1999 Physics C Solutions **Distribution of points**

Mech. 1 (15 points)

(a) 5 points

For conservation of momentum — 1 point

$mv_0 = (m+M_0)v$

$v = \dfrac{mv_0}{m+M_0}$

For conservation of mechanical energy — 1 point

$K + U = K' + U'$ or $\frac{1}{2}M_{\text{total}}v^2 = M_{\text{total}}gh$

For calculating h in terms of ℓ — 1 point

$h = \ell(1 - \cos\theta)$

For substituting for M_{total} and v in the energy equation — 1 point

$$\frac{1}{2}(m + M_0)\left(\frac{mv_0}{m + M_0}\right)^2 = (m + M_0)g\ell(1 - \cos\theta)$$

For the correct answer — 1 point

$$v_0 = \frac{m+M_0}{m}\sqrt{2g\ell(1-\cos\theta)}$$

(b) 4 points

For Newton's second law (not awarded if net force was set equal to zero) — 1 point

$F_{\text{net}} = ma$

For any equation that indicated that the tension minus the weight is <u>not</u> zero — 1 point

$T - M_{\text{total}}g = M_{\text{total}}a$

For an expression for the centripetal force — 1 point

$a = \dfrac{v^2}{r}$ or $\dfrac{v^2}{\ell}$

$$T - M_{\text{total}}g = M_{\text{total}}\frac{v^2}{\ell}$$

For correctly substituting for M_{total} and v — 1 point

$$(m + M_0)\frac{1}{\ell}\left(\frac{mv_0}{m + M_0}\right)^2 = T - (m + M_0)g$$

Solving for T

$$T = (m+M_0)g(3-2\cos\theta)$$

Distribution of points

Mech. 1 (continued)

(c) 4 points

For all of the following: — 4 points

1) A practical procedure that uses some or all of the apparatus listed and would work
2) Recognition of any assumptions that must be made
3) Indication of the proper mathematical computation using the variables measured

Two points were awarded if the description of the procedure was not complete but it would work, or if the mathematical work did not clearly specify the variables used, or any combination of the above.

No points were awarded if the procedure would not be feasible in a laboratory situation with the apparatus listed, or if the procedure was merely a repeat of that outlined in part (a).

(d) 2 points

$F_{net} = ma = -bv$

For expressing the acceleration as the time derivative of the speed, $a = \frac{dv}{dt}$ — 1 point

$$\int_{v_0}^{v} \frac{dv}{v} = \int_0^t -\frac{b}{m}\,dt$$

$$\ln\left(\frac{v}{v_0}\right) = -\frac{bt}{m}$$

$$v = v_0 e^{-bt/m}$$

$$\int_0^{\ell} ds = \int_0^t v_0 e^{-bt/m}\,dt$$

For a general expression for the length of the dart in the block as a function of time or for the expression for the total distance L — 1 point

$$\ell = \frac{mv_0}{b}\left(1 - e^{-bt/m}\right)$$

$$L = \frac{mv_0}{b}$$

 Distribution of points

Mech. 1 (continued)

(d) (continued)

Alternate Solution 1 — *Alternate points*

For indicating that work equals the change in kinetic energy — *1 point*

$$\int F\,dx = \frac{1}{2}mv_0^2$$

$$\left|F_{average}\right| = \frac{bv_0}{2}$$

$$\int_0^L \frac{bv_0}{2}\,dx = \frac{1}{2}mv_0^2$$

$$\frac{bv_0}{2}L = \frac{1}{2}mv_0^2$$

For the correct expression for the total distance L — *1 point*

$$L = \frac{mv_0}{b}$$

Alternate Solution 2 — *Alternate points*

$$\mathbf{F}_{\text{net}}\,\Delta t = \Delta\mathbf{p}$$

For the above expression — *1 point*

$$\int_0^\infty -bv\,dt = -mv_0$$

Using $v = \dfrac{ds}{dt}$ and the fact that s goes from zero to L as time goes from zero to infinity

$$\int_0^L -b\,ds = -mv_0$$

$$-bL = -mv_0$$

For the correct expression for the total distance L — *1 point*

$$L = \frac{mv_0}{b}$$

1999 Physics C Solutions **Distribution of points**

Mech. 2 (15 points)

