(\frac{1}{2D}\right)$

E & M 3. (continued)

Therefore:

$$D = \frac{B^2 \ell^2}{mR}$$

$v = v_0$ when $t = 0$, so $v_0 = Ce^{-0}$ and $C = v_0$

$$v = v_0 e^{-B^2 \ell^2 t/mR}$$ (1 point)

(d) 2 points

From energy conservation, the resistor will eventually dissipate all the kinetic energy of the rod

$$E_{diss} = \frac{1}{2} m v_0^2$$ 2 points

(One point was awarded for any other correct, relevant statement regarding energy or power)