# 2001 AP Physics Solutions

General Notes

1. The solutions contain the most common method(s) of solving the free-response questions, and the allocation of points for these solutions. Other methods of solution also receive appropriate credit for correct work.
2. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded.
   An exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g. a speed faster than the speed of light in vacuum.
3. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth one point, and a solution contains the application of the equation to the problem but does not separately list the basic equation, the point is still awarded.

Question 1 (15 points)

(a) 4 points
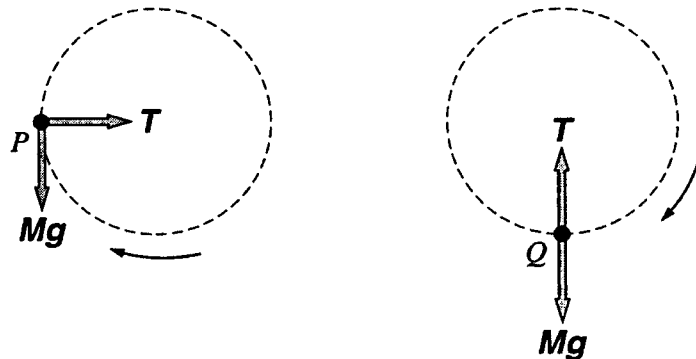
For each correctly drawn and appropriately labeled force

One point was deducted for each incorrect force, to a maximum of the number of points earned for correct forces (i.e. lowest score awarded is zero).

The upward force in the figure on the right did not receive credit if it was labeled as the centripetal force, since in this case the net centripetal force is the sum of two physical forces (tension and weight).

(b) 4 points

For setting the net force equal to some physical force, such as gravity or a tension   1 point

For correctly identifying that the only force is that due to gravity   1 point

$F_{net} = Ma = Mg$

For substituting the expression for centripetal acceleration   1 point

$$\frac{Mv_{min}^2}{R} = Mg$$

For the correct answer   1 point

$$v_{min} = \sqrt{Rg}$$

(Alternate solution)   *(Alternate points)*

For using the expression for centripetal acceleration   *1 point*

$$a = \frac{v^2}{R}$$

For recognizing that the condition for minimum speed at point Z is that the centripetal acceleration equals the acceleration due to gravity   *2 points*

$$g = \frac{v^2}{R}$$

For the correct answer   *1 point*

$$v_{min} = \sqrt{Rg}$$

(c) 5 points

For setting the net force equal to some physical force, such as gravity or a tension	1 point
For indicating that the net force is the difference between the tension and gravity	1 point
For indicating that the tension is larger than gravity	1 point

$Ma = T - Mg$

For substituting the expression for centripetal acceleration   1 point

$$M\frac{v_{max}^2}{R} = T_{max} - Mg$$

For the correct answer   1 point

$$v_{max} = \sqrt{\frac{R}{M}(T_{max} - Mg)}$$

(d) 2 points

For any indication that the velocity of the ball is straight up	1 point
For any indication that the ball would be slowing down	1 point

Question 2 (15 points)

(a) 4 points

For any indication that conservation of momentum is applicable   1 point

$p_{before} = p_{after}$

$m_A v_{Ai} = m_B v_{Bf} + m_A v_{Af}$

Solving for v_{Bf}

$$v_{Bf} = \frac{m_A(v_{Ai} - v_{Af})}{m_B}$$

For correct substitution of values before the collision   1 point
For correct substitution of values after the collision, including $v_{Af} = -0.70$ m/s   1 point

$$v_{Bf} = \frac{(0.10 \text{ kg})(1.4 \text{ m/s} + 0.70 \text{ m/s})}{0.50 \text{ kg}}$$

For the correct answer   1 point

$v_{Bf} = 0.42$ m/s

(b) 5 points

For any statement of an equation that can be used to find the time t for the ball to fall   1 point
For example:

$$h = h_0 + v_0 t + \frac{1}{2} a t^2$$

For correct substitution   1 point

$$0 = 1.2 \text{ m} + 0(t) + \frac{1}{2}\left(-9.8 \text{ m/s}^2\right) t^2$$

For the correct value for t 1 point

$t = 0.49$ s (same value obtained using $g = 10$ m/s^2)

