Setup:

The staff of a service center for electronic equipment includes three technicians who specialize in repairing many different makes of disk drives for desktop computers. It was desired to study the effects of technician (Factor A) and make of disk drive (Factor B) on the service time. Three makes of disk drives were randomly chosen for consideration. The data in the dataset (equipment.csv) show the number of minutes required to complete the repair job in a study where each technician was randomly assigned to five jobs on each make of disk drive. The columns in the dataset include (in order): minutes; technician; make of drive; job number.

The data for this study consists of two nominal variables, *technician* and *make of disk drive*, where each value of one nominal variable is found in combination with each value of other nominal variable. Also, it contains a measurement (dependent) variable, *service time*, for each technician randomly assigned to five jobs on each make of disk drive. So, a two-way ANOVA with replication seems reasonably the best way to analyse the given data. Further, since *technician* (Factor *A*) is a fixed way and *make of disk drive* (Factor *B*) is a random way, therefore, this is a case of a mixed study.

The data is analysed in SAS, by arranging in a particular way as shown in Table 1, to develop a two-way mixed ANOVA model. The dependent variable (*service time*) is in the first column (minutes), and each factor has a column containing a code to represent the different levels. Second column (Factor *A*) has codes 1 to 3 for technicians and third column (Factor *B*) has codes 1 to 3 for makes of disk drives. The last column has codes 1 to 5 for jobs (replications).

Table 1 Input Data

Obs	minutes	Factor A	Factor B	job	Obs	minutes	Factor A	Factor B	job
1	62	1	1	1	**24**	66	2	2	4
2	48	1	1	2	**25**	51	2	2	5
3	63	1	1	3	**26**	55	2	3	1
4	57	1	1	4	**27**	58	2	3	2
5	69	1	1	5	**28**	50	2	3	3
6	57	1	2	1	**29**	69	2	3	4
7	45	1	2	2	**30**	49	2	3	5
8	39	1	2	3	**31**	59	3	1	1
9	54	1	2	4	**32**	65	3	1	2
10	44	1	2	5	**33**	55	3	1	3
11	59	1	3	1	**34**	52	3	1	4
12	53	1	3	2	**35**	70	3	1	5
13	67	1	3	3	**36**	58	3	2	1
14	66	1	3	4	**37**	63	3	2	2
15	47	1	3	5	**38**	70	3	2	3
16	51	2	1	1	**39**	53	3	2	4
17	57	2	1	2	**40**	60	3	2	5
18	45	2	1	3	**41**	47	3	3	1
19	50	2	1	4	**42**	56	3	3	2

Obs	minutes	Factor A	Factor B	job	Obs	minutes	Factor A	Factor B	job
20	39	2	1	5	**43**	51	3	3	3
21	61	2	2	1	**44**	44	3	3	4
22	58	2	2	2	**45**	50	3	3	5
23	70	2	2	3					

The two-way ANOVA tests three effects: the main effect of each factor, and the interaction effect between the two factors. The mixed ANOVA full model for two-factor study, where Factor *A* is fixed and Factor *B* is random, can be stated as shown below. α_i, β_j represent the "main effect" of the *i*-th and *j*-th levels of Factor *A* and Factor *B*, respectively, $(\alpha\beta)_{ij}$ is the "interaction effect" in that combination of levels, and ε_{ijk} is the error term of the *k*-th observation in that combination.

$$Y_{ijk} = \mu_{\cdot\cdot} + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk}$$

where:

$i = 1, 2, 3;\ \ j = 1, 2, 3;\ \ k = 1, 2, 3, 4, 5$

$Y_{ijk} \overset{ind}{\sim} N(\mu_{\cdot\cdot} + \alpha_i + \beta_j + (\alpha\beta)_{ij}, \sigma^2)$

$\mu_{\cdot\cdot}$ is a constant

α_i are constants subject to the restriction $\sum_i \alpha_i = 0$

$\beta_j \overset{iid}{\sim} N(0, \sigma_\beta^2)$

$(\alpha\beta)_{ij} \sim N(0, \frac{a-1}{a}\sigma_{\alpha\beta}^2)$, subject to the restriction $\sum_i (\alpha\beta)_{ij} = 0$ for all *j*

$\varepsilon_{ijk} \overset{iid}{\sim} N(0, \sigma^2)$

β_j, $(\alpha\beta)_{ij}$, and ε_{ijk} are pairwise independent

Checking the assumptions of two-way ANOVA model

Before proceeding with detail analyses of the interactions and main effects, we should first examine the residuals of the model. Figure 1 shows the residuals against the fitted values plot which is used to see whether the equal variance assumption on the residuals of ANOVA model is met. In this case, it appears that the points are randomly scattered with no obvious patterns, which means that the equal variance assumption appears reasonable.