(a) 3 points

For indicating that the equation for gravitational force is applicable — 1 point

$$F = -\frac{Gm_1m_2}{r^2}$$

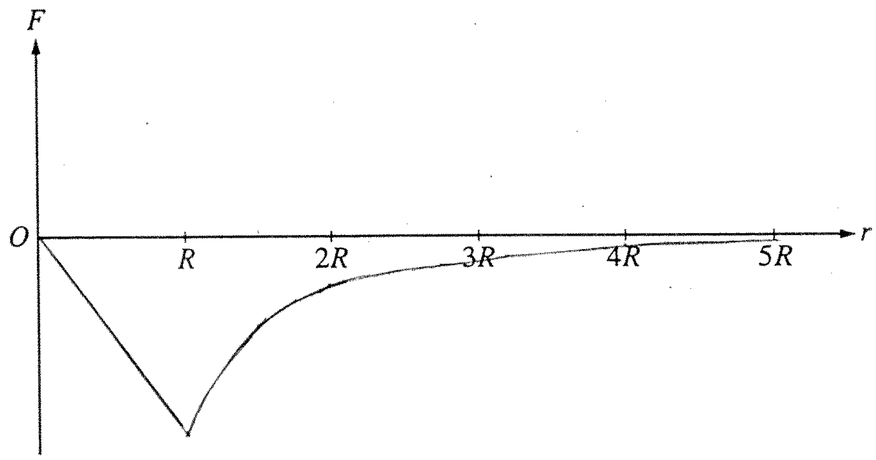
For using the proper expression for the mass of the planet enclosed by the radius — 1 point

$$F = -\frac{Gm\left(\rho\frac{4}{3}\pi r^3\right)}{r^2}$$

For proper cancellation of terms to show the final result — 1 point

$$F = -\left(\frac{4}{3}\pi G\rho m\right)r$$

(b) 4 points



For drawing a straight line from the origin to a distance R, and not going past R — 1 point

For having the maximum magnitude occur at R — 1 point

For having the curve from R to $5R$ decreasing in magnitude with proper curvature and appearing to reach an asymptote — 1 point

For recognizing that the force is always negative, i.e. the graph is always below the x-axis — 1 point

Mech. 2 (continued)

(c) 3 points

In this and all subsequent parts, either C or $\frac{4}{3}\pi G\rho m$ could be used.

For indicating the integral that needs to be calculated to determine the work — 1 point

$$W = \int F\,dr = \int -Cr\,dr$$

For using the proper limits on the integral (R to zero, not r) — 1 point

$$W = \int_R^0 -Cr\,dr$$

$$W = -C\frac{r^2}{2}\Big|_R^0$$

For the correct answer — 1 point

$$W = \frac{CR^2}{2}$$

Alternate Solution	*Alternate points*
For recognition that the work is the area under the curve, which is triangular	*1 point*
For using the corrects limits (zero to R)	*1 point*
For the correct answer	*1 point*

$$W = \frac{CR^2}{2}$$

(d) 2 points

For using conservation of energy or work-energy relationship — 1 point

$$\Delta K = \Delta U = W$$

$$\frac{1}{2}mv^2 = \frac{CR^2}{2}$$

For the correct answer — 1 point

$$v = \sqrt{\frac{CR^2}{m}}$$

An alternate solution indicating the potential energy as that of a harmonic oscillator also received full credit.

(e) 2 points

For indicating that the ball will move from the center to the surface of the planet — 1 point

For indicating that the ball will stop at the surface, return to the center, and continue oscillating in this manner, with no damping — 1 point

Describing the motion as simple harmonic oscillation with no damping earned full credit

Mech. 2 (continued)

(f) 1 point

For showing a proper application of Newton's first law — 1 point

$F = ma$

$$Cr = m\frac{d^2 r}{dt^2}$$

Alternately, one could relate the time to the period of oscillation, $T = 2\pi\sqrt{\dfrac{3}{4\pi G\rho}}$,

i.e. the time is one-fourth this period. The above equation was required; a more general form was not acceptable.