For substituting the value of v from part (a) into a correct equation for the horizontal   1 point
   distance (i.e. having initial position and acceleration both zero)

$d = v t = (0.42 \text{ m/s})(049 \text{ s})$

For the correct answer   1 point

$d = 0.21$ m

(c)   3 points

For any statement of the equation for speed   1 point
$v_C = d/t$
For substituting the time from part (b) or correctly recalculating it   1 point
$v_C = 0.15 \text{ m}/0.49 \text{ s}$
For the correct answer   1 point
$v_C = 0.31 \text{ m/s}$

(d)   3 points

For an indication that the momentum in the y-direction is conserved   1 point
$\mathbf{p}_{Ay} = -\mathbf{p}_{Cy}$
For an indication that the y-component of ball C's momentum was calculated   1 point
$p_{Ay} = m_C v_C \sin 30°$

$p_{Ay} = (0.10 \text{ kg})(0.31 \text{ m/s}) \sin 30°$
For the correct answer   1 point
$p_{Ay} = 0.015 \text{ kg} \cdot \text{m/s}$

Question 3 (15 points)

(a)
  i. 3 points

For using a scalar sum of potentials due to each charge   1 point

$$V = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i}{r_i}$$

For correctly substituting charges and distance (either as $s/\sqrt{2}$ or a generic distance   1 point
notation, such as r)

$$V = \frac{1}{4\pi\epsilon_0}\left(\frac{-Q}{r} + \left(\frac{-Q}{r}\right) + \frac{Q}{r} + \frac{Q}{r} \right)$$

For the correct answer   1 point
$$V = 0$$

  ii. 3 points

For taking the vector sum of fields due to each charge   1 point
For using an appropriate expression for calculating the field   1 point

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}\hat{\mathbf{r}} \quad OR \quad \mathbf{E} = \frac{\mathbf{F}}{q}$$

Defining diagonal axes, arbitrarily labeled t and u, with the t-axis along the line connecting
the positive charges, and the u-axis along the line connecting the negative charges:

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{r^2}\hat{\mathbf{t}} - \frac{Q}{r^2}\hat{\mathbf{t}} + \left(\frac{-Q}{r^2}\hat{\mathbf{u}}\right) - \left(\frac{-Q}{r^2}\hat{\mathbf{u}}\right) \right)$$

For the correct answer   1 point
$$E = 0$$

(b)
  i. 2 points

Using a scalar sum of potentials due to each charge:

$$V = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i}{r_i}$$

For correctly substituting charges and distance   (either as $s/\sqrt{2}$ or a generic distance   1 point
notation, such as r)

$$V = \frac{1}{4\pi\epsilon_0}\left(\frac{-Q}{r} + \left(\frac{-Q}{r}\right) + \frac{Q}{r} + \frac{Q}{r} \right)$$

For the correct answer   1 point
$$V = 0$$

An incorrect answer equal to that in part (a) received one point, for realizing that swapping
the charges does not affect the scalar sum

Note: The three answers above can easily be determined from symmetry considerations without
calculation, so full credit was awarded for writing just the correct answer. Incorrect work or ambiguous
statements with a correct answer could only receive the answer point.

Question 3 (continued)

(b) (continued)
  ii. 4 points

The field from each individual charge points along a diagonal, with an x-component to the right. The vertical components cancel in pairs, and the x-components are equal in magnitude.

For taking the vector sum of fields due to each charge   1 point

For correctly using the x-components (i.e. writing cos 45° or $\sqrt{2}/2$)   1 point

$$E = 4\frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\frac{\sqrt{2}}{2}$$

For substituting $r^2 = s^2/2$ 1 point

$$E = 4\frac{1}{4\pi\epsilon_0}\frac{2Q}{s^2}\frac{\sqrt{2}}{2}$$

For the correct answer   1 point

$$E = \frac{\sqrt{2}Q}{\pi\epsilon_0 s^2} \quad \text{or} \quad \text{equivalent}$$

(c) 2 points

For choosing arrangement 1   1 point
For an appropriate justification   2 points
For example: The force of attraction on the upper right charge is greater in arrangement 1 because the two closest charges are both positive, whereas in arrangement 2 one is positive and one is negative.

