AP® Physics C
1997 Scoring Guidelines

Distribution of points

Mech. 1 (15 points)

(a) 3 points

For using a correct method to calculate the constant A — 1 point

Left-hand graph: Substituting a correct point into the force equation

Right-hand graph: Substituting a correct point into the force equation, or calculating the slope of the line

Example: Slope of the line of the right-hand graph

Calculating the slope from points on the line

$$A = \frac{7.6\text{ N} - 2.5\text{ N}}{0.3\text{ m}^{1/2} - 0.1\text{ m}^{1/2}} = \frac{5.1\text{ N}}{0.2\text{ m}^{1/2}}$$

$$A = 25.5\ \text{N/m}^{1/2}$$

For the correct numerical value (within the range 23-27) — 1 point

For the correct units — 1 point

(b)

i. 4 points

For using the integral form of relationship between work and force — 1 point

$$W = \int \mathbf{F} \cdot ds$$

Writing this equation for the one-dimensional case

$$W = \int F(s)\, ds$$

For correctly substituting the expression for the force — 1 point

For the correct limits — 1 point

$$W = \int_0^x Ax^{1/2} dx$$

For the correct answer — 1 point

$W = \frac{2}{3} Ax^{3/2}$ (or equivalent answer with value of A substituted)

ii. 2 points

For indicating that the work is equal to the area under the left-hand graph — 1 point

For a correct explanation of how to obtain the area from the graph — 1 point

e.g. counting the grid boxes or taking the area of a combination of rectangles and triangles

Distribution of points

Mech. 1 (continued)

(c) 6 points

For using conservation of energy — 1 point

The initial kinetic energy of the ball is equal to the potential energy stored in the spring

$$\frac{1}{2}m\upsilon_i^2 = U_s$$

For substituting 0.2 J for the potential energy — 1 point

Solving for the initial speed and making appropriate substitutions

$$\upsilon_i = \sqrt{\frac{2(0.2\text{ J})}{0.1\text{ kg}}} = \sqrt{4}\text{ m/s}$$

$$\upsilon_i = 2\text{ m/s}$$

Using the kinematic equation for the vertical motion of the ball

$$y = y_0 + \upsilon_{y0}t - \frac{1}{2}gt^2$$

For realizing that the initial velocity is zero — 1 point

The final height is also zero: solving for the time t that it takes the ball to fall

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(1.3\text{ m})}{9.8\text{ m/s}^2}} \quad \left(\text{may also use } g = 10\text{ m/s}^2\right)$$

For the correct value of the time of the fall — 1 point

$t = 0.52$ s (or 0.51 s using $g = 10$ m/s^2)

For a correct equation for the horizontal distance, with zero initial values — 1 point

$$d = \upsilon_i t$$

For substituting the values obtained above — 1 point

$$d = (2\text{ m/s})(0.52\text{ s})$$

$$d = 1\text{ m}$$

Distribution of points

Mech. 2 (15 points)

(a) 2 points

For indicating that the total mass is the sum of the car's mass plus the mass of sand that has fallen up to that time — 1 point

$M = M_0 + (\Delta M/\Delta t)t$

For the correct answer — 1 point

$M = M_0 + Ct$

(b) 2 points

For using conservation of momentum — 1 point

$M_0 v_0 = Mv$

Substituting the expression for the total mass from part (a)

$M_0 v_0 = (M_0 + Ct)v$

For the correct answer — 1 point

$v = M_0 v_0/(M_0 + Ct)$

(c)

i. 1 point

For calculating the initial kinetic energy of the empty car — 1 point

$K_i = M_0 v_0^2/2$

ii. 2 points

$K_f = Mv^2/2$

For using the expressions for mass and velocity found in parts (a) and (b), with $t = T$ — 1 point

For the correct answer — 1 point

$$K_f = \frac{(M_0 + CT)}{2}\left(\frac{M_0 v_0}{M_0 + CT}\right)^2 \quad \text{OR} \quad K_f = \frac{M_0^2 v_0^2}{2(M_0 + CT)}$$

Mech. 2 (continued)

(c) (continued)

iii. 2 points

For indicating that kinetic energy is not conserved — 1 point

For a correct explanation — 1 point

Examples: It is dissipated in the inelastic collisions of the sand and car.

