

Answer:  $k = 6$   
 $m = 8$

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$$\sum_{i=1}^n i = \frac{n}{2} (1+n), \text{ so } 1+2+\dots+k-1 = \frac{k-1}{2} (1+k-1)$$

$$k+1+k+2+\dots+m = \frac{m-k}{2} (m+k+1)$$

$$\frac{k-1}{2} (1+k-1) = \frac{m-k}{2} (m+k+1)$$

$$k^2 - k = m^2 + m - k^2 - k$$

$$2k^2 = m^2 + m$$

$$k = \sqrt{\frac{m^2 + m}{2}}$$

$$\text{or } - \sqrt{\frac{m^2 + m}{2}}$$

rejected because  $k > 1$

Test (6, 8):

$$6 = \sqrt{\frac{64+8}{2}}$$

$$6 = \sqrt{36}$$

Also, note  $\frac{dk(m)}{dm} = \frac{d}{dm} \sqrt{\frac{m^2 + m}{2}}$   
 $= \frac{m + \frac{1}{2}}{2\sqrt{\frac{m^2 + m}{2}}}$

$$\lim_{m \rightarrow \infty} \frac{dk(m)}{dm} = \sqrt{2}$$