1999 Physics C Solutions

Distribution of points

Mech. 3 (15 points)

(a) 5 points

For indicating that the net torque is zero, or that the clockwise and counterclockwise torques are equal — 1 point

For a correct expression for the torque exerted by the rod — 1 point

$\tau_{\text{rod}} = mgR\sin\theta_0$

For a correct expression for the torque exerted by the block — 1 point

$\tau_{\text{block}} = 2mg(2R)\sin\theta_0 = 4mgR\sin\theta_0$

For a correct expression for the torque exerted by the string — 1 point

$\tau_{\text{string}} = TR$

For adding the counterclockwise torques and setting the sum equal to the clockwise torque (this point not awarded for just one torque) — 1 point

$TR = 4mgR\sin\theta_0 + mgR\sin\theta_0$

$T = 5mg\sin\theta_0$

Only four points could be earned if the wrong trigonometric function was used.
Only three points could be earned if no trigonometric function was used.

(b)
i. 4 points

For indicating that the rotational inertia is the sum of the inertias of the disk, rod, and block — 1 point

For calculating the total rotational inertia — 1 point

$$I = I_{\text{disk}} + I_{\text{rod}} + I_{\text{block}}$$

$$= \frac{3}{2}mR^2 + \frac{4}{3}mR^2 + 2m(2R)^2$$

$$= \frac{65}{6}mR^2$$

$\alpha = \tau_{\text{net}}/I$

For substituting the value of torque from part (a) — 1 point

$$\alpha = \frac{5mgR\sin\theta_0}{\frac{65}{6}mR^2}$$

For an answer consistent with the values use for torque and rotational inertia — 1 point

$$\alpha = \frac{6g\sin\theta_0}{13R}$$

Mech. 3 (continued)

(b) (continued)

ii. 1 point

Expressing the linear acceleration in terms of the angular acceleration

$a = \alpha r$

For substituting the value of α and the correct radius, $2R$ — 1 point

$$a = \frac{12g\sin\theta_0}{13}$$

(c) 5 points

For indicating that energy is conserved — 1 point

For indicating that the potential energy of two bodies (the rod and the block) changes — 1 point

$\Delta U = mgh_{\text{rod}} + mgh_{\text{block}}$

For the correct expressions for these two potential energies — 1 point

$\Delta U = mgR\cos\theta_0 + 2mg(2R)\cos\theta_0$

For indicating the correct kinetic energy when the rod is horizontal — 1 point

$$K = \frac{1}{2}I\omega^2$$

Equating the kinetic and potential energies, and solving for the angular speed

$$\frac{1}{2}\left(\frac{65}{6}mR^2\right)\omega^2 = mgR\cos\theta_0 + 4mgR\cos\theta_0$$

$$\omega = \sqrt{\frac{12g\cos\theta_0}{13R}}$$

For using the relationship between linear and angular speed, and substituting ω and the correct radius, $2R$ — 1 point

$\upsilon = \omega r$

$$\upsilon = \left(\sqrt{\frac{12g\cos\theta_0}{13R}}\right)(2R) = 4\sqrt{\frac{3gR\cos\theta_0}{13}}$$

Alternate methods of solution included use of the following proper integrations

$$\omega^2 = 2\int_{\theta_0}^{\pi/2} \alpha\, d\theta$$

$$K = \int_{\theta_0}^{\pi/2} \tau\, d\theta$$

1999 Physics C Solutions | **Distribution of points**

E & M 1 (15 points)