The system's mass increases while its momentum remains constant: according to the equation for K_f, $K = p_0^2/2m$ and thus kinetic energy decreases.

(d)

i. 1 point

The normal force before $t = 0$ equals the weight of the empty car.

For the correct answer — 1 point

$N = M_0 g$

ii. 4 points

The normal force for $0 < t < T$ equals the weight of the car plus the weight of the sand that has accumulated in the car plus the force required to stop the vertical motion of the sand.

$N = M_0 g + W_s + F_s$

For a correct expression for the weight of the sand — 1 point

$W_s = Ctg$

For calculating F_s in terms of the sand's change in momentum — 1 point

$$F_s = \frac{\Delta p}{\Delta t} = \frac{\Delta M}{\Delta t}\upsilon_y = C\upsilon_y$$

To find υ_y, one can apply kinematic equations or conservation of energy for the falling sand

$$\upsilon_y^2 = \upsilon_0^2 + 2gh \text{ (or equivalent)} \quad \text{OR} \quad \frac{1}{2}m\upsilon_y^2 = mgh$$

For the correct expression for the final vertical velocity of the sand — 1 point

$\upsilon_y = \sqrt{2gh}$

For the correct complete expression for the normal force — 1 point

$N = (M_0 + Ct)g + C\sqrt{2gh}$

Mech. 2 (continued)

(d) (continued)

iii. 1 point

The normal force after $t = T$ equals the weight of the car plus the accumulated sand.

For the correct answer 1 point

$N = (M_0 + CT)g$

Mech. 3 (15 points)

(a) 4 points

For using conservation of energy — 1 point

For including both linear and rotational energy — 1 point

$U = K_{tran} + K_{rot}$

$$MgH = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

For correctly substituting for ω — 1 point

$$MgH = \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\frac{v^2}{R^2}$$

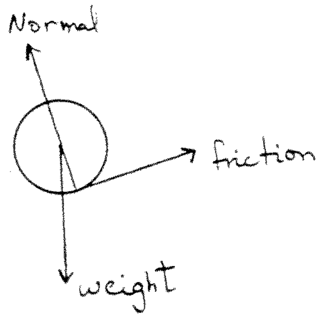
$$MgH = \frac{1}{2}Mv^2 + \frac{1}{4}Mv^2$$

$$MgH = \frac{3}{4}Mv^2$$

For the correct answer — 1 point

$$v = \sqrt{\frac{4}{3}gH}$$

(b) 3 points



For forces with correct direction, point of application, and label (1 point each) — 3 points

One point was deducted for any extraneous forces.

One point was awarded if all forces had correct direction and label but did not all begin at the correct point.

Mech. 3 (continued)

(c) 2 points

Using an appropriate kinematic equation
$v_f^2 = v_i^2 + 2as$
For one correct substitution into this equation — 1 point
For the other two correct substitutions — 1 point
$\frac{4}{3}gH = 0 + 2a\left(\frac{H}{\sin\theta}\right)$
Solving for the acceleration
$a = \frac{2}{3}g\sin\theta$

(Alternate solution) — *(Alternate points)*
Using Newton's second law and the equivalent rotational equation
$F_{net} = Ma$ and $\tau = I\alpha$
For proper substitutions in either of the above equations — *1 point*
$Mg\sin\theta - f = Ma$ and $fR = \left(\frac{1}{2}MR^2\right)\left(\frac{a}{R}\right)$
Solving the rotational equation above for the frictional force
$f = \frac{1}{2}Ma$
For substituting this frictional force into the Newton's law equation — *1 point*
$Mg\sin\theta - \frac{1}{2}Ma = Ma$
$Mg\sin\theta = \frac{3}{2}Ma$
$a = \frac{2}{3}g\sin\theta$

