AP® Physics C
1982 Scoring Guidelines

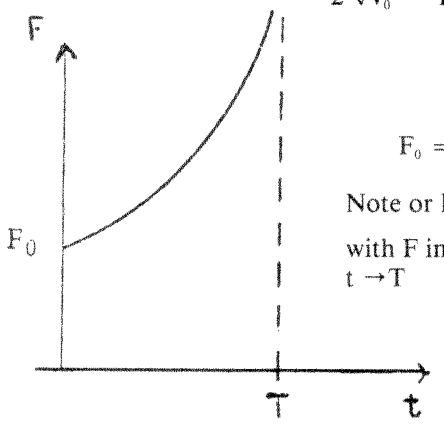
Solution	*Distribution of Points*
1. a) 5 points	
During the slide, mechanical energy is conserved,	1 point
$mgH = \frac{1}{2}mv^2$	2 points
Substituting $H = L \cdot \sin\theta = 3$ meters gives	1 point
$v = \sqrt{60}$ m/s	1 point

Solution	*Distribution of Points*
b) 6 points	
Between release and bottom stopping point, $\Delta K = K_f - K_o = 0$. Thus $\Delta U = 0$	
$\Delta U_g + \Delta U_{spring} = 0$ *or* $\lvert \Delta U_g \rvert = \lvert \Delta U_{spring} \rvert$	1 point
$\lvert \Delta U_g \rvert = mgh = mg\ell \sin\theta$ with $\ell = x + 6$	2 points
$\Delta U_{spring} = \frac{1}{2}kx^2$	2 points
Substitute and solve to get $100(x + 6) = 100x^2$ or $x^2 - x - 6 = 0$ whose positive root is $x = 3$ meters	1 point
c) 4 points	
No; at contact, the spring does not exert any force.	1 point
As the spring compression is increased, the spring force increases.	1 point
v will be maximum when the acceleration is zero. This occurs when the net force is zero.	1 point
Equating the component of gravity and the spring force: $mg \sin\theta = kx$	
Solving for x gives $x = \frac{1}{2}$ m	1 point
Total	15 points

2. a) 5 points	
The kinetic energy is given by $K = \frac{1}{2}Mv^2$	1 point
Substituting $v = \sqrt{v_0^2 - Rt/M}$ gives $K = \frac{1}{2}M(v_0^2 - \frac{Rt}{M})$	1 point
To find the time rate of change of kinetic energy, we differentiate, getting $\frac{dK}{dt}$	1 point
$\frac{dK}{dt} = \frac{1}{2}M(0 - \frac{R}{M}) = -\frac{R}{2}$	2 points
b) 3 points	
Setting v (or K) equal to zero:	1 point
$v_0^2 = \frac{RT}{M}$. Solving for T	1 point
gives $T = \frac{Mv_0^2}{R}$	1 point
c) 4 points	
Since $a = \frac{dv}{dt}$, we differentiate to get	2 points
$a = \frac{1}{2}(v_0^2 - \frac{Rt}{M})^{-1/2}(-\frac{R}{M})$	2 points

d) 3 points

Since F = Ma, we have $F = \dfrac{-R}{2\sqrt{v_0^2 - Rt/M}}$ 1 point



$F_0 = \dfrac{R}{2v_0}$ 1 point

Note or label nonzero F_0 at t = 0

with F increasing without limit as $t \to T$ 1 point

Total 15 points

82 M 3. a) 5 points

Using $L = I\omega$ 2 points

with $I = \Sigma mr^2 = (2m)\ell^2 + m(2\ell)^2$ 1 point

$= 6m\ell^2$ 1 point

we substitute to get $L = 6m\ell^2\omega_0$ 1 point

b) 5 points

The frictional force will be found by $f = \mu mg$ 1 point

and the torque will be given by $\Gamma = \Sigma\, R \times f$ 2 points

Combining gives $\Gamma = -[\ell(\mu \cdot 2mg) + 2\ell(\mu \cdot mg)]$ 1 point

$= -4\mu mg\ell$ 1 point

c) 5 points

Because the frictional torque is constant, the angular acceleration is constant and $\omega = \omega_0 + \alpha t$ 1 point

Since $\Gamma = I\alpha$ 1 point

$\alpha = \dfrac{\Gamma}{I} = \dfrac{-4\mu mg\ell}{6m\ell^2} = -\dfrac{2\mu g}{3\ell}$ 1 point

Setting ω equal to zero and solving for time 1 point

gives $t = -\dfrac{\omega_0}{\alpha} = \dfrac{3\omega_0\ell}{2\mu g}$ 1 point

Total 15 points

1982 C: E&M

1. a) 5 points

The potential V due to a point charge is $V = kq/R$ 1 point

Because potential adds as a scalar quantity, the total $V = V_1 + V_2 + V_3$ 1 point

$= \dfrac{2kq}{\sqrt{a^2 + x^2}} - \dfrac{kq}{x}$ 1 point

Solution	*Distribution of Points*
To find the point where $V = 0$, solve and find	
$\frac{1}{x} = \frac{2}{\sqrt{a^2 + x^2}}$	1 point
$a^2 + x^2 = 4x^2$	
$\therefore x = \pm a/\sqrt{3}$	1 point
b) 7 points	
In general, $\lvert\vec{E}\rvert = E = kq/R^2$ for a point charge.	1 point
By symmetry, $E_y = 0$	2 points
$E_x = E \cos\theta$	1 point
with $\cos\theta = x/\sqrt{a^2 + x^2}$	1 point
$\therefore E_x = E_{1x} + E_{2x} + E_{3x}$	1 point
$= \frac{2kq\,x}{(a^2 + x^2)^{3/2}} - \frac{kq}{x^2}$	1 point
c) 3 points	
By definition, $\phi_E = \oint\!\!\oint \vec{E} \cdot d\vec{A}$ and $\oint\!\!\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\epsilon_0}$ using Gauss's law.	2 points
In this situation, $Q_{enclosed} = +q + q - q = q$	
$\therefore \phi_E = q/\epsilon_0 = 4\pi kq$	1 point
Total	15 points