(a) 4 points

For using the relationship between potential and charge — 1 point

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$$

Solving for Q:

$$Q = 4\pi\epsilon_0 Vr$$

For correct substitutions for the potential and radius — 1 point

$$Q_0 = 4\pi\epsilon_0(-2000\text{ V})(0.20\text{ m}) \quad \text{or} \quad (-2000\text{ V})(0.20\text{ m})/\left(9\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)$$

$$Q_0 = -1600\pi\epsilon_0\,\text{C} \quad \text{or} \quad -4.4\times10^{-8}\ \text{C}$$

For the correct magnitude of Q_0 — 1 point

For the negative sign — 1 point

(b) 5 points

i. For indicating that the electric field is zero — 1 point

ii. The charge on the sphere can be treated as a point charge at its center

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q_0}{r^2}$$

$$E = \left(9\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{4.4\times10^{-8}\ \text{C}}{r^2}\right)$$

$$E = \frac{396}{r^2}\frac{\text{N}}{\text{C}} \quad \text{or} \quad \frac{400}{r^2}\frac{\text{N}}{\text{C}} \quad \text{where } r \text{ is in meters}$$

For any of the above expressions for E — 1 point

iii. For indicating that the electric field is zero — 1 point

iv. For indicating that the electric field is zero — 1 point

For having all four answers correct OR for some mention of using the enclosed charge OR for some mention of Gauss' law — 1 point

E & M 1 (continued)

(c) 3 points

$$\Delta V = V_b - V_a = -\int_a^b E\, dr$$

For recognition of the need to take the difference of the potentials at radii a and b, or for writing the definite integral (with limits) — 1 point

$$|\Delta V| = \frac{Q_0}{4\pi\epsilon_0}\int_a^b \frac{dr}{r^2}$$

$$= \frac{Q_0}{4\pi\epsilon_0}\left(\frac{1}{r}\right)\Big|_a^b$$

$$|\Delta V| = \frac{Q_0}{4\pi\epsilon_0}\left(\frac{1}{b} - \frac{1}{a}\right)$$

For correct substitution of variables or numerical values for Q_0, a, and b — 1 point

For the correct answer — 1 point

$$|\Delta V| = \frac{5Q_0}{8\pi\epsilon_0} \quad \text{or} \quad 1000\text{ V}$$

(Alternate solution) — *(Alternate points)*

For recognition of the need to take the difference of the potentials at radii a and b — *1 point*

$$\Delta V = V_b - V_a$$

$$\Delta V = \frac{Q_0}{4\pi\epsilon_0}\left(\frac{1}{r_b}\right) - \frac{Q_0}{4\pi\epsilon_0}\left(\frac{1}{r_a}\right)$$

For correct substitution of Q_0, a, and b — *1 point*

$$\Delta V = \frac{Q_0}{4\pi\epsilon_0}\left(\frac{1}{b} - \frac{1}{a}\right)$$

For the correct answer — *1 point*

$$|\Delta V| = \frac{5Q_0}{8\pi\epsilon_0} \quad \text{or} \quad 1000\text{ V}$$

(Alternate solution) — *(Alternate points)*

$$V = \frac{Q}{C}$$

For using the above relationship — *1 point*

For substituting Q_0 from part (a) and C from part (d) alternate solution — *1 point*

For the correct answer — *1 point*

$$|\Delta V| = \frac{5Q_0}{8\pi\epsilon_0} \quad \text{or} \quad 1000\text{ V}$$

 Distribution of points

E & M 1 (continued)

(d) 2 points

$$C = \frac{Q_0}{V}$$

For using the above relationship — 1 point

For substituting Q_0 from part (a) and ΔV from part (c) — 1 point

$$C = \frac{4.4 \times 10^{-8}\ \text{C}}{1000\ \text{V}}$$

$$C = 4.4 \times 10^{-11}\ \text{F}$$

(Alternate solution) — *(Alternate points)*

For writing the equation for the capacitance of the spherical capacitor — *1 point*

$$C = \frac{4\pi\epsilon_0 ab}{b - a}$$

$$C = \frac{(0.02\ \text{m})(0.04\ \text{m})}{\left(9 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(0.04\ \text{m} - 0.02\ \text{m})}$$