Note: Full credit was also awarded for a clear argument using the fact that the rotational energy is half the translational energy (which can be seen from part (a)). This means that the translational kinetic energy is equal to 2/3 of the total energy. The translational kinetic energy is also equal to the work done by the net force along the plane. Equating 2/3 the total energy, MgH, to the work, $Ma(H/\sin\theta)$, gives an acceleration equal to $\frac{2}{3}g\sin\theta$.

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Mech. 3 (continued)

(d) 2 points

Applying Newton's second law to the motion of the center of mass
$Ma = Mg \sin\theta - f$

For proper use of the normal force in the expression for the frictional force — 1 point
$f = \mu N = \mu Mg \cos\theta$

Substituting for f and a in Newton's law
$$M\left(\frac{2}{3}g \sin\theta\right) = Mg \sin\theta - \mu Mg \cos\theta$$

Canceling the factors of M and g, and solving for μ
$$\frac{2}{3}\sin\theta - \sin\theta = -\mu\cos\theta$$
$$\frac{1}{3}\sin\theta = \mu\cos\theta$$

For the correct answer — 1 point
$$\mu = \frac{1}{3}\tan\theta$$

(Alternate solution) — *(Alternate points)*

Determining the expression for the frictional force, in terms of the acceleration, from the torque equation (see the alternate solution to part (c))
$$f = \frac{1}{2}Ma$$

For proper use of the normal force in the expression for the frictional force — *1 point*
$f = \mu N = \mu Mg \cos\theta$

Substituting the expressions for the frictional force and the acceleration
$$\mu Mg \cos\theta = \frac{1}{2}M\left(\frac{2}{3}g \sin\theta\right)$$

For the correct answer — *1 point*
$$\mu = \frac{1}{3}\tan\theta$$

IC

Mech. 3 (continued)

(e)

i. 2 points

For indicating that the translational speed at the bottom of the incline is greater than in part (a)	1 point
For a reasonable explanation	1 point

Examples:

There is less energy transferred to the rotational motion, so more goes into translational motion and thus the speed is greater.

The frictional force is less, so by Newton's second law (see the first equation in part (d)) the translational acceleration is greater. Thus the cylinder gains more speed.

ii. 2 points

For indicating that the total kinetic energy at the bottom of the incline is less	1 point
For a reasonable explanation	1 point

Examples:

Since the cylinder slides, some energy is dissipated as heat.

There is a non-conservative force (friction) exerted.

Distribution of points

E & M 1 (15 points)

(a)

i. 3 points

For using Ohm's law	1 point
$V = IR$	
For obtaining a reasonable value of the initial current, I_0, from the graph ($\approx$ 9.3 A)	1 point
Substituting I_0 and the resistance	
$V = (9.3\ \text{A})(3.3\ \Omega)$	
For the correct answer	1 point
$V \approx 31\ \text{V}$	

ii. 2 points

For indicating that the battery's open-circuit voltage would be greater	1 point
For a correct explanation	1 point
Example: A real battery has internal resistance, so when it is connected in a circuit only part of the open-circuit voltage appears across the external components.	

(b) 4 points

Using the given expression for the current	
$I = I_0 e^{-kt}$	
For taking the natural log of the exponential function	1 point
$\ln(I) = \ln(I_0) - kt\ln(e)$	
$\ln(I/I_0) = -kt$	
$k = -\ln(I/I_0)/t$	
For correctly reading a point on the graph for $t > 0$	1 point
For this example, $I = 7.5$ A at $t = 0.5$ hr	
Substituting	
$k = -\ln(7.5/9.3)/(0.5\ \text{hr})$	
For the correct numerical answer	1 point
For the correct units	1 point
$k \approx 0.4\ \text{hr}^{-1}$	