Question 4 (15 points)

(a)   3 points

For using Snell's law OR indicating that the index of refraction is the slope of the graph   1 point
$n_1 \sin\theta_1 = n_2 \sin\theta_2$

For substituting or otherwise indicating use of data from the given graph   1 point

$(1)(0.5) = n_2(0.8)$ OR $\text{slope} = \dfrac{0.8}{0.5}$

For the correct answer   1 point
$n_2 = 1.60$

(Alternate solution)   *(Alternate points)*
For using the expression for the critical angle   *1 point*

$\sin\theta_c = \dfrac{n_2}{n_1}$

For using data from the graph   *1 point*
The critical angle occurs when $\sin\theta_2 = 1$. From the graph, $\sin\theta_2 = \sin\theta_c = 0.625$

$0.625 = \dfrac{1}{n_1}$

For the correct answer   *1 point*
$n_2 = 1.60$

Two points were awarded for inverting the values from the graph to obtain $n_2 = 0.625$.

(b)
  i.   2 points

For using the correct equation with the correct substitutions   1 point
$f = v/\lambda$

$f = \left(3 \times 10^8 \text{ m/s}\right)\!\big/\!\left(675 \times 10^{-9} \text{ m}\right)$

For the correct answer   1 point
$f = 4.44 \times 10^{14} \text{ Hz}$

  ii.   2 points

For using the correct equation with the correct substitutions   1 point
$v = c/n$

$v = \left(3 \times 10^8 \text{ m/s}\right)\!\big/ 1.60$

For the correct answer   1 point
$v = 1.88 \times 10^8 \text{ m/s}$

If an incorrect answer was carried through to obtain a speed greater than 3×10^8 m/s, only one point was awarded for the solution. The second point could be earned if there was some indication that the student realized that the value must be incorrect, because it could not be greater than the speed of light.

Question 4 (continued)

(b) (continued)
  iii. 2 points

  For using the correct equation with correct substitutions (consistent with previous answers)   1 point
  $$\lambda = v/f$$
  $$\lambda = \left(1.88 \times 10^8 \ \text{m/s}\right)\Big/\left(4.44 \times 10^{14} \ \text{Hz}\right)$$
  For the correct answer   1 point
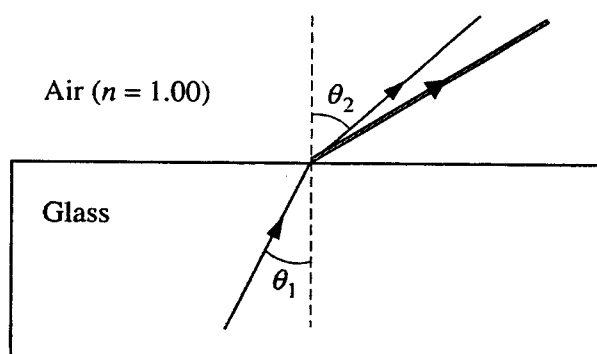 $$\lambda = 423 \times 10^{-9} \ \text{m} = 423 \ \text{nm}$$

<u>Units point</u>: For correct units on two of the three answers in part (b)   1 point