For the correct answer — *1 point*

$$C = 4.4 \times 10^{-11}\ \text{F}$$

For correct units on two answers and no incorrect units — 1 point

1999 Physics C Solutions	Distribution of Points

E & M 2 (15 points)

(a) 5 points

For using Faraday's law for a loop	1 point
$\mathcal{E} = -\dfrac{d\phi}{dt}$ or $\mathcal{E} = -\dfrac{\Delta\phi}{\Delta t}$	
For relating magnetic flux to magnetic field and area	1 point
$\dfrac{d\phi}{dt} = A\dfrac{dB}{dt}$ or $\dfrac{\Delta\phi}{\Delta t} = A\dfrac{\Delta B}{\Delta t}$	
For using the proper expression for the area of a loop	1 point
$A = \pi r^2$	
$\mathcal{E} = \pi r^2 \dfrac{dB}{dt}$ or $\mathcal{E} = \pi r^2 \dfrac{\Delta B}{\Delta t}$	
For using the correct radius, i.e. the radius of the field	1 point
$\mathcal{E} = \pi(0.6\text{ m})^2(0.40\text{ T/s})$	
For computing the correct answer	1 point
$\mathcal{E} = 0.45\text{ V}$	

(b) 3 points

For any statement of Ohm's law	1 point
$V = IR$	
Solving for the current:	
$I = V/R = \mathcal{E}/R$	
$I = (0.45\text{ V})/(5.0\ \Omega)$	
For computing the correct answer	1 point
$I = 0.090\text{ A}$	
For indicating a clockwise direction for the current	1 point

(c) 3 points

For relating the energy dissipated to the power in the resistor	1 point
$E = \int P\,dt$ or $E = Pt$	
For an expression for electric power	1 point
$P = I^2R$ or $\dfrac{V^2}{R}$ or IV	
Example using $P = I^2R$:	
$E = I^2Rt$	
$E = (0.090\text{ A})^2(5.0\ \Omega)(15\text{ s})$	
For computing the correct answer	1 point
$E = 0.61\text{ J}$	

 Distribution of Points

E & M 2 (continued)

(d) 3 points

For stating that the brightness of the bulb will be less	1 point
For indicating that the reduction in brightness is due to a decrease in current or a decrease in the emf	1 point
For indicating that the decrease in current or emf, or the reduction in brightness, is due to a decrease in the area of the loop or a decrease in the changing flux	1 point

For using correct units with three numerical answers	1 point

Distribution of points

E & M 3 (15 points)

(a) 3 points

The charge on any section of the ring is equidistant from a point on the x-axis, so one can write an equation in terms of the single distance r

For a correct expression of the potential	1 point

$$= \frac{1}{4\pi\epsilon_0}\frac{dq}{r} \quad \text{or} \quad \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$$

For a correct expression for the distance of the charge from location x	1 point

$$r = \sqrt{x^2 + R^2}$$

For the correct answer	1 point

$$= \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{x^2 + R^2}}$$

Alternate solution	*Alternate points*
For correctly expressing the potential as an integral of the electric field	*1 point*

$$dV = -\int E\,dr$$

For a correct expression for the field	*1 point*

$$dV = -\int \frac{1}{4\pi\epsilon_0}\frac{Qx}{\left(x^2 + R^2\right)^{3/2}}\,dx$$

For correctly integrating to get the final answer	*1 point*

$$= \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{x^2 + R^2}}$$

(b)

i. 3 points

$$E = -\frac{dV}{dr}$$

For using the above relationship	1 point
For taking the derivative with respect to x	1 point
For using the expression for V obtained in part (a)	1 point

$$E_x = -\frac{d}{dx}\left(\frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{x^2 + R^2}}\right)$$

$$E_x = \frac{1}{4\pi\epsilon_0}\frac{Qx}{\left(x^2 + R^2\right)^{3/2}}$$

E & M 3 (continued)

(b) (continued)

i. (continued)

Alternate solution — *Alternate points*

Calculating the field by integration:

$E = \int dE_x = \int dE \cos\theta$, where θ is the angle between the x-axis and the distance vector **r**