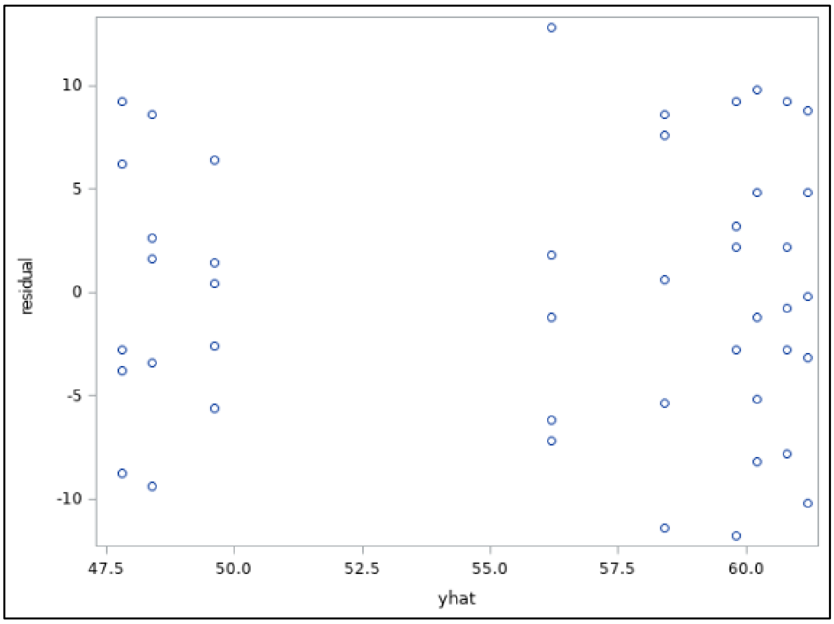

Figure 1 Residuals against fitted values plot

Next, in Figure 2, the Q-Q plot for residuals is shown to check whether the normality assumption of residuals is met. From the plot, it appears that we may have minor departures from normality at the ends. However, upon considering the Shapiro-Wilk test in Table 2, the *p*-value of 0.2712 suggests that the residuals are approximately normally distributed, so this assumption also appears reasonable.

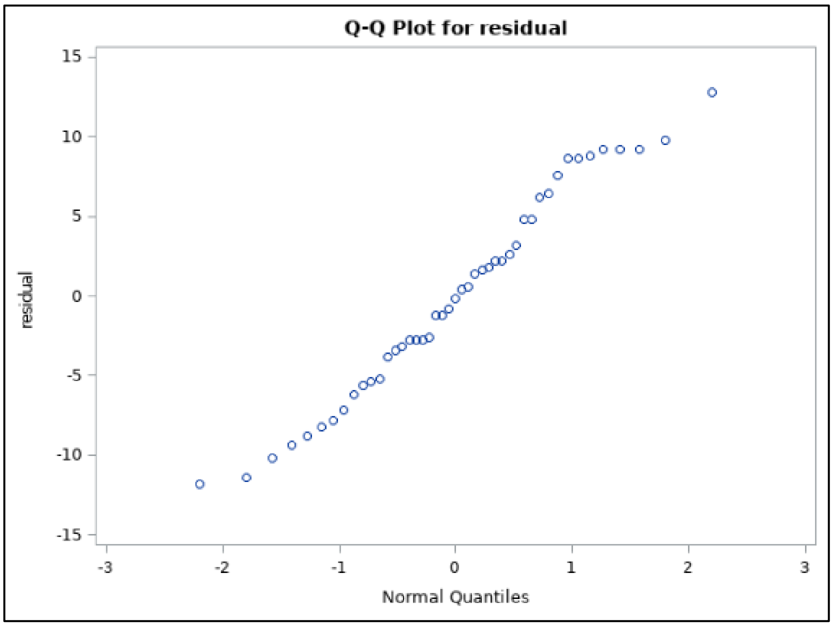

Figure 2 Q-Q Plot for Residuals

Table 2 Results of Normality Tests for Residuals

Tests for Normality				
Test	**Statistic**		**p Value**	
Shapiro-Wilk	**W**	0.969217	**Pr < W**	0.2712
Kolmogorov-Smirnov	**D**	0.084082	**Pr > D**	>0.1500
Cramer-von Mises	**W-Sq**	0.0449	**Pr > W-Sq**	>0.2500
Anderson-Darling	**A-Sq**	0.368368	**Pr > A-Sq**	>0.2500

Since, the residual plots didn't reveal any particular patterns, so our model assumptions seem reasonable here. The data is then displayed as shown in Figure 3 for an initial visual investigation of any trends in service time between the three technicians and across the three makes of disk drives.