2. a) 3 points

Ampere's Law states that for any closed path, $\oint \vec{B} \cdot d\vec{\ell} = \mu_0\, i_{enclosed}$	1 point
For a circular path of radius r, centered on the wire, $\oint \vec{B} \cdot d\vec{\ell} = B \cdot 2\pi r$	1 point
Solving for B gives $B = \frac{\mu_0 i}{2\pi r}$	1 point
b) 6 points	
Since B is not constant, the flux must be calculated by integration over the loop, using $\Phi_B = \oint\!\!\oint \vec{B} \cdot d\vec{A}$	2 points
The integral can be evaluated by dividing the loop into narrow strips, all of width ℓ, and integrating over r:	
$\Phi_B = \int_a^{a+b} B(r)\ell\, dr$	1 point
Substituting $B = \frac{\mu_0 i}{2\pi r}$ gives $\Phi_B = \frac{\mu_0 i\ell}{2\pi} \int_a^{a+b} \frac{dr}{r}$	1 point
Evaluating the integral gives $\Phi_B = \frac{\mu_0 i\ell}{2\pi} \ell n\, r \Big]_a^{a+b}$	1 point
and substituting the limits gives	
$\Phi_B = \frac{\mu_0 i\ell}{2\pi} \ell n \frac{a+b}{a}$	1 point

c) 6 points

By Lenz's Law, the induced current i must oppose the change that produced it. — 1 point

Since at time $t = \pi/\omega$, the current in the long wire is produced by a field through the loop that is changing from out of the page to into the page, the induced current must cause a field through the loop that is directed out of the page: it must be a counter-clockwise current. — 1 point

The induced emf follows from Faraday's Law:

$\mathcal{E} = - d\Phi_B/dt$ — 1 point

From part b), $\Phi_B = \frac{\mu_0 \ell}{2\pi} \ell n \frac{a+b}{a} i_m \sin \omega t$ — 1 point

and so $-\frac{d\Phi_B}{dt} = -\frac{\mu_0 \ell}{2\pi} \ell n \frac{a+b}{a} i_m \cos \omega t$ — 1 point

When $t = \pi/\omega$, $\cos \omega t = \cos \pi = -1$

and so

$\mathcal{E} = -\frac{d\Phi_B}{dt} = \frac{\mu_0 i_m \ell \omega}{2\pi} \ell n \frac{a+b}{a}$ — 1 point

Total 15 points

3. a) 3 points

After the switch has been closed for a long time, the current will have ceased changing, so the inductor voltage $V_L = L\frac{di}{dt} = 0$. Therefore, the inductor can be ignored in this part of the problem. — 1 point

The current can be calculated by using Ohm's Law, $V = iR$ — 1 point

Since the total resistance is just R, $i_A = \mathcal{E}/R$ — 1 point

b) 2 points

After the switch is opened, the two resistors are in series, and their combined resistance is 2R. — 1 point

The current is $i_B = \mathcal{E}/2R$ — 1 point

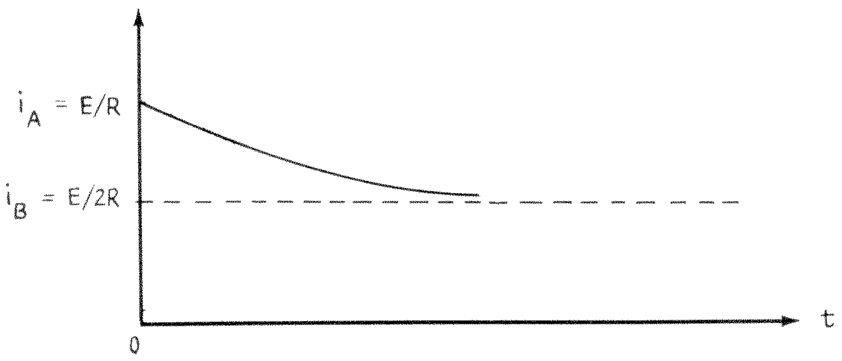
c) 3 points

When $t = 0$, the current is $i_A = \mathcal{E}/R$

It decreases after the switch has been opened. — 1 point

Eventually it approaches the limiting value $i_B = \mathcal{E}/2R$ — 1 point

The decrease is exponential, as shown below. — 1 point



Solution	*Distribution of Points*
d) 4 points	
The battery emf must equal the sum of the potential differences across the inductor: $\mathcal{E} = V_R + V_L$.	1 point
The sum of the potential differences across the resistors is $V_R = 2Ri$	1 point
The potential difference across the inductance is $V_L = L\dfrac{di}{dt}$	1 point
The differential equation for the current is therefore $\mathcal{E} = 2Ri + L\dfrac{di}{dt}$	1 point
e) 3 points	
The expression for i(t) must involve an exponential function of the form $e^{-t/T}$	1 point
The time constant $T = L/2R$	1 point
The solution must be the sum of a constant and a term involving $e^{-2Rt/L}$. It must satisfy the boundary conditions $i(0) = \mathcal{E}/R$, and $i(t) \to \mathcal{E}/2R$ as $t \to \infty$. By inspection, $i(t) = \dfrac{\mathcal{E}}{2R}(1 + e^{-2Rt/L})$	1 point
Total	15 points