For using the horizontal component of the field — *1 point*

For using a correct expression of Coulomb's law — *1 point*

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$$

For indicating that the integral is over the charge — *1 point*

$$E_x = \int \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}\cos\theta$$

Substituting $\cos\theta = x/r$ and $r = \sqrt{x^2 + R^2}$

$$E_x = \frac{1}{4\pi\epsilon_0}\frac{Q}{\left(x^2 + R^2\right)^{3/2}}\int_0^Q dq$$

$$E_x = \frac{1}{4\pi\epsilon_0}\frac{Qx}{\left(x^2 + R^2\right)^{3/2}}$$

ii. 1 point

For any indication that the y- and z-components are zero or cancel — 1 point

E & M 3 (continued)

(c)

i. 2 points

For taking the derivative of E with respect to x and setting it equal to zero — 1 point

$$\frac{dE_x}{dx} = \frac{d}{dx}\left(\frac{1}{4\pi\epsilon_0}\frac{Qx}{\left(x^2+R^2\right)^{3/2}}\right) = 0$$

$$\frac{Q}{4\pi\epsilon_0}\left(\frac{1}{\left(x^2+R^2\right)^{3/2}} + \left(-\frac{3}{2}\right)\frac{2x^2}{\left(x^2+R^2\right)^{5/2}}\right) = 0$$

$$\frac{1}{\left(x^2+R^2\right)^{3/2}} - \frac{3x^2}{\left(x^2+R^2\right)^{5/2}} = 0$$

$$\frac{1}{\left(x^2+R^2\right)^{3/2}} = \frac{3x^2}{\left(x^2+R^2\right)^{5/2}}$$

$$1 = \frac{3x^2}{x^2+R^2}$$

$$3x^2 = x^2 + R^2$$

$x = \pm\dfrac{R}{\sqrt{2}}$ and the maximum occurs at the positive value of x

For the correct answer — 1 point

$$x = +\frac{R}{\sqrt{2}}$$

ii. 1 point

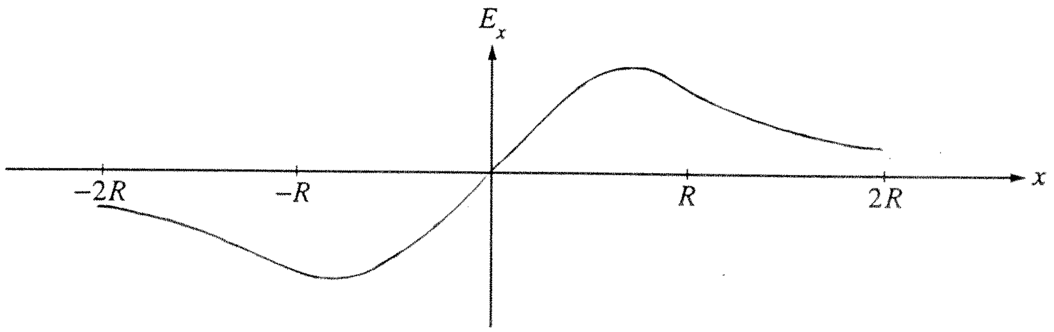
For substituting the answer from part (c)i into the given expression for the electric field — 1 point

$$E_{x\,\max} = \frac{1}{4\pi\epsilon_0}\frac{Q\left(R/\sqrt{2}\right)}{\left(\left(R/\sqrt{2}\right)^2 + R^2\right)^{3/2}}$$

$$E_{x\,\max} = \frac{1}{4\pi\epsilon_0}\frac{Q}{3\sqrt{3}R^2}$$

E & M 3 (continued)

(d) 3 points



For a curve in the first quadrant displaying a single positive maximum	1 point
For a curve passing through the origin	1 point
For the negative reflection of the first quadrant curve in the third quadrant	1 point

(e) 2 points

For any statement that describes the subsequent motion as oscillating, periodic etc. 2 points

One point was awarded for a statement that only described the electron as moving toward the ring or along the x-axis.