(c)
  i. 1 point

  For correctly indicating on the figure that θ_2 increases   1 point
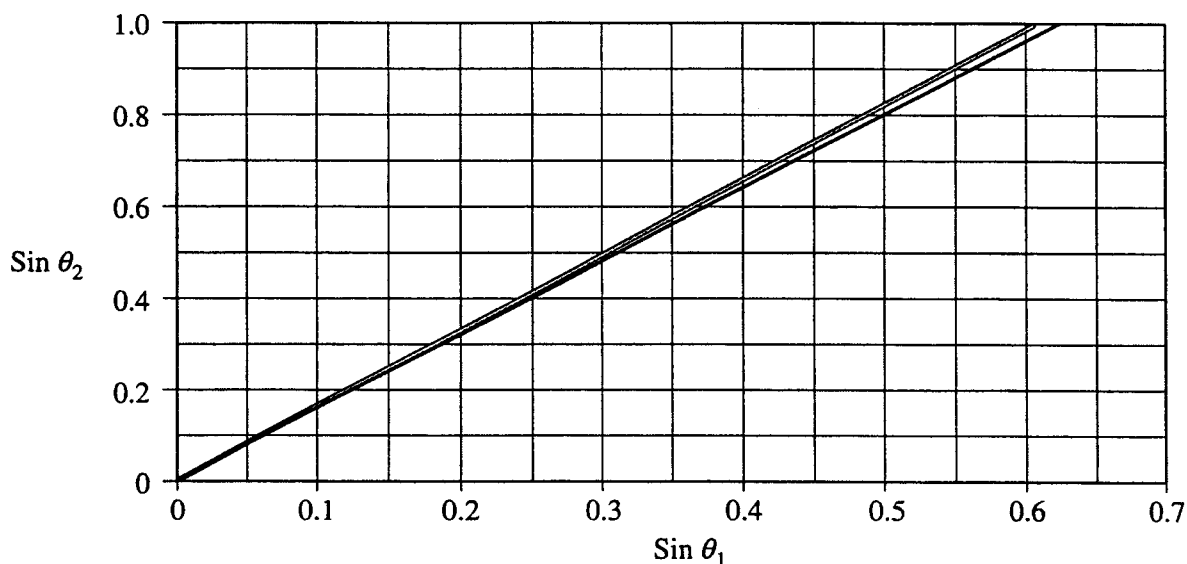
Question 4 (continued)

(c) (continued)
ii. 2 points

For a straight line that goes through the origin   1 point
For a steeper slope than the given line   1 point

Full credit was awarded for a line with less steep slope if it was consistent with the answer to
     part (a)

(d) 2 points

For the correct equation for the critical angle   1 point

$$\sin\theta_c = \frac{n_2}{n_1}$$

$$\sin\theta_c = \frac{1}{1.66} = 0.602$$

For the correct answer   1 point

$\theta_c = 37°$ or   0.624 radians

Question 5 (10 points)

The resistor needs to be connected in a circuit with the power supply, with the ammeter and voltmeter used to take the necessary measurements. Either the given data needs to be extrapolated to the range of interest, or a point at 100°C is measured by immersing the resistor in the hot water bath so that interpolation can be used. In either case a best-fit line should be drawn. Measurements of voltage and current are then taken with the resistor immersed in the liquid of unknown temperature, and are used to calculate a value for resistance. The line on the graph is then used to determine the temperature corresponding to that value of resistance.

(a) 3 points

For including all four circuit elements in a circuit diagram (power supply, resistor, voltmeter, and ammeter)	1 point
For connecting the power supply, ammeter, and resistor in a complete series circuit	1 point
For connecting the voltmeter in parallel with the power supply or resistor	1 point

Clear labels must be included to receive credit.
One earned point was deducted if the circuit that was drawn would not work.

The brief outline of the steps to be followed was scored along with part (b), since it is natural to combine these into one discussion.

(b) 5 points

For using the boiling water bath to obtain another data point and then interpolating the data One of these points was awarded for simply extrapolating the data, since this is a less precise method.	2 points
For indicating that measurements of the voltage and the current must be taken when the resistor is immersed in the liquid of unknown temperature	1 point
For indicating that the resistance must be calculated from the measurements (if an equation is included it must be correct)	1 point
For using the graph to obtain a temperature corresponding to the calculated resistance	1 point

Question 5 (continued)

(c) 2 points

For any reasonable answer clearly stated as an assumption   2 points
    One point was awarded for a less complete but still reasonable answer, or a true
    statement that is not a relevant assumption.

Examples of full-credit answers:
    Resistance versus temperature must be linear throughout the range of interest.
    The temperature of the boiling water is 100°C (only awarded if hot water bath was
        used in part (b) to allow interpolation).
    Heat from the resistor does not significantly affect the temperature of the liquid
        it is immersed in.
    Resistance of the connecting wires is negligible, with an explanation of how this would
        affect the measurements.

