Linear Algebra - Worksheet

Read this article before beginning the exercises: Linear Algebra Explained in 4 Pages

This assignment consists of 3 parts:

- Matrix Dimensions
- Vector Operations
- Matrix Operations

After completing the exercises by hand, use Python to check your work.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 7 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 5 & -1 \\ 9 & 1 \\ 6 & 0 \end{bmatrix} \quad D = \begin{bmatrix} 3 & -2 & -1 \\ 1 & 2 & 3 \end{bmatrix}$$

$$u = \begin{bmatrix} 6 & 2 & -3 & 5 \end{bmatrix}$$
 $v = \begin{bmatrix} 3 & 5 & -1 & 4 \end{bmatrix}$ $w = \begin{bmatrix} 1 \\ 8 \\ 0 \\ 5 \end{bmatrix}$

1. Matrix Dimensions

Write the dimensions of each matrix.

1.6)
$$w$$
 4x1

2. Vector Operations

Perform the following operations. Assume $\alpha = 6$.

$$2.1) \vec{u} + \vec{v} = \frac{\overset{(6,2,-3,5)}{+(3,5,-1,4)}}{\overset{(9,7,-4,9)}{-}}$$

$$(2.2) \ \vec{u} - \vec{v} =$$
 [3,-3,-2,1]

$$2.3) \; \alpha \vec{u} = \qquad \hbox{[36,12,-18,30]}$$

$$(2.4) \ \vec{u} \cdot \vec{v} = 18+10+3+20 = 51$$

$$||\vec{u}|| = \operatorname{sqrt}(36+4+9+25) = \operatorname{sqrt}(74)$$

3. Matrix Operations

Evaluate each of the following expressions, if it is defined; else fill in with "not defined." Do your work by hand on scratch paper.

CT = [[5,9,6], 3D = [[9,-6,-3],

[-1,1,0]] [3,6,9]]

$$3.1) A + C =$$
 not defined

3.2)
$$A - C^T = \begin{bmatrix} [-4, -7, -3] \\ [3,6,4] \end{bmatrix}$$

3.3)
$$C^T + 3D = \begin{bmatrix} [14,3,3], \\ [2,7,9] \end{bmatrix}$$

$$3.4) BA = {}^{-1} {}^{-5} {}^{-1} {}^{2} 74$$

$$3.5) BA^T =$$
 not defined

Optional

$$3.6) BC = \text{not defined}$$

$$3.7) CB =$$
 $\begin{array}{c} 5 & -6 \\ 9 & -8 \\ 6 & -6 \end{array}$

3.8)
$$B^4 = {\begin{array}{*{20}{c}} {\mathsf{B}} \cdot \mathsf{B} = \mathsf{1} & \mathsf{-2} & \mathsf{\cdot B} = \mathsf{[1} & \mathsf{-3} & \mathsf{\cdot B} = \mathsf{[1} & \mathsf{-4} \\ {\mathsf{0}} & \mathsf{1} & {\mathsf{0}} & \mathsf{1]} & {\mathsf{0}} & \mathsf{1]} \\ \end{array}}$$

$$3.9) AA^T = {14 28 \atop 28 69}$$

$$3.10) \ D^T D = \begin{array}{ccc} 10 & -4 & 0 \\ -4 & 8 & 8 \\ 0 & 8 & 10 \end{array}$$