AP® Physics C
1992 Scoring Guidelines

Mech 1.

(a) 2 points

Conserving energy:

$$\left.\begin{aligned}\frac{1}{2}mv^2 &= mgH \\ v &= \sqrt{2gH}\end{aligned}\right\}$$ For either of these equations (Second equation can also be obtained from kinematic equation for free fall) — 1 point

$= \sqrt{2(10\text{ m/s}^2)(5\text{ m})}$

$v = 10$ m/s (or $v = 9.9$ m/s using $g = 9.8$ m/s^2) — 1 point

(b) 4 points

For any indication that the collision occurs when the clay and the ball are at the same height — 1 point

Kinematic equation for height of the ball:

$h_b = v_0 t - \frac{1}{2}gt^2$ — 1 point

Kinematic equation for height of the clay:

$h_c = H - \frac{1}{2}gt^2$ — 1 point

Setting equations equal and cancelling like terms:

$v_0 t = H$

$t = H/v_0$

$= (5\text{ m})/(10\text{ m/s})$

$t = 0.50$ s (or $t = 0.51$ s using $g = 9.8$ m/s^2) — 1 point

(c) 2 points

Substituting t into equation for height of clay:

$h_c = 5\text{ m} - \frac{1}{2}(10\text{ m/s})(0.50\text{ s})^2$ — 1 point

$h_c = 3.8$ m (or $h_c = 3.7$ m using $g = 9.8$ m/s^2) — 1 point

Distribution of points

Mech 1. (continued)

(c) (continued)

(Alternate Solution) — (Alternate points)

Substituting values into equation for height of ball:

$h_b = (10\ \text{m/s})(0.50\ \text{s}) - \frac{1}{2}(10\ \text{m/s}^2)(0.50\ \text{s})^2$ — (1 point)

$h_b = 3.8\ \text{m}$ (same using $g = 9.8\ \text{m/s}^2$) — (1 point)

(d) 3 points

Kinematic equation for speed of ball:

$v_b = v_0 - gt$ — 1 point

$= 10\ \text{m/s} - (10\ \text{m/s}^2)(0.50\ \text{s})$

$v_b = 5.0\ \text{m/s}$ (or $v_b = 4.9\ \text{m/s}$ using $g = 9.8\ \text{m/s}^2$) — 1 point

Kinematic equation for speed of clay:

$v_c = gt$

$= (10\ \text{m/s}^2)(0.50\ \text{s})$

$v_c = 5.0\ \text{m/s}$ (same using $g = 9.8\ \text{m/s}^2$) — 1 point

(e) 4 points

For any statement of conservation of momentum — 1 point

For correct equation:

$m_b v_b - m_c v_c = (m_b + m_c)v_t$ — 1 point

$$v_t = \frac{(m_b v_b - m_c v_c)}{(m_b + m_c)}$$

$$= \frac{9m(5.0\ \text{m/s}) - m(5.0\ \text{m/s})}{10m}$$

$v_t = 4.0\ \text{m/s}$ (or $v_t = 3.9\ \text{m/s}$ using $g = 9.8\ \text{m/s}^2$) — 1 point

For direction "up," either explicitly stated or obvious from notation — 1 point

Distribution of points

Mech 2.

(a) 2 points

$\tau = \sum \mathbf{F} \times \mathbf{r}$ — 1 point

$= (3M + M)g\ell - Mg\ell$

$\tau = 3Mg\ell$ — 1 point

(b) 4 points

$I = \sum mR^2$ — 1 point

$= 4M\ell^2 + M\ell^2$

$= 5\ m\ell^2$ — 1 point

$\alpha = \frac{\tau}{I}$ — 1 point

Substituting for τ and I,

$$\alpha = \frac{3Mg\ell}{5M\ell^2}$$

$\alpha = \frac{3g}{5\ell}$ — 1 point

(solution continues on next page)

Distribution of points

Mech 2. (continued)

(c) 4 points

For any statement of conservation of energy — 1 point

For this case:

$\Delta U_{bug} + \Delta U_{left\ sphere} + \Delta U_{right\ sphere} = \Delta K_{rotational}$

The left sphere loses as much potential energy as the right sphere gains, so only the bug's contribution to the change in potential energy must be calculated.