Examples of partial-credit answers
    Resistance is proportional to temperature.
    Resistance of the connecting wires is negligible, with no explanation.
    Reference to impact of voltmeter or ammeter on the measurement
    No heat transferred to or from the environment

Question 6 (10 points)

(a)   3 points

For using the definition of pressure

$$P = \frac{F}{A}$$

Calculating the additional pressure due to the weight of the block:

$$\Delta P_2 = \frac{(2.50 \text{ kg})(9.8 \text{ m/s}^2)}{1.20 \times 10^{-2} \text{ m}^2} = 2.04 \times 10^3 \text{ Pa} \quad (\text{or } 2.08 \times 10^3 \text{ Pa using } g = 10 \text{ m/s}^2)$$

For adding the additional pressure to the pressure in state 1

$$P_2 = P_1 + \Delta P_2$$

$$P_2 = 1.02 \times 10^5 \text{ Pa} + 2.04 \times 10^3 \text{ Pa}$$

For the correct answer

$$P_2 = 1.04 \times 10^5 \text{ Pa}$$

(Alternate solution)
For determining the force on the piston in state 1

$$F_1 = P_1 A$$

$$F_1 = (1.02 \times 10^5 \text{ Pa})(1.20 \times 10^{-2} \text{ m}^2) = 1224 \text{ N}$$

Calculating the force due to the block:

$$\Delta F_2 = (2.50 \text{ kg})(9.8 \text{ m/s}^2) = 24.5 \text{ N}$$

For adding these two forces

$$F_2 = F_1 + \Delta F_2$$

$$F_2 = 1224 \text{ N} + 24.5 \text{ N} = 1250 \text{ N}$$

Calculating the pressure in state 2:

$$P_2 = \frac{F_2}{A} = \frac{1250 \text{ N}}{1.20 \times 10^{-2} \text{ m}^2}$$

For the correct answer

$$P_2 = 1.04 \times 10^5 \text{ Pa}$$

1 point

1 point

1 point

(Alternate points)
1 point

1 point

1 point

Question 6 (continued)

(b) 1 point

At constant temperature, pressure times volume is a constant:

$P_1V_1 = P_2V_2$

Solving for V_2:

$$V_2 = \frac{P_1V_1}{P_2}$$

For correct substitutions   1 point

$$V_2 = \frac{\left(1.02 \times 10^5 \text{ Pa}\right)\left(1.50 \times 10^{-3} \text{ m}^3\right)}{1.04 \times 10^5 \text{ Pa}}$$

$V_2 = 1.47 \times 10^{-3} \text{ m}^3$

(Alternate solution)   *(Alternate points)*

Applying the ideal gas law to state 1:

$P_1V_1 = nRT_1$

Solving for n:

$$n = \frac{P_1V_1}{RT_1}$$

$$n = \frac{\left(1.02 \times 10^5 \text{ Pa}\right)\left(1.50 \times 10^{-3} \text{ m}^3\right)}{(8.31 \text{ J/mol} \cdot \text{K})(273 \text{ K})} = 6.74 \times 10^{-2} \text{ mol}$$

Applying the ideal gas law to state 2:

$P_2V_2 = nRT_2$

Solving for V_2:

$$V_2 = \frac{nRT_2}{P_2}$$

For correct substitutions   *1 point*

$$V_2 = \frac{\left(6.74 \times 10^{-2} \text{ mol}\right)(8.31 \text{ J/mol} \cdot \text{K})(273 \text{ K})}{1.04 \times 10^5 \text{ Pa}}$$

$V_2 = 1.47 \times 10^{-3} \text{ m}^3$

(c) 2 points

For indicating the process is isobaric   1 point
For any explanation indicating that the pressure does not change from state 1 to state 2   1 point

(d) 2 points

For indicating 'yes" (the process is isobaric)   1 point
For any explanation indicating that the pressure does not change from state 4 to state 1   1 point

