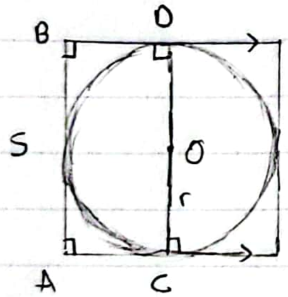
12. Shamid

Answer: $\frac{1}{3}$

Let the larger cube have a side length of s.

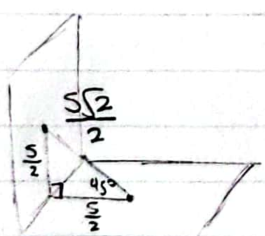
The cube and octahedron are dual platonic solids so the octahedron has vertices on the incenter of the cube's faces.



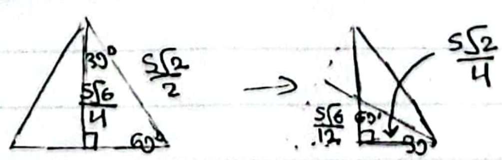

$\overline{AB} = \overline{CD}$

radius of circle $O = \frac{\overline{CD}}{2} = \frac{s}{2}$

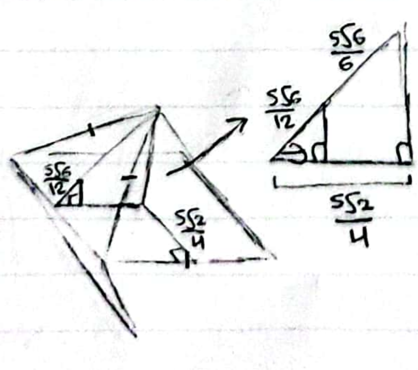
Each side of the octahedron is equal to the distance between two of those incenters.



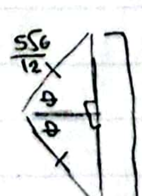

The incenter of the octahedron's faces can then be found as the intersection of its angle bisectors.




The smaller cube has vertices on those incenters, so its side length can be described as the distance between two of them.




$$\cos\theta = \frac{\frac{s\sqrt{2}}{4}}{\frac{s\sqrt{6}}{4}} = \frac{1}{\sqrt{3}}$$




$$2 \cdot \frac{s\sqrt{6}}{12} \cdot \sin\left(\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)\right)$$

$$= \frac{s}{3}$$