E & M 1 (continued)

(c)

i. 1 point

For using the appropriate expression for power — 1 point

$P = I^2R$

At $t = 0$, $I = I_0$

Substituting

$P = I_0^2R$

ii. 2 points

For substituting the expression for current into the power equation — 1 point

$P = \left(I_0 e^{-kt}\right)^2 R$

For the correct answer — 1 point

$P = I_0^2 R\, e^{-2kt}$

iii. 3 points

For expressing the total energy dissipated as the integral of the power — 1 point

$U_{diss} = \int P\, dt$

For setting up the integral with the correct limits ($t = 0$ to $t = \infty$), and substituting the expression for power — 1 point

$$U_{diss} = \int_0^\infty I_0^2 R\, e^{-2kt}\, dt$$

For correctly evaluating the integral — 1 point

$$U_{diss} = I_0^2 R\left(-\frac{1}{2k}\right)e^{-2kt}\Big|_0^\infty = -\frac{I_0^2 R}{2k}(0-1)$$

$$U_{diss} = \frac{I_0^2 R}{2k}$$

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Distribution of points

E & M 2 (15 points)

(a) 3 points

Using Gauss' law

$$\int \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\varepsilon_0}$$

$$E \cdot 4\pi R^2 = \frac{Q}{\varepsilon_0}$$

For the correct magnitude — 2 points

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}$$

For the correct direction — 1 point

The field is to the right.

This is because the electric field resulting from a positive charge points away from the charge.

One point was awarded for an unsuccessful attempt to use Gauss' law.

(b) 1 point

Using the definition of flux

$$\phi = \int \mathbf{E} \cdot d\mathbf{A}$$

Gauss' law can be used to substitute for the integral, or the integral can be evaluated

For a correct answer — 1 point

$$\phi = \frac{Q}{\varepsilon_0} \quad \text{or} \quad 4\pi R^2 E$$

E & M 2 (continued)

(c) 3 points

The new field is just the sum of the fields from the sphere and the point charge
$\mathbf{E}_{tot} = \mathbf{E}_s + \mathbf{E}_p$

For calculating the field from the point charge — 1 point

$$E_p = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2} \quad \text{downward}$$

Determining the magnitude of the resultant field

$$E_{tot} = \frac{1}{4\pi\varepsilon_0}\sqrt{\left(\frac{Q}{R^2}\right)^2 + \left(\frac{Q}{R^2}\right)^2}$$

For the correct magnitude of the total field — 1 point

$$E = \frac{\sqrt{2}Q}{4\pi\varepsilon_0 R^2}$$

For the correct direction of this field — 1 point

The resultant field is directed at a 45° angle from the horizontal, down and to the right.

(d)

i. 1 point

For indicating that charges q_2 and q_3 contribute to the net electric flux through the surface — 1 point

ii. 1 point

For indicating that all four charges contribute to the field at point P_1 — 1 point

iii. 2 points

For indicating that the answers to i. and ii. are different — 1 point

For a correct explanation (at the very least showing recognition that the field and flux are different quantities) — 1 point

Examples:

The electric field is the sum of the individual fields, while the flux is the sum of components over the whole surface.

The net contributions to the flux from the charges outside the surface are zero. They each have an inward and an outward contribution that cancel.

Only the enclosed charges contribute to the flux.

E & M 2 (continued)

(e) 2 points

For answering "No"; zero net charge enclosed by a surface does not necessarily mean zero field on points on the surface — 1 point

For a correct explanation — 1 point

Examples:

Charges may exist outside the surface that would contribute to a field on the surface.

A dipole inside the surface would cause a net field at some points.

(f) 2 points

For answering "Yes"; a surface with no field at any point encloses a zero net charge — 1 point

For a correct explanation — 1 point

Examples:

A zero net field means that the net flux is also zero. Since only charges inside the surface contribute to the flux, there can be no net charge inside the surface.