$\Delta U_{bug} = mgh = 3Mg\ell$ — 1 point

$\Delta K_{rotational} = \frac{1}{2}I\omega^2 = \frac{1}{2}(5M\ell^2)\omega^2$ — 1 point

Equating these two expressions:

$3Mg\ell = \frac{1}{2}(5M\ell^2)\omega^2$

$\omega^2 = 6Mg\ell/5M\ell^2 = 6g/5\ell$

$\omega = \sqrt{\frac{6g}{5\ell}}$ — 1 point

(d) 2 points

$$\underbrace{L = I\omega \qquad \text{OR} \qquad L = 5Mv\ell = 5M\omega\ell^2}$$ — 1 point

$L = 5M\ell^2\sqrt{\frac{6g}{5\ell}}$

$= \sqrt{30M^2g\ell^3}$ — 1 point

(solution continues on next page)

Mech 2. (continued)

(e) 3 points

Net force at the bottom must equal centripetal force required to keep bug moving in the circle at angular speed ω.

General equation for centripetal force:

$$F_c = \frac{mv^2}{r}$$ 1 point

$$v = \omega r, \text{ so } v^2 = \omega^2 r^2 = \frac{6g}{5\ell}\ell^2 = \frac{6}{5}g\ell$$

$$F_c = \frac{(3M)\left(\frac{6}{5}g\ell\right)}{\ell} = \frac{18}{5}Mg$$

Equating net force and centripetal force, where T is the force exerted on the bug by the sphere:

$$T - 3Mg = F_c$$

$$T = F_c + 3Mg$$ 1 point

$$= \frac{18}{5}Mg + 3Mg$$

$$T = \frac{33}{5}Mg = 6.6\ Mg$$

Force is directed upward 1 point

Distribution of points

Mech 3.

(a) 4 points

$E_{tot} = K + U$ 1 point

$K = \frac{1}{2}mv^2$ 1 point

$U = -\frac{GMm}{r}$ (point also awarded for + sign) 1 point

Substituting appropriate values:

$$E_{tot} = \frac{1}{2}(1000\ \text{kg})(7.1 \times 10^3\ \text{m/s})^2 - \frac{(6.67 \times 10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(6 \times 10^{24}\ \text{kg})(1000\ \text{kg})}{(1.2 \times 10^7\ \text{m})}$$

$E_{tot} = -8.1 \times 10^9$ J 1 point

(b) 2 points

$$\underbrace{L = mvr \qquad \text{OR} \qquad L = I\omega = (mv^2)(v/r) = mvr}$$ 1 point

Substituting appropriate values:

$L = (1000\ \text{kg})(7.1 \times 10^3\ \text{m/s})(1.2 \times 10^7\ \text{m})$

$L = 8.5 \times 10^{13}\ \text{kg}\cdot\text{m}^2/\text{s}$ 1 point

(c) 3 points

For any statement of conservation of angular momentum 1 point

$mv_a r_a = mv_b r_b$

$v_b = v_a r_a / r_b$

$= (7.1 \times 10^3\ \text{m/s})(1.2 \times 10^7\ \text{m})/(3.6 \times 10^7\ \text{m})$ 1 point

$v_b = 2.4 \times 10^3$ m/s 1 point

Mech 3. (continued)

(c) (continued)

(Alternate Solution) (Alternate points)

For any statement of conservation of energy (1 point)

$$\frac{1}{2}mv_b^2 - \frac{GMm}{r_b} = E_{tot}$$

$$v_b^2 = \frac{2E_{tot}}{m} + \frac{2GM}{r_b}$$

$$= \frac{2(-8.1 \times 10^9\ \text{J})}{10^3\ \text{kg}} + \frac{2(6.67 \times 10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(6 \times 10^{24}\ \text{kg})}{3.36 \times 10^7\ \text{m}}$$ (1 point)

$$= 6.0 \times 10^6\ \text{m}^2/\text{s}^2$$

$$v_b = 2.4 \times 10^3\ \text{m/s}$$ (1 point)

(d) 3 points

Equating centripetal and gravitational forces:

$$\left.\begin{array}{l} \dfrac{mv^2}{r} = \dfrac{GMm}{r^2} \\ \\ v = \sqrt{GM/r} \end{array}\right\}$$ For either of these equations 1 point

$$= \sqrt{(6.67 \times 10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(6 \times 10^{24}\ \text{kg})/(1.2 \times 10^7\ \text{m})}$$ 1 point

$$v = 5.8 \times 10^3\ \text{m/s}$$ 1 point

Mech 3. (continued)

