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C—Mechanics—1

Points

a) 6 points

The frictional force f is given by

$$f = \mu N,$$ 2 points

where μ is the coefficient of sliding friction and N is the normal force. The normal force N is the only radial force acting on the mass m. Hence, from Newton's Second Law,

$$N = m\,a_c = m(v^2/R),$$ 3 points

where a_c is the centripetal acceleration. Thus

$$f = \mu m\, v^2/R$$ 1 point

b) 4 points

The frictional force f is the only tangential force acting on the mass m. Hence, from Newton's Second Law,

$$f = -\,m\,a_t$$ 2 points

where the minus sign accounts for the direction of the frictional force. Thus

$$m\,\mu v^2/R = -\,m a_t$$ 1 point

$$a_t = \mu v^2/R$$ 1 point

c) 5 points

The time t_s required for the block to come to ~~rest~~ $V_0/3$ can be obtained from knowledge of the tangential acceleration in part (b)

$$a_t = \frac{dv}{dt} = -\mu v^2/R$$ 2 points

Separating the variables gives

$$\frac{dv}{v^2} = \frac{\mu}{R}dt$$

and imposing limits and integrating

$$\int_{v_o}^{\cancel{o}\ v_0/3} \frac{dv}{v^2} = -\frac{\mu}{R}\int_{o}^{t_s} dt,$$ 1 point

or

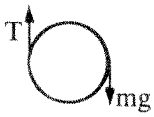
$$t_s = \frac{2R}{\mu v_o}$$ 2 points

C–Mechanics–2

Points

a) 3 points

The two forces acting on the cylinder are the tension T and the gravitational force mg.



For the two vectors	1 point
For correct labels	2 points
(− 1) loss of credit per extra irrelevant force	

b) 9 points

The acceleration can be found by two methods: by calculating the torque or by considering the energy. Moreover, each method has two or three variations.

First method, Variation 1: Torque about the center of mass
By considering the rotation about the center of mass, the tension must be calculated.

$\tau = I\alpha$	1 point
$TR = \frac{1}{2} MR^2 \cdot (a/R)$	3 points
$T = (1/2)Ma$	1 point
$Mg - T = Ma$	2 points
$Mg = (3/2)Ma$	1 point
$a = (2/3)g$	1 point

Variation 2: Torque about the rim
By considering the rotation about the point of contact of the tape, the moment of inertia about that point must be calculated.

$\tau = I\alpha$	1 point
$Mg\,R = I_{rim} \cdot (a/R)$	4 points
$I_{rim} = (1/2)MR^2 + MR^2$ (Parallel axis theorem)	2 points
$Mg\,R = (3/2)MR^2 \cdot (a/R)$	1 point
$a = (2/3)g$	1 point

Second Method: Energy

$Mgy = (1/2)I_{cm}\,\omega^2 + (1/2)Mv^2$	3 points
$Mgy = (1/2)(MR^2/2)(v/R)^2 + (1/2)Mv^2$	2 points
$v^2 = (4/3)gy$	1 point

Variation 1: The derivative with respect to t yields

$2va = (4/3)gv$	2 points
$a = (2/3)g$	1 point

Variation 2: By comparing the kinematic relationship

$v_f{}^2 = v_i{}^2 + 2as$ or $v^2 = 2ay$	2 points

with the result

$v^2 = (4/3)gy$. Again, $a = 2/3g$	1 point

c) 3 points

The center of the cylinder moves straight down because there are no horizontal forces.	3 points

	Points
a) 5 points	
The final speed V of the block M can be obtained from conservation of linear momentum.	
The initial momentum = the final momentum	1 point
or	
$m\,v_o = m\,v_o/3 + MV$	2 points
or solving for V	
$V = (2/3)\frac{m}{M}v_o$	2 points
b) 3 points	
From the definition of kinetic energy, one may write	
$\Delta\,KE = (1/2)mv_f^{\,2} - (1/2)mv_i^{\,2}$	1 point
Substituting the initial and final velocities gives	
$\Delta\,KE = (1/2)m\left(\frac{v_o}{2}\right)^2 - (1/2)mv_o^{\,2}$	
$= -\,(3/8)mv_o^{\,2}$	2 points
or	
$(3/4)(1/2\,mv_o^{\,2}) = 75\%\ KE_{initial}$	
c) 4 points	
From the definition of work, for a constant force F applied over a distance L, one has	
$W = FL$	1 point
The work-kinetic energy theorem states	
$W = \Delta\,KE$	
Therefore, for the case given	
$FL_o = (1/2)m\,(v_o/3)^2 - (1/2)m\,(v_o)^2$	1 point
and in order to stop the bullet	
$FL_s = (1/2)m(0)^2 - (1/2)m\,(v_o)^2$	1 point
Solving for L_s in terms of L_o yields	
$L_s = 9/8\,L_o$	1 point

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An alternative solution employing the kinematic relationship