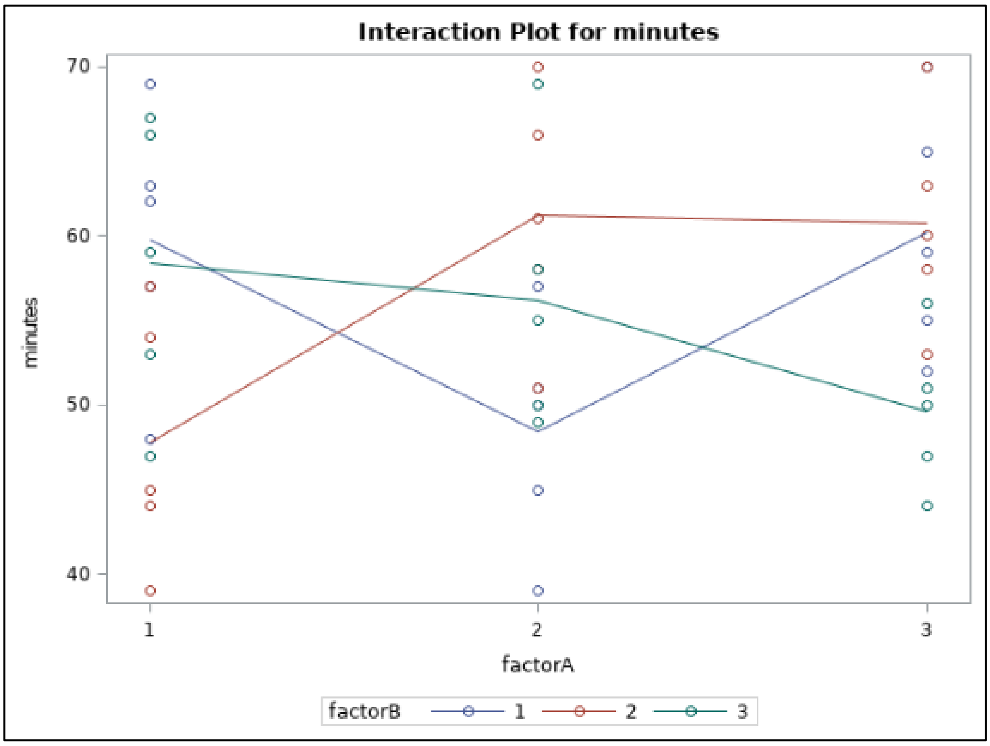

Figure 3 Interaction Plot

Since the lines representing the three makes of disk drives in the plot are not parallel, we might expect the interaction term between technician and make of disk drive to be significant in the two-way ANOVA model (the lines would be approximately parallel if there were no interaction). So, how service time changes with technician depends on the make of disk drive, and vice versa. From the plot we see that technician 1 is better at repairing drive 2, technician 2 is good at repairing drive 1 and technician 3 excels at repairing drive 3.

To fit the two-way ANOVA model, we use PROC GLM method in SAS. ANOVA table for the two-way mixed model is shown in Table 3 below.

Table 3 ANOVA table for two-way mixed effect ANOVA model

Analysis of Variance					
Source	**DF**	**Sum of Squares**	**Mean Squares**	**F Value**	**Pr(>F)**
A	2	24.5778	12.2889	0.04	0.9607
B	2	28.3111	14.1556	0.27	0.7633
AB	4	1215.2889	303.8222	5.84	0.0010
Error	36	1872.4000	52.0111		
Total	44	3140.5778			

It is recommended that the first effect which should be examined (even though it usually appears last in the list of effects in an ANOVA output table), is the interaction effect. If it is significant, then we should probably ignore the two main effects tests. If the interaction effect is significant, this means that the effect of one factor on the dependent variable is different depending on the level of the other factor.

So, let us now perform a hypothesis test to determine whether the two factors *technician* and *make of disk drive* interact. Let us assume a significance level of $\alpha = 0.05$ for the test.

- Hypotheses:

$$H_0: \sigma^2_{\alpha\beta} = 0 \qquad \text{versus} \qquad H_1: \sigma^2_{\alpha\beta} > 0$$

- Test Statistic: From ANOVA table,

$$F = \frac{MSAB}{MSE} = 5.84$$

- Critical Region:

$$\text{Reject } H_0 \text{ if } F > F_{0.05}(4,36) = 2.63$$

- p-value: From ANOVA table,

$$p - \text{value} = \text{P}[F > 5.84] = 0.0010$$

- Decision: Since 5.84 > 2.63 and p-value is smaller than $\alpha = 0.05$, Reject H_0.
- Conclusion: There is enough evidence to conclude that technician and make of disk drive interact with each other significantly.