(e) 2 points

Applying conservation of energy:

$$E_{tot} = K_a + U_a = U_\infty = 0$$

$$\frac{1}{2}mv^2 - \frac{GMm}{r} = 0$$

$$v = \sqrt{2GM/r}$$

For any of these equations — 1 point

$$= \sqrt{2(6.67 \times 10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(6.0 \times 10^{24}\ \text{kg})/(1.2 \times 10^7\ \text{m})}$$

$$v = 8.2 \times 10^3\ \text{m/s}$$ — 1 point

For correct units for either energy or angular momentum — 1 point

E & M 1.

(a) 3 points

Integrate volume charge density:

$Q = \int \rho \, dV$ — 1 point

For correct limits on integral: — 1 point

$$Q = \int_0^a \rho 4\pi r^2 \, dr$$

For correct substitution:

$$Q = \int_0^a 4\pi\beta r^3 \, dr$$ — 1 point

$$= 4\pi\beta \left.\frac{r^4}{4}\right|_0^a = \pi\beta(a^4 - 0)$$

$$Q = \beta\pi a^4$$

(b)

i. 2 points

For $r > a$, sphere can be treated as a point charge located at the center of the sphere

$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$ — 1 point

Substituting $Q = \beta\pi a^4$:

$$E = \frac{\beta a^4}{4\epsilon_0 r^2}$$ — 1 point

Distribution of points

E & M 1. (continued)

(b) (continued)

ii. 2 points

$r = a$ is the limiting case of (i) above, so merely need to substitute $r = a$ into the expression in (i).

$$E = \frac{\beta a^4}{4\epsilon_0 r^2}$$

$$E = \frac{\beta a^2}{4\epsilon_0}$$ 2 points

iii. 3 points

For applying Gauss' Law in some recognizable form — 1 point

$\int \mathbf{E}\cdot d\mathbf{s} = \frac{1}{\epsilon_0} q$, where q is the charge inside the region

For $\int E\cdot ds = E 4\pi r^2$ — 1 point

$$E\, 4\pi r^2 = \frac{1}{\epsilon_0}\int_0^r \rho\, dV = \frac{4\pi}{\epsilon_0}\int_0^r \rho R^2\, dR$$

$$= \frac{4\pi}{\epsilon_0}\int_0^r \beta R^3\, dR$$

$$= \frac{\pi\beta}{\epsilon_0} r^4$$

$$E = \frac{\beta r^2}{4\epsilon_0}$$ 1 point

(Alternate Solution) — (Alternate points)

Field is due only to the charge q inside the region of radius r

$q = \beta\pi r^4$ from part (a) — (1 point)

The charge q can be treated as being concentrated at origin, so substituting into Coulomb's law:

$$E = \frac{1}{4\pi\epsilon_0}\frac{\beta\pi r^4}{r^2}$$ (1 point)

$$E = \frac{\beta r^2}{4\epsilon_0}$$ (1 point)

E & M 1. (continued)

(c)

i. 2 points

This is limiting case of $r > a$, where sphere can be treated as a point charge located at sphere's center.

Generally:

$$V = \frac{Q}{4\pi\epsilon_0 r}$$ 1 point

$$= \frac{\beta a^4}{4\epsilon_0 r}$$

For $r = a$:

$$V = \frac{\beta a^3}{4\epsilon_0}$$ 1 point

(Alternate solution) (Alternate points)

$$V = -\int_\infty^a \mathbf{E}\cdot d\mathbf{l}$$ (1 point)

$$= -\int_\infty^a \frac{\beta a^4}{4\epsilon_0 r^2}\, dr$$ [substituting **E** from part (b)(i)]

$$V = \frac{\beta a^3}{4\epsilon_0}$$ (1 point)

ii. 3 points

$$V(r = 0) = -\int_a^0 \mathbf{E}\cdot d\mathbf{r} + V(r = a)$$ 1 point

$$= \int_0^a \frac{\beta}{4\epsilon_0} r^2\, dr + V(r = a)$$ 1 point

$$= \frac{\beta}{4\epsilon_0} \left.\frac{r^3}{3}\right|_0^a + V(r = a)$$

$$= \frac{\beta a^3}{12\epsilon_0} + \frac{\beta a^3}{4\epsilon_0} = \frac{4\beta a^3}{12\epsilon_0}$$

$$V(r = 0) = \frac{\beta a^3}{3\epsilon_0}$$ 1 point

Distribution of points

E & M 1. (continued)