If the field is zero, then the integral in Gauss' law is zero, and thus the enclosed charge is zero.

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E & M 3 (15 points)

(a) 2 points

For using Ampere's law — 1 point

$$\int \mathbf{B} \cdot d\ell = \mu_0 I$$

For correctly evaluating the line integral — 1 point

$$B\,2\pi r = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

(b) 2 points

For indicating that the magnetic field through the loop is perpendicular to the page — 1 point

For indicating the correct perpendicular direction, the $+z$-direction — 1 point

(c) 4 points

Using the definition of flux

$$\phi = \int \mathbf{B} \cdot d\mathbf{A}$$

For correctly expressing the element of area — 1 point

$$\phi = \int B\ell\, dr$$

For substituting the expression for the field from part (a) — 1 point

For indicating the limits of integration (s to $s + w$) — 1 point

$$\phi = \frac{\mu_0 I\ell}{2\pi} \int_s^{s+w} \frac{dr}{r}$$

For correctly integrating to obtain a natural log — 1 point

$$\phi = \frac{\mu_0 I\ell}{2\pi} \ln r \Big|_s^{s+w}$$

$$\phi = \frac{\mu_0 I\ell}{2\pi}\left(\ln\left(s+w\right) - \ln s\right)$$

$$\phi = \frac{\mu_0 I\ell}{2\pi} \ln \frac{s+w}{s}$$

E & M 3 (continued)

(d) 2 points

For indicating that the current in the loop is counterclockwise — 1 point
For a correct explanation — 1 point
For example:
As the loop moves away, the flux from the wire decreases. The induced current acts to maintain the flux, and creates a field that points out of the page. By the right-hand rule, the current is then counterclockwise.

(e) 2 points

For indicating that the net magnetic force on the moving loop acts in the $-y$-direction — 1 point
For a correct explanation — 1 point
For example:
The short sections of the loop (width w), experience equal forces in opposite directions. The long section of loop (length ℓ) closer to the wire experiences a force toward the wire. The other long section experiences a force away from the wire that is less in magnitude, so the net force is toward the wire.

(f) 3 points

For using Ohm's law — 1 point

$$I = \frac{\mathcal{E}}{R}$$

For using Faraday's law — 1 point

$$\mathcal{E} = -\frac{d\phi}{dt}$$

Substituting the flux from part (c)

$$\mathcal{E} = -\frac{d}{dt}\left(\frac{\mu_0 I \ell}{2\pi}\left[\ln(s+w) - \ln s\right]\right)$$

$$\mathcal{E} = -\frac{\mu_0 I \ell}{2\pi}\left(\frac{1}{s+w} - \frac{1}{s}\right)\frac{ds}{dt} = -\frac{\mu_0 I \ell}{2\pi}\left(\frac{s-(s+w)}{s(s+w)}\right)\frac{ds}{dt}$$

$$\mathcal{E} = \frac{\mu_0 I \ell}{2\pi}\left(\frac{w}{s(s+w)}\right)v$$

For the correct answer — 1 point

$$I_{\text{loop}} = \frac{\mu_0 I \ell v}{2\pi R}\left(\frac{w}{s(s+w)}\right)$$

E & M 3 (continued)

(f) (continued)

(Alternate solution) *(Alternate points)*

For using Ohm's law *1 point*

$$I = \frac{\mathcal{E}}{R}$$

For expressing the total emf in terms of the emf on each long segment of wire *1 point*

$$\mathcal{E} = B_1 \ell \upsilon - B_2 \ell \upsilon$$

$$\mathcal{E} = \ell \upsilon \left(\frac{\mu_0 I}{2\pi s} - \frac{\mu_0 I}{2\pi(s+w)} \right)$$ (or an equivalent expression with $s = s_0 + \upsilon t$)

For the correct answer *1 point*

$$I_{\text{loop}} = \frac{\mu_0 I \ell \upsilon}{2\pi R} \left(\frac{w}{s(s+w)} \right)$$