(e) 2 points

At constant pressure, volume divided by temperature is constant

$$\frac{V_1}{T_1} = \frac{V_4}{T_4}$$

Solving for V_4 :

$$V_4 = \frac{V_1 T_4}{T_1}$$

For correctly converting the temperatures to Kelvin   1 point
For correct substitutions   1 point

$$V_4 = \frac{\left(1.50 \times 10^{-3} \text{ m}^3\right)(373 \text{ K})}{273 \text{ K}}$$

$$V_4 = 2.05 \times 10^{-3} \text{ m}^3$$

(Alternate solution)   *(Alternate points)*
Applying the ideal gas law to state 4:

$$P_4 V_4 = nRT_4$$

Solving for V_4 :

$$V_4 = \frac{nRT_4}{P_4}$$

For correctly converting the temperature to Kelvin   *1 point*
For correct substitutions (using n from part (b) and $P_4 = P_1$)   *1 point*

$$V_4 = \frac{\left(6.74 \times 10^{-2} \text{ mol}\right)(8.31 \text{ J/mol} \cdot \text{K})(373 \text{ K})}{1.02 \times 10^5 \text{ Pa}}$$

$$V_4 = 2.05 \times 10^{-3} \text{ m}^3$$

Question 7 (10 points)

(a)   2 points

For any statement of conservation of mass

$3 \times$ (deuterium mass) = helium mass + hydrogen mass + neutron mass + mass defect

Substituting:

$3(2.0141 \text{ u}) = 4.0026 \text{ u} + 1.0078 \text{ u} + 1.0087 \text{ u} + \Delta m$

For the correct answer

$\Delta m = 0.0232 \text{ u}$

(b)   3 points

For indicating that mass-energy equivalence is applicable
For correctly converting the mass in order to get an answer in joules
For correct substitution and answer

$$E = (\Delta m)c^2 = (0.0232 \text{ u})\left(\frac{1.66 \times 10^{-27} \text{ kg}}{1 \text{ u}}\right)\left(3 \times 10^8 \text{ m/s}\right)^2 = 3.47 \times 10^{-12} \text{ J}$$

or

$$E = (\Delta m)c^2 = (0.0232 \text{ u})\left(\frac{931 \text{ Mev}/c^2}{1 \text{ u}}\right)\left(3 \times 10^8 \text{ m/s}\right)^2\left(\frac{1.6 \times 10^{-13} \text{ J}}{1 \text{ MeV}}\right) = 3.46 \times 10^{-12} \text{ J}$$

(c)   2 points

The number of reactions required is the total energy divided by the energy per reaction

For correctly using the energy from part (b)

$$10^{20} \text{ J}/\left(3.47 \times 10^{-12} \text{ J/reaction}\right) = 2.89 \times 10^{31} \text{ reactions}$$

Number of deuterium atoms $= \left(2.89 \times 10^{31} \text{ reactions}\right)\left(\frac{3 \text{ deuterium}}{\text{reaction}}\right)$

For the correct answer, including the factor of 3

Number of deuterium atoms $= 8.66 \times 10^{31}$

Question 7 (continued)

(d) 3 points

There are a number of different logical paths that can lead to a correct solution, all of which
could earn full credit. The following is just one example of a correct method.

Deriving an expression for the number of deuterium atoms that a given mass of seawater
would yield:

Number of deuterium atoms $= 0.015\%$(number of H atoms in needed amount of H_2O)

$$= 0.015\% \frac{(\text{total mass of H needed})}{(\text{mass of one H})}$$

$$= 0.015\% \frac{(\text{total mass of } H_2O \text{ needed})(\text{fraction of mass that is H})}{(\text{mass of one H})}$$

$$= 0.015\% \frac{(\text{total mass of } H_2O \text{ needed})}{(\text{mass of one H})} \frac{(\text{mass of H atoms in one } H_2O \text{ molecule})}{(\text{mass of } H_2O \text{ molecule})}$$

Solving for the total mass of water needed:

Total mass of H_2O needed $=$

$$\frac{(\text{number of deuterium atoms})(\text{mass of one H})(\text{mass of } H_2O \text{ molecule})}{0.015\%(\text{mass of H atoms in one } H_2O \text{ molecule})}$$

For correctly using 0.015% 1 point
For the correct ratio of masses (2 hydrogen atoms to one water molecule)   1 point
Substituting and adding the conversion to kilograms:

$$\text{Total mass of } H_2O = \frac{(8.66 \times 10^{31} \text{ atoms})(1.008 \text{ u})(18 \text{ u})}{(0.00015)(2 \text{ u})} \left(\frac{1.66 \times 10^{-27} \text{ kg}}{1 \text{ u}} \right)$$

For the correct answer   1 point
Total mass of $H_2O = 8.69 \times 10^9$ kg