(c) (continued)

(Alternate solution) (Alternate points)

$$V = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r}$$ (1 point)

$$V = \frac{1}{4\pi\epsilon_0}\int_0^a 4\pi\beta r^2\, dr$$ (1 point)

$$= \frac{\beta}{\epsilon_0}\int_0^a r^2\, dr$$

$$V = \frac{\beta a^3}{3\epsilon_0}$$ (1 point)

E & M 2.

(a)

i. 2 points

Potential difference across capacitor equals that across battery.

$V_b = Q/C$

$Q = CV_b$ — 1 point

$= (2 \times 10^{-6}\ F)(2 \times 10^{3}\ V)$

$Q = 4 \times 10^{-3}\ C$ — 1 point

ii. 2 points

$U = \frac{1}{2}CV^2$ (or equivalent) — 1 point

$= \frac{1}{2}(2 \times 10^{-6}\ F)(2 \times 10^{3}\ V)^2$

$U = 4\ J$ — 1 point

(b) 2 points

Immediately after the switch is closed, there is no charge on the 6-μF capacitor, so the potential difference across the resistor equals that across the 2-μF capacitor, i.e. 2,000 V.

$V = IR$

$I = V/R$ — 1 point

$= (2000\ V)/(1 \times 10^{6}\ \Omega)$

$I = 2 \times 10^{-3}\ A$ — 1 point

E & M 2. (continued)

(c) 4 points

In equilibrium, charge is no longer moving, and thus there is no potential difference across the resistor. Therefore the capacitors have the same potential difference:

$V_2 = V_6$, where subscripts refer to the 2-μF and 6-μF capacitors — 1 point

$$\frac{Q_2}{C_2} = \frac{Q_6}{C_6}$$

$$C_6 Q_2 = C_2 Q_6$$

But charge is conserved so

$Q_2 + Q_6 = Q$, where $Q = 4 \times 10^{-3}$ C from part (a) — 1 point

Solve these two simultaneous equations for Q_2 and Q_6.
For example, substituting for Q_6 in the first equation:

$C_6 Q_2 = C_2(Q - Q_2)$ — 1 point

$$Q_2 = C_2 Q/(C_6 + C_2)$$

$$= (2 \times 10^{-6}\ \text{F})(4 \times 10^{-3}\ \text{C})/(8 \times 10^{-6}\ \text{F})$$

$$Q_2 = 1 \times 10^{-3}\ \text{C}$$

Substituting for Q_2 in the second equation

$$Q_6 = Q - Q_2$$

$$= 4 \times 10^{-3}\ \text{C} - 1 \times 10^{-3}\ \text{C}$$

$$Q_6 = 3 \times 10^{-3}\ \text{C}$$

For correct answers for Q_2 and Q_6 — 1 point

(Alternate solution) — (Alternate points)

$C_{tot} = C_2 + C_6$ — (1 point)

$V = \dfrac{Q_{tot}}{C_{tot}} = \dfrac{4 \times 10^{-3}}{(2 \times 10^{-6}\ \text{F}) + (6 \times 10^{-6}\ \text{F})} = 500\ \text{V}$ — (2 points)

$$\left.\begin{aligned} Q_2 &= C_2 V = (2 \times 10^{-6}\ \text{F})(500\ \text{V}) = 1 \times 10^{-3}\ \text{C} \\ Q_6 &= C_6 V = (6 \times 10^{-6}\ \text{F})(500\ \text{V}) = 3 \times 10^{-3}\ \text{C} \end{aligned}\right\}$$ (1 point)

E & M 2. (continued)