Since, the interaction was significant, it is not meaningful to test for main effects. Whenever interactions are present, we cannot look at the levels of the factors individually because they have some intrinsic relationship with each other; to do so would be misleading. The main effects of the independent variables don't have their usual interpretations in this case. It is difficult to state the independent effect of Factor *A* (*technician*) because the nature and magnitude of the effect depends on the particular level of Factor *B* (*make of disk drive*). If no evidence of an interaction effect would have been found, then we could have proceeded to testing the main effects of the independent variables.

Even if we wanted to test for main effects in this case, we see from ANOVA table that the *p*-values are 0.9607 and 0.7633 for Factor *A* and Factor *B*, respectively. Thus, we would have concluded that both main effects are not significant because of the largish p-values. However, a test for any main factor alone disregards information about the influence of the other main factor(s). This weakens the ability to detect any real individual influence. For our dataset, this means that the individual main factor tests aren't very meaningful, but we could still use the Tukey intervals as estimates of the unknown model parameters.

The point estimates and Tukey's simultaneous 95% confidence limits for differences between means of fixed factor *A* are shown in Table 4 below. We see that all simultaneous CI are wide and none of the differences between means are significant i.e. there is no significant difference between the technicians on service time of the disk drives.

Table 4 Tukey's simultaneous 95% CI for fixed factor *A*

Least Squares Means for Effect factorA				
i	j	Difference Between Means	Simultaneous 95% Confidence Limits for LSMean(i)-LSMean(j)	
1	2	0.066667	-6.370139	6.503472
1	3	-1.533333	-7.970139	4.903472
2	3	-1.600000	-8.036806	4.836806

Conclusion

A two-way mixed effect ANOVA was carried out on dependent variable *service time* by nominal variables *technician* and *make of disk drive*. Model assumptions such as normality and equal variances of residuals were analysed and the assumptions were met. There was a statistically significant interaction between the effects of technician and make of disk drive on service time [$F = 5.84$, p-value = 0.0010].

The presence of interaction between *technician* and *make of disk drive* means that the way *service time* changes for different technicians depends on the makes of disk drives. Similarly, the way *service time* changes for different makes of disk drives depends on technicians. Overall, from the interaction plot we see that technician 1 is better at repairing drive 2, technician 2 is good at repairing drive 1 and technician 3 excels at repairing drive 3.

Tukey's post hoc tests were carried out for the fixed factor and we concluded that there was no significant difference between the technicians on service time of the disk drives.

APPENDIX

SAS Code:

```
/* Input Data */
DATA equipment;
INPUT minutes factorA $ factorB $ job $ @@;
DATALINES;
62 1 1 1 48 1 1 2 63 1 1 3 57 1 1 4 69 1 1 5
57 1 2 1 45 1 2 2 39 1 2 3 54 1 2 4 44 1 2 5
59 1 3 1 53 1 3 2 67 1 3 3 66 1 3 4 47 1 3 5
51 2 1 1 57 2 1 2 45 2 1 3 50 2 1 4 39 2 1 5
61 2 2 1 58 2 2 2 70 2 2 3 66 2 2 4 51 2 2 5
55 2 3 1 58 2 3 2 50 2 3 3 69 2 3 4 49 2 3 5
59 3 1 1 65 3 1 2 55 3 1 3 52 3 1 4 70 3 1 5
58 3 2 1 63 3 2 2 70 3 2 3 53 3 2 4 60 3 2 5
47 3 3 1 56 3 3 2 51 3 3 3 44 3 3 4 50 3 3 5
;

/* Print Data */
PROC PRINT DATA = equipment;
RUN;

/* Mixed Model: FULL MODEL ANOVA TABLE TO INVESTIGATE INTERACTIONS */
PROC GLM DATA = equipment;
	CLASS factorA factorB;
	MODEL minutes = factorA|factorB;
	random factorB;
	LSMEANS factorA/ADJUST=TUKEY CL ALPHA=0.05 OUT = lsmeans;
	test h = factorA e = factorA*factorB;
	OUTPUT out = results r = residual p = yhat;
RUN;

/* Print Fitted Values */
PROC PRINT DATA = lsmeans;
RUN;

/* Print Residuals */
PROC PRINT DATA = results;
RUN;

/* Produce Residual vs Yhat */
PROC SGPLOT DATA = results;
	 SCATTER X = yhat Y = residual;
RUN;

/* Produce Residual vs Expected Value (Q-Q Plot) */
PROC UNIVARIATE DATA = results NORMAL;
```

```
    VAR residual;
    QQPLOT;
RUN;
```