$v_f^2 = v_o^2 + 2as$ 1 point

can be employed where

$a = F/m$ 1 point

and

$FL_o = (1/2)m\,(v_o/3)^2 - (1/2)m(v_o)^2$ 1 point

Thus, if $v_o = 0$, $s = L_s$ and

$$L_s = 9/8\ L_o$$ 1 point

d) 3 points

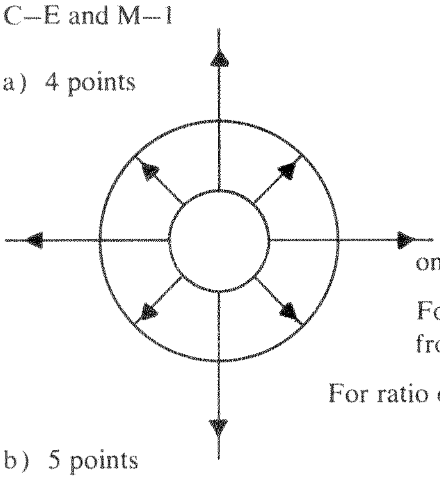
When the block is free to move, the constant force acts over a greater distance in the fixed inertial system; hence from the work-kinetic energy theorem there will be a greater change (negative) in the bullet's kinetic energy, and it will emerge with a lower speed. 3 points

Essentially the same argument can be made by stating that a certain amount of work is required for the bullet to tunnel through the block. This amount is the same if the block is moving or fixed. In the former case, however, the block also carries off some kinetic energy. Thus, as the energy of the system is a constant, the kinetic energy of the bullet is less.

C–E and M–1

Points

a) 4 points



For symmetry — 1 point

For lines starting only on inner, ending only on outer — 1 point

For outside lines directed away from sphere — 1 point

For ratio of 2:1 (between: outside) — 1 point

b) 5 points

The net flux Φ is given by

$$\Phi = 4\pi k Q_{\text{enclosed}} = \frac{Q}{\epsilon_o}$$ 1 point

Since the field is a constant for a given value of r, Gauss' Law yields

$$\int \vec{E} \cdot \vec{dS} = E \cdot 4\pi r^2 = 4\pi k \cdot 2Q$$ 3 points

Solving for the field, one obtains

$$E = \frac{2kQ}{r^2} \cdot (R < r < 3R)$$ 1 point

c) 4 points

Two different methods of attack can be employed.

First Method: From the definition of ΔV

$$\Delta V = \pm \int \vec{E} \cdot \vec{dr}$$ 1 point

$$\Delta V = \pm \int \frac{2kQ}{r^2}\, dr$$ 1 point

$$\Delta V = \mp \left. \frac{2kQ}{r} \right]_{R}^{3R}$$ 1 point

$$\Delta V = \frac{2kQ}{R} - \frac{2kQ}{3R} = \frac{4}{3}\frac{kQ}{R}$$ 1 point

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Second Method: From knowledge of the potential due to a point (spherical distribution) charge

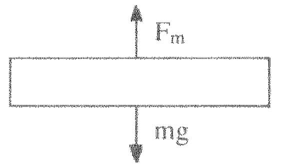
$V = \dfrac{2kQ}{R}$ (inner)	1 point
$V = \dfrac{2kQ}{3R}$ (outer)	1 point
Justification for above	1 point
$\Delta V = \dfrac{2kQ}{R} - \dfrac{2kQ}{3R} = \dfrac{4}{3}\dfrac{kQ}{R}$	1 point

d) 2 points

No charge on the center sphere, + Q on outside sphere	2 points
(No explanation required)	
For only stating they're at same potential	1 point

Points

a) 4 points



Credit was given for arbitrary labels, but vectors must be equal in magnitude and opposite in direction.

b) 4 points

Equating the magnetic force **BiL** to the gravitational force, one obtains

$$mg = BiL$$ 3 points

or

$$i = mg/BL$$ 1 point

c) 4 points

The electromotive force is given by

$$\text{Emf} = \frac{\Delta \Phi}{\Delta t}$$ 2 points

$$= B\left(\frac{\Delta A}{\Delta t}\right)$$ 1 point

$$= \frac{BLv_o \Delta t}{\Delta t} = BLv_o$$ 1 point

For only stating Emf $= BLv_o$ 2 points

An alternate approach that received full credit was

$$\vec{E} = \vec{v} \times \vec{B}$$

$$\text{Emf} = \int E dl = v_o BL$$

d) 3 points

From Ohm's Law one has

$$R = V/I$$ 1 point

$$\left.\begin{aligned} &= \frac{BLv_o}{mg/BL} \\ &= \frac{B^2L^2v_o}{mg} \end{aligned}\right\}$$ 2 points

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	Points
a) 1 point	
In order to move in a circular path (the speed is constant, because the force is perpendicular to the velocity), the force must be directed toward the center of the circle.	1 point
b) 3 points	
Because the particle initially experiences an upward force as it leaves region I (that is, when the $\vec{E}$ field no longer acts on it), then the magnetic force in region I must also be upward. Further, as the	1 point
particle is not deflected in either direction in region I, then the electric force must be downward.	1 point
As the $\vec{E}$ field is up, the charge must be negative.	1 point
or	
Applying either the Lorentz force law $\vec{F} = q\,\vec{v} \times \vec{B}$ or the "right hand rule," one obtains that in region II the initial deflection for a positive particle would be down. Thus, because it is initially deflected up, it must be negative.	
c) 3 points	
The magnetic force acts up and the electric force acts down.	3 points
d) 3 points	
As there is no acceleration in region I, the net force must be zero, or the magnetic force is equal to the electric force.	1 point
$Bqv = qE$	2 points
or	
$v = E/B$	
e) 5 points	
Since the particle experiences a uniform circular acceleration in region II under the magnetic force, one may write	
$F = ma$	
$Bqv = m\,v^2/R$	2 points
Solving for m, one obtains	
$m = BqR/v$	1 point
Substituting for v from part (d) gives	1 point
$m = B^2qR/E$	1 point