(d) 4 points

$$U = \frac{1}{2}\frac{Q^2}{C}$$

$$U_t = \frac{1}{2}\left(\frac{Q_2{}^2}{C_2} + \frac{Q_6{}^2}{C_6}\right)$$ 1 point

$$= \frac{1}{2}\left(\frac{(1 \times 10^{-3}\ \mathrm{C})^2}{2 \times 10^{-6}\ \mathrm{F}} + \frac{(3 \times 10^{-3}\ \mathrm{C})^2}{6 \times 10^{-6}\ \mathrm{F}}\right)$$ 1 point

$$= \frac{1}{2}\left(\frac{1}{2} + \frac{3}{2}\right)\ \mathrm{J}$$

$$U_t = 1\ \mathrm{J}$$ 1 point

(Alternate solution) (Alternate points)

$$U = \frac{Q_{tot}{}^2}{2C_{tot}}$$ (1 point)

$$= \frac{(4 \times 10^{-3}\ \mathrm{C})^2}{2[(2 \times 10^{-6}\ \mathrm{F}) + (6 \times 10^{-6}\mathrm{F})]}$$ (1 point)

$$U = 1\ \mathrm{J}$$ (1 point)

This energy is less than in part (a) ii. Part of the energy was converted to heat in the resistor or radiated out. 1 point

For at least 2 correct units in answers and no incorrect units. 1 point

Distribution of points

E & M 3.

(a) 2 points

The magnetic field is directed out of the page. — 2 points
Curled arrows in the right direction also acceptable.

(b)

i. 6 points

$B = \frac{\mu_0 I}{2\pi r}$ (or derivation using Ampere's law) — 1 point

$= \frac{\mu_0 \alpha(1 - \beta t)}{2\pi r}$ — 1 point

$\phi = \int \mathbf{B}\cdot d\mathbf{A}$ — 1 point

$dA = c\ dr$ — 1 point

Substituting into integral for ϕ:

$$\phi = \int_a^b \frac{\mu_0 \alpha}{2\pi r}(1 - \beta t)c\ dr$$

$$= \frac{\mu_0 \alpha c}{2\pi}(1 - \beta t)\int_a^b \frac{dr}{r}$$

$$= \frac{\mu_0 \alpha c}{2\pi}(1 - \beta t)\ \ln r\Big|_a^b$$ — 1 point

$$\phi = \frac{\mu_0 \alpha c}{2\pi}(1 - \beta t)\ \ln\left(\frac{b}{a}\right)$$ — 1 point

ii. 3 points

$$\varepsilon = -\frac{d\phi}{dt}$$ — 1 point

$$= -\frac{\mu_0 \alpha c}{2\pi}\ln\left(\frac{b}{a}\right)\frac{d}{dt}(1 - \beta t)$$ — 1 point

$$= -\frac{\mu_0 \alpha c}{2\pi}(-\beta)\ln\left(\frac{b}{a}\right)$$

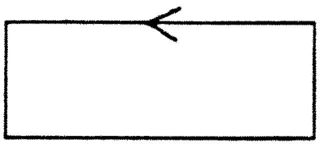
$$\varepsilon = \frac{\mu_0 \alpha c \beta}{2\pi}\ln\left(\frac{b}{a}\right)$$ — 1 point

E & M 3. (continued)

(c)

i. 1 point

$\beta t < 1$

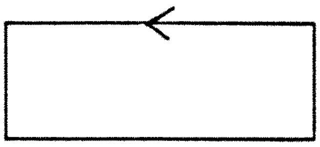


1 point

When $\beta t < 1$, the magnetic field within the loop points out of the page and is decreasing in magnitude. The induced current will thus create a field within the loop that is out of the page in order to oppose this change.

ii. 1 point

$\beta t > 1$



1 point

When $\beta t > 1$, the magnetic field within the loop points into the page and is increasing in magnitude. The induced current will thus create a field within the loop that is again out of the page in order to oppose this change.

E & M 3. (continued)

(d) 2 point

Net force is sum of the forces on each of the four pieces of the rectangle. In each case, the force pulls outward on the side of the loop.

The magnitudes of the forces on the short sides of the rectangle are the same, so the forces on these two sides cancel. The forces on the long sides do not.

$F_{mag} = I\ell B$

For the side closer to the wire: $F_1 = \alpha c \frac{\mu_0 \alpha}{2\pi a}$, down

For the side farther from the wire: $F_2 = \alpha c \frac{\mu_0 \alpha}{2\pi b}$, up

Since $a < b$, $F_2 < F_1$.

Therefore the net force is down, or toward the wire (Answer must be consistent with part (c) to receive credit) — 2 points

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